Architecture THM. Integration of the equilibrium equations

Summary:

This note presents the arguments and variable data-processing used in the routines THM. This note starts with a summary presentation of the equations, which does not replace Doc. R, only reference in the field.

Contents

1 Variational writings of the equilibrium equations

1.1 Mechanics

One leaves the following differential writing:

$$\mathrm{Div}\,\sigma+r\,\mathbf{F}^{m}=0 \qquad \text{éq 1.1-1}$$

We will further see we always adopt the decomposition $\sigma=\sigma'+\sigma_p I$, where σ' indicate the effective constraint.

It is thus with the load of the module of integration of the equilibrium equations to make the sum: $\sigma=\sigma'+\sigma_p I$.
One will then write a variational form of [éq 1.1-1] at time t^+.

$$\boxed{\begin{cases}\sigma^+=\sigma'^+ +\sigma_p^+ I\\ \int_\Omega \sigma^+ . \varepsilon(\mathbf{v})=\int_\Omega r^+\mathbf{F}^{m^+}.\mathbf{v}+\int_{\partial\Omega}\mathbf{f}^{ext^+}.\mathbf{v} \qquad \forall\,\mathbf{v}\in U_{ad}\end{cases}} \quad \text{éq 1.1-2}$$

1.2 Hydraulics

One leaves the following differential writing:

$$\frac{dm}{dt}+\mathrm{Div}(\mathbf{M})=0 \qquad \text{éq 1.2-1}$$

It is considered that there can be two components, and for each one of them two phases.

More precisely, variables $m_1, \mathbf{M}_1$ and $m_2, \mathbf{M}_2$ refer each one to a component of conservative mass.

One poses by principle:

$$m_1=m_1^1+m_1^2; \qquad \mathbf{M}_1=\mathbf{M}_1^1+\mathbf{M}_1^2$$
$$m_2=m_2^1+m_2^2; \qquad \mathbf{M}_2=\mathbf{M}_2^1+\mathbf{M}_2^2$$

What we will write:

$$m_{constituant}=\sum_{nb\ phase\ du\ constituant} m_{constituant}^{phase}$$
$$\mathbf{M}_{constituant}=\sum_{nb\ phase\ du\ constituant} \mathbf{M}_{constituant}^{phase}$$

In the applications, one could for example have:

2 components: air and water
2 phases for water
1 phase for the air

One would have then:

$m_1^1 \, et \, \mathbf{M}_1^1$: contribution of mass and liquid water flow

$m_1^2 \, et \, \mathbf{M}_1^2$: contribution of mass and vapor flow

$m_2^1 \, et \, \mathbf{M}_2^1$: contribution of mass and flow of dry air

$m_2^2 \, et \, \mathbf{M}_2^2$: non-existent

It is considered that there are two pressures. No assumption is made on what the pressures mean $p_1 \, et \, p_2$, that will depend on the laws of behavior and the way which one will choose to write them: one could for example choose:

$$p_1 = pression\ capillaire\left(p(gaz) - p(liquide)\right)$$
$$p_2 = pression\ de\ gaz\left(vapeur + gaz\right)$$

One will write then a variational form of [éq 1.2-1].

$$-\int_\Omega d\frac{(m_1^1+m_1^2)}{dt}\pi_1 + \int_\Omega (\mathbf{M}_1^1+\mathbf{M}_1^2).\nabla\pi_1 = \int_{\partial\Omega}(\mathbf{M}_{1\text{ext}}^1+\mathbf{M}_{1\text{ext}}^2).\pi_1 \qquad \forall \pi_1 \in P_{1\text{ad}} \qquad \text{éq 1.2-2}$$

$$-\int_\Omega d\frac{(m_2^1+m_2^2)}{dt}\pi_2 + \int_\Omega (\mathbf{M}_2^1+\mathbf{M}_2^2).\nabla\pi_2 = \int_{\partial\Omega}(\mathbf{M}_{2\text{ext}}^1+\mathbf{M}_{2\text{ext}}^2).\pi_2 \qquad \forall \pi_2 \in P_{1\text{ad}} \qquad \text{éq 1.2-3}$$

After discretization by theta method:

$$\boxed{\begin{aligned} &-\int_\Omega (m_1^{1+}+m_1^{2+})\pi_1 + \theta\Delta t\int_\Omega (\mathbf{M}_1^{1+}+\mathbf{M}_1^{2+}).\nabla\pi_1 = \\ &\quad -\int_\Omega (m_1^{1-}+m_1^{2-})\pi_1 - (1-\theta)\Delta t\int_\Omega (\mathbf{M}_1^{1-}+\mathbf{M}_1^{2-}).\nabla\pi_1 \\ &\quad +\Delta t\int_{\partial\Omega}(\mathbf{M}_{1ext}^{1\theta}+\mathbf{M}_{1\text{ext}}^{2\theta}).\pi_1 \qquad \forall\pi_1\in P_{1\text{ad}} \end{aligned}} \qquad \text{éq 1.2-4}$$

$$\boxed{\begin{aligned} &-\int_\Omega (m_2^{1+}+m_2^{2+})\pi_2 + \theta\Delta t\int_\Omega (\mathbf{M}_2^{1+}+\mathbf{M}_2^{2+}).\nabla\pi_2 = \\ &\quad -\int_\Omega (m_2^{1-}+m_2^{2-})\pi_2 - (1-\theta)\Delta t\int_\Omega (\mathbf{M}_2^{1-}+\mathbf{M}_2^{2-}).\nabla\pi_2 \\ &\quad +\Delta t\int_{\partial\Omega}(\mathbf{M}_{2ext}^{1\theta}+\mathbf{M}_{2\text{ext}}^{2\theta}).\pi_2 \qquad \forall\pi_2\in P_{2\text{ad}} \end{aligned}} \qquad \text{éq 1.2-5}$$

Note:

In the framework of saturated modeling HM permanent, the term $\frac{dm_1^1}{dt}$ *disappears from the writing of the conservation of the fluid mass. The latter is written simply:*

$$\text{Div}(\mathbf{M}_1^1) = 0$$

The corresponding variational form is written:

$$\int_\Omega \mathbf{M}_1^1.\nabla\pi_1 = \int_{\partial\Omega}\mathbf{M}_{1\text{ext}}^1.\pi_1 \quad \forall\pi_1\in P_{1\text{ad}}$$

1.3 Thermics

We introduce the enthali of each phase of each component: $h_{c\ m}^{p}$

We note: np_c the number of phases of the component C.

We adopt the rule of summation of the dumb indices:

$$h_{c\ m}^{p}\mathbf{M}_c^p=\sum_{i=1}^{np_c} h_{cm}^{i}\mathbf{M}_c^i \qquad h_{c\ m}^{p}\frac{dm_c^p}{dt}=\sum_{i=1}^{np_c} h_{cm}^{i}\frac{dm_c^i}{dt}$$

The equation of thermics (or energy) is written:

$$\frac{dQ'}{dt}+h_{c\ m}^{p}\frac{dm_c^p}{dt}+\mathrm{Div}\left(h_{c\ m}^{p}\mathbf{M}_c^p+\mathbf{q}\right)=R+\mathbf{M}_c^p.\mathbf{F}^m \quad \text{éq 1.3-1}$$

One will then write a variational form of [éq 1.3-1] without injecting the hydraulic equilibrium equation there:

$$\boxed{\begin{array}{l}\int_\Omega \frac{dQ'}{dt}\tau+\int_\Omega h_{c\ m}^{p}\frac{dm_c^p}{dt}\tau-\int_\Omega\left(h_{c\ m}^{p}\mathrm{M}_c^p+\mathrm{q}\right).\nabla\tau=\int_\Omega\left(R+\mathrm{M}_c^p.\mathrm{F}\right)\tau-\int_{\partial\Omega}\left(h_{c\ m}^{p}\mathrm{M}_{c\,ext}^{p}+\mathrm{q}_{ext}\right).\tau\\ \quad\forall\tau\in T_{ad}\end{array}} \quad \text{éq1.3-2}$$

The discretization of [éq 1.3-2] by theta method leads to:

$$\boxed{\begin{array}{c}\int_\Omega\left(Q'^{+}-Q'^{-}\right)\tau-\theta\Delta t\int_\Omega\left(\left(h_{c\ m}^{p+}\mathbf{M}_c^{p+}+\mathbf{q}^{+}\right)\right)\nabla\tau\left(1-\theta\right)\Delta t\int_\Omega\left(\left(h_{c\ m}^{p-}\mathbf{M}_c^{p-}+\mathbf{q}^{-}\right)\right)\nabla\tau+\ldots\\ +\theta\int_\Omega h_{c\ m}^{p+}\left(m_{c\ m}^{p+}-m_{c\ m}^{p-}\right)\tau+\left(1-\theta\right)\int_\Omega h_{c\ m}^{p-}\left(m_{c\ m}^{p+}-m_{c\ m}^{p-}\right)\tau \quad =\\ \theta\Delta t\int_\Omega \mathbf{M}_c^{p+}.\mathbf{F}^m\tau+\left(1-\theta\right)\Delta t\int_\Omega\mathbf{M}_c^{p-}.\mathbf{F}^m\tau+\Delta t\int_\Omega R^\theta\tau-\Delta t\int_\Omega\left(h_{c\ m}^{p}\mathbf{M}_{ext}^{p\theta}+\mathbf{q}_{ext}^{\theta}\right).\tau\\ \forall\tau\in T_{ad}\end{array}} \quad \text{éq 1.3-3}$$

One notices in the equation [éq 1.3-3] a term of contribution of heat by the flow of fluid at the edge of the field: $\int_{\partial\Omega}\left(h_{c\ m}^{p}\mathbf{M}_{c\,ext}^{p\theta}+\mathbf{q}_{ext}^{\theta}\right).\tau$.

One will be able in makes consider that the conditions of heat flux define directly:

$$\tilde{\mathbf{q}}_{ext}^{\theta}=h_{c\ m}^{p}\mathbf{M}_{c\,ext}^{p\theta}+\mathbf{q}_{ext}^{\theta}$$

2 Laws of behavior

2.1 Mechanics

2.1.1 General writing

$$\begin{cases}\sigma^{+}=\sigma^{+}(\varepsilon^{+},p_1^{+},p_2^{+},T^{+};\varepsilon^{-},p_1^{-},p_2^{-},T^{-},\sigma^{-},\chi^{-})\\ \chi^{+}=\chi^{+}(\varepsilon^{+},p_1^{+},p_2^{+},T^{+};\varepsilon^{-},p_1^{-},p_2^{-},T^{-},\sigma^{-},\chi^{-})\end{cases}$$ éq 2.1.1-1

2.1.2 Case of the effective constraints

In the case of the assumption of the effective constraints, this function will break up in the form:

$\sigma=\sigma'+\sigma_p I$

σ' est le tenseur des contraintes effectives:

σ_p est un scalaire

$$\begin{cases}\sigma'^{+}=\sigma'^{+}(\varepsilon^{+},T^{+};\varepsilon^{-},T^{-},\sigma'^{-},\chi_\sigma^{-})\\ \chi_\sigma^{+}=\chi_\sigma^{+}(\varepsilon^{+},T^{+};\varepsilon^{-},T^{-},\sigma'^{-},\chi_\sigma^{-})\end{cases}$$ éq 2.1.2-1

$$\begin{cases}\sigma_p^{+}=\sigma_p^{+}(p_1^{+},p_2^{+};p_1^{-},p_2^{-},\chi_H^{-})\\ \chi_H^{+}=\chi_H^{+}(p_1^{+},p_2^{+};p_1^{-},p_2^{-},\chi_H^{-})\end{cases}$$ éq 2.1.2-2

It is noticed that in this decomposition:

1) the dependence compared to thermics was left in the effective constraints; typically, it is thought that the laws on the effective constraints are written as in thermo mechanical classic:

$$\sigma'^{+}=\sigma'^{+}(\varepsilon^{+}-\alpha^{+}T^{+};\varepsilon^{-}-\alpha^{-}T^{-},\sigma'^{-},\chi_\sigma^{-})$$

1) one distinguished the internal variables relating to the law from behavior on the effective constraints, which one wrote χ_σ , internal variables of hydraulic origin which one wrote χ_H and internal variables of thermal origin which one wrote χ_T (see following paragraphs).

2.1.3 Choice of the constraints

Because of rather frequent use of the assumption of the effective constraints, one decides that the vector of the constraints for the mechanical part contains in all the cases the tensor of the effective constraints σ' and the scalar σ_p . In the case general where the assumption of the effective constraints is not retained, one will have simply: . $\sigma_p=0$

It is thus with the load of the module of integration of the equilibrium equations to make the sum: $\sigma=\sigma'+\sigma_p I$.

2.2 Hydraulics

The hydraulic law of behavior will provide the following relations:

$$\left.\begin{cases} m_c^{p+} = m_c^{p+}\left(\varepsilon^+, p_1^+, p_2^+, T^+; \varepsilon^-, p_1^-, p_2^-, T^-, m_d^{q-}, \mathbf{M}_d^{q-}, \chi_H^-\right) \\ \mathbf{M}_c^{p+} = \mathbf{M}_c^{p+}\begin{pmatrix} \varepsilon^+, p_1^+, \nabla p_1^+, p_2^+, \nabla p_2^+, T^+, \nabla T^+; \\ \varepsilon^-, p_1^-, \nabla p_1^-, p_2^-, \nabla p_2^-, T^-, \nabla T^-, \mathbf{M}_d^{q-}, \chi_H^- : \mathbf{F}^{m+} \end{pmatrix} \\ \chi_H^+ = \chi_H^+\left(\varepsilon^+, p_1^+, p_2^+, T^+; \varepsilon^-, p_1^-, p_2^-, T^-, m_1^-, m_2^-, \chi_H^-\right) \end{cases}\right\} \forall c \, et \, \forall p \quad de\ 1\ à\ np_c \qquad \text{éq 2.2-1}$$

It is noticed that the field of gravity is a data of the hydraulic law of behavior by what the evolution of the vector of flow follows of the relations of the type: $\mathbf{M} = \lambda_H \rho^{fl}\left[-\nabla P + \rho^{fl}\mathbf{F}^m\right]$.

