

Note of implementation of calculations of rotors

Summary:

This document constitutes an aid for users for the realization of the studies of revolving machines in *Code_Aster*. It must enable him to conclude a calculation of revolving machines (static analyses, modal, harmonic and transitory).

The list of the CAS-tests available of revolving machines in *Code_Aster* is put in appendix.

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1 Setting in equation

1.1 Equations of the movement

This part aims to present the essential aspects of a calculation of line of trees. For that, we will study the simple model of rotor of Figure 1.1-a.

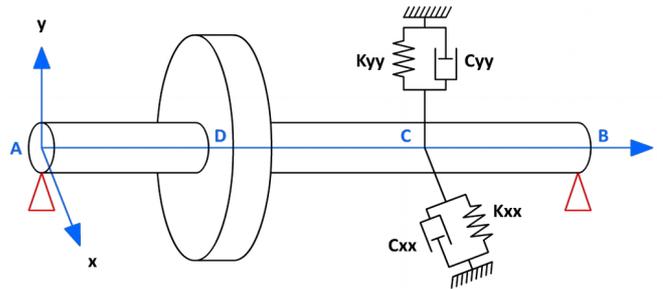


Figure 1.1-a : Simplified model

The rotor of axis of rotation z is modelled by a beam which comprises a disc with the third its length. It rests, on the one hand, on two infinitely rigid supports at each end and, on the other hand, on a linear stage in the directions x and y located at two thirds of the beam.

The expressions of the kinetic energies of the tree, the discs and the stages are necessary to characterize the latter. The tree moreover is characterized by its deformation energy. We use also the forces due to the stages in order to calculate virtual work, and thus to obtain the corresponding efforts exerted on the tree. We will present here only the principal equations of behavior, without necessarily detailing calculations to arrive there.

The general equations of the system are thus obtained according to the following diagram: one calculates for the elements of the system, the kinetic energy T , the deformation energy U and the virtual work of the external efforts, then one applies the equations of Lagrange in the form:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = F_{q_i} \quad \text{with } 1 < i < N \quad \text{éq 1.1-1}$$

where N is the number of degrees of freedom, q_i are the generalized independent coordinates and F_{q_i} generalized forces.

It is supposed that the disc is indeformable, and one thus limits oneself to his characterization by his kinetic energy. Thus, the disc is characterized by its matrix of inertia in its centre of gravity:

$$[I] = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \quad \text{éq 1.1-2}$$

The tree is presented like a beam to constant circular section and is characterized by its energies kinetic and of deformation.

The terms of stiffnesses and damping are supposed to be known (cf sub-section 2.2.1 for more details on the identification of these coefficients). The forces exerted by the stage on the tree can thus be expressed like:

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \quad \text{éq 1.1-3}$$

Figure 1.1-b present the model and the notations adopted for the stages while Figure 1.1-a present a schematic view of the line of trees.

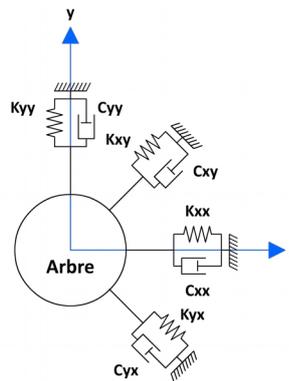


Figure 1.1-b : Linear modeling of the stages

Starting from the calculation of the kinetic energies and deformations of all the elements, and equation éq 1.1-1 of Lagrange, one obtains equations of behavior which can be rewritten in a matrix form (see equation éq 1.1-4):

$$M \cdot \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + C \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \Omega \cdot G \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + K \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad \text{éq 1.1-4}$$

Table 1.1-1

where M , C , G and K are, respectively, the square matrices of mass, damping, gyroscopy and stiffness. We voluntarily omit the detail of calculations and return to the work of Lalanne and Ferraris [1]. It will be noticed that the matrix of gyroscopy is multiplied by the number of revolutions of the rotor Ω . The dynamic behavior of the line of trees thus depends its number of revolutions. The homogeneous solution of this equation makes it possible to find the frequencies and clean modes of the rotor.

2 Digital modeling of the revolving machines

A study of revolving machines requires at the same time a modeling of the revolving part – the line of trees – and a modeling of the bodies of self-supporting quality. It is also possible to take into account the influence of the table of group, structure resulting from the civil engineer and on which the bodies of self-supporting quality rest, on the dynamic behavior of the line of trees.

2.1 Modeling of the revolving parts

The rotors lend themselves rather well to a telegraphic modeling. From a dynamic point of view, the lines of trees are flexible elements and it is necessary to take into account transverse shearing in the

calculation of the behavior. This is why, a modeling by finite elements of standard beam of Timoshenko, fascinating of account the gyroscopic effect, the deformation of shearing action and the inertia of rotation of the sections, is classically selected.

The theory of the beams does not make it possible however to correctly treat the abrupt changes of section. In the presence of an important change of section, the mass put concerned in the movement corresponds to the geometry of the section. On the other hand, with regard to the deformation, this one is limited and the use of dimensions of the structure for the definition of the sections of the elements leads to a strong overvaluation of the rigidity of the zone considered.

Code_Aster must thus allow, for each element of rotor, a dissociated treatment of the terms of mass and rigidity. With each section of an element are associated a section masses used for the calculation of the elementary terms of mass and a section stiffness used for the calculation of the elementary terms of rigidity. The current setting in data of the models of lines of trees is thus done by 2 dual geometries (diameter in stiffness/diameter in mass). This duality diameter of mass/diameter of stiffness is explained by the empirical rule of 17° . The definition of the mass and the stiffness of each element of rotor is based on two assignments different from the elementary characteristics. In a general way, the section mass corresponds to the geometry of the structure. The sections stiffness can not be in coherence with the geometry of the structure of the abrupt rotor in particular in the case of change of section.

In addition, a three-dimensional modeling of the rotors is also possible in *Code_Aster*. For more details on the procedure to be followed, the reader will be able to refer to the documents of validation [V5.03.108], [V5.03.109] and [V5.03.110].

The discs with wings are regarded as rigid and are modelled by equivalent masses and inertias. That makes it possible to neglect the local modes due to the wings of the rotors and to simplify the model of the structure. By way of an example, the schematic representation of a line of trees P4-P' 4 is visible below on the figure.

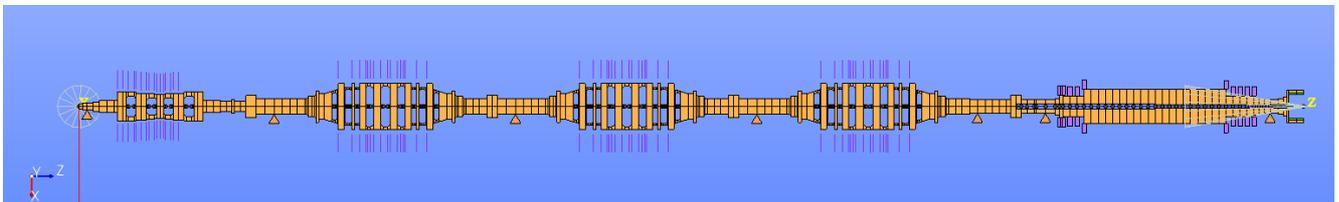


Figure 2.1-a: Representation of a model finite elements P4-P' 4

2.2 Modeling of the stages of self-supporting quality

In a turbine of the nuclear capacity, the line of trees is supported by stages which rest on the solid mass. Figure 2.2-a present the models and the notations adopted.

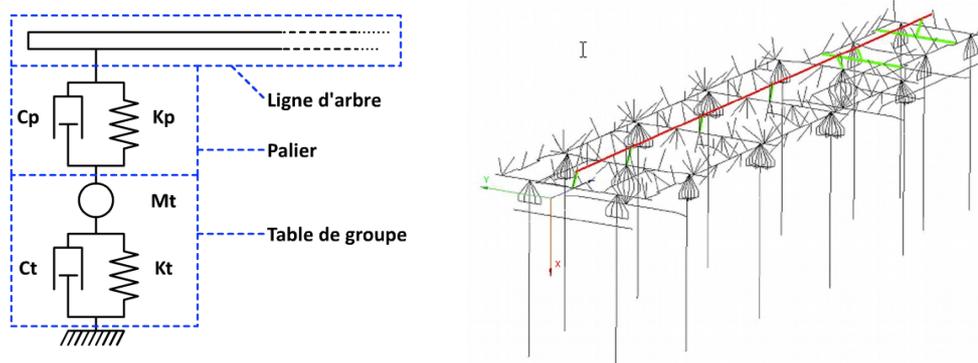


Figure 2.2-a : Representations of the stages and the table of group

2.2.1 Linear stages

In a first approximation, a linear behavior can be retained for the stages, this behavior being a function number of revolutions. In particular, with the assumption of small displacements, the coefficients of stiffness and damping can be calculated by linearizing the equations of Reynolds around the position of balance. Then, by calculating virtual work W_{paliers} external forces operating the tree, it comes:

$$\delta W_{\text{paliers}} = \begin{bmatrix} F_x & F_y \end{bmatrix} \begin{Bmatrix} \delta x \\ \delta y \end{Bmatrix}$$

with F_x and F_y components of the generalized force operating the stages.

