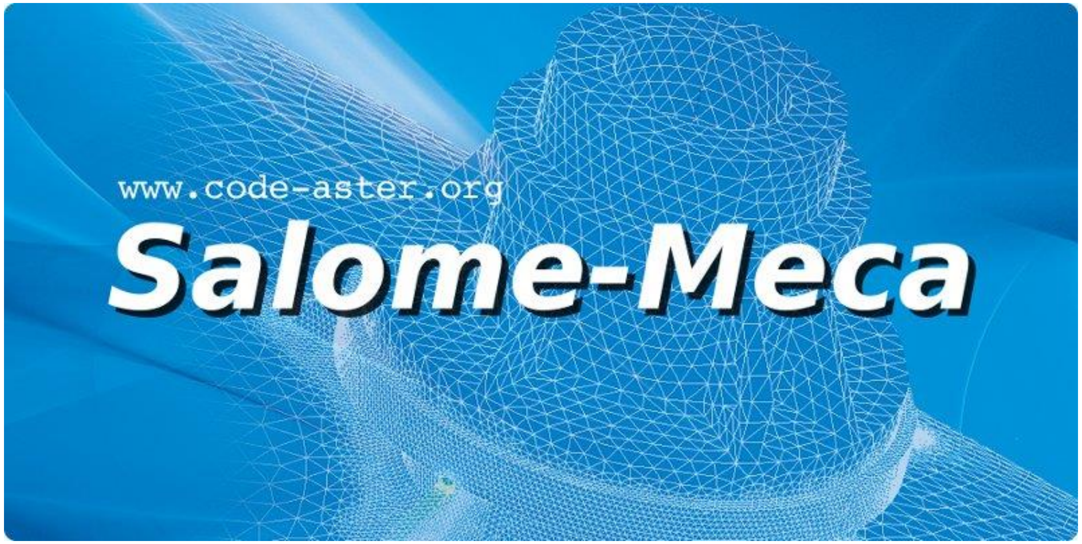
Damping



Code_Aster, Salome-Meca course material

Outline

- Damping in dynamic analysis
  - Types of modeling
  - Getting started with *Code_Aster* for dynamic analysis

- Modal calculation with damping
  - Solving the quadratic problem : principles
  - Methods and algorithms implemented in *Code_Aster*

Damping : theoretical issues

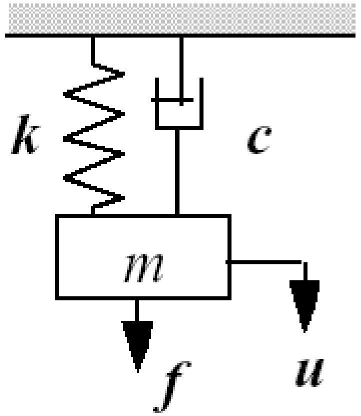
- Damping : energy dissipation within the different materials of the structure, in the connections between structural elements, and with the surrounding medium.

- Two simple models in Code_Aster
  - viscous damping : the dissipated energy is proportional to the speed of movement ;
  - hysteretic damping : the dissipated energy is proportional to the displacement & such that the damping force is of opposite sign to the speed.
- Variables to characterize damping
  - Loss rate : $$\eta = \frac{E_{d\ par\,cycle}}{2\pi E_{p\max}}$$
  - Reduced damping : $$\xi = \frac{\eta}{2}$$

Viscous damping : modelling principles

- Viscous damping : time or frequency definition

$$m\,\ddot{u} + c\,\dot{u} + k\,u = f$$



- Rayleigh damping

$$[C] = \alpha[K] + \beta[M]$$

  - Defined in the material properties **DEFI_MATERIAU**
  - Or easy to implement with the command **COMB_MATR_ASSE**
  - Useful for validating algorithms
  - Successfully introduced for transient analysis by modal recombination
  - Does not represent heterogeneity of the structure relative to the damping
  - Depends heavily on the identification of the coefficients α, β
  - Non physical model for dissipation, but required for some regulatory studies

Viscous damping : identification of coefficients

- Damping proportional inertia characteristics $\alpha = 0 \quad , \quad \beta = \beta_i$
  - Widely used for direct transient resolution
  - β can be identified with the experimental reduced damping ξ_i of the normal mode which contributes most to the response
  - Higher modes are very little damped and low frequency modes are very damped
- Damping proportional to stiffness characteristics $\alpha = \alpha_i \quad , \quad \beta = 0$
  - Widely used for direct transient resolution
  - α can be identified with the experimental reduced damping ξ_j of the normal mode which contributes most to the response
  - Higher modes are very damped and low frequency modes are very little damped
- Full proportional damping $\alpha = \alpha_i \quad , \quad \beta = \beta_i$
  - α, β are identified from two independent modes of frequencies ω_i, ω_j
  - In the interval $\left[\omega_i, \omega_j\right]$, the variation of the reduced damping remains low and outside modes will be over-damped

