Connections 3D – Beam, 2D – Beam

Summary:

This document explains the principle retained in *Code_Aster* to connect a modeling continuous medium 3D or 2D and a modeling beam.

In 3D, this connection results in 6 linear relations connecting displacements of the whole of nodes 3D (3 degrees of freedom per node) dependent with the node of beam with the 6 degrees of freedom of this node.

In 2D, this connection results in 3 linear relations connecting displacements of the whole of nodes 2D (2 degrees of freedom per node) dependent with the node of beam with the 3 degrees of freedom of this node.

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1 Presentation

The order employed to treat the connections between elements of beam and elements 2D or 3D is `AFFE_CHAR_MECA` [U4.44.01], keywords `LIAISON_ELEM` and `OPTION = '3D_POU'` or `'2D_POU'`. One will be able to refer to [R3.03.06] for the connection connection between elements of beam and elements of hull.

2 The connection 3D-beam

2.1 Objectives and excluded solutions

When one wishes to finely analyze part of a slim structure complexes [Figure 2.1-a], one can, to minimize the size of the grid to be handled, to want to represent the structure by a beam “far” from the part being analyzed. The goal of schematization by a beam is to bring boundary conditions realistic to the edges of the part modelled and with a grid in continuous medium 3D. The connection 3D - Beam must thus meet the following requirements:

P1 To be able to transmit the efforts of beam (torque) to the grid 3D

P2 Not to generate “parasitic” constraints (even of stress concentration), because it would then be necessary to place the connection far from the zone to be sufficiently analyzed so that these disturbances are attenuated in the zone of study.

P3 Not to support the conditions kinematics or the static conditions of connection one compared to the other. It must be equivalent to bring back a torque of effort or displacement to the limits of the field 3D.

P4 To admit unspecified behaviors on both sides of connection (elasticity, plasticity…) and to also allow a dynamic analysis.

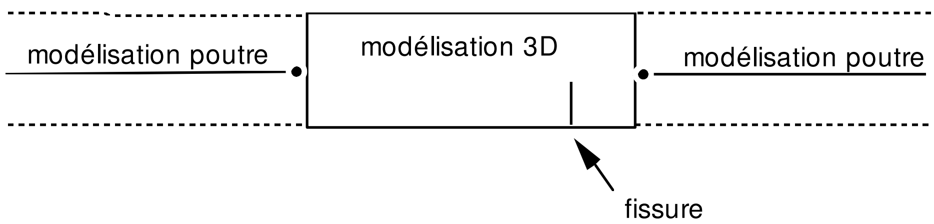


Figure 2.1-a

If these goals are achieved one will be able to also use the rules of connection to deal with the problem of the embedding of a beam in a solid mass 3D. However the distribution of the constraints in the solid mass around embedding will remain rather coarse and will have to be used with precaution. It is preferable to net the connection in 3D then to prolong the starter of the grid 3D of the section of beam by one of the elements of beam with connection 3D/Poutre [Figure 2.1-b].

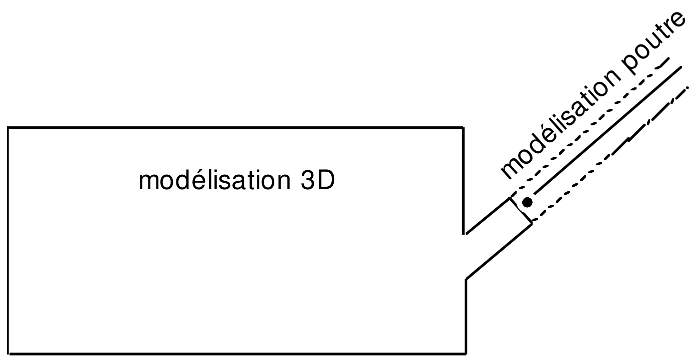


Figure 2.1-b

Within sight of objectives 1 to 4, one can eliminate two current techniques of connection right now:

1) the first which brings back all the connection to the treatment of conditions of connections between the points in opposite to the intersection of the neutral axis of the beam and the solid 3D. Except the difficulty in correctly defining the "specific" rotation of the material point pertaining to the solid 3D, one concentrates the efforts (concentrated reaction, couple) in this point and one breaks kinematic/static symmetry by privileging a particular kinematics.
2) the second solution which completely imposes a displacement of beam (NAVIER - BERNOULLI) on the points of the solid mass 3D being with the intersection of the solid 3D and the section of the beam. In elasticity, it is known that the assumption of indeformability of the sections in their plan is only one approximation. Correct from the energy point of view for the beam, it leads to stress concentrations in the vicinity of the limits of the section of junction for the solid 3D.

Note:

It goes without saying that all that is presented here is valid only on the assumption of the small disturbances (small displacements).

2.2 Orientation

We will leave the machine elements of the connection:

- the field of vector forced $\boldsymbol{\sigma}.n$ defined on the trail of the section S beam on the solid mass 3D, n being the normal with the plan of S,
- and the field of displacement $\mathbf{u}^{\mathbf{3D}}$ defined on this same field,

for the three-dimensional solid, like:

- the torque $\mathbf{T}$ efforts of beam in the geometrical centre of inertia G of S ,
- and the torque $\mathbf{D}$ displacements of beam in this same point,

for the beam.

These mechanical magnitudes are connected by:

- conditions of kinematic continuity,
- equilibrium conditions of the connection.

The first conditions are the conditions of connections to impose in an approach “displacement”, the seconds result from the weak formulation of balance via the virtual work of the actions of contact between beam and solid mass (which is not other than the expression of the “principle” of the action and the reaction writes for the interface S). On surface S , one has indeed for any virtual displacement $(\mathbf{v}, T, \mathbf{\Omega})$ licit:

$$\int s.n.v\,dS = \mathrm{F}.\mathrm{T} + \mathrm{M}.\mathbf{\Omega} \qquad \text{éq 2.2-1}$$

where:

- T et $\mathbf{\Omega}$ are respectively the translation and the infinitesimal rotation of the beam: $\mathbf{D}=(\mathrm{T}, \mathbf{\Omega})$
- F et M are respectively the resultant and the moment in the beam at the point of connection: $\mathbf{T}=(\mathrm{F}, \mathrm{M})$

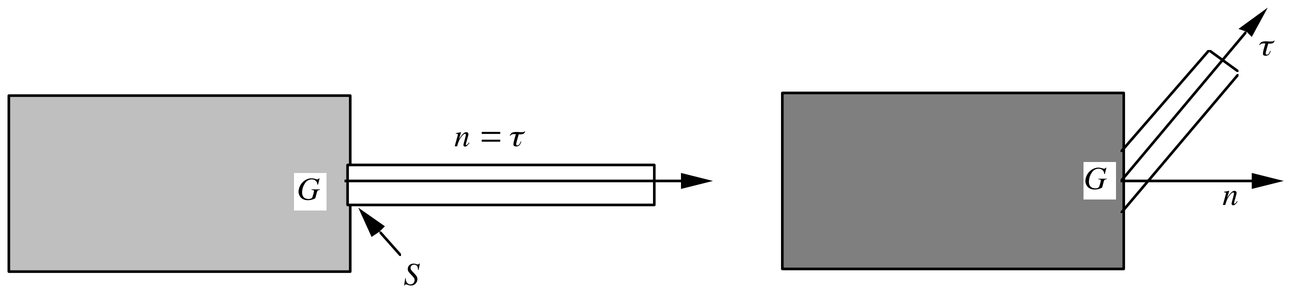
The first member of this equality will provide the scalar product thanks to which one will define the “component beam” of a field of displacement 3D defined on S . By using this scalar product, one will ensure the symmetry of the approach between conditions kinematics and statics of connection (P3) as well as the possibility of treating unspecified behaviors on both sides of connection (P4) since no aspect of behavior appears in the equality of balance used.

