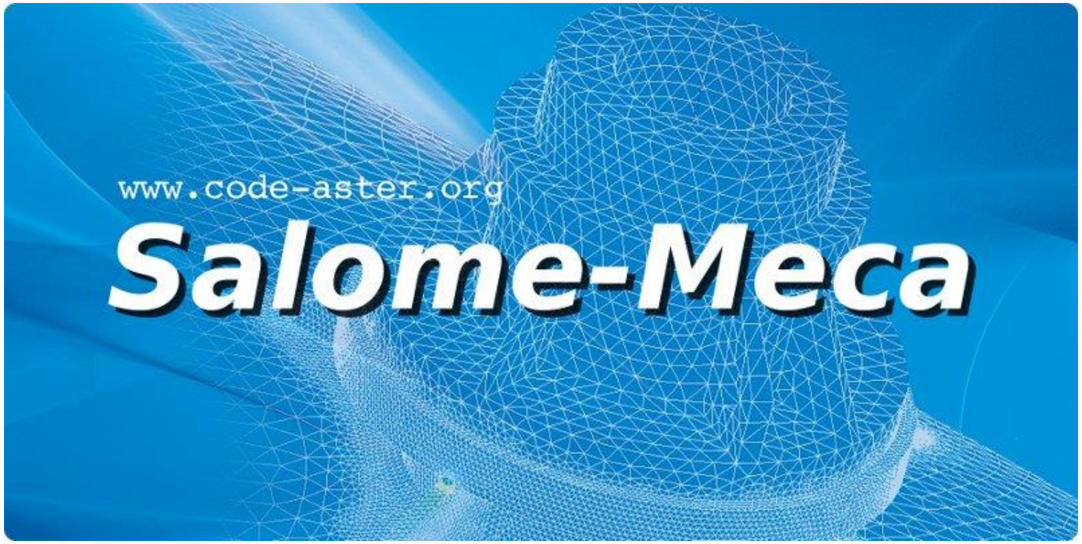
Loading path for a structure



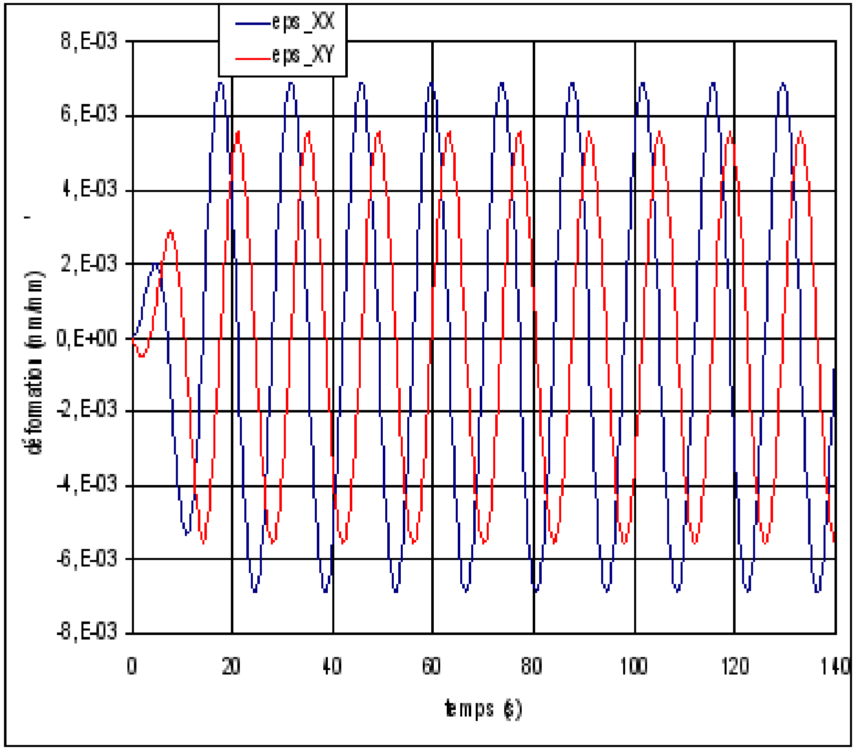
Code_Aster, Salome-Meca course material

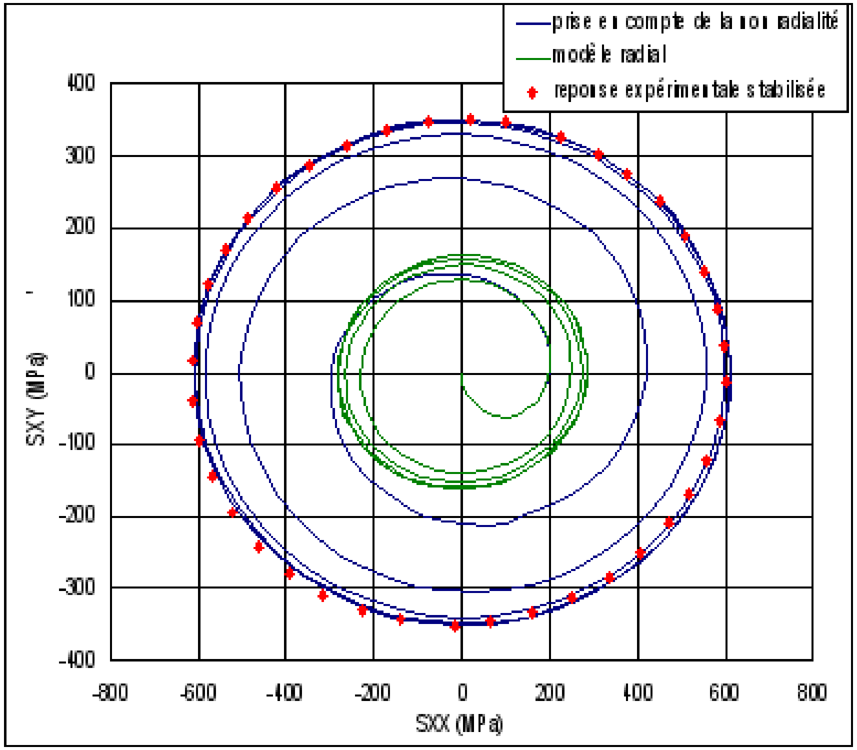
Problem to be solved

- The chosen model is suited to loading path ?
- What is the error in the integration of elastoplastic constitutive law ?
- How to avoid these errors?