2.3 Thermics

The laws of behavior will provide:

$$\begin{cases} Q'^+ = Q'^+\left(\varepsilon^+, p_1^+, p_2^+, T^+; \varepsilon^-, p_1^-, p_2^-, T^-, S'^-\right) \\ h_{c\ m}^{p+} = h_{c\ m}^{p+}\left(\varepsilon^+, p_1^+, p_2^+, T^+; \varepsilon^-, p_1^-, p_2^-, T^-, s_{dm}^{q-}\right) \quad \forall c \quad et \quad \forall p \quad de\ 1\ à\ np_c \\ \mathbf{q}^+ = \mathbf{q}^+\left(\varepsilon^+, p_1^+, p_2^+, T^+, \nabla T^+; \varepsilon^-, p_1^-, p_2^-, T^-, \nabla T^-, \mathbf{q}^-\right) \\ \chi_T^+ = \chi_T^+\left(\varepsilon^+, p_1^+, p_2^+, T^+, \nabla T^+; \varepsilon^-, p_1^-, p_2^-, T^-, \nabla T^-, \chi_T^-\right) \end{cases}$$
$$\text{Avec } h_{dm}^{q-} = \left(h_{1m}^{1-}, h_{1m}^{2-}, h_{2m}^{1-}, h_{2m}^{2-}\right) \qquad \text{éq 2.3-1}$$

2.4 Homogenized density

$$r^+ = r_0 + m_1^{1+} + m_1^{2+} + m_2^{1+} + m_2^{2+} \qquad \text{éq 2.4-1}$$

3 Generalized efforts

It arises of what is written higher than the generalized constraints are:

$$\begin{array}{l} \underline{\underline{\sigma}}', \sigma_p; \\ m_1^{1,} \mathbf{M}_1^{1,} h_{1m}^1; m_1^{2,} \mathbf{M}_1^{2,} h_{1m}^2; \\ m_2^{1,} \mathbf{M}_2^{1,} h_{2m}^1; m_2^{2,} \mathbf{M}_2^{2,} h_{2m}^2; \\ Q', \mathbf{q} \end{array}$$

The associated generalized deformations are:

$$\mathbf{u}, \underline{\underline{\varepsilon}}(\mathbf{u}) : p_{1,} \nabla p_1 : p_{2,} \nabla p_2 ; T, \nabla T$$

Note:

Within the framework of saturated modeling HM permanent, the generalized constraints do not contain the mass term of contribution.

4 Algorithm of resolution

4.1 Nonlinear algorithm of resolution of the equilibrium equations

In the case general of modeling (variable coefficients, desaturation, convection) the variational problem presented above is nonlinear compared to the fields of displacement, pressure and temperature. After discretization by finite elements, one obtains a nonlinear matric system. The matrix of resolution contains moreover one nonsymmetrical term and is treated like such (not symmetrization of this matrix to use methods of minimum). One uses in all the cases of modeling the nonlinear solvor of *Code_Aster* `STAT_NON_LINE` resting on a method of Newton-Raphson, described in [R5.03.01]. Its principle is the following (the equations corresponding to the treatment by dualisation of the boundary conditions are not indicated explicitly here).

The equilibrium equation thermo-poro-mechanics at the moment t^+, knowing at the previous moment $(\mathbf{u}_-, P_-, T_-)$, as well as the possible internal variables is written:

$$F_i(\mathbf{u}_+, P_+, T_+) = L_e(t^+) - G(\mathbf{u}_-, P_-, T_-) \quad ,$$

To find the solution of this nonlinear equation, a continuation is built:

- initialized by a prediction which gives $(\mathbf{u_0}, P_0, T_0) = (\mathbf{u}_-, P_-, T_-) + (\Delta\mathbf{u_0}, \Delta P_0, \Delta T_0)$:

$$DF_{i(\mathbf{u}_-, P_-, T_-)} \circ (\Delta\mathbf{u_0}, \Delta P_0, \Delta T_0) = L_e(t^+) - L_e(t^-)$$

- corrected by recurrence giving:

$$(\mathbf{u}_{n+1}, P_{n+1}, T_{n+1}) = (\mathbf{u}_n, P_n, T_n) + (\delta\mathbf{u}_{n+1}, \delta P_{n+1}, \delta T_{n+1})$$

$$DF_i \circ (\delta\mathbf{u_{n+1}}, \delta P_{n+1}, \delta T_{n+1}) = -F_i(\mathbf{u}_n, P_n, T_n) + L_e(t^+) - G(\mathbf{u}_-, P_-, T_-)$$

The following notations were adopted:

- $F_i(\mathbf{u}, P, T)$ contains the work of deformation, the contributions to the current moment of the terms of hydraulic and thermal dissipation expressed within θ - method, and of the variations of fluid contribution of mass and entropy;
- DF_i appoint the tangent operator, who can not be updated with each iteration in $(\mathbf{u}_n, P_n, T_n)$, according to a compromise cost calculation-speed of convergence; convergence is checked by a test on the relative standard of the difference of reiterated successive (via the keyword `INCO_GLOB_RELA`);
- $G(\mathbf{u}_-, P_-, T_-)$ contains the contributions to the previous moment of the terms of hydraulic and thermal dissipation expressed within θ - method, and of the variations of fluid contribution of mass and entropy;
- $L_e(t)$ indicate the virtual work of the "dead" forces external and hydraulic external contributions and of heat expressed by θ - method.

WITH convergence with the iteration $n+1$, an actualization of the fields is operated. $(\mathbf{u}_+, P_+, T_+) = (\mathbf{u}_{n+1}, P_{n+1}, T_{n+1})$

In the version present of algorithm THM, we decided to gather all the terms including those due to the following forces and those of time less:

While posing:

$$\boxed{-R_i(\mathbf{u_n}, P_n, T_n) = -F_i(\mathbf{u}_n, P_n, T_n) - G(\mathbf{u}_-, P_-, T_-)} \quad ,$$

thus $DF_i = DR_i$

one has finally:

$$\boxed{DF_i \circ (\delta \mathbf{u_{n+1}}, \delta P_{n+1}, \delta T_{n+1}) = -R_i(\mathbf{u}_n, P_n, T_n) + L_e(t^+)}$$

The algorithm general of balance will be written then, for a step of time:

Initializations:

Calculation of $L_e(t^+)$ (option CHAR_MECA)

Calculation of $DF_{i(\mathbf{u}_-, P_-, T_-)}$ (option RIGI_MECA-TANG)

Calculation of $(\Delta \mathbf{u_0}, \Delta P_0, \Delta T_0)$ by: $DF_{i(\mathbf{u}_-, P_-, T_-)} \circ (\Delta \mathbf{u_0}, \Delta P_0, \Delta T_0) = L_e(t^+) - L_e(t^-)$

Iterations of balance of Newton N

If option FULL_MECA :

Calculation of $DF_{i(\mathbf{u}^+, P^+, T^+)}$ and $-R_i(\mathbf{u}_n^+, P_n^+, T_n^+)$:

Update stamps tangent: $DF_i = DF_{i(\mathbf{u}_n^+, P_n^+, T_n^+)}$

If option RAPH_MECA

Calculation of $- R_i(\mathbf{u}_n^+, P_n^+, T_n^+)$

Calculation of $(\delta \mathbf{u}_{n+1}, \delta P_{n+1}, \delta T_{n+1})$ by:

$$DF_i \circ (\delta \mathbf{u_{n+1}}, \delta P_{n+1}, \delta T_{n+1}) = -R_i(\mathbf{u}_n^+, P_n^+, T_n^+) + L_e(t^+)$$

Actualization :

$$(\mathbf{u}_{n+1}^+, P_{n+1}^+, T_{n+1}^+) = (\mathbf{u}_n^+, P_n^+, T_n^+) + (\delta \mathbf{u}_{n+1}, \delta P_{n+1}, \delta T_{n+1})$$

IF test convergence OK

fine Newton: no next time

If not

N = n+1

4.2 Buckle on the elements, the points of Gauss

As in all the codes of finite elements, the terms are calculated by loop on the elements and buckles on the points of Gauss:

$$R_i(\mathbf{u}_n^+, P_n^+, T_n^+) = \sum_{el} \sum_g w_g^{el} R_{g\ i}^{el}(\mathbf{u}_n^+, P_n^+, T_n^+)$$
$$DF_{i(\mathbf{u}_n^+, P_n^+, T_n^+)} = \sum_{el} \sum_g w_g^{el} DF^{el}_{g\ i(\mathbf{u}_n^+, P_n^+, T_n^+)}$$

Let us note: $\{X^{el}\}$ the vector of the nodal unknown factors, on a finite element el

for example $$\{X^{el}\}=\left.\begin{matrix}u\\v\\w\\p_1\\p_2\\T\end{matrix}\right\}\text{noeud 1}\quad\left.\begin{matrix}u\\v\\w\\p_1\\p_2\\T\end{matrix}\right\}\text{noeud 2}\quad\left.\begin{matrix}u\\v\\w\\p_1\\p_2\\T\end{matrix}\right\}\text{noeud 3}$$

In this paragraph, to simplify the presentation, we suppose that we deal with supporting finite element of the ddl of displacement, two ddl of pressure and a ddl of temperature.

Let us note $\{\mathrm{E}_g^{el}\}$ the vector of the deformations generalized at the point of Gauss *G* element *el*
For example:

$$\{\mathrm{E}_g^{el}\}=\begin{Bmatrix}\mathbf{u}\\ \varepsilon(\mathbf{u})\\ p_1\\ \nabla p_1\\ p_2\\ \nabla p_2\\ T\\ \nabla T\end{Bmatrix}$$

We note $\{\Sigma_g^{el}\}$ the vector of constraints generalized for the point of Gauss *G* element *el*

For example, and always in the most complete case:

$$\{\Sigma_g^{el}\}=\begin{Bmatrix} \underline{\underline{\sigma}}' \\ \sigma_p \\ m_1^1 \\ \mathbf{M}_1^1 \\ h_{1m}^1 \\ m_1^2 \\ \mathbf{M}_1^2 \\ h_{1m}^2 \\ m_2^1 \\ \mathbf{M}_2^1 \\ h_{2m}^1 \\ m_2^2 \\ \mathbf{M}_2^2 \\ h_{2m}^2 \\ Q' \\ \mathbf{q} \end{Bmatrix}$$

The routines finite elements calculate the matrix: $[B]_g^{el}$ defined by:

$$\{E_g^{el}\}=[B]_g^{el}\{X\}$$

The algorithm will become then:

Initializations:

Calculation of $L_e(t^+)$ (option CHAR_MECA)

Calculation of $DF_{i(\mathbf{u}_-, P_-, T_-)}$ (option RIGI_MECA-TANG)

Calculation of $(\Delta\mathbf{u}_0, \Delta P_0, \Delta T_0)$ by: $DF_{i(\mathbf{u}_-, P_-, T_-)} \circ (\Delta\mathbf{u}_0, \Delta P_0, \Delta T_0) = L_e(t^+) - L_e(t^-)$

Iterations of balance of Newton N

Buckle elements *el*

Buckle points of gauss *G*

Calculation $[B]_g^{el}$

Calculation $\{E_g^{el-}\} = [B]_g^{el}\{X^-\}$ and $\{E_{gn}^{el+}\} = [B]_g^{el}\{X_n^+\}$

Calculation $\{\Sigma_{gn}^{el+}\}$, $-R_{ig}^{el}(\mathbf{u}_n^+, P_n^+, T_n^+)$ and $DF_{g\ i(\mathbf{u}_n^+, P_n^+, T_n^+)}^{el}$ (according to options) from:
$\{E_g^{el-}\}, \{E_g^{el+}\}, \{\Sigma_g^{el-}\}, \{E_g^{el+}\}, [B]_g^{el}$

Calculation of $(\delta\mathbf{u}_{n+1}, \delta P_{n+1}, \delta T_{n+1})$ by:

$$DF_i \circ (\delta\mathbf{u}_{n+1}, \delta P_{n+1}, \delta T_{n+1}) = -R_i(\mathbf{u}_n^+, P_n^+, T_n^+) + L_e(t^+)$$

Actualization :

$$(\mathbf{u}_{n+1}^+, P_{n+1}^+, T_{n+1}^+) = (\mathbf{u}_n^+, P_n^+, T_n^+) + (\delta\mathbf{u}_{n+1}, \delta P_{n+1}, \delta T_{n+1})$$

IF test convergence OK

fine Newton: no next time

If not

N = n+1

4.3 Vectors and matrices according to the options: routine EQUTHM

The framed central part of the algorithm presented Ci above is carried out by a generic routine EQUTHM. We give in appendix a chart of the call of this routine.

This routine is parameterized according to the equations present (mechanics, hydraulics with 1 or 2 pressures, thermics). The work carried out by this routine is parameterized by the option.

The term $-R_i(\mathbf{u}_n, P_n, T_n)$ will be calculated by the options RAPH_MECA and FULL_MECA. This term includes the following forces of volume: it will be considered that the following forces will be integrated into the options RAPH_MECA, FULL_MECA and RIGI_MECA_TANG. If the user data do not comprise forces of volume, the vector $\mathbf{F}^{m^+}$ will be simply null.

The presentations made in the two following paragraphs are made in the case more the general where one has an equation of mechanics, two equations of hydraulics and an equation of thermics. Routine EQUTHM will calculate or not the various terms according to description that him equations present will be made.

Indices *G* and *el* from now on are omitted, but it is clear that what is described applies to each point of Gauss of each element.

Note:

Within the framework of saturated modeling HM permanent, a routine similar to the routine EQUTHM was established (routine EQUTHP), which takes account of specificities of the equations of permanent modeling (not of mass contribution).