By supposing that the rotor is perfectly rigid, that displacements of the tree are limited in the vicinity of a position of static balance (subscripted 0) and by considering the framework of the small disturbances (small displacements (x, y) and low speeds (\dot{x}, \dot{y})), the components of the forces to the stages are written:

$$\begin{cases} F_x(x+x_0, y+y_0, \dot{x}, \dot{y}) = F_x(x_0, y_0, 0, 0) + x \left[\frac{\partial F_x}{\partial x} \right]_0 + y \left[\frac{\partial F_x}{\partial y} \right]_0 + \dot{x} \left[\frac{\partial F_x}{\partial \dot{x}} \right]_0 + \dot{y} \left[\frac{\partial F_x}{\partial \dot{y}} \right]_0 + \dots \\ F_y(x+x_0, y+y_0, \dot{x}, \dot{y}) = F_y(x_0, y_0, 0, 0) + x \left[\frac{\partial F_y}{\partial x} \right]_0 + y \left[\frac{\partial F_y}{\partial y} \right]_0 + \dot{x} \left[\frac{\partial F_y}{\partial \dot{x}} \right]_0 + \dot{y} \left[\frac{\partial F_y}{\partial \dot{y}} \right]_0 + \dots \end{cases}$$

with $k_{ij} = -\frac{\partial F_i}{\partial x_j}$ and $c_{ij} = -\frac{\partial F_i}{\partial \dot{x}_j}$ who correspond to the stiffnesses and depreciation due to the oil film. These coefficients can be calculated in manner according to the geometry of the stage, which one can find in the literature specialized for very simple geometries of stages. While being limited to the first order, the forces exerted by the fluid on the rotor, following displacements of the center of the rotor, can be put in matric form:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = - \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} - \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix}$$

with the terms of matrices stiffness and damping described by linear functions per pieces the number of revolutions. The values of the coefficients are usually calculated by EDYOS.

2.2.2 Nonlinear stages

In general, the nonlinear behavior of the stages can be approximate by a linear representation of the equation of Reynolds around a point of operation. However, according to the REX of the EDF studies, for certain fluid stages, where the amplitudes of the vibrations become excessive, it is preferable to take into account a nonlinear behavior, in particular at the time of the passage critical velocities where the oil film is strongly crushed.

In the case of the turbine generator sets, the line of trees is supported by hydrodynamic bearings which cannot be regarded passive but as elements influencing the dynamic behavior of the line of trees.

Indeed, the oil film has properties of stiffness and damping which vary according to the mode of operation of the turbine, *i.e.* eccentricity of the rotor and number of revolutions (in particular at the critical velocities), and control the stability of the unit.

The reactions to the stages are obtained after integration of the field of pressure calculated starting from the nonlinear equation of Reynolds. The simultaneous resolution of the equations of the movement of the rotor and the hydrodynamic behavior of each stage can prove to be complex and expensive in computing times.

The taking into account of non-linear stages for the calculation of revolving machines is carried out by the coupling enters *Code_Aster* and EDYOS via YACS.

2.3 Modeling of the table of group

The stages, via the landing chairs, are contingent on a structure resulting from Génie Civil: the table of group. This table, made up mainly of beams and cross-pieces, is relatively delicate to model. In the majority of the studies revolving machines, one is satisfied with a very simple modeling. One has three types of modeling:

- support rigid
- simplified support
- generalized support

2.3.1 Rigid support

In a modeling of type “rigid support”, the table of group is not modelled and the stage (linear or non-linear) is directly connected to an embedded node. The table of group is then indeformable and does not allow the transmission of vibrations between the stages. It is then enough simply to impose boundary conditions of type embedding on the nodes connected to the stages.

2.3.2 Simplified support

In a modeling of type “simplified support”, the table of group is modelled by discrete elements characterized by a system mass-arise-shock absorber (cf. Figure 2.2-a on the left). It is enough to identify the dynamic characteristics of the table of group, and to assign them to the discrete elements. The dynamic stiffness of the table of group is then integrated in modeling, however, this approach does not authorize transmission of vibrations between the stages via the table of group.

2.3.3 Generalized support

In a modeling of type “generalized support”, the behavior of the support is defined by a modal base calculated as a preliminary, for example starting from a modeling 1D, 3D or hybrid 1D-3D of the table of group and its static parts (cf. Figure 2.2-a on the right). The taking into account of the generalized solid mass can be done either by direct calculation or by under-structuring of the type Craig-Bampton or Mac-Neal (cf Doc. [U2.06.04] – Note of construction of the scale models in dynamics). By way of an example, CAS-test SDLV132 [U2.04.132] (cf annexes) the setting illustrates in work of a modal calculation of line of trees with its generalized solid mass, this last being taken into account by under-structuring. The procedure employed also makes it possible to take into account technological specificities (condenser related to the external bodies of turbine, for example).

3 Setting in data of the studies “revolving machines”

In a classical way, the setting in data of the studies of revolving machines passes by the use of the operators `AFFE_MODELE`, to inform modeling, `AFFE_CARA_ELEM`, to inform the characteristics of the elements, `AFFE_CHAR_MECA`, to define the boundary conditions. One proposes to describe these stages in the continuation. The calculation of the elementary matrices and their assemblies will be also approached.

3.1 Assignment of the models

After the reading of the grid by the operator `LIRE_MAILLAGE`, the creation of a simple model of the rotor represented by Figure 1.1-a is done by the operator `AFFE_MODELE` as follows:

```
MODELE=AFFE_MODELE (MAILLAGE=MAIL,  
                    AFPE= (_F (GROUP_MA=' ROTOR',  
                               PHENOMENE=' MECANIQUE',  
                               MODELISATION=' POU_D_T',),  
                           _F (GROUP_MA= ('DISC'),  
                               PHENOMENE=' MECANIQUE',  
                               MODELISATION=' DIS_TR',),),),);
```

As indicated previously, the elements retained for the modeling of the rotors are generally beams of Timoshenko `'POU_D_T'`. The modeling of the discs and wings is generally carried out by `'DIS_TR'`.

3.2 Assignment of the elementary characteristics

After the definition (`DEFI_MATERIAU`) and the assignment (`AFFE_MATERIAU`) materials, the description of characteristics geometrical for the setting in data of the model (it rotor, stages and possibly the support) is carried out via the operator `AFFE_CARA_ELEM`. As a preliminary, characteristics of the bodies of self-supporting qualities (stages, obstinate) and, possibly, those of the support must be described.

Note:

In the presence of abrupt changes of section, it is preferable to build two structures of the type `cara_elem`, for rigidity and the mass. It is important to also note that, for dynamic calculations, the produced structures can be included in a loop Python since they depend on the number of revolutions.

3.2.1 Definition of the bodies of self-supporting quality

The operator `DEFI_FONCTION` can be used to describe the variation of the characteristics of the bodies of self-supporting qualities according to the number of revolutions. For example, the stage represented on Figure 1.1-a, is informed by the following function describing the variation of the stiffness in the direction x :

```
KXX1=DEFI_FONCTION (NOM_PARA=' INST',  
                   NOM_RESU=' KXX1',  
                   VALE= (1500.0,8600000000.0, 750.0,13000000000.0),),  
                   PROL_DROITE=' CONSTANT', PROL_GAUCHE=' CONSTANT', VERIF='  
NON',);
```

In a similar way, one informs the mass and damping coefficients (for the thrusts) in all the directions.

One uses these characteristics to inform the discrete elements representing the influence of the stages (`K_TR_L`, `A_TR_L` and `M_TR_L`).

3.2.2 Definition of the simplified support

The operator `DEFI_FONCTION` can be used to describe the variation of the characteristics (stiffness, damping, mass) of the support simplified according to the number of revolutions, a way similar to the definition of the bodies of self-supporting quality.

The discrete elements are then also used `K_TR_L`, `A_TR_L` and `M_TR_L` to describe the influence of the support simplified on the behavior of the line of trees.

3.3 Boundary conditions

The boundary conditions are classically indicated via the operator `AFFE_CHAR_MECA`.

For the groups of nodes indicating them **linear stages**, one blocks the degrees of freedom according to: `DRX`, `DRY`, `MARTINI`, `DRZ`, `DZ`. When it **stage is non-linear**, one adds to the conditions preceding them blockings according to `DX` and `DZ`. In addition, **if behaviour in torsion of the line of trees is not evaluated**, one blocks the nodes of the line of trees according to `DRZ`. Same manner, **if behaviour in compression of the line of trees is not evaluated**, one blocks the nodes of the line of trees according to `DZ`. **In the presence of a crack**, degrees of freedom `DX`, `DY`, `DRX` and `DRY` `MARTINI` groups of nodes framing the crack are blocked.

The boundary conditions also make it possible to take account of the chain of the line of trees (destriping under the effect of the actual weight).

3.3.1 Example: taking into account of chain of the line of trees

For calculations integrating the chain of the line of trees, one imposes the altimetry of the various nodes of the stages. These boundary conditions in displacement for the nodes stages (with or without chain) are carried out using the keyword `DDL_IMPO`, in the order `AFFE_CHAR_MECA`. For example, the boundary conditions for the study of a line of trees P4-P' 4 in inflection with taking into account of the chain can be as follows:

```
POIDS=AFFE_CHAR_MECA (MODELE=MODELE,  
                      DDL_IMPO= (_F (GROUP_NO=' NO_LA', DRZ=0, DZ=0.),  
                                   _F (GROUP_NO=' NO_PAL', DRX=0., DRY=0., DRZ=0.,  
DZ=0.),),  
                      _F (GROUP_NO=' SUP', DX=0., DY=0., DZ=0.,  
DRX=0., DRY=0., DRZ=0.),),  
                      _F (NOEUD=' N2', DY=-0.0160,)),  
                      _F (NOEUD=' N40', DY=-0.0049,)),  
                      _F (NOEUD=' N110', DY= 0.0040,)),  
                      _F (NOEUD=' N180', DY= 0.0020,)),  
                      _F (NOEUD=' N247', DY=-0.0059,)),  
                      _F (NOEUD=' N258', DY=-0.0089,)),  
                      _F (NOEUD=' N291', DY=-0.0210,)),),  
PESANTEUR= (_F (GRAVITE=9.81,  
                DIRECTION= (1. , 0. , 0. ,),),),);
```