Problem to be solved

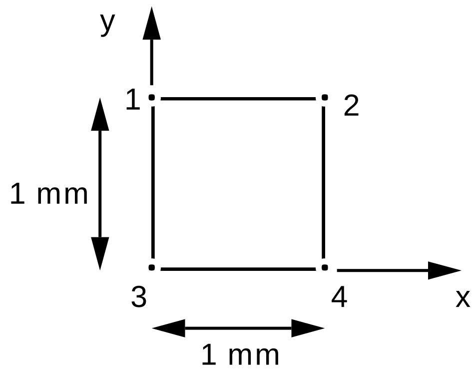
- Know the limitations of theoretical models used
  - Traction loading + 90 ° phase shifted torsion

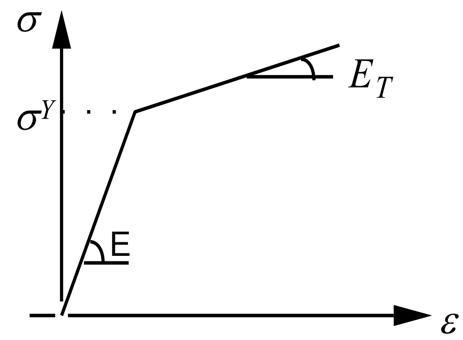




A (very) simple problem

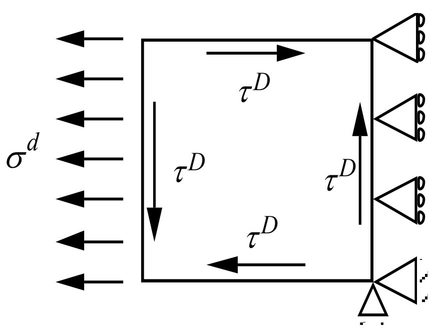
- Material point subjected to a loading path
  - Elastoplasticity with linear isotropic hardening