4.3.1 Residue or nodal force: options `RAPH_MECA` and `FULL_MECA`

One will distribute the terms of the variational formulation according to the following principle:

If E_g^{*el} indicate a virtual field of deformation, $\mathrm{E}_g^{*elT}=(\mathbf{v},\varepsilon(\mathbf{v}),\pi_1,\nabla\pi_1,\pi_2,\nabla\pi_2,\tau,\nabla\tau)$ calculated starting from a vector of displacement nodal virtual: $\{X^{*el}\}$

$$\mathrm{E}_g^{*elT}\cdot R_{ig}^{el}(\mathbf{u}_+,P_+,T_+)=R_1\mathbf{v}+R_2\varepsilon(\mathbf{v})+R_3\pi_1+R_4\nabla\pi_1+R_5\pi_2+R_6\nabla\pi_2+R_7\tau+R_8\nabla\tau$$

One has then:

Index	R	associated with
1	$-\left(m_1^{1+}+m_1^{2+}+m_2^{1+}+m_2^{2+}\right)\mathbf{F}^{m^+}$	$\mathbf{v}$
2	$\sigma'^{+}+\sigma_p^+ I$	$\varepsilon(\mathbf{v})$
3	$-m_1^{1+}-m_1^{2+}+m_1^{1-}+m_1^{2-}$	π_1
4	$\theta\Delta t\left(\boldsymbol{M}_1^{1+}+\boldsymbol{M}_1^{2+}\right)+(1-\theta)\Delta t\left(\boldsymbol{M}_1^{1-}+\boldsymbol{M}_1^{2-}\right)$	$\nabla\pi_1$
5	$-m_2^{1+}-m_2^{2+}+m_2^{1-}+m_2^{2-}$	π_2
6	$\theta\Delta t\left(\boldsymbol{M}_2^{1+}+\boldsymbol{M}_2^{2+}\right)+(1-\theta)\Delta t\left(\boldsymbol{M}_2^{1-}+\boldsymbol{M}_2^{2-}\right)$	$\nabla\pi_2$
7	$Q'^{+}-Q'^{-}$ $(\theta h_{1m}^{1+}+(1-\theta)h_{1m}^{1-})(m_1^{1+}-m_1^{1-})+(\theta h_{1m}^{2+}+(1-\theta)h_{1m}^{2-})(m_1^{2+}-m_1^{2-})$ $(\theta h_{2m}^{1+}+(1-\theta)h_{2m}^{1-})(m_2^{1+}-m_2^{1-})+(\theta h_{2m}^{2+}+(1-\theta)h_{2m}^{2-})(m_2^{2+}-m_2^{2-})$ $-\Delta t\,\theta(\mathbf{M}_1^{1+}+\mathbf{M}_1^{2+}+\mathbf{M}_2^{1+}+\mathbf{M}_2^{2+})\cdot\mathbf{F}^m-\Delta t(1-\theta)(\mathbf{M}_1^{1-}+\mathbf{M}_1^{2-}+\mathbf{M}_2^{1-}+\mathbf{M}_2^{2-})\cdot\mathbf{F}^m$	τ
8	$-\theta\Delta t\left(h_{1m}^{1+}\boldsymbol{M}_1^{1+}+h_{1m}^{2+}\boldsymbol{M}_1^{2+}+h_{2m}^{1+}\boldsymbol{M}_2^{1+}+h_{2m}^{2+}\boldsymbol{M}_2^{2+}+\mathbf{q}^+\right)$ $-(1-\theta)\Delta t\left(h_{1m}^{1+}\boldsymbol{M}_1^{1+}+h_{1m}^{2+}\boldsymbol{M}_1^{2+}+h_{2m}^{1+}\boldsymbol{M}_2^{1+}+h_{2m}^{2+}\boldsymbol{M}_2^{2+}+\mathbf{q}^+\right)$	$\nabla\tau$

From there one will define the vector nodal residue $\{\boldsymbol{V}_g^{el}\}$ such as:

$$\{X^{*el}\}^T\cdot\{\boldsymbol{V}_g^{el}\}=E_g^{*el^T}\cdot R_{ig}^{el}(\mathbf{u}_+,P_+,T_+)$$

$\{\boldsymbol{V}_g^{el}\}$ will be calculated by:

$$\boxed{\{\boldsymbol{V}_g^{el}\}=\left[\boldsymbol{B}_g^{el}\right]^T\cdot\{R\}}$$

Note:

Within the framework of saturated modeling HM permanent, routine EQUTHP never assembles the terms R3 and R5.

4.3.2 Loading: options CHAR_MECA

This chapter is here only for memory because the routine EQUTHM will not deal with these terms.

One will distribute the terms of the variational formulation according to the following principle:

$$E_g^{*el^T}\cdot L_{eg}^{el}(t+)=L_1\mathbf{v}+L_2\varepsilon(\mathbf{v})+L_3\pi_1+L_4\nabla\pi_1+L_5\pi_2+L_6\nabla\pi_2+L_7\tau+L_8\nabla\tau$$

Index	L	standard element	associated with
1	$\mathbf{f}^{ext^+}$	edge	$\mathbf{v}$
3	$\Delta t\left(\boldsymbol{M}_{1\text{ext}}^{1\theta}+\boldsymbol{M}_{1\text{ext}}^{2\theta}\right)$	edge	π_1
5	$\Delta t\left(\boldsymbol{M}_{2\text{ext}}^{1\theta}+\boldsymbol{M}_{2\text{ext}}^{2\theta}\right)$	edge	π_2
7	$\Delta t\,R^\theta$ $-\Delta t\left(\mathbf{q}_{ext}^\theta+\left(h_{1\text{m}}^{1\theta}\mathbf{M}_{1\text{ext}}^{1\theta}+h_{1\text{m}}^{2\theta}\mathbf{M}_{1\text{ext}}^{2\theta}\right)\right)$ $-\Delta t\left(h_{2\text{m}}^{1\theta}\mathbf{M}_{2\text{ext}}^{1\theta}+h_{2\text{m}}^{2\theta}\mathbf{M}_{2\text{ext}}^{2\theta}\right)$ $=-\Delta t\,\tilde{\mathbf{q}}_{ext}^{\theta}$	volume edge	τ

4.3.3 Tangent operator: options FULL_MECA, RIGI_MECA_TANG

Notice on the matric notations:

In what follows, if X *indicate a vector of components* X^i *and* Y *a vector of components* Y^i *,* $\left[\frac{\partial X}{\partial Y}\right]$ *a matrix will indicate of which the element* $(ligne:i\,,colonne:j)$ *is* $\frac{\partial X^i}{\partial Y^j}$ *.*

To calculate the tangent operator DF_i , the following quantities will be calculated:

$[\mathbf{DRDE}]$ =

DR1U	DR1E	DR1P1	DR1GP1	DR1P2	DR1GP2	DR1T	DR1GT
DR2U	DR2E	DR2P1	DR2GP1	DR2P2	DR2GP2	DR2T	DR2GT
DR3U	DR3E	DR3P1	DR3GP1	DR3P2	DR3GP2	DR3T	DR3GT
DR4U	DR4E	DR4P1	DR4GP1	DR4P2	DR4GP2	DR4T	DR4GT
DR5U	DR5E	DR5P1	DR5GP1	DR5P2	DR5GP2	DR5T	DR5GT
DR6U	DR6E	DR6P1	DR6GP1	DR6P2	DR6GP2	DR6T	DR6GT
DR7U	DR7E	DR7P1	DR7GP1	DR7P2	DR7GP2	DR7T	DR7GT
DR8U	DR8E	DR8P1	DR8GP1	DR8P2	DR8GP2	DR8T	DR8GT

Where one noted:

$$DRiU=\frac{\partial F_i}{\partial \underline{u}}$$
$$DRiE=\frac{\partial F_i}{\partial \underline{\underline{\varepsilon}}}$$
$$DRiP1=\frac{\partial F_i}{\partial p_1}$$
$$DRiP2=\frac{\partial F_i}{\partial p_2}$$
$$DRiGP1=\frac{\partial F_i}{\partial \underline{\nabla p_1}}$$
$$DRiGP2=\frac{\partial F_i}{\partial \underline{\nabla p_2}}$$
$$DRiT=\frac{\partial F_i}{\partial T}$$
$$DRiGT=\frac{\partial F_i}{\partial \underline{\nabla T}}$$

To do these calculations one considers that the laws of behavior will provide, for the corresponding options, all the derivative following:

$$
[\mathbf{DSDE}]=\left|\begin{matrix}
\frac{\partial\sigma'}{\partial\mathbf{u}} & \frac{\partial\sigma'}{\partial\varepsilon} & \frac{\partial\sigma'}{\partial p_1} & \frac{\partial\sigma'}{\partial\nabla p_1} & \frac{\partial\sigma'}{\partial p_2} & \frac{\partial\sigma'}{\partial\nabla p_2} & \frac{\partial\sigma'}{\partial T} & \frac{\partial\sigma'}{\partial\nabla T}\\
\frac{\partial\sigma_p}{\partial\mathbf{u}} & \frac{\partial\sigma_p}{\partial\varepsilon} & \frac{\partial\sigma_p}{\partial p_1} & \frac{\partial\sigma_p}{\partial\nabla p_1} & \frac{\partial\sigma_p}{\partial p_2} & \frac{\partial\sigma_p}{\partial\nabla p_2} & \frac{\partial\sigma_p}{\partial T} & \frac{\partial\sigma_p}{\partial\nabla T}\\
\frac{\partial m_1^1}{\partial\mathbf{u}} & \frac{\partial m_1^1}{\partial\varepsilon} & \frac{\partial m_1^1}{\partial p_1} & \frac{\partial m_1^1}{\partial\nabla p_1} & \frac{\partial m_1^1}{\partial p_2} & \frac{\partial m_1^1}{\partial\nabla p_2} & \frac{\partial m_1^1}{\partial T} & \frac{\partial m_1^1}{\partial\nabla T}\\
\frac{\partial\mathbf{M}_1^1}{\partial\mathbf{u}} & \frac{\partial\mathbf{M}_1^1}{\partial\varepsilon} & \frac{\partial\mathbf{M}_1^1}{\partial p_1} & \frac{\partial\mathbf{M}_1^1}{\partial\nabla p_1} & \frac{\partial\mathbf{M}_1^1}{\partial p_2} & \frac{\partial\mathbf{M}_1^1}{\partial\nabla p_2} & \frac{\partial\mathbf{M}_1^1}{\partial T} & \frac{\partial\mathbf{M}_1^1}{\partial\nabla T}\\
\frac{\partial h_{1m}^1}{\partial\mathbf{u}} & \frac{\partial h_{1m}^1}{\partial\varepsilon} & \frac{\partial h_{1m}^1}{\partial p_1} & \frac{\partial h_{1m}^1}{\partial\nabla p_1} & \frac{\partial h_{1m}^1}{\partial p_2} & \frac{\partial h_{1m}^1}{\partial\nabla p_2} & \frac{\partial h_{1m}^1}{\partial T} & \frac{\partial h_{1m}^1}{\partial\nabla T}\\
\frac{\partial m_1^2}{\partial\mathbf{u}} & \frac{\partial m_1^2}{\partial\varepsilon} & \frac{\partial m_1^2}{\partial p_1} & \frac{\partial m_1^2}{\partial\nabla p_1} & \frac{\partial m_1^2}{\partial p_2} & \frac{\partial m_1^2}{\partial\nabla p_2} & \frac{\partial m_1^2}{\partial T} & \frac{\partial m_1^2}{\partial\nabla T}\\
\frac{\partial\mathbf{M}_1^2}{\partial\mathbf{u}} & \frac{\partial\mathbf{M}_1^2}{\partial\varepsilon} & \frac{\partial\mathbf{M}_1^2}{\partial p_1} & \frac{\partial\mathbf{M}_1^2}{\partial\nabla p_1} & \frac{\partial\mathbf{M}_1^2}{\partial p_2} & \frac{\partial\mathbf{M}_1^2}{\partial\nabla p_2} & \frac{\partial\mathbf{M}_1^2}{\partial T} & \frac{\partial\mathbf{M}_1^2}{\partial\nabla T}\\
\frac{\partial h_{1m}^2}{\partial\mathbf{u}} & \frac{\partial h_{1m}^2}{\partial\varepsilon} & \frac{\partial h_{1m}^2}{\partial p_1} & \frac{\partial h_{1m}^2}{\partial\nabla p_1} & \frac{\partial h_{1m}^2}{\partial p_2} & \frac{\partial h_{1m}^2}{\partial\nabla p_2} & \frac{\partial h_{1m}^2}{\partial T} & \frac{\partial h_{1m}^2}{\partial\nabla T}\\
\frac{\partial m_2^1}{\partial\mathbf{u}} & \frac{\partial m_2^1}{\partial\varepsilon} & \frac{\partial m_2^1}{\partial p_1} & \frac{\partial m_2^1}{\partial\nabla p_1} & \frac{\partial m_2^1}{\partial p_2} & \frac{\partial m_2^1}{\partial\nabla p_2} & \frac{\partial m_2^1}{\partial T} & \frac{\partial m_2^1}{\partial\nabla T}\\
\frac{\partial\mathbf{M}_2^1}{\partial\mathbf{u}} & \frac{\partial\mathbf{M}_2^1}{\partial\varepsilon} & \frac{\partial\mathbf{M}_2^1}{\partial p_1} & \frac{\partial\mathbf{M}_2^1}{\partial\nabla p_1} & \frac{\partial\mathbf{M}_2^1}{\partial p_2} & \frac{\partial\mathbf{M}_2^1}{\partial\nabla p_2} & \frac{\partial\mathbf{M}_2^1}{\partial T} & \frac{\partial\mathbf{M}_2^1}{\partial\nabla T}\\
\frac{\partial h_{2m}^1}{\partial\mathbf{u}} & \frac{\partial h_{2m}^1}{\partial\varepsilon} & \frac{\partial h_{2m}^1}{\partial p_1} & \frac{\partial h_{2m}^1}{\partial\nabla p_1} & \frac{\partial h_{2m}^1}{\partial p_2} & \frac{\partial h_{2m}^1}{\partial\nabla p_2} & \frac{\partial h_{2m}^1}{\partial T} & \frac{\partial h_{2m}^1}{\partial\nabla T}\\
\frac{\partial m_2^2}{\partial\mathbf{u}} & \frac{\partial m_2^2}{\partial\varepsilon} & \frac{\partial m_2^2}{\partial p_1} & \frac{\partial m_2^2}{\partial\nabla p_1} & \frac{\partial m_2^2}{\partial p_2} & \frac{\partial m_2^2}{\partial\nabla p_2} & \frac{\partial m_2^2}{\partial T} & \frac{\partial m_2^2}{\partial\nabla T}\\
\frac{\partial\mathbf{M}_2^2}{\partial\mathbf{u}} & \frac{\partial\mathbf{M}_2^2}{\partial\varepsilon} & \frac{\partial\mathbf{M}_2^2}{\partial p_1} & \frac{\partial\mathbf{M}_2^2}{\partial\nabla p_1} & \frac{\partial\mathbf{M}_2^2}{\partial p_2} & \frac{\partial\mathbf{M}_2^2}{\partial\nabla p_2} & \frac{\partial\mathbf{M}_2^2}{\partial T} & \frac{\partial\mathbf{M}_2^2}{\partial\nabla T}\\
\frac{\partial h_{2m}^2}{\partial\mathbf{u}} & \frac{\partial h_{2m}^2}{\partial\varepsilon} & \frac{\partial h_{2m}^2}{\partial p_1} & \frac{\partial h_{2m}^2}{\partial\nabla p_1} & \frac{\partial h_{2m}^2}{\partial p_2} & \frac{\partial h_{2m}^2}{\partial\nabla p_2} & \frac{\partial h_{2m}^2}{\partial T} & \frac{\partial h_{2m}^2}{\partial\nabla T}\\
\frac{\partial Q'}{\partial\mathbf{u}} & \frac{\partial Q'}{\partial\varepsilon} & \frac{\partial Q'}{\partial p_1} & \frac{\partial Q'}{\partial\nabla p_1} & \frac{\partial Q'}{\partial p_2} & \frac{\partial Q'}{\partial\nabla p_2} & \frac{\partial Q'}{\partial T} & \frac{\partial Q'}{\partial\nabla T}\\
\frac{\partial\mathbf{q}}{\partial\mathbf{u}} & \frac{\partial\mathbf{q}}{\partial\varepsilon} & \frac{\partial\mathbf{q}}{\partial p_1} & \frac{\partial\mathbf{q}}{\partial\nabla p_1} & \frac{\partial\mathbf{q}}{\partial p_2} & \frac{\partial\mathbf{q}}{\partial\nabla p_2} & \frac{\partial\mathbf{q}}{\partial T} & \frac{\partial\mathbf{q}}{\partial\nabla T}
\end{matrix}\right|
$$

In fact, in these expressions, the derivative compared to U are all worthless, but we keep the writing taking into account the definition of the matrices $[B]_g^{el}$ that we adopted.