3.4 Calculation of the elementary matrices and assembly

3.4.1 Matrices of stiffness, mass and mechanical cushioning

Calculation elementary matrices of stiffness and mass is done via the operator `CALC_MATR_ELEM`, options `RIGI_MECA` and `MASS_MECA` :

```
RIGI_ELE=CALC_MATR_ELEM (OPTION=' RIGI_MECA',  
                        MODELE=MODELE,  
                        CHAM_MATER=CHMAT,  
                        CARA_ELEM=CARELEM,  
                        CHARGE=CONDLIM,);  
  
MASS_ELE=CALC_MATR_ELEM (OPTION=' MASS_MECA',  
                        MODELE=MODELE,  
                        CHAM_MATER=CHMAT,  
                        CARA_ELEM=CARELEM,
```

```
CHARGE=CONDLIM, ) ;
```

Then a concept `sd-nume_ddl` must be created to generate the classification of degrees of freedom:
`NUMEDDL=NUMÉRIQUE_DDL (MATR_RIGI=RIGI_ELE,) ;`

The elementary matrices are then assembled with the operator `ASSE_MATRICE` :

```
RIGIDITE=ASSE_MATRICE (MATR_ELEM=RIGI_ELE,  
NUME_DDL=NUMEDDL, ) ;
```

```
MASSE=ASSE_MATRICE (MATR_ELEM=MASS_ELE,  
NUME_DDL=NUMEDDL, ) ;
```

Note:

All three operations described above can be carried out via the macro-order `ASSEMBLY`.

The matrix of mechanical cushioning (in the stages) is generated in the same way:

```
AMOR_ELE=CALC_MATR_ELEM (OPTION=' AMOR_MECA',  
MODELE=MODELE,  
CHAM_MATER=CHMAT,  
CARA_ELEM=CARELEM,  
RIGI_MECA=RIGI_ELE,  
MASS_MECA=MASS_ELE,  
CHARGE=CONDLIM, ) ;
```

```
AMOR=ASSE_MATRICE (MATR_ELEM=AMOR_ELE,  
NUME_DDL=NUMEDDL, ) ;
```

3.4.2 Gyroscopic matrices

The gyroscopic effect is available in *Code_Aster* for the voluminal elements, discrete elements and elements of right of type Euler and Timoshenko, possibly multifibre beam with warping.

For the elements of beam and the elements discrete, the matrices of gyroscopic damping and stiffness are calculated as follows:

```
GYRO_ELE=CALC_MATR_ELEM (OPTION=' MECA_GYRO',  
MODELE=MODELE,  
CHAM_MATER=CHMAT,  
CARA_ELEM=CARELEM, ) ;
```

```
GYRO=ASSE_MATRICE (MATR_ELEM=GYRO_ELE,  
NUME_DDL=NUMEDDL, ) ;
```

```
RIGY_ELE=CALC_MATR_ELEM (OPTION=' RIGI_GYRO',  
MODELE=MODELE,  
CHAM_MATER=CHMAT,  
CARA_ELEM=CARELEM, ) ;
```

```
RIGY=ASSE_MATRICE (MATR_ELEM=RIGY_ELE,  
NUME_DDL=NUMEDDL, ) ;
```

For the voluminal elements, one must initially define a unit loading of rotation (number of revolutions equalizes to 1 tr/min) around the axis of rotation wished as follows:

```
CL_ROTATION=AFFE_CHAR_MECA (MODELE=MODELE,  
ROTATION=_F (VITESSE=1,
```

```
AXE= (nx, ny, nz),,);
```

With, for example, $n_x=0$, $n_y=0$ and $n_z=1$ for a rotation around axis Z. **The user will take care that the definite vector (n_x, n_y, n_z) is normalized to 1.**

Then, the matrices of gyroscopic damping and stiffness are calculated using the load `CL_ROTA` as follows:

```
GYRO_ELE=CALC_MATR_ELEM (OPTION=' MECA_GYRO',  
                          MODELE=MODELE,  
                          CHAM_MATER=CHMAT,  
                          CHARGE=CL_ROTA,);
```

```
GYRO=ASSE_MATRICE (MATR_ELEM=GYRO_ELE,  
                   NUME_DDL=NUMEDDL,);
```

```
RIGY_ELE=CALC_MATR_ELEM (OPTION=' RIGI_GYRO',  
                          MODELE=MODELE,  
                          CHAM_MATER=CHMAT,  
                          CHARGE=CL_ROTA,);
```

```
RIGY=ASSE_MATRICE (MATR_ELEM=RIGY_ELE,  
                   NUME_DDL=NUMEDDL,);
```

The contributions of the matrices of gyroscopic damping and stiffness must be respectively multiplied by the speed and the angular acceleration, expressed in rad/s and rad/s/s, and are combined after assembly with the matrices of damping and rigidity classics by the operator `COMB_MATR_ASSE` as follows:

```
OM=1500.*pi/30. # to obtain the number of revolutions in rad/s  
GYOM=COMB_MATR_ASSE (COMB_R= (_F (MATR_ASSE=GYRO, COEF_R=OM,)),  
                    _F (MATR_ASSE=AMOR, COEF_R=1.,)),)
```

For more details on the use of damping and the gyroscopic stiffness in *Code_Aster*, the reader can refer to documentation [U2.06.31].

4 Calculations of the type “revolving machines”

It is possible to carry out several types of calculation for studies of revolving machines in *Code_Aster*:

- **static analysis**,
- **modal analysis**,
- **transitory analysis**,
- **harmonic analysis**.

The operators used currently for calculations of revolving the machines type are the static operator of calculation (`MECA_STATIQUE`), the operator of modal calculation (`CALC_MODES`), the single operator allowing to carry out harmonic calculations and transitory dynamic calculations on physical basis or reduced basis (`DYNA_VIBRA`).

These calculations can be realized **in the presence of a crack**: one will also indicate how to take into account a crack in calculations of revolving machines.

One will describe finally **method of reference** used to evaluate the behavior of a AWG at the time of one **accidental situation** characterized by the loss of a final wing.

4.1 Static analysis

A first elementary validation of the digital model of line of trees built can be realized, for example, by a static calculation of deformation under the action of the weight. LE loading mechanical is carried out then via the operator `AFFE_CHAR_MECA`.

For example, the calculation of the efforts resulting from the configuration chain of line of trees is done via the operator `MECA_STATIQUE` [U4.51.01]:

```
STAT = MECA_STATIQUE (MODEL = MODEL,  
                      CHAM_MATER = CHMAT,  
                      CARA_ELEM = CARELEM,  
                      EXCIT = (_F (CHARGE= WEIGHT, ), ), );
```

One uses then the operator of postprocessing `CALC_CHAMP`, with the option `REAC_NODA` to obtain the efforts with the supports stages in this configuration :

```
STAT = CALC_CHAMP (reuse = STAT,  
                  RESULT = STAT,  
                  FORCE = ('REAC_NODA'), )
```

The reactions of support on the level of the nodes stages under actual weight make it possible to know the load supported by each stage. Using a study EDYOS, it remains to determine the corresponding position and the linearized coefficients.

4.2 Modal calculation

Modal calculation provides the clean solutions (frequencies, clean modes) of the revolving machines, which makes it possible to obtain for example the critical velocities. According to the size of the model considered (many degrees of freedom), modal calculation can be carried out in two different ways: by **direct calculation** or by **calculation on a basis of projection**.

4.2.1 Direct modal calculation

For the systems with a reduced number of degrees of freedom, a direct modal calculation can be carried out with the operator `CALC_MODES` and with one or the other of the algorithms of resolution (algorithm `QZ` algorithm `SORENSEN`). For example:

```
MODEG = CALC_MODES (MATR_RIGI = RIGIDITY,  
                   MATR_MASS = MASS,  
                   MATR_AMOR = GYOM,  
                   OPTION = 'CENTER',  
                   CALC_FREQ = _F (FREQ = 1. ,  
                                   NMAX_FREQ = 16)  
                   SOLVEUR_MODAL=_F (METHODE=' SORENSEN'))
```

According to whether one seeks to determine integrality or part of the spectrum, one will respectively use the algorithms of modal resolution of type `QZ` or `SORENSEN`.

4.2.2 Modal calculation by projection on a basis of Ritz

To carry out a modal calculation for systems with a large number of degrees of freedom (more than 300 degrees of freedom), two stages are necessary:

- Construction of a base of Ritz made up of the clean modes of the associated conservative system enriched by static modes
- Modal calculation on the matrices of mass, stiffness and damping projected in the base of Ritz and restitution in the physical base

4.2.2.1 Construction of the base of Ritz

To build the base of projection, one must:

- **to calculate the clean modes of the conservative system associated (without damping) with boundary conditions of the support-supported type, carried out by blocking the nodes stages with dimensions rotor.** For example, one can use the operator `CALC_MODES` :

```
BASEAP = CALC_MODES (MATR_RIGI = KSYM,  
                    MATR_MASS = MSYM,  
                    OPTION      = 'CENTER',  
                    CALC_FREQ   = _F (FREQ = 1. ,  
                                       NMAX_FREQ = 16)  
                    SOLVEUR_MODAL=_F (METHODE=' SORENSEN'))
```

- **to calculate the static modes where a static mode corresponds to the static response of the line of trees to a unit displacement imposed on the node stage in the directions x and y .** For example, to calculate a static mode corresponding to a stage, one can use the operator `MODE_STATIQUE`:

```
MDST = MODE_STATIQUE (MATR_RIGI = K,  
                     MATR_MASS = M,  
                     MODE_STAT = _F (NODE = 'P1',  
                                       AVEC_CMP = ('DX', 'DY')))
```