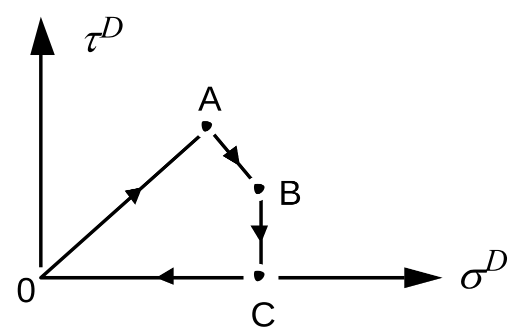




E = 195000 MP
$\nu = 0.3$
$\sigma^Y = 181$ MPa
$E_T = 1930$ MPa

  - Loading





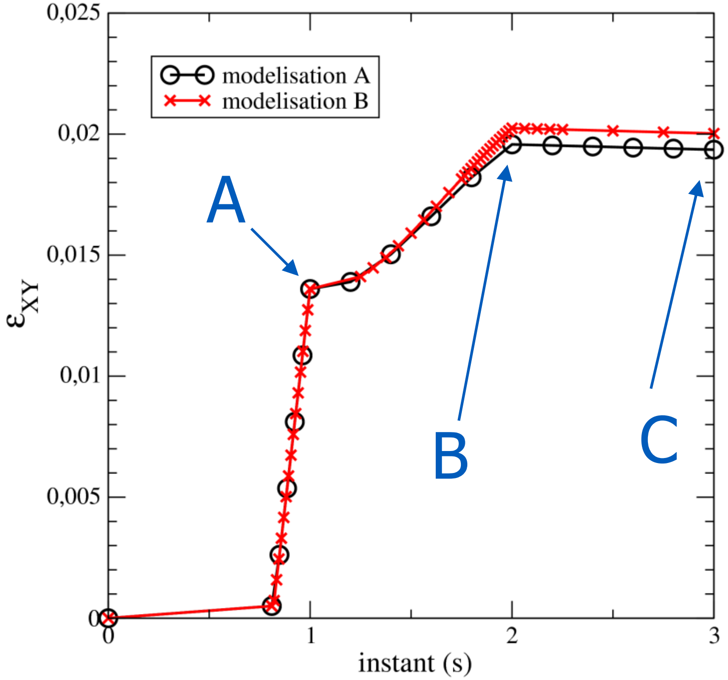
	σ^D (MPa)	τ^D (MPa)
A	151.2	93.1
B	257.2	33.1
C	259.3	0

Time discretization error

- 3 methods of resolution
  - Analytical resolution -> reference values for ε_{xy}
  - Model A : coarse time discretization
  - Model B : fine time discretization

(between two time steps, the cumulated plastic strain is constrained not to increase no more than 0.1 10^{-2})

	Relative error of ε_{XY}	
Time step	Model A	Model B
1 s (point A)	0,003 %	0,003%
2 s (point B)	4,3%	1,1 %



Implicit discretization

	Continuous	Discrete
Constitutive law	$\varepsilon(t) \to \sigma(t)$ $\begin{pmatrix}\varepsilon \\ \alpha\end{pmatrix} \xrightarrow{\dot{\varepsilon}} \dot{\alpha}$ $\sigma = \sigma(\varepsilon, \alpha)$	$\varepsilon(t_i) = \varepsilon_i \to \sigma(t_i) = \sigma_i$ $\begin{pmatrix}\varepsilon_i \\ \alpha_i\end{pmatrix} \xrightarrow{\Delta\varepsilon} \Delta\alpha$ $\sigma_{i+1} = \sigma(\varepsilon_{i+1}, \alpha_{i+1}, \varepsilon_i, \alpha_i)$
Isotropic hardening + implicit scheme	1) $\sigma = A:(\varepsilon - \varepsilon^p) \quad R = R(p)$ 2) $\dot{\varepsilon}^p = \frac{3}{2}\dot{\lambda}\frac{\tilde{\sigma}}{\sigma_{eq}} \quad \dot{p} = \dot{\lambda}$ 3) $\dot{\lambda} f = 0 \quad \dot{\lambda} \geq 0 \quad f \leq 0$	1) $\sigma^+ = A:(\varepsilon^+ - \varepsilon^{p+}) \quad R^+ = R(p^+)$ 2) $\frac{\Delta\varepsilon^p}{\Delta t} = \frac{3}{2}\frac{\Delta\lambda}{\Delta t}\frac{\tilde{\sigma}^+}{\sigma^+_{eq}} \quad \frac{\Delta p}{\Delta t} = \frac{\Delta\lambda}{\Delta t}$ 3) $\Delta\lambda f^+ = 0 \quad \Delta\lambda \geq 0 \quad f^+ \leq 0$