The call to the laws of behavior will provide the pieces of the matrix $[\mathbf{DSDE}]$ according to the equations present:

$$[\mathbf{DMECDE}]=\begin{bmatrix}\frac{\partial\sigma'}{\partial\varepsilon}\\ \frac{\partial\sigma_p}{\partial\varepsilon}\end{bmatrix}\ ;\ [\mathbf{DMECP1}]=\begin{bmatrix}\frac{\partial\sigma'}{\partial p_1} & \frac{\partial\sigma'}{\partial\nabla p_1}\\ \frac{\partial\sigma_p}{\partial p_1} & \frac{\partial\sigma_p}{\partial\nabla p_1}\end{bmatrix}\ ;\ [\mathbf{DMECP2}]=\begin{bmatrix}\frac{\partial\sigma'}{\partial p_2} & \frac{\partial\sigma'}{\partial\nabla p_2}\\ \frac{\partial\sigma_p}{\partial p_2} & \frac{\partial\sigma_p}{\partial\nabla p_2}\end{bmatrix}\ ;\ [\mathbf{DMECDT}]=\begin{bmatrix}\frac{\partial\sigma'}{\partial T} & \frac{\partial\sigma'}{\partial\nabla T}\\ \frac{\partial\sigma_p}{\partial T} & \frac{\partial\sigma_p}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP11DE}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial\varepsilon}\\ \frac{\partial\mathbf{M}_1^1}{\partial\varepsilon}\\ \frac{\partial h_{1m}^1}{\partial\varepsilon}\end{bmatrix}\ ;\ [\mathbf{DP11P1}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial p_1} & \frac{\partial m_1^1}{\partial\nabla p_1}\\ \frac{\partial\mathbf{M}_1^1}{\partial p_1} & \frac{\partial\mathbf{M}_1^1}{\partial\nabla p_1}\\ \frac{\partial h_{1m}^1}{\partial p_1} & \frac{\partial h_{1m}^1}{\partial\nabla p_1}\end{bmatrix}\ ;\ [\mathbf{DP11P2}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial p_2} & \frac{\partial m_1^1}{\partial\nabla p_2}\\ \frac{\partial\mathbf{M}_1^1}{\partial p_2} & \frac{\partial\mathbf{M}_1^1}{\partial\nabla p_2}\\ \frac{\partial h_{1m}^1}{\partial p_2} & \frac{\partial h_{1m}^1}{\partial\nabla p_2}\end{bmatrix}\ ;\ [\mathbf{DP11DT}]=\begin{bmatrix}\frac{\partial m_1^1}{\partial T} & \frac{\partial m_1^1}{\partial\nabla T}\\ \frac{\partial\mathbf{M}_1^1}{\partial T} & \frac{\partial\mathbf{M}_1^1}{\partial\nabla T}\\ \frac{\partial h_{1m}^1}{\partial T} & \frac{\partial h_{1m}^1}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP12DE}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial\varepsilon}\\ \frac{\partial\mathbf{M}_1^2}{\partial\varepsilon}\\ \frac{\partial h_{1m}^2}{\partial\varepsilon}\end{bmatrix}\ ;\ [\mathbf{DP12P1}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial p_1} & \frac{\partial m_1^2}{\partial\nabla p_1}\\ \frac{\partial\mathbf{M}_1^2}{\partial p_1} & \frac{\partial\mathbf{M}_1^2}{\partial\nabla p_1}\\ \frac{\partial h_{1m}^2}{\partial p_1} & \frac{\partial h_{1m}^2}{\partial\nabla p_1}\end{bmatrix}\ ;\ [\mathbf{DP12P2}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial p_2} & \frac{\partial m_1^2}{\partial\nabla p_2}\\ \frac{\partial\mathbf{M}_1^2}{\partial p_2} & \frac{\partial\mathbf{M}_1^2}{\partial\nabla p_2}\\ \frac{\partial h_{1m}^2}{\partial p_2} & \frac{\partial h_{1m}^2}{\partial\nabla p_2}\end{bmatrix}\ ;\ [\mathbf{DP12DT}]=\begin{bmatrix}\frac{\partial m_1^2}{\partial T} & \frac{\partial m_1^2}{\partial\nabla T}\\ \frac{\partial\mathbf{M}_1^2}{\partial T} & \frac{\partial\mathbf{M}_1^2}{\partial\nabla T}\\ \frac{\partial h_{1m}^2}{\partial T} & \frac{\partial h_{1m}^2}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP21DE}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial\varepsilon}\\ \frac{\partial\mathbf{M}_2^1}{\partial\varepsilon}\\ \frac{\partial h_{2m}^1}{\partial\varepsilon}\end{bmatrix}\ ;\ [\mathbf{DP21P1}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial p_1} & \frac{\partial m_2^1}{\partial\nabla p_1}\\ \frac{\partial\mathbf{M}_2^1}{\partial p_1} & \frac{\partial\mathbf{M}_2^1}{\partial\nabla p_1}\\ \frac{\partial h_{2m}^1}{\partial p_1} & \frac{\partial h_{2m}^1}{\partial\nabla p_1}\end{bmatrix}\ ;\ [\mathbf{DP21P2}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial p_2} & \frac{\partial m_2^1}{\partial\nabla p_2}\\ \frac{\partial\mathbf{M}_2^1}{\partial p_2} & \frac{\partial\mathbf{M}_2^1}{\partial\nabla p_2}\\ \frac{\partial h_{2m}^1}{\partial p_2} & \frac{\partial h_{2m}^1}{\partial\nabla p_2}\end{bmatrix}\ ;\ [\mathbf{DP21DT}]=\begin{bmatrix}\frac{\partial m_2^1}{\partial T} & \frac{\partial m_2^1}{\partial\nabla T}\\ \frac{\partial\mathbf{M}_2^1}{\partial T} & \frac{\partial\mathbf{M}_2^1}{\partial\nabla T}\\ \frac{\partial h_{2m}^1}{\partial T} & \frac{\partial h_{2m}^1}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DP22DE}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial\varepsilon}\\ \frac{\partial\mathbf{M}_2^2}{\partial\varepsilon}\\ \frac{\partial h_{2m}^2}{\partial\varepsilon}\end{bmatrix}\ ;\ [\mathbf{DP22P1}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial p_1} & \frac{\partial m_2^2}{\partial\nabla p_1}\\ \frac{\partial\mathbf{M}_2^2}{\partial p_1} & \frac{\partial\mathbf{M}_2^2}{\partial\nabla p_1}\\ \frac{\partial h_{2m}^2}{\partial p_1} & \frac{\partial h_{2m}^2}{\partial\nabla p_1}\end{bmatrix}\ ;\ [\mathbf{DP22P2}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial p_2} & \frac{\partial m_2^2}{\partial\nabla p_2}\\ \frac{\partial\mathbf{M}_2^2}{\partial p_2} & \frac{\partial\mathbf{M}_2^2}{\partial\nabla p_2}\\ \frac{\partial h_{2m}^2}{\partial p_2} & \frac{\partial h_{2m}^2}{\partial\nabla p_2}\end{bmatrix}\ ;\ [\mathbf{DP22DT}]=\begin{bmatrix}\frac{\partial m_2^2}{\partial T} & \frac{\partial m_2^2}{\partial\nabla T}\\ \frac{\partial\mathbf{M}_2^2}{\partial T} & \frac{\partial\mathbf{M}_2^2}{\partial\nabla T}\\ \frac{\partial h_{2m}^2}{\partial T} & \frac{\partial h_{2m}^2}{\partial\nabla T}\end{bmatrix}$$

$$[\mathbf{DTDE}]=\begin{bmatrix}\frac{\partial Q'}{\partial\varepsilon}\\ \frac{\partial\mathbf{q}}{\partial\varepsilon}\end{bmatrix}\ ;\ [\mathbf{DTDP1}]=\begin{bmatrix}\frac{\partial Q'}{\partial p_1} & \frac{\partial Q'}{\partial\nabla p_1}\\ \frac{\partial\mathbf{q}}{\partial p_1} & \frac{\partial\mathbf{q}}{\partial\nabla p_1}\end{bmatrix}\ ;\ [\mathbf{DTDP2}]=\begin{bmatrix}\frac{\partial Q'}{\partial p_2} & \frac{\partial Q'}{\partial\nabla p_2}\\ \frac{\partial\mathbf{q}}{\partial p_2} & \frac{\partial\mathbf{q}}{\partial\nabla p_2}\end{bmatrix}\ ;\ [\mathbf{DTDT}]=\begin{bmatrix}\frac{\partial Q'}{\partial T} & \frac{\partial Q'}{\partial\nabla T}\\ \frac{\partial\mathbf{q}}{\partial T} & \frac{\partial\mathbf{q}}{\partial\nabla T}\end{bmatrix}$$

In addition, by deriving the expression from the residue compared to the constraints, one defines:

$$[\mathbf{DRDS}]=\begin{bmatrix}
\frac{\partial R_1}{\partial \sigma'} & \frac{\partial R_1}{\partial \sigma_p} & \frac{\partial R_1}{\partial m_1^1} & \frac{\partial R_1}{\partial \mathbf{M}_1^1} & \frac{\partial R_1}{\partial h_{1m}^1} & \frac{\partial R_1}{\partial m_1^2} & \frac{\partial R_1}{\partial \mathbf{M}_1^2} & \frac{\partial R_1}{\partial h_{1m}^2} & \frac{\partial R_1}{\partial m_2^1} & \frac{\partial R_1}{\partial \mathbf{M}_2^1} & \frac{\partial R_1}{\partial h_{2m}^1} & \frac{\partial R_1}{\partial m_2^2} & \frac{\partial R_1}{\partial \mathbf{M}_2^2} & \frac{\partial R_1}{\partial h_{2m}^2} & \frac{\partial R_1}{\partial Q'} & \frac{\partial R_1}{\partial \mathbf{q}} \\
\frac{\partial R_2}{\partial \sigma'} & \frac{\partial R_2}{\partial \sigma_p} & \frac{\partial R_2}{\partial m_1^1} & \frac{\partial R_2}{\partial \mathbf{M}_1^1} & \frac{\partial R_2}{\partial h_{1m}^1} & \frac{\partial R_2}{\partial m_1^2} & \frac{\partial R_2}{\partial \mathbf{M}_1^2} & \frac{\partial R_2}{\partial h_{1m}^2} & \frac{\partial R_2}{\partial m_2^1} & \frac{\partial R_2}{\partial \mathbf{M}_2^1} & \frac{\partial R_2}{\partial h_{2m}^1} & \frac{\partial R_2}{\partial m_2^2} & \frac{\partial R_2}{\partial \mathbf{M}_2^2} & \frac{\partial R_2}{\partial h_{2m}^2} & \frac{\partial R_2}{\partial Q'} & \frac{\partial R_2}{\partial \mathbf{q}} \\
\frac{\partial R_3}{\partial \sigma'} & \frac{\partial R_3}{\partial \sigma_p} & \frac{\partial R_3}{\partial m_1^1} & \frac{\partial R_3}{\partial \mathbf{M}_1^1} & \frac{\partial R_3}{\partial h_{1m}^1} & \frac{\partial R_3}{\partial m_1^2} & \frac{\partial R_3}{\partial \mathbf{M}_1^2} & \frac{\partial R_3}{\partial h_{1m}^2} & \frac{\partial R_3}{\partial m_2^1} & \frac{\partial R_3}{\partial \mathbf{M}_2^1} & \frac{\partial R_3}{\partial h_{2m}^1} & \frac{\partial R_3}{\partial m_2^2} & \frac{\partial R_3}{\partial \mathbf{M}_2^2} & \frac{\partial R_3}{\partial h_{2m}^2} & \frac{\partial R_3}{\partial Q'} & \frac{\partial R_3}{\partial \mathbf{q}} \\
\frac{\partial R_4}{\partial \sigma'} & \frac{\partial R_4}{\partial \sigma_p} & \frac{\partial R_4}{\partial m_1^1} & \frac{\partial R_4}{\partial \mathbf{M}_1^1} & \frac{\partial R_4}{\partial h_{1m}^1} & \frac{\partial R_4}{\partial m_1^2} & \frac{\partial R_4}{\partial \mathbf{M}_1^2} & \frac{\partial R_4}{\partial h_{1m}^2} & \frac{\partial R_4}{\partial m_2^1} & \frac{\partial R_4}{\partial \mathbf{M}_2^1} & \frac{\partial R_4}{\partial h_{2m}^1} & \frac{\partial R_4}{\partial m_2^2} & \frac{\partial R_4}{\partial \mathbf{M}_2^2} & \frac{\partial R_4}{\partial h_{2m}^2} & \frac{\partial R_4}{\partial Q'} & \frac{\partial R_4}{\partial \mathbf{q}} \\
\frac{\partial R_5}{\partial \sigma'} & \frac{\partial R_5}{\partial \sigma_p} & \frac{\partial R_5}{\partial m_1^1} & \frac{\partial R_5}{\partial \mathbf{M}_1^1} & \frac{\partial R_5}{\partial h_{1m}^1} & \frac{\partial R_5}{\partial m_1^2} & \frac{\partial R_5}{\partial \mathbf{M}_1^2} & \frac{\partial R_5}{\partial h_{1m}^2} & \frac{\partial R_5}{\partial m_2^1} & \frac{\partial R_5}{\partial \mathbf{M}_2^1} & \frac{\partial R_5}{\partial h_{2m}^1} & \frac{\partial R_5}{\partial m_2^2} & \frac{\partial R_5}{\partial \mathbf{M}_2^2} & \frac{\partial R_5}{\partial h_{2m}^2} & \frac{\partial R_5}{\partial Q'} & \frac{\partial R_5}{\partial \mathbf{q}} \\
\frac{\partial R_6}{\partial \sigma'} & \frac{\partial R_6}{\partial \sigma_p} & \frac{\partial R_6}{\partial m_1^1} & \frac{\partial R_6}{\partial \mathbf{M}_1^1} & \frac{\partial R_6}{\partial h_{1m}^1} & \frac{\partial R_6}{\partial m_1^2} & \frac{\partial R_6}{\partial \mathbf{M}_1^2} & \frac{\partial R_6}{\partial h_{1m}^2} & \frac{\partial R_6}{\partial m_2^1} & \frac{\partial R_6}{\partial \mathbf{M}_2^1} & \frac{\partial R_6}{\partial h_{2m}^1} & \frac{\partial R_6}{\partial m_2^2} & \frac{\partial R_6}{\partial \mathbf{M}_2^2} & \frac{\partial R_6}{\partial h_{2m}^2} & \frac{\partial R_6}{\partial Q'} & \frac{\partial R_6}{\partial \mathbf{q}} \\
\frac{\partial R_7}{\partial \sigma'} & \frac{\partial R_7}{\partial \sigma_p} & \frac{\partial R_7}{\partial m_1^1} & \frac{\partial R_7}{\partial \mathbf{M}_1^1} & \frac{\partial R_7}{\partial h_{1m}^1} & \frac{\partial R_7}{\partial m_1^2} & \frac{\partial R_7}{\partial \mathbf{M}_1^2} & \frac{\partial R_7}{\partial h_{1m}^2} & \frac{\partial R_7}{\partial m_2^1} & \frac{\partial R_7}{\partial \mathbf{M}_2^1} & \frac{\partial R_7}{\partial h_{2m}^1} & \frac{\partial R_7}{\partial m_2^2} & \frac{\partial R_7}{\partial \mathbf{M}_2^2} & \frac{\partial R_7}{\partial h_{2m}^2} & \frac{\partial R_7}{\partial Q'} & \frac{\partial R_7}{\partial \mathbf{q}} \\
\frac{\partial R_8}{\partial \sigma'} & \frac{\partial R_8}{\partial \sigma_p} & \frac{\partial R_8}{\partial m_1^1} & \frac{\partial R_8}{\partial \mathbf{M}_1^1} & \frac{\partial R_8}{\partial h_{1m}^1} & \frac{\partial R_8}{\partial m_1^2} & \frac{\partial R_8}{\partial \mathbf{M}_1^2} & \frac{\partial R_8}{\partial h_{1m}^2} & \frac{\partial R_8}{\partial m_2^1} & \frac{\partial R_8}{\partial \mathbf{M}_2^1} & \frac{\partial R_8}{\partial h_{2m}^1} & \frac{\partial R_8}{\partial m_2^2} & \frac{\partial R_8}{\partial \mathbf{M}_2^2} & \frac{\partial R_8}{\partial h_{2m}^2} & \frac{\partial R_8}{\partial Q'} & \frac{\partial R_8}{\partial \mathbf{q}}
\end{bmatrix}$$