The approach:

One will break up the field of displacement 3D into a part “beam” and a “complementary” part. This will lead us to rather naturally define the conditions of kinematic connection between beam and solid 3D like the equality of the displacement (torque) of beam and of the beam part of the field of displacement 3D [(§ 2.3)]. Once this made, the equality [éq 2.2-1] will enable us to interpret in static term the conditions of connection and to thus reach the conditions of static connection [(§2.4)].

2.3 Decomposition of displacement 3D on the interface

The junction enters the three-dimensional solid B and the beam of section S is supposed to be plane and of normal n parallel with the tangent τ with the beam at the point of contact G , geometrical centre of inertia of the section S [Figure 2.3-a].



(a) Normale au solide = tangente à la poutre (b) Normale au solide ° tangente à la pout

Figure 2.3-a

One thus excludes the case (b) where the beam “does not leave” by perpendicular to surface the solid. It should well be understood that this restriction is necessary to the coherence of the connection such as it is considered here since the theory of the beams knows only cuts normal with average fibre: the equilibrium condition [éq 2.2-1] does not have any direction if S is not the cross-section of the beam. If this condition is violated, one will be able to modify the grid to carry out it as diagram Ci - indicates it below.

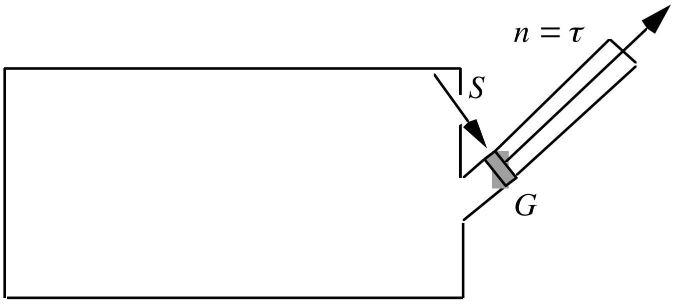
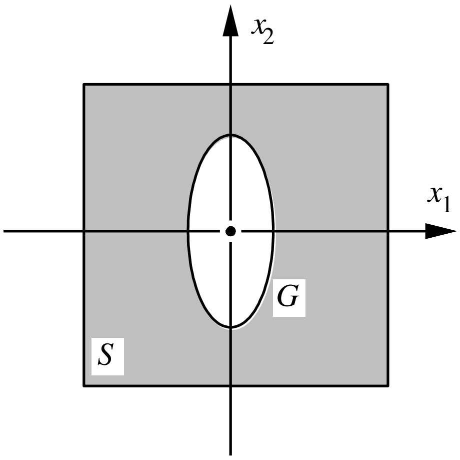


Figure 2.3-b

One notes:



- $(G, \mathbf{e}_1 \mathbf{e}_2)$ one **principal reference mark of geometrical inertia** of S having for origin the centre of inertia G, and (x_1, x_2) associated coordinates,
- n or $\mathbf{e}_3$ the normal with the plan S, outgoing with the solid mass 3D,
- $\boldsymbol{\varepsilon}^{\alpha\beta 3}=(\mathbf{e}_\alpha, \mathbf{e}_\beta, \mathbf{e}_3)$ the alternate shape of the mixed product of the basic vectors,

  finally $\mathbf{I}$ the geometrical tensor of inertia of S (diagonal in the reference mark $(\mathbf{e}_1 \mathbf{e}_2)$) and $A=|S|$ the surface of the section S .

Let us recall that the tensor of inertia $\mathbf{I}$ can be defined in an equivalent way by a linear application (mixed representative):

$$\mathbf{I}(\mathbf{U})=\int_S \mathbf{GM}(x)\wedge(\mathbf{U}\wedge\mathbf{GM}(x))\,dx$$

or a symmetrical bilinear application (covariant representative):

$$\mathbf{I}(\mathbf{U},\mathbf{V})=\int_S (\mathbf{U}\wedge\mathbf{GM}(x)).(\mathbf{V}\wedge\mathbf{GM}(x))\,dx$$

These two expressions will be useful, in the reference mark $(G, \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3)$ the matrix representative of $\mathbf{I}$ is:

$$[\mathbf{I}]=\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_1+I_2 \end{bmatrix}$$

with I_α geometrical moment of inertia of S compared to the axis $(G, \mathbf{e}_\alpha)$. By convention the Greek indices take values 1 or 2.