Choice of the scheme

Implicit order 1 scheme

Error

- 1

$$\Delta p = \int_{t^-}^{t^+} \frac{dp}{dt} dt = \int_{\lambda^-}^{\lambda^+} d\lambda = \Delta\lambda$$

OK

- 2

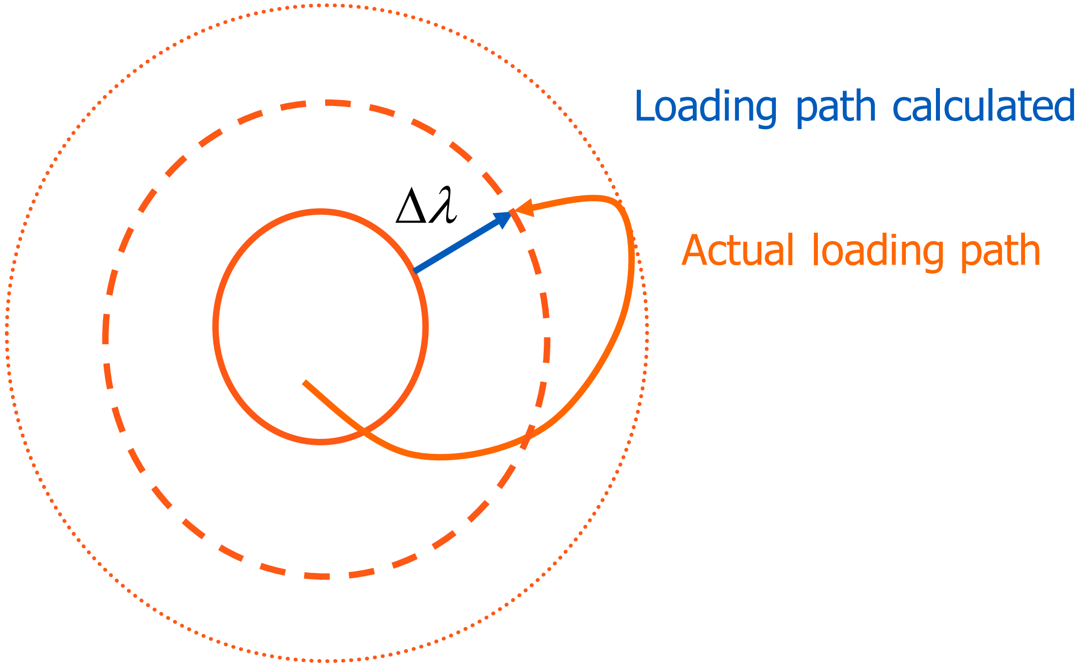
$$\Delta\varepsilon^p = \int_{t^-}^{t^+} \frac{d\varepsilon^p}{dt} dt = \int_{t^-}^{t^+} \frac{d\lambda}{dt} n(\lambda) dt = \int_{\lambda^-}^{\lambda^+} n(\lambda) d\lambda$$

$$\approx \int_{\lambda^-}^{\lambda^+} \left[n(\lambda^+) + n'(\lambda^+).(\lambda - \lambda^+) \right] d\lambda = n(\lambda^+)\Delta\lambda - n'(\lambda^+).\frac{\Delta\lambda^2}{2}$$

Error!

Error

- 3 : if $\dot{\lambda} \geq 0 \quad f \leq 0$ from t$^-$ to t$^+$: OK, except if : load + undload in 1 single time step.



Error

- Main source of error : $$-n'(\lambda^{+}).\frac{\Delta\lambda^{2}}{2}$$
- Exact integration
  - Permanent load during the loading step
  - The normal to the criterion is not reversing ("radial" load)
- Refine the time step outside these two cases!
- But how to detect?