All these quantities not being inevitably calculated, one will note:

$$[\mathbf{DR1DS}]=\begin{bmatrix}\frac{\partial R_1}{\partial \sigma^{'+}} & \frac{\partial R_1}{\partial \sigma_p^+}\end{bmatrix} \quad ; \quad [\mathbf{DR1P11}]=\begin{bmatrix}\frac{\partial R_1}{\partial m_1^{1+}} & \frac{\partial R_1}{\partial \mathbf{M}_1^{1+}}\end{bmatrix} ou \begin{bmatrix}\frac{\partial R_1}{\partial m_1^{1+}} & \frac{\partial R_1}{\partial \mathbf{M}_1^{1+}} & \frac{\partial R_1}{\partial \sigma_{1m}^{1+}}\end{bmatrix}$$

$$[\mathbf{DR1P12}]=\begin{bmatrix}\frac{\partial R_1}{\partial m_1^{2+}} & \frac{\partial R_1}{\partial \mathbf{M}_1^{2+}}\end{bmatrix} ou \begin{bmatrix}\frac{\partial R_1}{\partial m_1^{2+}} & \frac{\partial R_1}{\partial \mathbf{M}_1^{2+}} & \frac{\partial R_1}{\partial h_{1m}^{2+}}\end{bmatrix}$$

$$[\mathbf{DR1P21}]=\begin{bmatrix}\frac{\partial R_1}{\partial m_2^{1+}} & \frac{\partial R_1}{\partial \mathbf{M}_2^{1+}}\end{bmatrix} ou \begin{bmatrix}\frac{\partial R_1}{\partial m_2^{1+}} & \frac{\partial R_1}{\partial \mathbf{M}_2^{1+}} & \frac{\partial R_1}{\partial h_{2m}^{1+}}\end{bmatrix}$$

$$[\mathbf{DR1P22}]=\begin{bmatrix}\frac{\partial R_1}{\partial m_2^{2+}} & \frac{\partial R_1}{\partial \mathbf{M}_2^{2+}}\end{bmatrix} ou \begin{bmatrix}\frac{\partial R_1}{\partial m_2^{2+}} & \frac{\partial R_1}{\partial \mathbf{M}_2^{2+}} & \frac{\partial R_1}{\partial h_{2m}^{2+}}\end{bmatrix}$$

$$[\mathbf{DR1DT}]=\begin{bmatrix}\frac{\partial R_1}{\partial Q^{'+}} & \frac{\partial R_1}{\partial \mathbf{q}^+}\end{bmatrix}$$

In the same way:

$$[\mathbf{DR8DS}],[\mathbf{DR8P11}],[\mathbf{DR8P12}],[\mathbf{DR8P21}],[\mathbf{DR8P22}],[\mathbf{DR8DT}]$$

It is then clear that:

$$\boxed{[\mathbf{DRDE}]=[\mathbf{DRDS}]\cdot[\mathbf{DSDE}]}$$

And the contribution of the point of Gauss to the tangent matrix $\mathrm{DF}^{el}_{g\,i(u_n^+,P_n^+,T_n^+)}$ is obtained by:

$$\boxed{\left[\mathbf{DF}^{el}_{g\,i(u_n^+,P_n^+,T_n^+)}\right]=\left[\mathbf{B}_g^{el}\right]^T\cdot[\mathbf{DRDE}]\cdot\left[\mathbf{B}_g^{el}\right]}$$

5 Outline general

`STAT_NON_LINE`

finite element

Routine `TE …`

Knows unknown:

- T, P1, P2, U

Buckle points of gauss

Calculate deformations :

$\underline{\underline{\varepsilon}}(\mathbf{u})$,

$p_1, p_2, \nabla p_1, \nabla p_2$,

$T, \nabla T$

Calculate $[B]_g^{el}$

Cal `EQUTHM`

Routine `EQUTHM`

Arg In:
nature unknown factors
Cal behavior

Comp méca
Comp hydrau
Comp ther

Assemble contribution
not Gauss with residue

and/or M tgte

$\underline{\underline{\sigma}}', \sigma_p$;

$m_1^1, \mathbf{M}_1^1, h_{1m}^1$;

$m_1^2, \mathbf{M}_1^2, h_{1m}^2$;

$m_2^1, \mathbf{M}_2^1, h_{2m}^1$;

$m_2^2, \mathbf{M}_2^2, h_{2m}^2$;

$Q', \mathbf{q}$

6 Specifications of under generic program EQUTHM

6.1 Arguments of the routine

ARGUMENTS OF ENTRY: IN		
COMPOR	Description of the behavior	
OPTION	Option to be calculated	
NDIM	dimension spaces	2 or 3
NDDL	Full number of degrees of freedom of the appealing element	
DIMDEF	dimension of the table of the deformations generalized at the point of Gauss	
DIMCON	dimension of the table of the constraints generalized at the point of Gauss	
NVIMEC	Many internal variables "mechanical"	
ADVIME	Address of the mechanical internal variables in the table of the internal variables at the point of Gauss	
NVIHY	Many internal variables "hydraulic"	
ADVIHY	Address of the hydraulic internal variables in the table of the internal variables at the point of Gauss	
NVITM	Many internal variables "thermal"	
ADVITM	Address of the thermal internal variables in the table of the internal variables at the point of Gauss	
B (1: dimdef, 1: nddl)	Matrix $[B]_g^{el}$	
DEFGEP (1: dimdef)	Values of deformations generalized at the point of Gauss time more	
DEFGEM (1: dimdef)	Values of deformations generalized at the point of Gauss time less	
CONGEM (1: dimcon)	Values of constraints generalized at the point of Gauss time less	
VINTM (1: nvimec+nvihy+nvitm)	Values of the internal variables at the point of Gauss time less	
MECA (1: 5)	YAMEC = MECA (1)	logic if 1 there is an equation of mechanics

	ADDEME = MECA (2)	Address in the tables of the deformations at the point of Gauss DEFGEP and DEFGEM deformations corresponding to mechanics
	ADCOME = MECA (3)	Address in the tables of the constraints at the point of Gauss CONGEP and CONGEM constraints corresponding to the equation ieq
	NDEFME = MECA (4)	Many mechanical deformations
	NCONME = MECA (5)	Many mechanical constraints
PRESS1 (1: 5)	YAP1 = CLOSE 1(1)	logic if 1 there is an equation constituting 1
	NBPHA1 = CLOSE 1(2)	many phases for component 1
	ADDEP1 = CLOSE 1(3)	Address in the tables of the deformations at the point of Gauss DEFGEP and DEFGEM deformations corresponding to the first pressure
	ADCP11 = CLOSE 1(4)	Address in the tables of the constraints at the point of Gauss CONGEP and CONGEM constraints corresponding to the first phase of the first component
	ADCP12 = CLOSE 1(5)	Address in the tables of the constraints at the point of Gauss CONGEP and CONGEM constraints corresponding to the second phase of the first component
	NDEFP1 = CLOSE 1(6)	Many deformations pressure 1
	NCONP1 = CLOSE 1(7)	Many constraints for each phase of component 1
PRESS2 (1: 5)	YAP2 = CLOSE 2(1)	logic if 1 there is an equation constituting 2
	NBPHA2 = CLOSE 2(2)	many phases for component 2
	ADDEP2 = CLOSE 2(3)	Address in the tables of the deformations at the point of Gauss DEFGEP and DEFGEM deformations corresponding to PRE2
	ADCP21 = CLOSE 2(4)	Address in the tables of the constraints at the point of Gauss CONGEP and CONGEM constraints corresponding to the first phase of the second component
	ADCP22 = CLOSE 2(5)	Address in the tables of the constraints at the point of Gauss CONGEP and CONGEM constraints corresponding to the second phase of the second component
	NDEFP2 = CLOSE 2(6)	Many deformations pressure 2
	NCONP2 = CLOSE 2(7)	Many constraints for each phase of component 2
TEMPLE (1: 5)	YATE = TEMPLE (1)	logic if 1 there is an equation of thermics
	ADDETE = TEMPLE (2)	Address in the tables of the deformations at the point of Gauss

		DEFGEP and DEFGEM deformations corresponding to thermics
	ADCOTE = TEMPLE (3)	Address in the tables of the constraints at the point of Gauss CONGEP and first CONGEM constraints corresponding to thermics
	NDEFT = TEMPLE (4)	Thermal number of deformations
	NCONT = TEMPLE (5)	Number of thermal stresses
ARGUMENTS OF EXIT: OUT		
CONGEP (1: dimcon)	Values of constraints generalized at the point of Gauss time more	
VINTP (1: nvimec+nvihy+nvitm)	Values of the internal variables at the point of Gauss time more	
V (1: nddl)	$\{\mathbf{V}_g^{el}\}=[\mathbf{B}_g^{el}]^T\{R\}$	
CHECHMATE (1: nddl, 1: nddl)	$[\mathbf{DF}^{el}_{g\,i(u_n^i,P_n^i,T_n^i)}]=[\mathbf{B}_g^{el}]^T\cdot[\mathbf{DRDE}]\cdot[\mathbf{B}_g^{el}]$	
TABLES OF WORK		
R (1: dimdef)		
DRDS (1: dimdef, 1: dimcon)		
DSDE (1: dimcon, 1: dimdef)		

6.2 Addressing in the tables of strain and stress

6.2.1 Addressing in the deformations

6.2.1.1 Deformations time less

Part (local name in routine COMTHM)	Significance	Address in DEFGEM
DEMECM	$\mathbf{u}, \underline{\underline{\varepsilon}}(\mathbf{u})$	ADDEME
DEP1M	$p_{1,} \nabla p_1$	ADDEP1
DEP2M	$p_{2,} \nabla p_2$	ADDEP2
DETM	$T, \nabla T$	ADDETE

6.2.1.2 Deformations time more

Part (local name in routine COMTHM)	Significance	Address in DEFGEP
DEMECP	$\mathbf{u}, \underline{\underline{\varepsilon}}(\mathbf{u})$	ADDEME
DEP1P	$p_{1,} \nabla p_1$	ADDEP1
DEP2P	$p_{2,} \nabla p_2$	ADDEP2
DETP	$T, \nabla T$	ADDETE

6.2.2 Addressing in the constraints

6.2.2.1 Constraints time less

Part (local name in routine COMTHM)	Significance	Address in CONGEM
COMECM	$\underline{\underline{\sigma}}', \sigma_p$	ADCOME
CP11M	$m_1^{1,}\mathbf{M_1^1}$ ou $m_1^{1,}\mathbf{M_1^{1,}}h_{1m}^1$	ADCP11
CP12M	$m_1^{2,}\mathbf{M_1^2}$ ou $m_1^{2,}\mathbf{M_1^{2,}}h_{1m}^2$	ADCP12
CP21M	$m_2^{1,}\mathbf{M_2^1}$ ou $m_2^{1,}\mathbf{M_2^{1,}}h_{2m}^1$	ADCP21
CP22M	$m_2^{2,}\mathbf{M_2^2}$ ou $m_2^{2,}\mathbf{M_2^{2,}}h_{2m}^2$	ADCP22
COTM	$Q', \mathbf{q}$	ADCOTE