One uses then the operator `DEFI_BASE_MODAL` to build the base of projection made up of the clean modes of the system deadened nouveau riches of the static modes. One thus obtains:

```
BASSET HOUND = DEFI_BASE_MODAL (RITZ = (_F (MODE_MECA= (BASEAP, MDST)),  
                                NUME_REF = NDDL)
```

The operator `NUME_DDL_GENE` finally allows to establish the relation between the degrees of freedom in the physical base and the degrees of freedom in the base of projection.

```
NUMGE = NUME_DDL_GENE (BASE = BASSET HOUND, STORAGE = 'FULL')
```

4.2.2.2 Modal calculation of the reduced matrices and restitution on physical basis

One projects then the matrices of stiffness, mass and, possibly, damping on the basis of Ritz built previously by using the operator `PROJ_MATR_BASE`. For example, the matrix of reduced mass is obtained by:

```
MG = PROJ_MATR_BASE (BASE = BASSET HOUND, MATR_ASSE = M, NUME_DDL_GENE =  
NUMGE)
```

Once the built reduced matrices, the operator is used `CALC_MODES` for the resolution of the problem to the eigenvalues reduced by fixing the method `QZ` :

```
MODG = CALC_MODES ( MATR_RIGI = KG,  
                   MATR_MASS = MG,  
                   MATR_AMOR = CG,  
                   OPTION      = 'CENTER',  
                   CALC_FREQ   = _F (FREQ = 1. ,
```

```
NMAX_FREQ = 16)  
SOLVEUR_MODAL = _F (METHODE=' QZ '))
```

The return to the physical coordinates is done finally via the operator `REST_GENE_PHYS`:

```
MODP = REST_GENE_PHYS (RESU_GENE = MODG,  
                        TOUT_CHAM = 'YES',  
                        TOUT_ORDRE = 'YES')
```

4.2.3 Modeling of damping in linear dynamics

Besides the damping brought by the hydrodynamic bearings, there exists another source of damping in the revolving machines. It is the damping of the material (concrete of Génie Civil, for example). There exist several modelings of damping:

- viscous damping
- damping hysteretic (known as also “structural damping”)

For more details, the reader is returned to the reference material [R5.05.04].

With regard to the studies of revolving machines, it is also of use to carry out linear analyses by using the methods of dynamic response by projection on modal basis, with a modal base of real clean modes. In this case, damping is used for digital stabilization or then to simulate experimental results (retiming).

In *Code_Aster*, it is possible to introduce modal damping coefficients. In the operators `DEFI_BASE_MODAL` and `DYNA_VIBRA`, the keyword `AMOR_REDUIT` and/or `LIST_AMOR` allow to add reduced depreciation (percentages of damping criticizes) for the modes even if in the beginning they are real modes.

In the case of a calculation by dynamic under-structuring, reduced depreciation is defined for each substructure separately (operator `MACR_ELEM_DYNA` [U4.65.01]).

4.3 Transitory analysis

The transitory analysis aims to determine the transitory dynamic response to an unspecified temporal loading.

4.3.1 Definition of a turning effort of standard unbalance in transient

To create an effort of type unbalance, it is necessary to create an effort of module proportional to ω^2 and to carry out of it a decomposition in a component as a cosine and a component in sine following the two radial directions d_x and D_y . One must thus define the function cosine for the load application in the direction d_x and the function sine for the load application in the direction D_y . One uses for that the operators `FORMULA` and `AFFE_CHAR_MECA`. For example, if one applies an unbalance of amplitude 0.05 kg.m of worthless phase (unbalance colinéaire to x), one writes:

```
FCM0 = FORMULA (VALE = 'OM*OM*cos (OM*INST+0.0)', NOM_PARA=' INST')  
FH0 = AFFE_CHAR_MECA (MODEL = MODEL,  
                       FORCE_NODALE = _F (NODE = 'N6Z1',  
                                           FX = 0.05))  
  
FCM1 = FORMULA (VALE = 'OM*OM*sin (OM*INST+0.0)', NOM_PARA=' INST')  
FH0 = AFFE_CHAR_MECA (MODEL = MODEL,  
                       FORCE_NODALE = _F (NODE = 'N6Z1',  
                                           FY = 0.05))
```

4.3.2 Calculation of the transitory answer to the unbalance

It is possible to determine the answer transoire either on physical basis or on modal basis. In both cases, the single operator `DYNA_VIBRA` is used.

4.3.2.1 Transitory answer on physical basis

In the case of the transitory answer on physical basis, the single operator `DYNA_VIBRA` fact call to the historical operator `DYNA_LINE_TRAN`. For example, if one wishes to calculate the transitory answer of a line of trees between 0 and 5 S, one will be able to write:

```
TRP = DYNA_VIBRA (TYPE_CALCUL = 'TRAN',  
                 BASE_CALCUL = 'PHYS',  
                 MATR_MASS = M,  
                 MATR_RIGI = K,  
                 MATR_AMOR = GOM,  
                 SCHEMA_TEMPS = _F (SCHEMA=' NEWMARK', BETA=0.25,  
GAMMA=0.5, ),  
                 INCREMENT = _F (INST_INIT=0.0, INST_FIN= 5, PAS =0.00025, ),  
                 EXCIT = ( _F (VECT_ASSE=FHa0, FONC_MULT=FCM0, ),  
                          _F (VECT_ASSE=FHa1, FONC_MULT=FCM1)) )
```

4.3.2.2 Transitory answer on modal basis

The construction of the base of projection is made same manner as for the realization of a modal calculation by projection on a basis of Ritz (clean modes of the associated conservative system enriched by the static modes). One will however take care to also calculate the generalized coordinates of the effort in the base of projection via the operator `PROJ_VECT_BASE`.

Same manner as previously, to calculate the transitory answer by projection on modal basis, one uses the operator `DYNA_VIBRA` who calls in this case upon the historical operator `DYNA_TRAN_MODAL`.

```
TRG=DYNA_VIBRA (TYPE_CALCUL = 'TRAN',  
                 BASE_CALCUL = 'GENE',  
                 VITESSE_VARIABLE = 'NOT',  
                 MATR_MASS = MG,  
                 MATR_RIGI = KG,  
                 MATR_AMOR = CG,  
                 EXCIT = ( _F (VECT_ASSE_GENE=FHG0, FONC_MULT=FCM0, ),  
                          _F (VECT_ASSE_GENE=FHG1, FONC_MULT=FCM1, ) ),  
                 SCHEMA_TEMPS=_F (SCHEMA=' EULER', ),  
                 INCREMENT=_F (INST_INIT=0.0, INST_FIN= 5, PAS =1.5625E-05, ) );
```

One will use then the operator `REST_GENE_PHYS` for the restitution on physical basis.

4.3.2.3 Typical case of a transient speed

The modeling of the transitory deceleration of a revolving machine requires the taking into account of the dependence of the matrices of gyroscopic damping and stiffness (options `MECA_GYRO` and `RIGI_GYRO`) and of the revolving forces of unbalance at the speed and the angular acceleration.

In the case of instruction known in advance (i.e. imposed or controlled speed), the slow transients ("slow" evolution by stages number of revolutions) can be treated as follows:

- one defines a step number of revolutions for the update of the matrices and the efforts (number of constant revolutions per blocks),
- one carries out transitory calculations by interval speed,
- one builds the solution by successive continuations.

Always in the case of instruction known in advance (that is to say imposed or controlled speed), Lbe transitory rapids ("fast" evolution number of revolutions) of speed can be treated by adding the keyword `VITESSE_VARIABLE = 'YES'` in the single operator `DYNA_VIBRA`. One must then define the laws speed and angular acceleration imposed by formulas or functions in `Code_Aster` as follows:

```
NTRIMdeb = 0.0;
OMdeb = 2 * pi * NTRIMdeb/60.0; # number of initial revolutions in rad/s
tdeb = 0.0; # initial moment in S
NTRIMfin = 5000.0;
OMfin = 2 * pi * NTRIMfin/60.0; # number of final revolutions in rad/s
tfin = 12.0; # final moment in S
```

Example of linear law of evolution number of revolutions

```
With = (OMdeb*tfin-OMfin*tdeb)/(tfin-tdeb);
B = (OMdeb-OMfin)/(tdeb-tfin);
C = PHIdeb - A*tdeb - 0.5*B* (tdeb ** 2);
VITE=FORMULE (VALE=' A+B*INST',
              NOM_PARA=' INST',);
ACCE=FORMULE (VALE=' B',
              NOM_PARA= ' INST',);
```

Example of law of exponential evolution number of revolutions

```
alpha = 3.273e-01;
A = ((exp (alpha*tfin) *OMfin) - (exp (alpha*tdeb) *OMdeb))/(exp (alpha*tfin)
- exp (alpha*tdeb));
B = (OMfin-OMdeb)/(exp (- alpha*tfin) - exp (- alpha*tdeb));
C = PHIdeb - A*tdeb - ((B/alpha) *exp (- alpha*tdeb));
VITE=FORMULE (VALE=' A+B*exp (- alpha*INST) ',
              NOM_PARA=' INST',);
ACCE=FORMULE (VALE=' -alpha*B*exp (- alpha*INST) ',
              NOM_PARA=' INST',);
```

Then, one feeds the operator of calculation by the angular laws `QUICKLY` and `ACCE` as well as the assembled gyroscopic matrices `AMGE` and `GYGE` under the keyword `VITE_ROTA`, `ACCE_ROTA`, `MATR_GYRO` and `MATR_RIGY`.

```
TRG = DYNA_VIBRA (TYPE_CALCUL = 'TRAN',
                 BASE_CALCUL = 'GENE',
                 VITESSE_VARIABLE = 'YES',
                 VITE_ROTA = QUICKLY,
                 ACCE_ROTA = ACCE,
                 MATR_MASS = MG,
                 MATR_RIGI = KG,
                 MATR_RIGY = KGYG,
                 MATR_AMOR = CG,
                 MATR_GYRO = GG,
                 EXCIT = (_F (VECT_ASSE_GENE=FHG0,
                              _F (VECT_ASSE_GENE=FHG1,
                                   FONC_MULT=FCM0),
                                   FONC_MULT=FCM1))
                 SCHEMA_TEMPS=_F (SCHEMA=' NEWMARK', BETA=0.25, GAMMA=0.5,)),
                 INCREMENT=_F (INST_INIT=tdeb, INST_FIN=tfin, PAS=pdtf))
```

For more details, the reader is returned to the document of validation [V2.02.144] of CAS-test SDLL144 implementing a transient number of revolutions like at the documents of use [U4.53.03] and [U4.53.21] of the operators `DYNA_TRAN_MODAL` and `DYNA_VIBRA`.