Viscous damping

- Global damping
  - Coefficients are defined with the material properties in **DEFI_MATERIAU**
    - **AMOR_ALPHA** and **AMOR_BETA**
  - Use in harmonic and transient analysis, physical or reduced bases
    - **DYNA_VIBRA** with the keyword **MATR_AMOR**
- Modal reduced damping
    - **DYNA_VIBRA(BASE_CALCUL='GENE'** with the keyword **AMOR_REDUIT**
    - **COMB_SISM_MODAL** with the keyword **AMOR_REDUIT**
- Localized or non-proportional damping
  - Discrete dampers are introduced with **AFFE_CARA_ELEM**

$$c_{elem\ i} = a_{elem\ i}$$

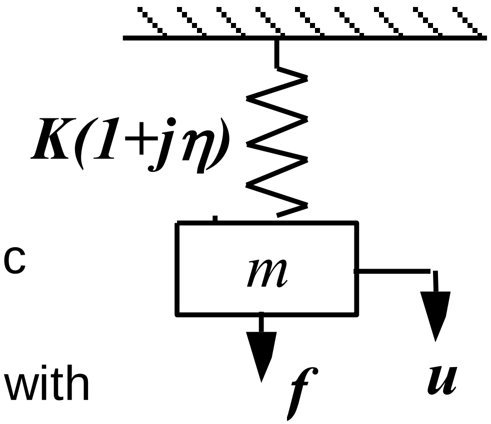
  - Material damping coefficients defined in **DEFI_MATERIAU** with **AMOR_ALPHA** and **AMOR_BETA** affected material by material in the model

$$c_{elem\ i} = \alpha_i k_{elem\ i} + \beta_i m_{elem\ i}$$

Hysteretic (or structural) damping : principles

- Valid only in frequential approach – noncausal model

$$-\omega^2 m u + k(1 + j\eta)u = f$$



- η is a constant
- Representation of the dissipation suitable for metallic materials
- Used to deal with harmonic responses of structures with viscoelastic materials
- The coefficient η is determined from a test with cyclic harmonic loading

$$\left.\begin{aligned}\sigma &= \sigma_0 \mathrm{e}^{\mathrm{j}\omega \mathrm{t}} \\ \varepsilon &= \varepsilon_0 \mathrm{e}^{\mathrm{j}(\omega t - \varphi)}\end{aligned}\right\} \quad E^* = \frac{\sigma}{\varepsilon} = \frac{\sigma_0}{\varepsilon_0}\mathrm{e}^{\mathrm{j}\varphi}$$

$$E^* = \frac{\sigma_0}{\varepsilon_0}(\cos\varphi + j\sin\varphi)$$

$$E^* = E_1(1 + j\eta) \text{ with } \eta = \frac{\mathrm{E}_1}{\mathrm{E}_2} = \tan\varphi$$

Hysteretic damping

- Used for harmonic analysis only
  DYNA_VIBRA(TYPE_CALCUL='HARM') keyword **MATR_RIGI**
- Global or homogeneous damping, identical on the whole model

$$[K_c] = (1 + j\eta)[K]$$

  - The damping coefficient is defined with **DEFI_MATERIAU**, keyword **AMOR_HYST**
  - The complex global matrix is built within **ASSEMBLAGE**
- Localized or non-proportional damping
  - Discrete stiffness elements are defined with **AFFE_CARA_ELEM**, option **AMOR_HYST**

$$k_{c_{discret\ i}} = \left(1 + j\eta_{discret\ i}\right) k_{discret\ i}$$

  - Material damping is defined with **DEFI_MATERIAU**, keyword **AMOR_HYST**

$$k_{c_{elem\ i}} = \left(1 + j\eta_{elem\ i}\right) k_{elem\ i}$$

- Transient analysis
  - calculation of eigenmodes and damping coefficient (quadratic **CALC_MODES**) and resolution on a real modal basis with identified modal damping coefficients (**DYNA_VIBRA(TYPE_CALCUL='TRANS',BASE_CALCUL='GENE')**)

Solving the quadratic problem

- Initial quadratic problem : finding λ & Φ such as

$$\left([\mathbf{K}]+\lambda^2[\mathbf{M}]+\lambda[\mathbf{C}]\right)\,\Phi=\mathbf{0}$$

- Linear form :

$$\underbrace{\left(-\lambda\begin{bmatrix}-\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{K}\end{bmatrix}\right.}_{\mathbf{M}_{quad}}+\underbrace{\left.\begin{bmatrix}\mathbf{C} & \mathbf{K}\\ \mathbf{K} & \mathbf{0}\end{bmatrix}\right)}_{\mathbf{K}_{quad}}\begin{bmatrix}\lambda\Phi\\ \Phi\end{bmatrix}=\mathbf{0}$$