6.2.2.2 Constraints time more

Part (local name in routine COMTHM)	Significance	Address in CONGEP
COMECP	$\underline{\underline{\sigma}}', \sigma_p$	ADCOME
CP11P	$m_1^{1,}\mathbf{M_1^1}$ ou $m_1^{1,}\mathbf{M_1^{1,}}h_{1m}^1$	ADCP11
CP12P	$m_1^{2,}\mathbf{M_1^2}$ ou $m_1^{2,}\mathbf{M_1^{2,}}h_{1m}^2$	ADCP12
CP21P	$m_2^{1,}\mathbf{M_2^1}$ ou $m_2^{1,}\mathbf{M_2^{1,}}h_{2m}^1$	ADCP21
CP22P	$m_2^{2,}\mathbf{M_2^2}$ ou $m_2^{2,}\mathbf{M_2^{2,}}h_{2m}^2$	ADCP22
COTP	$Q', \mathbf{q}$	ADCOTE

6.2.3 Addressing in the internal variables (example)

6.2.3.1 Internal variables at time less

Part (local name in routine COMTHM)	Significance	Address in VINTM
VIMEM	φ	ADVIME
VIHYM	S_{lq}, P_{vq}, P_{lq}	ADVIHY

6.2.3.2 Internal variables at time more

Part (local name in routine COMTHM)	Significance	Address in VINTP
VIMEP	φ	ADVIME
VIHYP	S_{lq}, P_{vq}, P_{lq}	ADVIHY

6.3 Addressing R, DRDS, DSDE

6.3.1 Addressing in R

Under part of R	Associated with	Address in R
R1	$\mathbf{v}$	ADDEME
R2	$\varepsilon(\mathbf{v})$	ADDEME+NDIM
R3	π_1	ADDEP1
R4	$\nabla\pi_1$	ADDEP1+1
R5	π_2	ADDEP2
R6	$\nabla\pi_2$	ADDEP2+1
R7	τ	ADDETE
R8	$\nabla\tau$	ADDETE+1

6.3.2 Addressing in DRDS

Part of the table DRDS	Significance	Address in DRDS
DR1DS	$\left[\frac{\partial R_1}{\partial\sigma'^+} \quad \frac{\partial R_1}{\partial\sigma_p^+}\right]$	ADDEME, ADCOME
DR2DS		ADDEME+NDIM-1, ADCOME
DR1P11	$\left[\frac{\partial R_1}{\partial m_1^{1+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_1^{1+}}\right] ou$ $\left[\frac{\partial R_1}{\partial m_1^{1+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_1^{1+}} \quad \frac{\partial R_1}{\partial h_{1m}^{1+}}\right]$	ADDEME, ADCP11
DR2P11		ADDEME+NDIM-1, ADCP11
DR1P12	$\left[\frac{\partial R_1}{\partial m_1^{2+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_1^{2+}}\right] ou$ $\left[\frac{\partial R_1}{\partial m_1^{2+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_1^{2+}} \quad \frac{\partial R_1}{\partial h_{1m}^{2+}}\right]$	ADDEME, ADCP12
DR2P12		ADDEME+NDIM-1, ADCP12
DR1P21	$\left[\frac{\partial R_1}{\partial m_2^{1+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_2^{1+}}\right] ou$ $\left[\frac{\partial R_1}{\partial m_2^{1+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_2^{1+}} \quad \frac{\partial R_1}{\partial h_{2m}^{1+}}\right]$	ADDEME, ADCP21
DR2P21		ADDEME+NDIM-1, ADCP21
DR1P22	$\left[\frac{\partial R_1}{\partial m_2^{2+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_2^{2+}}\right] ou$ $\left[\frac{\partial R_1}{\partial m_2^{2+}} \quad \frac{\partial R_1}{\partial\mathbf{M}_2^{2+}} \quad \frac{\partial R_1}{\partial h_{2m}^{2+}}\right]$	ADDEME, ADCP22
DR2P22		ADDEME+NDIM-1, ADCP22
DR1DT	$\left[\frac{\partial R_1}{\partial Q'^+} \quad \frac{\partial R_1}{\partial\mathbf{q}^+}\right]$	ADDEME, ADCOTE

DR2DT		ADDEME+NDIM-1,	ADCOTE
DR3DS		ADDEP1,	ADCOME
DR4DS		ADDEP1+1,	ADCOME
DR3P11		ADDEP1,	ADCP11
DR4P11		ADDEP1+1,	ADCP11
DR3P21		ADDEP1,	ADCP21
DR4P21		ADDEP1+ 1,	ADCP21
DR3DT		ADDEP1,	ADCOTE
DR4DT		ADDEP1+ 1,	ADCOTE
DR5DS		ADDEP2,	ADCOME
DR6DS		ADDEP2+ 1,	ADCOME
DR5P11		ADDEP2,	ADCP11
DR6P11		ADDEP2+ 1,	ADCP11
DR5P21		ADDEP2,	ADCP21
DR6P21		ADDEP2+1,	ADCP21
DR5DT		ADDEP2,	ADCOTE
DR6DT		ADDEP2+ 1,	ADCOTE
DR7DS		ADDETE,	ADCOME
DR8DS		ADDETE+ 1,	ADCOME
DR7P11		ADDETE,	ADCP11
DR8P11		ADDETE+ 1,	ADCP11
DR7P21		ADDETE,	ADCP21
DR8P21		ADDETE+ 1,	ADCP21
DR7DT		ADDETE,	ADCOTE
DR8DT		ADDETE+1,	ADCOTE

6.3.3 Addressing in DSDE

Part (local name with COMTHM)	Significance	Address in DSDE
DMECDE	$\begin{bmatrix} \frac{\partial \sigma'}{\partial \varepsilon} \\ \frac{\partial \sigma_p}{\partial \varepsilon} \end{bmatrix}$	ADCOME, ADDEME
DMECP1	$\begin{bmatrix} \frac{\partial \sigma'}{\partial p_1} & \frac{\partial \sigma'}{\partial \nabla p_1} \\ \frac{\partial \sigma_p}{\partial p_1} & \frac{\partial \sigma_p}{\partial \nabla p_1} \end{bmatrix}$	ADCOME, ADDEP1
DMECP2	$\begin{bmatrix} \frac{\partial \sigma'}{\partial p_2} & \frac{\partial \sigma'}{\partial \nabla p_2} \\ \frac{\partial \sigma_p}{\partial p_2} & \frac{\partial \sigma_p}{\partial \nabla p_2} \end{bmatrix}$	ADCOME, ADDEP2
DMECDT	$\begin{bmatrix} \frac{\partial \sigma'}{\partial T} & \frac{\partial \sigma'}{\partial \nabla T} \\ \frac{\partial \sigma_p}{\partial T} & \frac{\partial \sigma_p}{\partial \nabla T} \end{bmatrix}$	ADCOME, ADDETE

DP11DE	$\begin{bmatrix} \frac{\partial m_1^1}{\partial \varepsilon} \\ \frac{\partial \mathbf{M}_1^1}{\partial \varepsilon} \\ \frac{\partial h_{1m}^1}{\partial \varepsilon} \end{bmatrix}$	ADCP11, ADDEME
DP11P1	$\begin{bmatrix} \frac{\partial m_1^1}{\partial p_1} & \frac{\partial m_1^1}{\partial \nabla p_1} \\ \frac{\partial \mathbf{M}_1^1}{\partial p_1} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla p_1} \\ \frac{\partial h_{1m}^1}{\partial p_1} & \frac{\partial h_{1m}^1}{\partial \nabla p_1} \end{bmatrix}$	ADCP11, ADDEP1
DP11P2	$\begin{bmatrix} \frac{\partial m_1^1}{\partial p_2} & \frac{\partial m_1^1}{\partial \nabla p_2} \\ \frac{\partial \mathbf{M}_1^1}{\partial p_2} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla p_2} \\ \frac{\partial h_{1m}^1}{\partial p_2} & \frac{\partial h_{1m}^1}{\partial \nabla p_2} \end{bmatrix}$	ADCP11, ADDEP2
DP11DT	$\begin{bmatrix} \frac{\partial m_1^1}{\partial T} & \frac{\partial m_1^1}{\partial \nabla T} \\ \frac{\partial \mathbf{M}_1^1}{\partial T} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla T} \\ \frac{\partial h_{1m}^1}{\partial T} & \frac{\partial h_{1m}^1}{\partial \nabla T} \end{bmatrix}$	ADCP11, ADDETE
DP12DE	$\begin{bmatrix} \frac{\partial m_1^2}{\partial \varepsilon} \\ \frac{\partial \mathbf{M}_1^2}{\partial \varepsilon} \\ \frac{\partial h_{1m}^2}{\partial \varepsilon} \end{bmatrix}$	ADCP12, ADDEME
DP12P1	$\begin{bmatrix} \frac{\partial m_1^2}{\partial p_1} & \frac{\partial m_1^2}{\partial \nabla p_1} \\ \frac{\partial \mathbf{M}_1^2}{\partial p_1} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla p_1} \\ \frac{\partial h_{1m}^2}{\partial p_1} & \frac{\partial h_{1m}^2}{\partial \nabla p_1} \end{bmatrix}$	ADCP12, ADDEP1

DP12P2		$\begin{bmatrix} \frac{\partial m_1^2}{\partial p_2} & \frac{\partial m_1^2}{\partial \nabla p_2} \\ \frac{\partial \mathbf{M}_1^2}{\partial p_2} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla p_2} \\ \frac{\partial h_{1m}^2}{\partial p_2} & \frac{\partial h_{1m}^2}{\partial \nabla p_2} \end{bmatrix}$	ADCP12, ADDEP2
DP12DT		$\begin{bmatrix} \frac{\partial m_1^2}{\partial T} & \frac{\partial m_1^2}{\partial \nabla T} \\ \frac{\partial \mathbf{M}_1^2}{\partial T} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla T} \\ \frac{\partial h_{1m}^2}{\partial T} & \frac{\partial h_{1m}^2}{\partial \nabla T} \end{bmatrix}$	ADCP12, ADDETE
DP21DE		$\begin{bmatrix} \frac{\partial m_2^1}{\partial \varepsilon} \\ \frac{\partial \mathbf{M}_2^1}{\partial \varepsilon} \\ \frac{\partial h_{2m}^1}{\partial \varepsilon} \end{bmatrix}$	ADCP21, ADDEME
DP21P1		$\begin{bmatrix} \frac{\partial m_2^1}{\partial p_1} & \frac{\partial m_2^1}{\partial \nabla p_1} \\ \frac{\partial \mathbf{M}_2^1}{\partial p_1} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla p_1} \\ \frac{\partial h_{2m}^1}{\partial p_1} & \frac{\partial h_{2m}^1}{\partial \nabla p_1} \end{bmatrix}$	ADCP21, ADDEP1
DP21P2		$\begin{bmatrix} \frac{\partial m_2^1}{\partial p_2} & \frac{\partial m_2^1}{\partial \nabla p_2} \\ \frac{\partial \mathbf{M}_2^1}{\partial p_2} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla p_2} \\ \frac{\partial h_{2m}^1}{\partial p_2} & \frac{\partial h_{2m}^1}{\partial \nabla p_2} \end{bmatrix}$	ADCP21, ADDEP2
DP21DT		$\begin{bmatrix} \frac{\partial m_2^1}{\partial T} & \frac{\partial m_2^1}{\partial \nabla T} \\ \frac{\partial \mathbf{M}_2^1}{\partial T} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla T} \\ \frac{\partial h_{2m}^1}{\partial T} & \frac{\partial h_{2m}^1}{\partial \nabla T} \end{bmatrix}$	ADCP21, ADDETE

DP22DE	$\begin{bmatrix} \dfrac{\partial m_2^2}{\partial \varepsilon} \\ \dfrac{\partial \mathbf{M}_2^2}{\partial \varepsilon} \\ \dfrac{\partial h_{2m}^2}{\partial \varepsilon} \end{bmatrix}$	ADCP22, ADDEME
DP22P1	$\begin{bmatrix} \dfrac{\partial m_2^2}{\partial p_1} & \dfrac{\partial m_2^2}{\partial \nabla p_1} \\ \dfrac{\partial \mathbf{M}_2^2}{\partial p_1} & \dfrac{\partial \mathbf{M}_2^2}{\partial \nabla p_1} \\ \dfrac{\partial h_{2m}^2}{\partial p_1} & \dfrac{\partial h_{2m}^2}{\partial \nabla p_1} \end{bmatrix}$	ADCP22, ADDEP1
DP22P2	$\begin{bmatrix} \dfrac{\partial m_2^2}{\partial p_2} & \dfrac{\partial m_2^2}{\partial \nabla p_2} \\ \dfrac{\partial \mathbf{M}_2^2}{\partial p_2} & \dfrac{\partial \mathbf{M}_2^2}{\partial \nabla p_2} \\ \dfrac{\partial h_{2m}^2}{\partial p_2} & \dfrac{\partial h_{2m}^2}{\partial \nabla p_2} \end{bmatrix}$	ADCP22, ADDEP2
DP22DT	$\begin{bmatrix} \dfrac{\partial m_2^2}{\partial T} & \dfrac{\partial m_2^2}{\partial \nabla T} \\ \dfrac{\partial \mathbf{M}_2^2}{\partial T} & \dfrac{\partial \mathbf{M}_2^2}{\partial \nabla T} \\ \dfrac{\partial h_{2m}^2}{\partial T} & \dfrac{\partial h_{2m}^2}{\partial \nabla T} \end{bmatrix}$	ADCP22, ADDETE
DTDE	$\begin{bmatrix} \dfrac{\partial Q'}{\partial \varepsilon} \\ \dfrac{\partial \mathbf{q}}{\partial \varepsilon} \end{bmatrix}$	ADCOTE, ADDEME
DTDP1	$\begin{bmatrix} \dfrac{\partial Q'}{\partial p_1} & \dfrac{\partial Q'}{\partial \nabla p_1} \\ \dfrac{\partial \mathbf{q}}{\partial p_1} & \dfrac{\partial \mathbf{q}}{\partial \nabla p_1} \end{bmatrix}$	ADCOTE, ADDEP1
DTDP2	$\begin{bmatrix} \dfrac{\partial Q'}{\partial p_2} & \dfrac{\partial Q'}{\partial \nabla p_2} \\ \dfrac{\partial \mathbf{q}}{\partial p_2} & \dfrac{\partial \mathbf{q}}{\partial \nabla p_2} \end{bmatrix}$	ADCOTE, ADDEP2