4.4 Harmonic analysis

The harmonic analysis aims to determine the harmonic answer to a loading in permanent mode: this analysis supposes that the transient of loading is deadened. It also supposes that the behavior of the machine is linear and that the structure is subjected to harmonic loadings of the same pulsation of excitation.

4.4.1 Definition of a turning effort of standard unbalance in harmonic

To create an effort of type unbalance, it is necessary to create an effort of module proportional to ω^2 , and to carry out of it a decomposition in a component as a cosine and a component in sine following the two radial directions. In a harmonic calculation, the loading is the real part of the excitation and thus as a cosine. Like $\cos(\omega \cdot t - \pi/2) = \sin(\omega \cdot t)$, the loading following the other radial direction is thus affected of a phase of -90° besides dephasing.

Example of rotation along the axis OX :

```
balou = 0.05; # Value of the unbalance in kg.m  
PHI = 0.0; # angle enters the direction  $x$  and that of the unbalance at the initial moment
```

```
NTRIM = 15000.0; # number of revolutions in tours/min  
FREQ = (NTRIM/60.0); # rotational frequency in Hz  
OM = ((FREQ * 2) * pi); # number of revolutions in rad/s  
AMOGYRO=COMB_MATR_ASSE (COMB_R= (_F (MATR_ASSE=GYRO,  
                                     COEF_R=OM,)),  
                          _F (MATR_ASSE=AMOR,  
                               COEF_R=1.0,)),);
```

Creation of an unbalance of module $balou \times OM^2$ on the node N_DIS

```
bal_FY=AFFE_CHAR_MECA (MODELE=MODELE,  
                       FORCE_NODALE=_F (GROUP_NO=' N_DIS2',  
                                         FY= (balou * (OM ** 2))),);  
bal_FZ=AFFE_CHAR_MECA (MODELE=MODELE,  
                       FORCE_NODALE=_F (GROUP_NO=' N_DIS2',  
                                         FZ= (balou * (OM ** 2))),);
```

The derivation of the elementary vector corresponding to the mechanical loading (option `CHAR_MECA`) is done using the operator `CALC_VECT_ELEM`.

```
bal_FYe=CALC_VECT_ELEM (OPTION=' CHAR_MECA',  
                       CHARGE=bal_FY,);  
bal_FZe=CALC_VECT_ELEM (OPTION=' CHAR_MECA',  
                       CHARGE=bal_FZ,);
```

The assembly is done via the operator `ASSE_VECTEUR` according to obviously the same classification of the degrees of freedom as that of the assembled matrices.

```
bal_Yass=ASSE_VECTEUR (VECT_ELEM=bal_FYe,  
                      NUME_DDL=NUMEDDL,);  
bal_Zass=ASSE_VECTEUR (VECT_ELEM=bal_FZe,  
                      NUME_DDL=NUMEDDL,);
```

4.4.2 Calculation of the harmonic answer to the unbalance

In order to determine the critical velocities of revolutions, it is necessary to search the number of revolutions for which the amplitude of vibration is maximum. With this intention, the resolution of the harmonic problem of answer to an unbalance can be carried out by considering one after the other, all the frequencies of excitation contained in a list defined by the user. One supposes a synchronous loading with the number of revolutions of the machine and one carries out a sweeping of the frequencies of excitation (and thus number of revolutions) with a step of discretization. The value of this step is judiciously selected to make it possible to collect all the critical velocities in the studied beach.

4.4.2.1 Harmonic answer on physical basis

The calculation of the harmonic answer to an unbalance on physical basis is done via the single operator `DYNA_VIBRA` (`TYPE_CALCUL=' HARM'` and `BASE_CALCUL=' PHYS'`) and fact call to the historical operator `DYNA_LINE_HARM`.

4.4.2.2 Harmonic answer on modal basis

Harmonic calculation requires the resolution of a linear system, but this one is inexpensive. The interest of the modal reduction for a harmonic calculation is thus limited being given the speed of one such calculation. However, two calculations, on bases physics and modal, can be carried out and lead to the same results since the modal base is sufficiently representative.

Besides the constitution of the modal base and projection of the assembled matrices, it is necessary to project the mechanical loadings (unbalance in this case). The projection of the loading is carried out via the operator `PROJ_BASE`.

```
PROJ_BASE (BASE=MODES,  
          PROFIL=' PLEIN',  
          MATR_ASSE_GENE= (_F (MATRICE=CO ('MAGUS'),  
                               MATR_ASSE=MASSE, ),  
                           _F (MATRICE=CO ('RIGE'),  
                               MATR_ASSE=RIGIDITE, ),  
                           _F (MATRICE=CO ('AGGE'),  
                               MATR_ASSE=AMOGYRO, ), ),  
          VECT_ASSE_GENE= (_F (VECTEUR=CO ('BYGE'),  
                               VECT_ASSE=bal_Yass, ),  
                           _F (VECTEUR=CO ('BZGE'),  
                               VECT_ASSE=bal_Zass, ), ), );
```

A call to the single operator `DYNA_VIBRA` (`TYPE_CALCUL=' HARM'` and `BASE_CALCUL=' GENE'`) then allows to carry out the harmonic calculation of answer on modal basis.

4.5 Calculation of fissured rotor

The procedure of implementation of a calculation of fissured rotor is below exposed. In this section, one proposes to present the modifications to be made to a healthy model to take account of the presence of a crack.

4.5.1 Approach

The approach privileged to model a crack on a line of trees (in order to evaluate its impact on the dynamic behavior) is to introduce the law of corresponding stiffness into `DYNA_VIBRA`. Currently, it is

possible to define a crack only in the case of a transitory analysis on modal basis and in the case of the supports rigid and simplified.

For more details, CAS-test SDNL133 ([V5.02.133]) base of Code_Aster illustrate the approach suggested.

4.5.2 Determination of the law of stiffness of the crack

The reference document [R5.05.08] approach the manner of determining the equivalent stiffness of a crack. The calculation of the law of stiffness of the crack is done on a fissured cylindrical bar of unit size (diameter = 1 and length = 4 * diameter). During the generation of the grid of the bar fissured several shapes of cracks can be modelled (bottom right, semi-elliptic concave and convex).

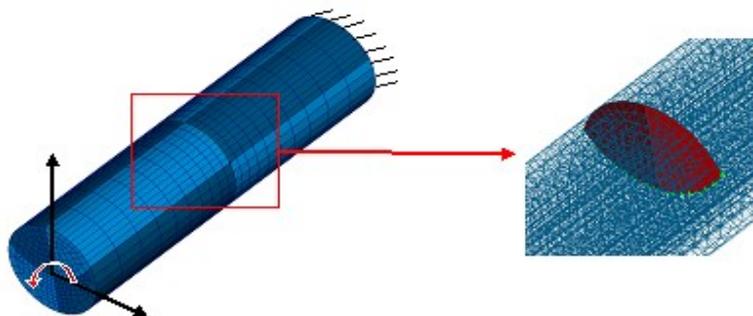


Figure 4.5.2-a: Modeling of the fissured test-tube

The fissured bar is embedded on an end and a moment is applied to the second end. The orientation of the moment evolves during calculation so as to traverse interval supplements between 0° and 360° with a step of 10° , for example.

A static calculation is then carried out on the bar in order to determine the law of stiffness of the crack like its derivative. With resulting from calculation, a Python treatment, capitalized in CAS-test SDNLL133D ([V5.02.133]) allows to determine the evolution of the stiffness as well as its derivative with a step of 0.2° . The results are presented in the form of file to 3 columns (angle, stiffness, derived from the stiffness). Figure 4.5.2-a watch an example of evolution of the stiffness as well as its derivative.

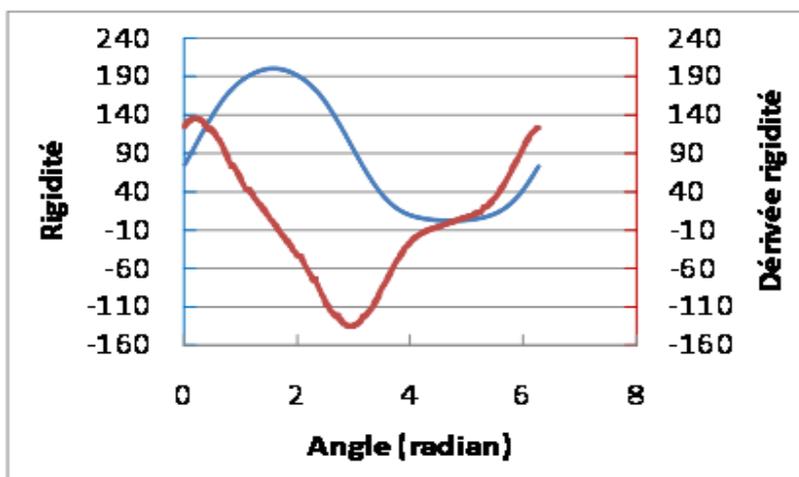


Figure 4.5.2-b: Example of law of stiffness of crack

As dimensions of the test-tube are unit, the stiffness as its derivative are adimensionnées. Their levels will be then adapted to dimensions of the tree during dynamic calculation.