Useful space for the fields of displacements and vectors forced definite on S is $V=L^2(S)^3$. Space is introduced $\mathbf{T}$ fields associated with a torque (defined by two vectors):

$$\mathbf{T}=\{\mathbf{v}\in V/\exists(\mathrm{T},\mathbf{\Omega})\text{ tel que } \mathbf{v}(M)=\mathrm{T}+\mathbf{\Omega}\wedge\mathbf{GM}\} \quad \text{éq 2.3-1}$$

For the fields of displacement of S, T is the translation of the section (or point G), $\mathbf{\Omega}$ infinitesimal rotation and fields $\mathbf{v}$ are displacements preserving the section S plane and not deformed (Assumptions of NAVIER-BERNOULLI).

For the fields of vectors forced, $|S|\mathrm{T}$ is the resultant F actions applied to S, and $\mathbf{I}(\mathbf{\Omega})$ is the resulting moment M in G. Fields $\mathbf{v}$ correspond then to distributions of constraints closely connected in the section. Indeed, one a:

$$\mathrm{F}(\sigma)\equiv\int_S s.n\,dS=\int_S \mathrm{T}\,dS=|S|\mathrm{T}$$
$$\mathrm{M}(\sigma)\equiv\int_S \mathbf{GM}(x)\wedge\boldsymbol{\sigma}.n\,dS=\int_S \mathbf{GM}(x)\wedge(\Omega\wedge\mathbf{GM})\,dS=\mathbf{I}(\mathbf{\Omega})$$

One used the fact here that G is geometrical centre of inertia thus: $\int_S x_\alpha dS=0$. The vectorial subspace $\mathbf{T}$ being of finished size (equal to 6) has additional orthogonal for the definite scalar product on V :

$$\mathbf{T}^\perp=\{\mathbf{v}\in V/\int_S \mathbf{v}.\mathbf{w}\,dS=0\;\forall\;\mathbf{w}\in\mathbf{T}\} \quad \text{éq 2.3-2}$$

Maybe, in a more explicit way:

$$\mathbf{T}^\perp=\{\mathbf{v}\in V/\int_S \mathbf{v}\,dS=0\text{ et }\int_S \mathbf{GM}\wedge\mathbf{v}\,dS=0\} \quad \text{éq 2.3-3}$$

Any field of V all in all breaks up in a single way of an element of $\mathbf{T}$ and of an element of $\mathbf{T}^\perp$.

$$\mathbf{u}=\mathbf{u}^\mathbf{p}+\mathbf{u}^\mathbf{s} \quad \mathbf{u}^\mathbf{p}\in\mathbf{T}, \quad \mathbf{u}^\mathbf{s}\in\mathbf{T}^\perp \quad \text{éq 2.3-4}$$

One has moreover the following property:

For any couple of field 3D $(\mathbf{u},\mathbf{v})$ defined on S ,

$$\begin{array}{l}\mathbf{u}=\mathbf{u}^{\mathbf{p}}+\mathbf{u}^{\mathbf{s}}\\ \mathbf{v}=\mathbf{v}^{\mathbf{p}}+\mathbf{v}^{\mathbf{s}}\end{array}\Rightarrow\int_S\mathbf{v}.\mathbf{w}\,dS=\int_S\mathbf{v}^{\mathbf{p}}.\mathbf{w}^{\mathbf{p}}dS+\int_S\mathbf{v}^{\mathbf{s}}.\mathbf{w}^{\mathbf{s}}dS \qquad \text{éq 2.3-5}$$

The following definition is thus natural:

Definition:

One calls component of displacement of beam of a field u defined on the section the component u^p of u on the subspace.

The calculation of the beam part of a field 3D $\mathbf{u}$ take place by using the property of orthogonal projection since $\mathbf{T}$ and $\mathbf{T}^{\perp}$ are orthogonal by definition.