  - Usual algorithm for normal modes

- Meaning of eigenvalues

$$\lambda_i=\xi_i\omega_i\pm j\omega_i\sqrt{1-\xi_i^2}\qquad \frac{\phi_i^{*T}C\phi_i}{\phi_i^{*T}M\phi_i}=2\,\mathrm{Re}\,(\lambda_i)\qquad \frac{\phi_i^{*T}K\phi_i}{\phi_i^{*T}M\phi_i}=|\lambda_i|^2$$

  - ω_i is the pulsation ξ_i is the damping of mode i

Some theory

- Spectral transform => different strategies
- Shift & Invert with complex shift σ

$$\left[\mathbf{K}_{quad}\right]\mathbf{u} = \lambda\left[\mathbf{M}_{quad}\right]\mathbf{u} \Rightarrow \underbrace{\left[\left(\mathbf{K}_{quad} - \sigma\mathbf{M}_{quad}\right)^{-1}\mathbf{M}\right]}_{\mathbf{A}}\mathbf{u} = \underbrace{\frac{1}{\lambda - \sigma}}_{\mu}\mathbf{u}$$

- For high damping : real arithmetic $\mathrm{Re}(\mathbf{A}) \Rightarrow \mu = \frac{1}{2}\left(\frac{1}{\lambda - \sigma} + \frac{1}{\lambda - \overline{\sigma}}\right)$
- For low damping : imaginary arithmetic $\mathrm{Im}(\mathbf{A}) \Rightarrow \mu = \frac{1}{2j}\left(\frac{1}{\lambda - \sigma} - \frac{1}{\lambda - \overline{\sigma}}\right)$
- More generally : complex approach $\mathbf{A} \Rightarrow \mu = \frac{1}{\lambda - \sigma}$

In *Code_Aster*

- Power iteration methods (inverse methods) in **`CALC_MODES`**
  - Only for a couple of frequencies
- Subspace iteration methods in **`CALC_MODES`**
  - **`METHODE='TRI_DIAG'`**
    - Real approach
      - shift = 0 => **`OPTION = 'PLUS_PETITE'`** or **`'CENTRE'`**
      - Non-zero shift => **`OPTION='CENTRE'`**
    - Imaginary approach
      - Non-zero shift => **`OPTION='CENTRE'`**
  - **`METHODE='SORENSEN'`**
    - Real approach
      - shift = 0 => **`OPTION = 'PLUS_PETITE'`** or **`'CENTRE'`**
      - Non-zero shift => **`OPTION='CENTRE'`**
    - Imaginary approach
      - Non-zero shift => **`OPTION='CENTRE'`**
    - Complex approach
      - Non-zero shift => **`OPTION='CENTRE'`**
- **Decomposition method in the complex space `METHOD='QZ'`**
  - **Very robust**
  - **Only for small problems : computes all the modes !**

- Nota Bene : No **`OPTION='BANDE'`** and no verifications on the number of eigenvalues

In three steps with modal reduction

- Problem to solve : (λ_c,Φ_c) such as $\left([\mathbf{K}]+\lambda_c^2[\mathbf{M}]+\lambda_c[\mathbf{C}]\right)\ \Phi_c=\mathbf{0}$
- First step : normal modes without damping
  - (λ,Φ) such as $$\left([\mathbf{K}]+\lambda^2[\mathbf{M}]\right)\ \Phi=\mathbf{0}$$
- Second step : modal reduction by projection
$$\mathbf{K}_g=\Phi^t\mathbf{K}\Phi \quad \mathbf{M}_g=\Phi^t\mathbf{M}\Phi \quad \mathbf{C}_g=\Phi^t\mathbf{C}\Phi$$
- Third step : quadratic problem on a reduced basis
$$\left(-\lambda_c\begin{bmatrix}-\mathbf{M}_g & \mathbf{0}\\ \mathbf{0} & \mathbf{K}_g\end{bmatrix}+\begin{bmatrix}\mathbf{C}_g & \mathbf{K}_g\\ \mathbf{K}_g & \mathbf{0}\end{bmatrix}\right)\begin{bmatrix}\lambda_c\Phi_g\\ \Phi_g\end{bmatrix}=\mathbf{0}$$
- Only for low damping
- Far more robust & effective method
  - use **METHOD='QZ'** for the third step
- Always use **PROFIL='PLEIN'** for numbering (and assembly)

  Default option in NUME_DDL and ASSEMBLAGE commands

End of presentation

Is something missing or unclear in this document?
Or feeling happy to have read such a clear tutorial?

Please, we welcome any feedbacks about Code_Aster training materials.
Do not hesitate to share with us your comments on the Code_Aster forum dedicated thread.