Evaluation of the error

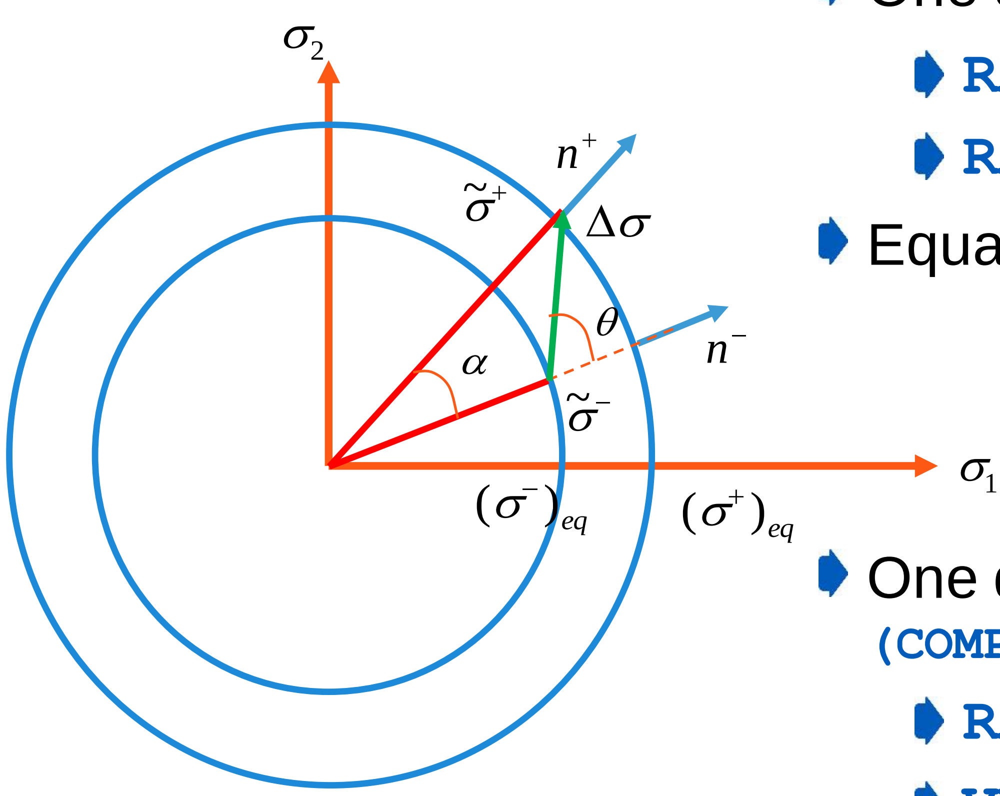
- Relative error on integration :

$$\eta = \frac{\left\|\delta\Delta\varepsilon^{p}\right\|}{\left\|\Delta\varepsilon^{p}\right\|} = \frac{\left\|n'(\lambda)\right\|\frac{\Delta\lambda^{2}}{2}}{\left\|n(\lambda)\right\|\Delta\lambda} = \frac{\left\|n'(\lambda)\right\|\frac{\Delta\lambda}{2}}{1}$$

$$\approx \left\|\frac{\Delta n}{\Delta\lambda}\right\|\frac{\Delta\lambda}{2} = \frac{1}{2}\left\|\Delta n(\lambda)\right\|$$

An indicator of loss of "radiality"

$$\eta = \frac{1}{2}\|\Delta n\| = \left|\sin\frac{\alpha}{2}\right| \approx \frac{|\sin\theta|}{2}$$



The error is measured by α and θ

- One define **DERA_ELGA** **(CALC_CHAMP)**
  - **RADI_T** = $\left|\sin\frac{\alpha}{2}\right|$
  - **RADI_V** = $(1-\cos\theta)$
- Equals 0 when the loading is radial
- One define **RESI_RADI_RELA** **(COMP_INCR)**
  - **RADI_T**
  - **VMIS_xx**

