Example: the Norton law

A very simple constitutive law: the viscoplastic Norton law. Let's dare the equations!

\[
\begin{align*}
\dot{\varepsilon}^v &= \dot{\rho} n \\
\dot{\rho} &= \left( \frac{\sigma_{eq}}{K} \right)^n \\
\sigma &= H (\varepsilon - \varepsilon^v)
\end{align*}
\]

The coefficients to be defined are: \(n > 0\) et \(\frac{1}{K}\)

\(H\) is the elasticity operator, \(\sigma_{eq}\) the Von Mises norm, \(n\) the normal vector to the loading surface.

Let's dare now the fortran! In addition to the classic routine of material coefficients reading, simply insert the following routine (called par LCDVIN):

```
SUBROUTINE NORTON(NVI,VINI,COEFT,NMAT,SIGI,DEPS,DVIN,IRET)
IMPLICIT NONE
C ==================================================================
C      MODELE VISCOPLASTIQUE DE NORTON
C ==================================================================
C     DERIVEES DE L'ENSEMBLE DES VARIABLES INTERNES DU MODELE
C    IN NVI  :  NOMBRE DE VARIABLES INTERNES
C    VINI   :  VARIABLES INTERNES A T
C    COEFT  :  COEFFICIENTS MATERIAU INELASTIQUE A T
C    NMAT   :  DIMENSION MAXI DE COEFT
C    SIGP   :  CONTRAINTES A L'INSTANT COURANT, AVEC SQRT(2)
C    DEPS   :  INCREMENT DE DEFORMATIONS, AVEC SQRT(2)
C    OUT:
C    DVIN   :  DERIVEES DES VARIABLES INTERNES A T
C    IRET   :  CODE RETOUR =0 SI OK, =1 SI PB
C     ----------------------------------------------------------------
INTEGER IRET,ITENS,NDI,NMAT,NVI,NDT
REAL*8 COEFT(NMAT),VINI(NVI),DVIN(NVI),SMX(6),SIGI(6)
REAL*8 DP,N,UNSURK,GRJ2V,EPSI,R8MIEM,LCNRTS,DEPS(6)
C     ----------------------------------------------------------------
IRET=0
C     INITIALISATION DES DERIVEES DES VARIABLES INTERNES A ZERO
CALL R8INIR(7,0.D0,DVIN,1)
C -- COEFFICIENTS MATERIAU
N      = COEFT(1)
UNSURK = COEFT(2)
C ZERO NUMERIQUE ABSOLU
EPSI=R8MIEM()
C---------- CALCU DU TENSEUR DEVIATORIQUE DES CONTRAINTES -----
CALL LCDEVI(SIGI , SMX )
C----------CALCU DU DEUXIEME INVARIANT DE CONTRAINE ---------
GRJ2V  = LCNRTS(SMX )
C------ EQUATION DONNANT LA DERIVEE DE LA DEF VISCO PLAST
IF (GRJ2V .GT. EPSI) THEN
  DP=(GRJ2V*UNSURK)**N
  IF (DP .GT. 1.D-10) THEN
    DO 12 ITENS=1,6
      DVIN(ITENS)=1.5D0*DP*SMX(ITENS)/GRJ2V
    12       CONTINUE
  DVIN(7)=DP
ENDIF
ENDIF
END
```

Creep test with the Norton law
Comparison of performances:

<table>
<thead>
<tr>
<th>Algo</th>
<th>CPU time</th>
<th>Nb time steps</th>
<th>Nb iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norton Implicit</td>
<td>83s</td>
<td>101</td>
<td>382</td>
</tr>
<tr>
<td>Norton Explicit (1/3 of calculation)</td>
<td>162s</td>
<td>1061</td>
<td>2275</td>
</tr>
<tr>
<td>Lemaitre implicit</td>
<td>23s</td>
<td>101</td>
<td>330</td>
</tr>
</tbody>
</table>

The implicit resolution is much more efficient than the explicit resolution (which does not lead beyond half of total time). The results are identical to the reference calculation (Lemaitre law), as shown by the maximum strain variation with time:

Fluage sur éprouvette entaille

Loi de Norton