4.5.3 Introduction of the law of the crack

The introduction of the crack into the setting in data is done by the integration of an ASCII file composed of three columns: the swing angle (from 0 to 2π), stiffness of the element fissured and it derived from the stiffness of the fissured element. For example, the first lines of the law of crack integrated in CAS-test SDNL133 are:

```
0.00000E+00  1.88326E+01  5.84082E+01
1.74533E-02  1.98750E+01  6.09235E+01
3.49066E-02  2.09592E+01  6.35813E+01
5.23599E-02  2.20944E+01  6.63547E+01
6.98132E-02  2.32755E+01  6.96498E+01
```

One then informs in ASTK the address of the ASCII file.

4.5.4 Modification of the command file

Initially it is necessary to assign the type of element to the mesh "CRACK" already created in the file grid:

```
MOD_ROT=AFFE_MODELE (MAILLAGE=ARBRE,
    AFFE= (... ,
        ... ,
        _F (GROUP_MA= ('CRACK'),
            PHENOMENE=' MECANIQUE',
            MODELISATION=' DIS_TR'),
        ... ,
    ) );
```

Properties of the element "CRACK" are then defined by duplicating the operator `AFFE_CARA_ELEM` already existing in the file of calculation on the healthy model. It is necessary to create 2 assignments of the elementary characteristics, a first with crack closed (or rigid) named `CARAELEM` and a second which takes account of the angular point of the tree towards crack (tree kneecap at this place) named `CARAELEF`. The law of stiffness of the crack is not integrated at this stage, it acts here to model the crack like a kneecap along the 2 bending axes x and y in order to determine the modal base. In `CARAELEM`, the masses and stiffnesses are defined as follows:

```
CARAELEM=AFFE_CARA_ELEM (MODELE=MOD_ROT,
    ... ,
    _F (CARA=' K_TR_D_L',
        GROUP_MA=' FISSURES,
        VALE= (raidmax, raidmax, raidmax, raidmax, raidmax,
raidmax),
    _F (CARA=' M_TR_D_L',
        GROUP_MA=' FISSURES,
        VALE= (0, 0, 0, 0, 0, 0),
    ... ,
    );
```

The rigidity of the crack 'raidmax' compared to that of the line D' tree is selected sufficiently high (100 larger).

In `CARAELEF`, the values of the stiffnesses are worthless along the 2 bending axes of the tree x and y (in order to take account of the presence of the crack):

```
CARAELEF=AFFE_CARA_ELEM (MODELE=MOD_ROT,  
    .../  
    _F (CARA=' K_TR_D_L',  
        GROUP_MA=' FISSURES,  
        VALE= (raidmax, raidmax, raidmax, 0.0, raidmax),  
    _F (CARA=' M_TR_D_L',  
        GROUP_MA=' FISSURES,  
        VALE= (0,0,0,0,0,0),  
    .../  
);
```

4.5.5 Determination of the modal base

The modal base of a rotor is made up by the clean modes of the associated conservative system when the crack is closed, enriched by the static modes corresponding to the whole of the static answers when a unit displacement in the directions DX, DY, DRX and DRY MARTINI is imposed on the nodes on the left and on the right of the open crack. For example, one must write:

```
#FISSURE FERMEE  
  
Modf = CALC_MODES (MATR_RIGI = KF,  
    MATR_MASS = M,  
    OPTION      = 'CENTER',  
    CALC_FREQ   = _F (FREQ = 1. ,  
                    NMAX_FREQ = 16)  
    SOLVEUR_MODAL = _F (METHODE=' QZ'))  
  
OPEN #FISSURE: construction of static modes  
  
MSTF0 = MODE_STATIQUE (MATR_RIGI = K,  
    MATR_MASS = M,  
    MODE_STAT = _F (GROUP_NO = ('FISG0', 'FISD0'),  
    AVEC_CMP = ('DX', 'DY', 'DRX', 'DRY MARTINI')))
```

The operator `DEFI_BASE_MODAL` must then be used to build the base of projection. In a last time, all the real matrices (mass, stiffness, damping and gyroscopy) are projected via the operator `PROJ_MATR_BASE` on the basis supplements RITZ (bases BMOD). For the calculation of the clean modes, final modal calculation is carried out starting from these projected, said matrices generalized, using the algorithm of resolution of the type QZ.

4.5.6 Realization of a transitory calculation

Initially, it is necessary to determine the multiplying coefficient of the law of stiffness which takes account of real dimensions of the rotor. This coefficient is given by the following equation:

$$COEFF = \frac{E \times I}{2 \times L}$$

with $I = \frac{\pi \times D^4}{64}$ and $L = 4 \times D$

E being the Young modulus of material, D the diameter of the tree, L its length and I its quadratic moment in inflection.

```
YOUNG = 208.e9;  
DIAM = 1,078;
```

```
LENGTH = (2.0 * DIAM);  
I = (((((pi * DIAM) * DIAM) * DIAM) * DIAM) / 64.0);  
COEFF = YOUNG*I/2/LONG
```

The laws of evolution of the stiffness and its derivative are then read and multiplied by this geometrical coefficient:

Law of stiffness :

```
F_K1=LIRE_FONCTION (UNITE=29,  
                    NOM_PARA=' ABSC',  
                    INDIC_PARA= [1.1],  
                    INDIC_RESU= [1.2],  
                    " LINEAR" PROL_DROITE=,  
                    " LINEAR" PROL_GAUCHE=,  
                    INTERPOL=' LIN');
```

Derived from the stiffness :

```
F_KP1=LIRE_FONCTION (UNITE=29,  
                    NOM_PARA=' ABSC',  
                    INDIC_PARA= [1.1],  
                    INDIC_RESU= [1.3],  
                    " LINEAR" PROL_DROITE=,  
                    " LINEAR" PROL_GAUCHE=,  
                    INTERPOL=' LIN')
```

Multiplication by the geometrical coefficient :

```
F_K= CALC_FONCTION (COMB= (_F (FONCTION=F_K1, COEF=COEFF))  
F_KP=CALC_FONCTION (COMB= (_F (FONCTION=F_KP1, COEF=COEFF))
```

The resolution of transitory calculation taking account of the presence of the crack is done then with the order DYNA_VIBRA :

```
TRANGEN = DYNA_VIBRA (MATR_MASS = MASGEN,  
                    MATR_RIGI = RIGGEN,  
                    MATR_AMOR = GYOGEN,  
                    BASE_CALCUL = 'GENE',  
                    TYPE_CALCUL = 'TRAN',  
                    SCHEMA_TEMPS = _F (SCHEMA=' EULER'),  
                    INCREMENT = _F (NOT = not, INST_FIN = tfin),  
                    EXCIT = (_F (VECT_ASSE = vgen_po, COEF_MULT= 1  
)),  
                    ROTOR_FISS = _F (GROUP_NO_G = 'FISS_G',  
                                     GROUP_NO_D = 'FISS_D',  
                                     K_PHI = F_K,  
                                     VITE_ROTA = -1*OM,  
                                     DK_DPFI = F_KP),  
                    FILING = _F (PAS_ARCH = 1000))
```

The crack is defined thanks to the keyword `ROTOR_FISS`, laws of stiffness (`K_PHI`) and of its derivative (`DK_DPFI`) are called in this keyword. The 2 nodes also should be indicated ('`FISS_G`' and '`FISS_D`') delimiting the crack.

4.6 Implementation of the method of calculating accidental

The implementation of the method of calculating accidental on the AWG following the accidental loss of a final wing was formalized recently [5] where the interested reader will be able to find more details. One is satisfied here to describe the great stages of implementation of such a calculation.

The method of reference is structured in two stages:

- the phase former at the beginning of wing,
- the consecutive phase at the beginning of a wing.

4.6.1 Phase former at the beginning of a wing

4.6.1.1 Static calculation under actual weight

The objective of static simulation under the effect of gravity is to determine the distribution of the efforts on the various stages of the AWG. The destriping is taken into account in the model of the AWG. This stage, a strong assumption of support of the line of trees on infinitely rigid stages is retained. Thus, the stiffnesses of the stages, at this stage unknown factors, are fixed at a very high value.

For this stage, the operator classically is used `MECA_STATIQUE` with boundary conditions of type destriping as that was presented in the paragraph 3.3.1.

4.6.1.2 Calculation of the linear dynamic coefficients of the stages

The objective of this stage is to determine the values of the linear dynamic coefficients associated with each stage (stiffness and damping), for several number of revolutions of the line of trees. These coefficients represent the effect of the oil film of the stages on the rotor, which appears in the normal plan with the axis of the line of trees. The calculation of the coefficients, and their use, is done with the assumption of small displacements of the rotor, therefore of a linear behavior of the fluid film. As starter of calculation, the effort taken again by a stage given under the effect of gravity, is provided to `EDYOS`. The geometry and operating the conditions of this stage must also be indicated.

4.6.1.3 Calculation of the linear harmonic answer to an unbalance

This calculation consists in evaluating the answer in harmonic mode of the AWG subjected to an unbalance corresponding at the beginning of one or more wings of the last stage of the one of flows of the BP bodies. The value of the unbalance is calculated on the basis of distance mass of a wing, between its centre of gravity and neutral fibre of the tree and number of revolutions of the AWG. The application is identical for several wings. The various possible positions of the unbalance are examined in order to determine which is more penalizing from a point of view of the efforts on the stages.

The objective of this calculation is multiple. It makes it possible to determine:

- the stages whose oil film is likely to have a non-linear behavior, that it will be necessary to take into account,
- positions of the unbalance, on the last stages of wings, more penalizing starting from the values of efforts raised with the stages,
- critical velocities of the line of trees,
- the characteristics of the model of simplified support which make it possible to obtain, at the critical velocities, of the harmonic answers under unbalance equivalent to those resulting from a simulation with a generalized support.