Sufficient conditions of radiality

- "Radiality" = when the threshold normal is not reversing

$$\sigma(x,t) = \alpha(t)\ \sigma_0(x)$$

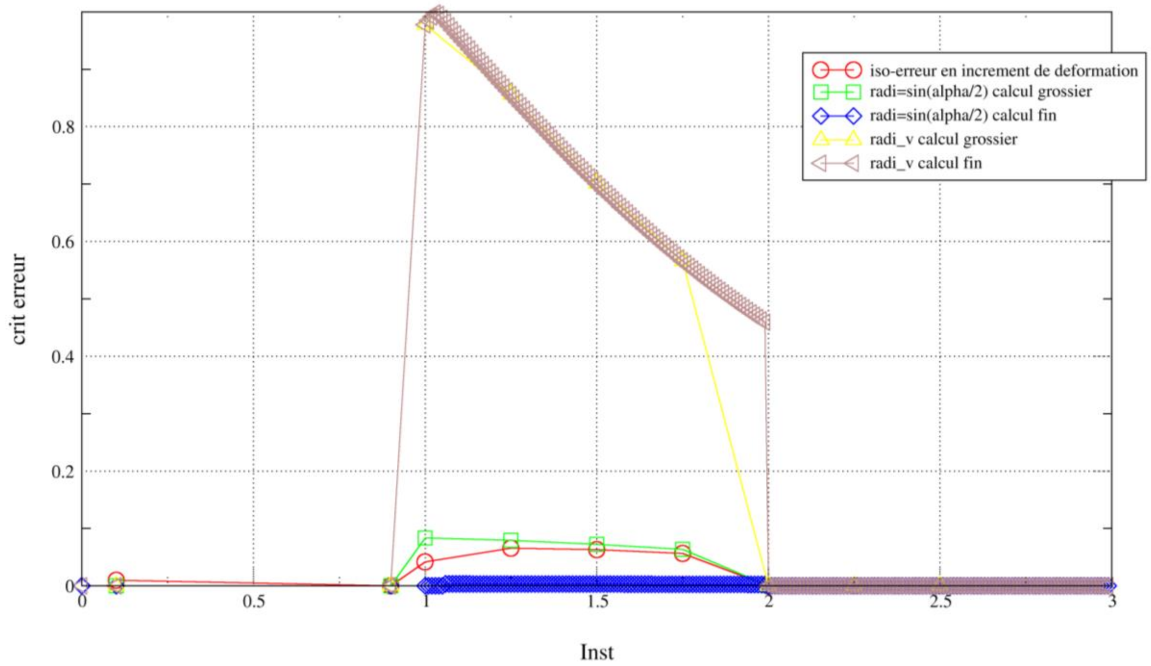
- Radiality conditions
  - External loads are proportional
  - Initial state undeformed and not hardened
  - Von Mises plasticity with isotropic hardening $R(p) = Cp^n$

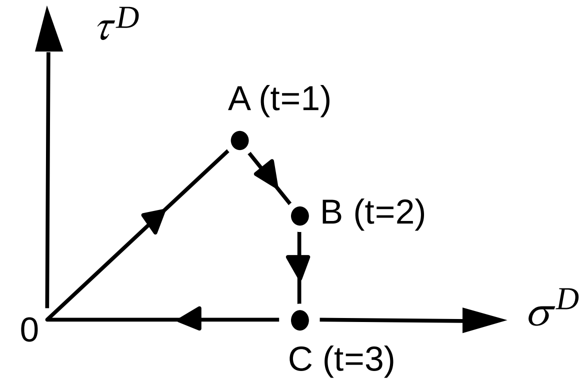
- Advantages
  - Exact local integration
  - Easy global convergence

- In the other cases, refine the time step !