DTDT		$\begin{bmatrix} \frac{\partial \sigma'}{\partial T} & \frac{\partial \sigma'}{\partial \nabla T} \\ \frac{\partial \mathbf{q}}{\partial T} & \frac{\partial \mathbf{q}}{\partial \nabla T} \end{bmatrix}$	ADCOTE, ADDETE

6.4 Algorithm routine `EQUTHM`

```
YAMEC = MECA (1)
ADDEME = MECA (2)
ADCOME = MECA (3)
NDEFME = MECA (4)
NCONME = MECA (5)

YAP1 = CLOSE 1(1)
NBPHA1 = CLOSE 1(2)
ADDEP1 = CLOSE 1(3)
ADCP11 = CLOSE 1(4)
ADCP12 = CLOSE 1(5)
NDEFP1 = CLOSE 1(6)
NCONP1 = CLOSE 1(7)
YAP2 = CLOSE 2(1)
NBPHA2 = CLOSE 2(2)
ADDEP2 = CLOSE 2(3)
ADCP21 = CLOSE 2(4)
ADCP22 = CLOSE 2(5)
NDEFP2 = CLOSE 2(6)
NCONP2 = CLOSE 2(7)

YATE = TEMPLE (1)
ADDETE = TEMPLE (2)
ADCOTE = TEMPLE (3)
NDEFT = TEMPLE (4)
NCONT = TEMPLE (5)

CAL COMTHM (
```

COMPOR	OPTION	NDIM	NDDL
DIMDEF	DIMCON	NVIMEC	NVIHY, NVITM
NDEFME	NDEFP1	NDEFP2	NDEFT
NCONME	NCONP1	NCONP2	NCONT
YAP1	NBPHA1	YAP2	NBPHA2
DEFGEM (ADDEME)	DEFGEM (ADDEP1)	DEFGEM (ADDEP2)	DEFGEM (ADDETE)
DEFGEP (ADDEME)	DEFGEP (ADDEP1)	DEFGEP (ADDEP2)	DEFGEP (ADDETE)
CONGEM (ADCOME)	CONGEM (ADCOTE)		
CONGEM (ADCP11)	CONGEM (ADCP12)	CONGEM (ADCP21)	CONGEM (ADCP21)
VINTM (ADVIME)	VINTM (ADVIHY)	VINTM (ADVITM)	
CONGEP (ADCOME)	CONGEP (ADCP11)	CONGEP (ADCP21)	CONGEP (ADCOTE)
VINTP (ADVIME)	VINTP (ADVIHY)	VINTP (ADVITM)	
DSDE (ADCOME, ADDEME)	DSDE (ADCOME, ADDEP1)	DSDE (ADCOME, ADDEP2)	DSDE (ADCOME, ADDETE)
DSDE (ADCP11, ADDEP1)	DSDE (ADCP11, ADDEME)	DSDE (ADCP11, ADDEP2)	DSDE (ADCP11, ADDETE)
DSDE (ADCP12, ADDEP1)	DSDE (ADCP12, ADDEME)	DSDE (ADCP12, ADDEP2)	DSDE (ADCP12, ADDETE)
DSDE (ADCP21, ADDEP2)	DSDE (ADCP21, ADDEME)	DSDE (ADCP21, ADDEP1)	DSDE (ADCP21, ADDETE)
DSDE (ADCP22,	DSDE (ADCP22,	DSDE (ADCP22, ADDEP1)	DSDE (ADCP22,

ADDEP2)	ADDEME)		ADDETE)
DSDE (ADCOTE, ADDETE)	DSDE (ADCOTE, ADDEME)	DSDE (ADCOTE, ADDEP1)	DSDE (ADCOTE, ADDEP2)

)

If FULL_MECA or RAPH_MECA

If YAMEC

Injection of the terms $\sigma'^{+}+\sigma_p^{+} I$ in R (ADDEME+NDIM-1)

Injection of the terms: $-r_0 \mathbf{F}^{m^+}$ in R (ADDEME)

If YAP1

Injection of the terms $-m_1^{1+}+m_1^{1-}$ ou $-m_1^{1+}-m_1^{2+}+m_1^{1-}+m_1^{2-}$ in R (ADDEP1)

Injection of the terms

$$\Delta t\,\theta\,\mathbf{M}_1^{1+}+(1-\theta)\Delta t\,\mathbf{M}_1^{1-} \text{ ou}$$
$$\theta\,\Delta t\left(\mathbf{M}_1^{1+}+\mathbf{M}_1^{2+}\right)+(1-\theta)\Delta t\left(\mathbf{M}_1^{1-}+\mathbf{M}_1^{2-}\right)$$

in R (ADDEP1+1)

IF YAMEC

Injection of the terms:

$-m_1^{1+}\mathbf{F}^{m^+}$ ou $-\left(m_1^{1+}+m_1^{2+}\right)\mathbf{F}^{m^+}$ in R (ADDEME)

If YATE

Injection of the terms:

$$\Delta t\left(\theta\,h_{1m}^{1+}+(1-\theta)h_{1m}^{1-}\right)\left(m_1^{1+}-m_1^{1-}\right)-\theta\,\Delta t\,\mathbf{M}_1^{1+}\boldsymbol{F}^m-(1-\theta)\Delta t\,\mathbf{M}_1^{1-}\boldsymbol{F}^m$$

ou

$$\Delta t\left(\theta h_{1m}^{1+}+(1-\theta)h_{1m}^{1-}\right)\left(m_1^{1+}-m_1^{1-}\right)+\Delta t\left(\theta\,h_{1m}^{2+}+(1-\theta)h_{1m}^{2-}\right)\left(m_1^{2+}-m_1^{2-}\right)$$
$$-\theta\,\Delta t\,\mathbf{M}_1^{1+}\boldsymbol{F}^m-(1-\theta)\Delta t\,\mathbf{M}_1^{1-}\boldsymbol{F}^m-\theta\,\Delta t\,\mathbf{M}_1^{2+}\boldsymbol{F}^m-(1-\theta)\Delta t\,\mathbf{M}_1^{2-}\boldsymbol{F}^m$$

in R (ADDETE)

Injection of the terms

$$-\theta\,\Delta t\,h_{1m}^{1+}\mathbf{M}_1^{1+}-(1-\theta)-\theta\,\Delta t\,h_{1m}^{1-}\mathbf{M}_1^{1-} \text{ ou}$$
$$-\theta\,\Delta t\left(h_{1m}^{1+}\mathbf{M}_1^{1+}+h_{1m}^{2+}\mathbf{M}_1^{2+}\right)-(1-\theta)\Delta t\left(h_{1m}^{1-}\mathbf{M}_1^{1-}+h_{1m}^{2-}\mathbf{M}_1^{2-}\right)$$

in R (ADDETE+1)

If YAP2

Injection of the terms $+m_2^{1+}-m_2^{1-}$ ou $+m_2^{1+}+m_2^{2+}-m_2^{1-}-m_2^{2-}$ in R (ADDEP2)

Injection of the terms

$$\Delta t\,\theta\,\mathbf{M}_2^{1+}+(1-\theta)\Delta t\,\mathbf{M}_2^{1-} \text{ ou}$$
$$\theta\,\Delta t\left(\mathbf{M}_2^{1+}+\mathbf{M}_2^{2+}\right)+(1-\theta)\Delta t\left(\mathbf{M}_2^{1-}+\mathbf{M}_2^{2-}\right)$$

in R (ADDEP2+1)

IF YAMEC

Injection of the terms:

$-m_2^{1+}\mathbf{F}^{m^+}$ ou $-\left(m_2^{1+}+m_2^{2+}\right)\mathbf{F}^{m^+}$ in R (ADDEME)

If YATE

Injection of the terms:

$$\Delta t\left(\theta\,h_{2m}^{1+}+(1-\theta)h_{2m}^{1-}\right)\left(m_2^{1+}-m_2^{1-}\right)-\theta\,\Delta t\,\mathbf{M}_2^{1+}\boldsymbol{F}^m-(1-\theta)\Delta t\,\mathbf{M}_2^{1-}\boldsymbol{F}^m$$

ou

$$\Delta t\left(\theta h_{2m}^{1+}+(1-\theta)h_{2m}^{1-}\right)\left(m_2^{1+}-m_2^{1-}\right)+\Delta t\left(\theta\,h_{2m}^{2+}+(1-\theta)h_{2m}^{2-}\right)\left(m_2^{2+}-m_2^{2-}\right)$$
$$-\theta\,\Delta t\,\mathbf{M}_2^{1+}\boldsymbol{F}^m-(1-\theta)\Delta t\,\mathbf{M}_2^{1-}\boldsymbol{F}^m-\theta\,\Delta t\,\mathbf{M}_2^{2+}\boldsymbol{F}^m-(1-\theta)\Delta t\,\mathbf{M}_2^{2-}\boldsymbol{F}^m$$

in R (ADDETE)

Injection of the terms
$-\theta\,\Delta t\, h_{2m}^{1+}\mathbf{M}_2^{1+}-(1-\theta)-\theta\,\Delta t\, h_{2m}^{1-}\mathbf{M}_2^{1-}$ ou
$-\theta\,\Delta t\left(h_{2m}^{1+}\mathbf{M}_2^{1+}+h_{2m}^{2+}\mathbf{M}_2^{2+}\right)-(1-\theta)\,\Delta t\left(h_{2m}^{1-}\mathbf{M}_2^{1-}+h_{2m}^{2-}\mathbf{M}_2^{2-}\right)$
in R (ADDETE+1)

<u>If YATE</u>

Injection of the terms: $Q'^{+}-Q'^{-}$ in R (ADDETE)

Injection of the terms $-\theta\,\Delta t\,\mathbf{q}^{+}-(1-\theta)\,\Delta t\,\mathbf{q}^{-}$ in R (ADDETE+1)

Accumulation in vector V:

$\{\mathbf{V}\}=\{\mathbf{V}\}+\left[\mathbf{B}_g^{el}\right]^T\{R\}$

IF RAPH_MECA or RIGI_MECA_TANG
- IF YAMEC
  - calculation of DR1DS and injection in DRDS (ADDEME, ADCOME)
  - calculation of DR2DS and injection in DRDS (ADDEME+NDIM-1, ADCOME)
  - IF YAP1
    - calculation of DR1P11 and injection in DRDS (ADDEME, ADCP11)
    - calculation of DR2P11 and injection in DRDS (ADDEME+NDIM-1, ADCP11)
    - IF NBPHA1 > 1
      - calculation of DR1P12 and injection in DRDS (ADDEME, ADCP12)
      - calculation of DR2P12 and injection in DRDS (ADDEME+NDIM-1, ADCP12)
  - IF YAP2
    - calculation of DR1P21 and injection in DRDS (ADDEME, ADCP21)
    - calculation of DR2P21 and injection in DRDS (ADDEME+NDIM-1, ADCP21)
    - IF NBPHA2 > 1
      - calculation of DR1P22 and injection in DRDS (ADDEME, ADCP22)
      - calculation of DR2P22 and injection in DRDS (ADDEME+NDIM-1, ADCP22)
  - IF YATE
    - calculation of DR1DT and injection in DRDS (ADDEME, ADCOTE)
    - calculation of DR2DT and injection in DRDS (ADDEME+NDIM-1, ADCOTE)
- IF YAP1
  - calculation of DR3P11 and injection in DRDS (ADDEP1, ADCP11)
  - calculation of DR4P11 and injection in DRDS (ADDEP1+1, ADCP11)
  - IF NBPHA1 > 1
    - calculation of DR3P12 and injection in DRDS (ADDEP1, ADCP12)
    - calculation of DR4P12 and injection in DRDS (ADDEP1+1, ADCP12)
  - IF YAMEC
    - calculation of DR3DS and injection in DRDS (ADDEP1, ADCOME)
    - calculation of DR4DS and injection in DRDS (ADDEP1+1, ADCOME)
  - IF YAP2
    - calculation of DR3P21 and injection in DRDS (ADDEP1, ADCP21)
    - calculation of DR4P21 and injection in DRDS (ADDEP1+ 1, ADCP21)
    - IF NBPHA2 > 1
      - calculation of DR3P22 and injection in DRDS (ADDEP1, ADCP22)
      - calculation of DR4P21 and injection in DRDS (ADDEP1+ 1, ADCP22)
  - IF YATE
    - calculation of DR3DT and injection in DRDS (ADDEP1, ADCOTE)
    - calculation of DR4DT and injection in DRDS (ADDEP1+ 1, ADCOTE)
- IF YAP2
  - calculation of DR5P21 and injection in DRDS (ADDEP2, ADCP21)
  - calculation of DR6P21 and injection in DRDS (ADDEP2+1, ADCP21)
  - IF NBPHA2 > 1
    - calculation of DR5P22 and injection in DRDS (ADDEP2, ADCP22)
    - calculation of DR6P22 and injection in DRDS (ADDEP2+1, ADCP22)
  - IF YAMEC
    - calculation of DR5DS and injection in DRDS (ADDEP2, ADCOME)
    - calculation of DR6DS and injection in DRDS (ADDEP2+ 1, ADCOME)
  - YAP1 thus:
  - calculation of DR5P11 and injection in DRDS (ADDEP2, ADCP11)
  - calculation of DR6P11 and injection in DRDS (ADDEP2+ 1, ADCP11)
  - IF NBPHA1 > 1
    - calculation of DR5P12 and injection in DRDS (ADDEP2, ADCP12)
    - calculation of DR6P12 and injection in DRDS (ADDEP2+ 1, ADCP12)
  - IF YATE
    - calculation of DR5DT and injection in DRDS (ADDEP2, ADCOTE)
    - calculation of DR6DT and injection in DRDS (ADDEP2+ 1, ADCOTE)
- IF YATE
  - calculation of DR7DT and injection in DRDS (ADDETE, ADCOTE)
  - calculation of DR8DT and injection in DRDS (ADDETE+1, ADCOTE)
  - IF YAMEC

calculation of DR7DS and injection in DRDS (ADDETE, ADCOME)
calculation of DR8DS and injection in DRDS (ADDETE+ 1, ADCOME)