If one notes $\mathbf{u}^{\mathbf{p}}=\mathrm{T}_{\mathrm{u}}+\boldsymbol{\Omega}_{\mathbf{u}}\wedge\mathbf{GM}$, then:

$$\left(\mathrm{T}_{\mathrm{u}},\boldsymbol{\Omega}_{\mathbf{u}}\right)=\underset{\left(\mathrm{T},\boldsymbol{\Omega}\right)}{\mathrm{Argmin}}\int_S\left(\mathbf{u}-\mathrm{T}-\boldsymbol{\Omega}\wedge\mathbf{GM}\right)^2 \qquad \text{éq 2.3-6}$$

One will on the way note the interpretation of the component beam of $\mathbf{u}$: it is the field of displacement of beam nearest to $\mathbf{u}$ within the meaning of least squares. The calculation of the minimum leads immediately to the characterization:

$$\mathrm{T}_{\mathrm{u}}=\frac{1}{|S|}\int_S\mathbf{u}\,dS\,,\qquad\boldsymbol{\Omega}_{\mathbf{u}}=\mathbf{I}^{-1}\left(\int_S\mathbf{GM}\wedge\mathbf{u}\,dS\right) \qquad \text{éq 2.3-7}$$

The kinematic condition of connection sought is thus the following linear constraint between the field 3D on S and elements of the torque of displacement of the beam in G :

$$|S|\mathbf{T}-\int_S\mathbf{u}\,dS\,,\qquad\mathbf{I}\left(\boldsymbol{\Omega}\right)-\int_S\mathbf{GM}\wedge\mathbf{u}\,dS=0 \qquad \text{éq 2.3-8}$$

2.4 Expression of the static condition of connection

While returning to the weak formulation of the balance of the interface [éq 2.2-1], one can deduce from them the requirements and sufficient from static connection. Indeed, one a:

$$\int_S\sigma.n.\mathbf{v}\,dS=\mathbf{R}.\mathbf{T}_{\mathbf{v}}+\mathbf{M}.\boldsymbol{\Omega}_{\mathbf{v}}\qquad\forall\,\mathbf{v}\in V \qquad \text{éq 2.4-1}$$

Thanks to the expressions [éq 2.3-7] and the decomposition of space V , and with the property [éq 2.3 - 5], there are immediately the three equations:

$$\begin{aligned} &\mathrm{F}=\int_S \sigma . n\, dS \\ &\mathrm{M}=\int_S \mathbf{GM}(x)\wedge\sigma . n\, dS \\ &(\sigma . n)^{\sigma}=0 \quad \text{ou de mani\`ere \'equivalente} \quad \int_S \sigma . n . \mathbf{v}\, dS=0 \quad \forall\, \mathbf{v}\in T^{\perp} \end{aligned} \qquad \text{éq 2.4-2}$$

The conditions of static connection are thus:

- transmission of the torque of the efforts of beam, (satisfied the P1 property),
- nullity of the complementary part (“not beam”) of the field of vector forced 3D on the section of the solid 3D (satisfied the P2 property).

One will also notice static and kinematic symmetry (P3 property) since the conditions of connection are also interpreted like:

- equality within the meaning of least squares between displacement 3D and the displacement of the beam,
- equality within the meaning of least squares between the field of vector forced and the end cells of the torque of the efforts of beam.

2.5 Establishment of the method of connection

For each connection, the user must define:

S: the trace of the section of the beam on the solid mass 3D: it does it by the keywords `MAILLE_1` and/or `GROUP_MA_1` ; i.e. it gives the list of the meshs (lma) surface (affected of elements “edge” of modeling 3D) which represents this section geometrically.

P: a node (keyword `NOEUD_1` or `GROUP_NO_1`) carrying the 6 classical degrees of freedom of beam: DX , DY , DZ , DRX , DRY , DRZ

Note:

- *the node P can be a node of element of beam or discrete element,*
- *the list of the meshs lma must represent **exactly** the section of the beam. It is an important constraint for the grid.*

For each node, the program calculates the coefficients of the 6 linear relations [éq 2.3-8] which connect:

- 6 degrees of freedom of the node P ,
- with the degrees of freedom of **all** nodes of lma .