In practice, this harmonic calculation is carried out in a classical way by a call to `DYNA_VIBRA` for a number of constant revolutions.

4.6.2 Consecutive phase at the beginning of a wing – calculation non-linear

At the time of the departure of one or more final wings on a BP body, an unbalance of strong amplitude appears, which generates strong vibrations of the line of trees. These vibrations can lead to displacements of the rotor which call into question the linear behavior of the oil film of certain stages, and to contacts between the rotor and the stator. For the BP bodies, allowances are at the level of the diaphragms and the mobile ends of wings.

The method exposed here includes the modeling of the non-linearity of the stages as long as the contact is not established. It is about a transitory calculation under the effect of gravity, then, once the stabilized answer, under the cumulated effect of an unbalance. The interaction between the non-linear behavior of the fluid film and the line of trees is simulated using a coupling enters `Code_Aster` and `EDYOS`. With each step of calculation, `Code_Aster` send the position and the speed of the rotor to `EDYOS`. This last returns the efforts due to the fluid film. `Code_Aster` calculate new the position and speed of the rotor which are sent to `EDYOS` with the following step.

For the positions of the unbalance more penalizing (determined by preceding harmonic calculation), in the model of AWG, a non-linear behavior is affected with the most requested stages. A decreasing list number of revolutions, representing the deceleration of the line of trees, is drawn up. This number of revolutions is imposed in calculations independent from/to each other because one is interested in the solution in established mode.

Two criteria of stop are associated with calculation:

- appearance of the contact rotor-stator, when displacements of the line of trees become higher or equal to allowances with the points of the stator in opposite,
- the attack a thickness of calculated the oil film lower than 10 μm . Indeed, when the thickness of this film reaches 2 μm , it becomes of the same order of magnitude as the roughness of surfaces. Under these conditions, an important shearing appears, which has an effect on thermics in this zone, and led to a fall of the viscosity of the fluid, which is not simulated by the code "stage". To be freed from any defect of representativeness, a margin is taken on the minimal thickness below which simulation is not valid any more, from where the value selected of 10 μm , below which calculation "stage" any more is not considered licit.

The objectives of this stage are thus the description:

- maximum efforts acting with the stages and the number of revolutions to which they appear,
- maximum efforts acting with the stages at the number of revolutions to which the first contact rotor-stator appears, if necessary.

These values are then confronted with the acceptable maximum efforts with the stages specified by the manufacturer. This comparison will have to take into account that the acceptable ones generally correspond to a static level, whereas the calculated efforts are of dynamic nature.

The transitory analysis in situation accidental is done exclusively on modal basis by considering a situation at number of constant revolutions. One thus carries out a succession of transitory calculations until the appearance of the one of the two criteria of stop. The explicit diagrams recommended for this transitory analysis are Euler, `Adapt_ordre_1` and `Adapt_ordre_2`.

5 Postprocessings for the studies "revolving machines"

For a modal calculation, it `Code_Aster` allows to calculate the modal sensitivities to the unbalance as well as the coefficients of participation. For harmonic calculations, `Code_Aster` allows to trace the evolution of the amplitude and phase of a degree of freedom according to the frequency of excitation, to trace for a frequency of excitation given the deformation of the rotor and the ellipses trajectories of

the nodes. It also makes it possible to determine the characteristics of the trajectory of the node for a frequency of excitation given.

5.1 Modal sensitivities to the unbalance

It can happen that a mode (position its critical velocity) is in a zone considered as "at the risk" without this last not posing a particular vibratory problem. Indeed, during the introduction of the concept of damping, the roots of the equation of behavior written in matrix form are not purely any more complex, and a real part appears:

$$r_i = -\frac{\epsilon_i \omega_i}{\sqrt{1 - \epsilon_i^2}} + j \omega_i$$

Where ϵ_i is the reduced damping of the mode and ω_i the Eigen frequency of the mode. One can then define the factor of sensitivity to the unbalance, M_i :

$$M_i = \frac{\left(\frac{\Omega}{\omega_i}\right)^2}{\sqrt{\left(1 - \left(\frac{\Omega}{\omega_i}\right)^2\right)^2 + 4\epsilon_i^2 \left(\frac{\Omega}{\omega_i}\right)^2}}$$

Where Ω is the number of revolutions of the line of trees.

This size makes it possible to determine or not the mode so yes is critical. Thus, a perfectly synchronous mode with the number of revolutions can prove to be inoffensive for the line of trees if its reduced damping has a value close to 1. This is why, it is considered that if the value of factor of sensitivity is lower than a certain threshold, then it does not have there a particular vibratory problem. This size can be calculated in the following way starting from the concept `MODE_MECA` :

```
TABLE=RECU_TABLE (CO=MODES,
                  NOM_PARA= ('NUMÉRIQUE_MODE', 'FREQ', 'AMOR_REDUIT'),);
tab=TABLE.EXTR_TABLE ()
fprop=tab.FREQ
amort=tab.AMOR_REDUIT
# Calculation for Ième mode
fp=fprop [I]
am=amort [I]
r=w/fp
Mn=r ** 2/sqrt ((1. - R ** 2) ** 2.+ (2.*am*r) ** 2)
```

5.2 Coefficients of participation

For a given mode, the coefficients of participation are the respective contributions of deformation energies of the rotor and the support to the total deformation energy of the system. The total deformation energy of the mode k is written:

$$E_k = \Phi_k^t K \Phi_k$$

By separating the degrees of freedom relating to the rotor and those relating to the support, energy is expressed in the form:

$$E_k = \begin{bmatrix} \Phi_{kr}^t & \Phi_{ks}^t \end{bmatrix} \left(\begin{bmatrix} K_r & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} K_{rr}(\Omega) & K_{rs}(\Omega) \\ K_{sr}(\Omega) & K_{ss}(\Omega) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_s \end{bmatrix} \right) \begin{bmatrix} \Phi_{kr} \\ \Phi_{ks} \end{bmatrix}$$

If one notes E_{kr} and E_{ks} energies of the rotor and the support, defined by:

$$E_{kr} = \Phi_{kr}^t K_r \Phi_{kr} \quad \text{and} \quad E_{ks} = \Phi_{ks}^t K_s \Phi_{ks}$$

The coefficients of participation relating to the rotor and support are respectively defined by:

$$\rho_{kr} = \frac{E_{kr}}{E_k} \quad \text{and} \quad \rho_{ks} = \frac{E_{ks}}{E_k}$$

The operator `POST_ELEM` gives access directly energies potential, kinetic and of elastic strain on all the mechanical system (option `TOUT=' OUI '`) or of the parts of the system (option `GROUP_MA=' ROTOR '` or `GROUP_MA=' MASSIF '`). The following example makes it possible to calculate these energies for the first 4 modes of the line of trees.

Note:

It is important to note that the calculation of energies is available only for the real modes.

```
EPOT_TOT=POST_ELEM (RESULTAT=MODES,  
                    NUME_MODE= (1,2,3,4),  
                    MODELE=MODELE,  
                    CARA_ELEM=CARELEM,  
                    CHAM_MATER=CHMAT,  
                    ENER_POT=_F (TOUT=' OUI '),  
                    );  
epot_tot=EPOT_TOT.EXTR_TABLE ()  
tot_epot=epot_tot.TOTALE  
  
EPOT_ROT=POST_ELEM (RESULTAT=MODES,  
                    NUME_MODE= (1,2,3,4),  
                    MODELE=MODELE,  
                    CARA_ELEM=CARELEM,  
                    CHAM_MATER=CHMAT,  
                    ENER_POT=_F (GROUP_MA=' ROTOR '),  
                    );  
epot_rot=EPOT_ROT.EXTR_TABLE ()  
rot_epot=epot_rot.TOTALE  
  
EPOT_MAS=POST_ELEM (RESULTAT=MODES,  
                    NUME_MODE= (1,2,3,4),  
                    MODELE=MODELE,  
                    CARA_ELEM=CARELEM,  
                    CHAM_MATER=CHMAT,  
                    ENER_POT=_F (GROUP_MA=' MASSIF '),  
                    );  
epot_mas=EPOT_MAS.EXTR_TABLE ()  
mas_epot=epot_mas.TOTALE
```

Obtaining the coefficients of participation can be made in Python in the same way that for the modal sensitivities to the unbalance M_n (cf paragraph 7.1).

```
# Calculation of the participations of the rotor and solid mass for the first mode  
tot_pot1=tot_epot [1]  
rot_pot1=rot_epot [1]  
mas_pot1=mas_epot [1]  
part_rot=rot_pot1/tot_pot1  
part_mas=mas_pot1/tot_pot1  
print 'part_rot', part_rot  
print 'part_mas', part_mas
```

5.3 Determination of the phases and amplitudes in harmonic

The deformation (modal, harmonic answer, etc) are characterized by complex displacements. This complex character of displacements corresponds to the introduction of dephasing between displacements of the nodes of the model. This dephasing is due to the damping and the gyroscopic effects related to rotation of the rotor. Starting from the concept result, side displacements are recovered in the form of functions as follows:

```
DY_DIS2=RECU_FONCTION (RESULTAT=DHAM,  
                        NOM_CHAM=' DEPL',  
                        NOM_CMP=' DX',  
                        GROUP_NO=' N_DIS2',);
```

```
DZ_DIS2=RECU_FONCTION (RESULTAT=DHAM,  
                        NOM_CHAM=' DEPL',  
                        NOM_CMP=' DY',  
                        GROUP_NO=' N_DIS2',);
```