practical example : TP1

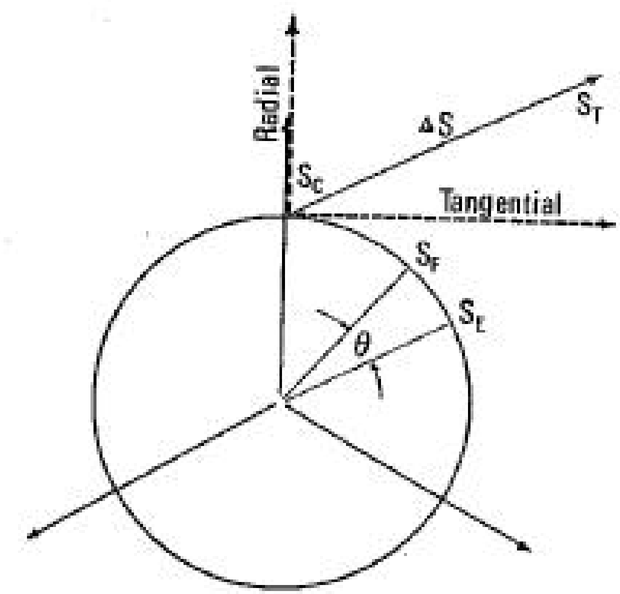
- Indicator of radiality : **DERA_ELGA** in **CALC_CHAMP**
- Component(s): **RADI_T** or **RADI_V**



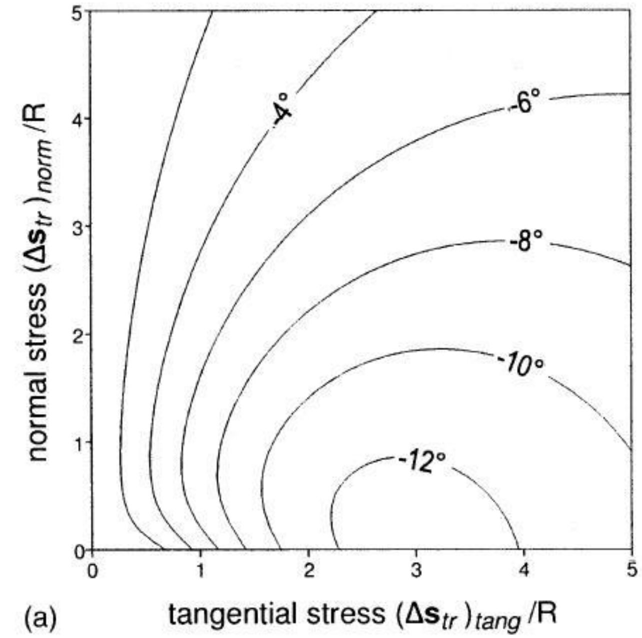


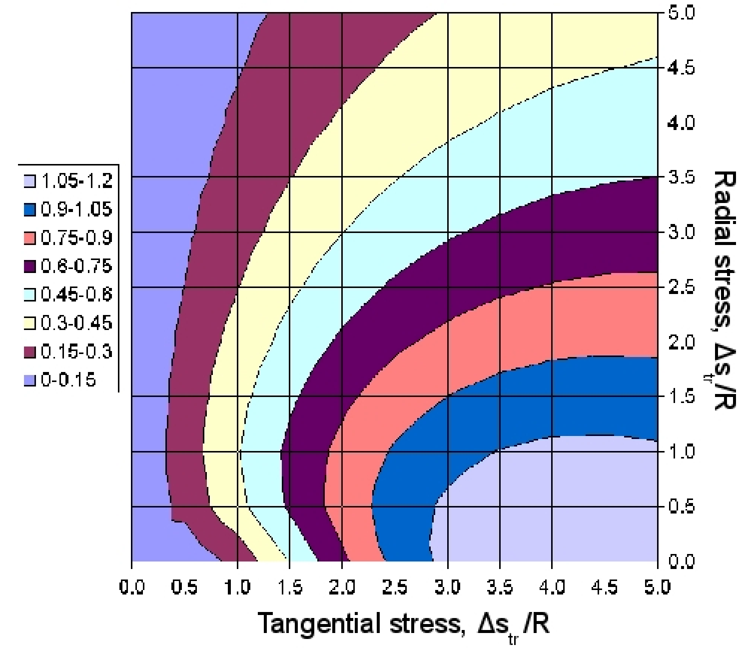
Example taken from literature

- Perfect Von-Mises elasto-plasticity



$$ERR = \frac{\sqrt{(\sigma_n - \sigma_{ex}):(\sigma_n - \sigma_{ex})}}{\sqrt{\sigma_{ex}:\sigma_{ex}}} \times 100\%$$





End of presentation

Is something missing or unclear in this document?
Or feeling happy to have read such a clear tutorial?

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