IF YAP1
- calculation of DR7P11 and injection in DRDS (ADDETE, ADCP11)
- calculation of DR8P11 and injection in DRDS (ADDETE+ 1, ADCP11)
- IF NBPHA1 > 1
  - calculation of DR7P12 and injection in DRDS (ADDETE, ADCP12)
  - calculation of DR8P12 and injection in DRDS (ADDETE+ 1, ADCP12)

IF YAP2
- calculation of DR7P21 and injection in DRDS (ADDETE, ADCP21)
- calculation of DR8P21 and injection in DRDS (ADDETE+ 1, ADCP21)
- IF NBPHA1 > 1
  - calculation of DR7P22 and injection in DRDS (ADDETE, ADCP22)
  - calculation of DR8P22 and injection in DRDS (ADDETE+ 1, ADCP22)

$$[\mathbf{DRDE}]=[\mathbf{DRDS}]\cdot[\mathbf{DSDE}]$$

$$\left[\mathbf{DF}^{el}_{g\,i(u_n^+,P_n^+,T_n^+)}\right]=\left[\mathbf{B}_g^{el}\right]^T\cdot[\mathbf{DRDE}]\cdot\left[\mathbf{B}_g^{el}\right]$$ accumulated in CHECHMATE

6.5 Arguments of the routine of call of the laws of behavior

```
SUBROUTINE COMTHM (
```

ARGUMENTS OF ENTRY: IN			
COMPOR	OPTION	NDIM	NDDL
DIMDEF	DIMCON	NVIMEC	NVIHY, NVITM
NDEFME	NDEFP1	NDEFP2	NDEFT
NCONME	NCONP1	NCONP2	NCONT
YAP1	NBPHA1	YAP2	NBPHA2
DEMECM $\mathbf{u},\underline{\underline{\varepsilon}}(\mathbf{u})$ time less	DEP1M $p_{1,}\nabla p_1$ time less	DEP2M $p_{2,}\nabla p_2$ time less	DETM $T,\nabla T$ time less
DEMECP $\mathbf{u},\underline{\underline{\varepsilon}}(\mathbf{u})$ time more	DEP1P $p_{1,}\nabla p_1$ time more	DEP2P $p_{2,}\nabla p_2$ time more	DETP $T,\nabla T$ time more
COMECM $\underline{\underline{\sigma}}',\sigma_p$ time less	COTM $Q',\mathbf{q}$ time less		
CP11M $m_1^{1,}\mathbf{M}_1^1$ or $m_1^{1,}\mathbf{M}_1^{1,}h_{1m}^1$ time less	CP12M $m_1^{2,}\mathbf{M}_1^2$ or $m_1^{2,}\mathbf{M}_1^{2,}h_{1m}^2$ time less	CP21M $m_2^{1,}\mathbf{M}_2^1$ or $m_2^{1,}\mathbf{M}_2^{1,}h_{2m}^1$ time less	CP21M $m_2^{2,}\mathbf{M}_2^2$ or $m_2^{1,}\mathbf{M}_2^{1,}h_{2m}^1$ time less
VIMEM internal variables méca time less	VIHYM hydro internal variables time less	VITMM internal variables therm time less	
ARGUMENTS OF EXIT: OUT			
COMECP $\underline{\underline{\sigma}}',\sigma_p$ time more	COTP $Q',\mathbf{q}$ time more		
CP11P $m_1^{1,}\mathbf{M}_1^1$ or $m_1^{1,}\mathbf{M}_1^{1,}h_{1m}^1$	CP12P $m_1^{2,}\mathbf{M}_1^2$ or $m_1^{2,}\mathbf{M}_1^{2,}h_{1m}^2$	CP21P $m_2^{1,}\mathbf{M}_2^1$ or $m_2^{1,}\mathbf{M}_2^{1,}h_{2m}^1$	CP21P $m_2^{2,}\mathbf{M}_2^2$ or $m_2^{1,}\mathbf{M}_2^{1,}h_{2m}^1$

time more	time more	time more	time more
VIMEP internal variables méca time more	VIHYP hydro internal variables time more	VITMP internal variables therm time more	
DMECDE $\begin{bmatrix} \frac{\partial \sigma'}{\partial \varepsilon} \\ \frac{\partial \sigma_p}{\partial \varepsilon} \end{bmatrix}$	DMECP1 $\begin{bmatrix} \frac{\partial \sigma'}{\partial p_1} & \frac{\partial \sigma'}{\partial \nabla p_1} \\ \frac{\partial \sigma_p}{\partial p_1} & \frac{\partial \sigma_p}{\partial \nabla p_1} \end{bmatrix}$	DMECP2 $\begin{bmatrix} \frac{\partial \sigma'}{\partial p_2} & \frac{\partial \sigma'}{\partial \nabla p_2} \\ \frac{\partial \sigma_p}{\partial p_2} & \frac{\partial \sigma_p}{\partial \nabla p_2} \end{bmatrix}$	DMECDT $\begin{bmatrix} \frac{\partial \sigma'}{\partial T} & \frac{\partial \sigma'}{\partial \nabla T} \\ \frac{\partial \sigma_p}{\partial T} & \frac{\partial \sigma_p}{\partial \nabla T} \end{bmatrix}$
DP11DE $\begin{bmatrix} \frac{\partial m_1^1}{\partial \varepsilon} \\ \frac{\partial \mathbf{M}_1^1}{\partial \varepsilon} \\ \frac{\partial h_{1m}^1}{\partial \varepsilon} \end{bmatrix}$	DP11P1 $\begin{bmatrix} \frac{\partial m_1^1}{\partial p_1} & \frac{\partial m_1^1}{\partial \nabla p_1} \\ \frac{\partial \mathbf{M}_1^1}{\partial p_1} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla p_1} \\ \frac{\partial h_{1m}^1}{\partial p_1} & \frac{\partial h_{1m}^1}{\partial \nabla p_1} \end{bmatrix}$	DP11P2 $\begin{bmatrix} \frac{\partial m_1^1}{\partial p_2} & \frac{\partial m_1^1}{\partial \nabla p_2} \\ \frac{\partial \mathbf{M}_1^1}{\partial p_2} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla p_2} \\ \frac{\partial h_{1m}^1}{\partial p_2} & \frac{\partial h_{1m}^1}{\partial \nabla p_2} \end{bmatrix}$	DP11DT $\begin{bmatrix} \frac{\partial m_1^1}{\partial T} & \frac{\partial m_1^1}{\partial \nabla T} \\ \frac{\partial \mathbf{M}_1^1}{\partial T} & \frac{\partial \mathbf{M}_1^1}{\partial \nabla T} \\ \frac{\partial h_{1m}^1}{\partial T} & \frac{\partial h_{1m}^1}{\partial \nabla T} \end{bmatrix}$
DP12DE $\begin{bmatrix} \frac{\partial m_1^2}{\partial \varepsilon} \\ \frac{\partial \mathbf{M}_1^2}{\partial \varepsilon} \\ \frac{\partial h_{1m}^2}{\partial \varepsilon} \end{bmatrix}$	DP12P1 $\begin{bmatrix} \frac{\partial m_1^2}{\partial p_1} & \frac{\partial m_1^2}{\partial \nabla p_1} \\ \frac{\partial \mathbf{M}_1^2}{\partial p_1} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla p_1} \\ \frac{\partial h_{1m}^2}{\partial p_1} & \frac{\partial h_{1m}^2}{\partial \nabla p_1} \end{bmatrix}$	DP12P2 $\begin{bmatrix} \frac{\partial m_1^2}{\partial p_2} & \frac{\partial m_1^2}{\partial \nabla p_2} \\ \frac{\partial \mathbf{M}_1^2}{\partial p_2} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla p_2} \\ \frac{\partial h_{1m}^2}{\partial p_2} & \frac{\partial h_{1m}^2}{\partial \nabla p_2} \end{bmatrix}$	DP12DT $\begin{bmatrix} \frac{\partial m_1^2}{\partial T} & \frac{\partial m_1^2}{\partial \nabla T} \\ \frac{\partial \mathbf{M}_1^2}{\partial T} & \frac{\partial \mathbf{M}_1^2}{\partial \nabla T} \\ \frac{\partial h_{1m}^2}{\partial T} & \frac{\partial h_{1m}^2}{\partial \nabla T} \end{bmatrix}$
DP21DE $\begin{bmatrix} \frac{\partial m_2^1}{\partial \varepsilon} \\ \frac{\partial \mathbf{M}_2^1}{\partial \varepsilon} \\ \frac{\partial h_{2m}^1}{\partial \varepsilon} \end{bmatrix}$	DP21P1 $\begin{bmatrix} \frac{\partial m_2^1}{\partial p_1} & \frac{\partial m_2^1}{\partial \nabla p_1} \\ \frac{\partial \mathbf{M}_2^1}{\partial p_1} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla p_1} \\ \frac{\partial h_{2m}^1}{\partial p_1} & \frac{\partial h_{2m}^1}{\partial \nabla p_1} \end{bmatrix}$	DP21P2 $\begin{bmatrix} \frac{\partial m_2^1}{\partial p_2} & \frac{\partial m_2^1}{\partial \nabla p_2} \\ \frac{\partial \mathbf{M}_2^1}{\partial p_2} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla p_2} \\ \frac{\partial h_{2m}^1}{\partial p_2} & \frac{\partial h_{2m}^1}{\partial \nabla p_2} \end{bmatrix}$	DP21DT $\begin{bmatrix} \frac{\partial m_2^1}{\partial T} & \frac{\partial m_2^1}{\partial \nabla T} \\ \frac{\partial \mathbf{M}_2^1}{\partial T} & \frac{\partial \mathbf{M}_2^1}{\partial \nabla T} \\ \frac{\partial h_{2m}^1}{\partial T} & \frac{\partial h_{2m}^1}{\partial \nabla T} \end{bmatrix}$

DP22DE	DP22P1	DP22P2	DP22DT
$\begin{bmatrix} \frac{\partial m_2^2}{\partial \varepsilon} \\ \frac{\partial \mathbf{M}_2^2}{\partial \varepsilon} \\ \frac{\partial h_{2m}^2}{\partial \varepsilon} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial m_2^2}{\partial p_1} & \frac{\partial m_2^2}{\partial \nabla p_1} \\ \frac{\partial \mathbf{M}_2^2}{\partial p_1} & \frac{\partial \mathbf{M}_2^2}{\partial \nabla p_1} \\ \frac{\partial h_{2m}^2}{\partial p_1} & \frac{\partial h_{2m}^2}{\partial \nabla p_1} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial m_2^2}{\partial p_2} & \frac{\partial m_2^2}{\partial \nabla p_2} \\ \frac{\partial \mathbf{M}_2^2}{\partial p_2} & \frac{\partial \mathbf{M}_2^2}{\partial \nabla p_2} \\ \frac{\partial h_{2m}^2}{\partial p_2} & \frac{\partial h_{2m}^2}{\partial \nabla p_2} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial m_2^2}{\partial T} & \frac{\partial m_2^2}{\partial \nabla T} \\ \frac{\partial \mathbf{M}_2^2}{\partial T} & \frac{\partial \mathbf{M}_2^2}{\partial \nabla T} \\ \frac{\partial h_{2m}^2}{\partial T} & \frac{\partial h_{2m}^2}{\partial \nabla T} \end{bmatrix}$
DTDE	DTDP1	DTDP2	DTDT
$\begin{bmatrix} \frac{\partial Q'}{\partial \varepsilon} \\ \frac{\partial \mathbf{q}}{\partial \varepsilon} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial Q'}{\partial p_1} & \frac{\partial Q'}{\partial \nabla p_1} \\ \frac{\partial \mathbf{q}}{\partial p_1} & \frac{\partial \mathbf{q}}{\partial \nabla p_1} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial Q'}{\partial p_2} & \frac{\partial Q'}{\partial \nabla p_2} \\ \frac{\partial \mathbf{q}}{\partial p_2} & \frac{\partial \mathbf{q}}{\partial \nabla p_2} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial Q'}{\partial T} & \frac{\partial Q'}{\partial \nabla T} \\ \frac{\partial \mathbf{q}}{\partial T} & \frac{\partial \mathbf{q}}{\partial \nabla T} \end{bmatrix}$

```
              )

REAL*8
       DEMECM (NDEFME), DEP1M (NDEFP1), DEP2M (NDEFP2), DETM (NDEFT)
       DEMECP (NDEFME), DEP1P (NDEFP1), DEP2P (NDEFP2), DETP (NDEFT)
       COMECM (NCONME), CP11M (NCONP1), CP21M (NCONP2), COTM (NCONT)

       VIMEM (NVIMEC), VIHYM (NVIHY), VITMM (NVITM)

       COMECP (NCONME), CP11P (NCONP1), CP21P (NCONP2), COTP (NCONT)
       VIMEP (NVIMEC), VIHYP (NVIHY), VITMP (NVITM)

       DMECDE (NCONME, NDEFME), DMECP1 (NCONME, NDEFP1),
       DMECP2 (NCONME, NDEFP2), DMECDT (NCONME, NDEFT)
       DP11DE (NCONP1, NDEFME), DP11P1 (NCONP1, NDEFP1),
       DP11P2 (NCONP1, NDEFP2), DP11DT (NCONP1, NDEFT)
       DP21DE (NCONP2, NDEFME), DP21P1 (NCONP2, NDEFP1,
       DP21P2 (NCONP2, NDEFP2, DP21DT (NCONP2, NDEFT)

       DP12DE (NCONP1, NDEFME), DP12P1 (NCONP1, NDEFP1),
       DP12P2 (NCONP1, NDEFP2), DP12DT (NCONP1, NDEFT)
       DP22DE (NCONP2, NDEFME), DP22P1 (NCONP2, NDEFP1,
       DP22P2 (NCONP2, NDEFP2, DP22DT (NCONP2, NDEFT)


       DTDE (NCONT2, NDEFME), DTDP1 (NCONT2, NDEFP1),
       DTDP2 (NCONT2, NDEFP2), DTDT (NCONT2, NDEFT)
```

7 Finite elements in THM

7.1 Attributes in the catalogues

To identify a finite element of type THM in the catalogue phenomenons_modelisation, the following attributes are used:

- Attribute TYPMOD2=' THM' to say that this element allows coupling THM;
- Attribute THER = 'YES'/'NOT' when one of thermics:
- Attribute MECA = 'YES'/'NOT' when one of mechanics:
- Attribute HYDR1 = '0', '1' or '2' according to the number of phases of the first component;
- Attribute HYDR2 = '0', '1' or '2' according to the number of phases of the second component.

Intentionally white left page