Amplitudes and phases of displacement following the axes x and y are obtained starting from complex modal displacements in the same directions as follows:

```
MOD_Y_D2=CALC_FONCTION (EXTRACTION=_F (FONCTION=DX_DIS2,  
                                         PARTIE=' MODULE',),);
```

```
MOD_Z_D2=CALC_FONCTION (EXTRACTION=_F (FONCTION=DY_DIS2,  
                                         PARTIE=' MODULE',),);
```

```
PHA_Y_D2=CALC_FONCTION (EXTRACTION=_F (FONCTION=DX_DIS2,  
                                         PARTIE=' PHASE',),);
```

```
PHA_Z_D2=CALC_FONCTION (EXTRACTION=_F (FONCTION=DY_DIS2,  
                                         PARTIE=' PHASE',),);
```

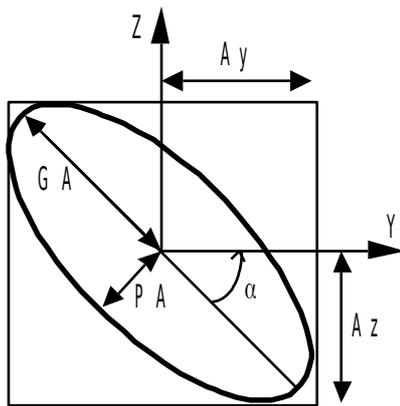
For the convenience of implementation of the later operations of postprocessing, these sizes are stored under format of tables.

```
TABMODY=CRÉA_TABLE (FONCTION=_F (FONCTION=MOD_X_D2,))  
TABMODZ=CRÉA_TABLE (FONCTION=_F (FONCTION=MOD_Y_D2,))  
TABPHAY=CRÉA_TABLE (FONCTION=_F (FONCTION=PHA_X_D2,))  
TABPHAZ=CRÉA_TABLE (FONCTION=_F (FONCTION=PHA_Y_D2,))
```

```
MODYDIS2=TABMODZ ['DX', 1]  
MODZDIS2=TABMODY ['DY', 1]  
PHAYDIS2=TABPHAZ ['DX', 1]  
PHAZDIS2=TABPHAY ['DY', 1]
```

5.4 Determination of the ellipses of the rotors

As for the clean modes, the trajectory of a node in the plan perpendicular to neutral fibre of rotor is thus an ellipse whose characteristics are in the following way defined:



Caractéristiques des trajectoires

Ay : amplitude suivant Y
Az : amplitude suivant Z
GA : demi grand-axe
PA : demi petit-axe
 α : angle entre Y et le grand-axe

Figure 5.4-a : Illustration of the trajectory of a mode

Then, one must determine the large-axis of the elliptic trajectory. From a vibratory point of view, it is the value of the half large-axis which represents the level (peak) vibratory to take into account for dimensioning or the monitoring.

```
dax=ellipse (MODXDIS2, MODYDIS2, PHAXDIS2, PHAYDIS2)
```

The function ellipse is defined as follows:

```
def ellipse (modx, mody, thetx, thety):
    thetx=thetx*pi/180.
    thety=thety*pi/180.
    num = modx ** 2*sin (2*thetx) + mody ** 2*sin (2*thety)
    denum = modx ** 2*cos (2*thetx) + mody ** 2*cos (2*thety)
    yew ABS (denum) < 1e-33:
        maxdemiaxe = 0.
    else:
        T=.5*atan (- num/denum)
        ux1 = modx*cos (T+thetx)
        uy1 = mody*cos (T+thety)
        ux2 = modx*cos (T+thetx+pi/2.)
        uy2 = mody*cos (T+thety+pi/2.)
        axy1 = sqrt (ux1 ** 2+uy1 ** 2)
        axy2 = sqrt (ux2 ** 2+uy2 ** 2)
        maxdemiaxe = max (axy1, axy2)
    return maxdemiaxe
```

The direction of course of the trajectory is called direction of precession. If the direction of course of the elliptic trajectory corresponds within the meaning of rotation of the rotor, the precession is known as direct. In the contrary case the precession is known as opposite or retrograde.

One supposes without it to show that, for the same mode, all the nodes have the same direction of precession. One speaks then about direct or retrograde mode.

In *Code_Aster*, the identification of the direction of precession is done according to the sign of the greatest orbit (it node whose main roads are maximum) in a mode, that is to say according to the sign of the sum of the signs of all the orbits. For more information on calculation of the direction of precession, the reader will be able to refer to the document [R7.10.03].

5.5 Diagram of Campbell

The diagram of Campbell is a chart allowing the follow-up of the natural frequencies of a system in rotation according to its number of revolutions as well as the zones of instability of these modes

([R7.10.03]). The natural frequencies and the modes of a system turning are obtained by the resolution of the dynamic equilibrium equation of a system of rotating shafts, without second member and including the effects of gyroscopic damping.

For this purpose, two macro-orders are developed in *Code_Aster*. The first macro-order, `CALC_MODE_ROTATION` [U4.42.51], the calculation of the frequencies and modes allows on the complete system according to the number of revolutions. The second macro-order, `IMPR_DIAG_CAMPBELL` [U4.52.52], allows to classify the modes in inflection, torsion and traction compression, to normalize these modes, to determine the direction of precession of the modes in inflection, to carry out the sorting of the frequencies according to various methods of follow-up of modes and, finally, to plot the diagram of Campbell.

6 Bibliographical references

- 1 Mr. LALANNE, G. FERRARIS, "Rotordynamics Prediction in Engineering", Second Edition, Wiley, 2001.
- 2 `CODE_ASTER`, www.code-aster.org .
- 3 CADYRO, instruction manual.
- 4 ROTORINSA, software finite elements intended to envisage the dynamic behavior of rotors in inflection, LaMCoS UMR5259, INSA-Lyon.
- 5 NR. GUILLOTEAU, I. NISTOR, NR. SELLALI-HARAIGUE, H-T63-2014-00306-FR "Project TAKES OFF: Method of reference for the accidental calculation of the lines of trees of turbines under Salome Meca"

7 Appendices

7.1 Table of the CAS-tests revolving machines in Code_Aster

Name of the test		SDLLI23			SDLLI24		SDLLI25	SDLLI26				SDLLI40	SDNLI33			
		has	B	C	has	B	has	has	B	C	D	has	has	B	C	D
Following rotor		x	x	x	z	z	z	x	x	x	z	x	x	y	x	
Element	Beam Euler	Yes	Yes	Yes												
	Timoshenko beam	Yes			Yes	Yes		Yes	Yes	Yes	Yes		Yes	Yes	Yes	Not
	voluminal 3D															
Disc		1	1		3	3	2	3	3	3	3	3	1	1	1	
Stage	rigid	2	2													
	linear			2	2	2	2	2	2	2	2	2	1	1	1	
	symmetrical	Yes	Yes	Not	Yes	Yes	Not	Yes	Yes	Yes	Yes	Yes	1	1	1	
Support	rigid	2	2										2	2	2	
	simplified			2												
	generalized				Not	Not	Not	Not	Not	Not	Not	Not				
Fréq. clean	QZ											Yes				
	Sorensen											Yes	Yes	Yes	Yes	
	2 stages											Yes				
	Campbell				Yes	Yes	Yes									
Harmonic	DYNA_LINE_HARM									Yes						
Transient	DYNA_LINE_TRAN							Yes								
	DYNA_TRAN_MODAL								Yes		Yes		Yes	Yes	Yes	
Crack		Not			Not	Not	Not	Not	Not	Not	Not	Not	Yes	Yes	Yes	
Validation	Not regression	Yes	Yes	Yes												
	External source				Yes	Yes	Yes					Yes				
	Analytical															
	Another Aster							Yes	Yes	Yes	Yes		Yes	Yes	Yes	
Remarks																Determination of the law of crack

Name of the test		SDLL141				SHLL102	SHLL103	SDLL127	SDLL128	SDLL129	SDLL132
		has	B	C	D	has	has	has	has	has	has
Following rotor		x	$x + y$	$x + y$	x	x	x	x	x	x	x
Element	Beam Euler	Yes	Yes		Yes						
	Timoshenko beam					Yes	Yes	Yes		Yes	Yes
	voluminal 3D										
Disc				distribu te	3	3		3	3	Not	
Stage	rigid	2	2	2	2	2	2				
	linear							2	2	2	2
	symmetrical					Yes	Not	Yes	VAr	VAr	Yes
Support	rigid	2	2	2	2	2	2				
	simplified										
	generalized					Not	Not	Not	Not	Not	Yes
Fréq. clean	QZ							Yes			
	Sorensen	Yes	Yes	Yes	Yes						
	2 stages										
	Campbell									Yes	
Harmonic	DYNA_LINE_HARM					Yes	Yes				
Transient	DYNA_LINE_TRAN										
	DYNA_TRAN_MODAL										
Crack		Not	Not	Not	Not	Not	Not	Not	Not	Not	
Validation	Not regression						Yes	Yes	Yes		Yes
	External source					Yes	Yes	Yes		Yes	Yes
	Analytical	Yes	Yes	Yes	Yes						
	Another Aster							Yes			Yes
Remarks					Distort variable section			Variable section			

Name of the test		SDNV108			SDNV109		SDNV110	
		has	B	C	has	B	has	B
Following rotor		z	z	z	x	x	x	x
Element	Beam Euler							
	Timoshenko beam							
	voluminal 3D	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Disc								
Stage	rigid							
	linear						2	2
	symmetrical						Yes	Yes
Support	rigid						2	2
	simplified							
	generalized							
Fréq. clean	QZ							
	Sorensen	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	2 stages							
Harmonic	Campbell							
	DYNA_LINE_HARM							
	DYNA_LINE_TRAN							
Transient	DYNA_TRAN_MODAL							
Crack		Not			Not	Not		
Validation	Not regression							
	External source	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	Analytical							
	Another Aster							
Remarks								