These linear relations will be dualisées, like all the linear relations resulting for example from the keyword `LIAISON_DDL` of `AFFE_CHAR_MECA`.

The calculation of the coefficients of the linear relations is done in several stages:

* calculation of elementary quantities on the elements of lma : (OPTION: CARA_SECT_POUT3)
    * surface = $\int_{elt} 1 ; \int_{elt} x ; \int_{elt} y ; \int_{elt} x2 ; \ldots$
* summation of these quantities on (S) from where the calculation of:
    * $A=|S|$
    * position of G
    * tensor of inertia $\mathbf{\Omega}$
* knowing G, elementary calculation on the elements of lma of: (OPTION: CARA_SECT_POUT4)

$$\int_{elt} Ni ; \int_{elt} xNi ; \int_{elt} yNi ; \int_{elt} \mathrm{zNi} \quad \text{où :} \quad \begin{array}{l} \mathbf{GM}=\{x, y, z\} \\ Ni=\text{fonctions de forme de l'élé}\square\text{ment} \end{array}$$

* "assembly" of the terms calculated above to obtain of each node of lma, coefficients of the terms of the linear relations.

3 The connection 2D-beam

3.1 Objective

As for the connection 3D/poutre, the objective is to be able to represent part of a slim structure complexes ([Figure 3.1-a]) by a beam of which the part to be analyzed is relatively "far". The goal of schematization by a beam is to bring boundary conditions realistic to the edges of the part modelled and with a grid in continuous medium 2D. These boundary conditions can be brought by a beam, but also by discrete elements. The connection 2D/poutre must thus meet the following requirements:

P1 To be able to transmit the efforts of beam to the grid 2D

P2 Not to generate "parasitic" constraints (even of stress concentration), because it would then be necessary to place the connection far from the zone to be sufficiently analyzed so that these disturbances are attenuated in the zone of study.

P3 Not to support the conditions kinematics or the static conditions of connection one compared to the other. It must be equivalent to bring back a torque of effort or displacement to the limits of the 2D field.

P4 To admit unspecified behaviors on both sides of connection (elasticity, plasticity…) and to also allow a dynamic analysis.

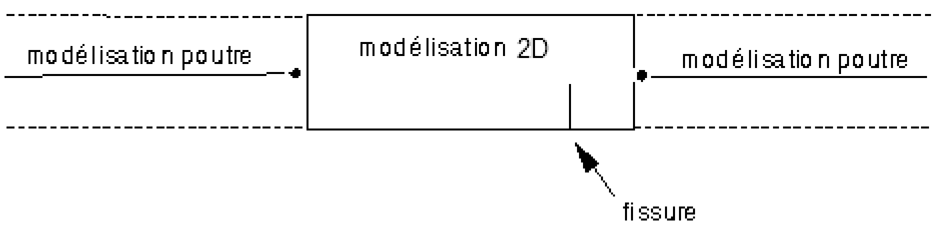


Figure 3.1-a

The approach:

As for the connection 3D/poutre, one breaks up the field of 2D displacement into a part "beam" and a "complementary" part. This leads us to define the conditions of kinematic connection between the beam and the structure 2D like the equality of the displacement of beam and the beam part of the field of 2D displacement. Once this made, one then interprets in static term the conditions of connection and one to reach the conditions of static connection thus.

The reader is invited to consult the paragraph 2 (The connection 3D-beam) who describes the method of the approach explicitly opposite. He will easily establish the link with the case 2D.

3.2 Establishment of the method of connection

For each connection, the user must define:

S: The edge of surface 2D: it is done by the keywords `MAILLE_1` and/or `GROUP_MA_1` ; i.e. it gives the list of the meshs (lma) linear (affected of elements "edge" of modeling 2D) which represents this section geometrically.

P: a node (keyword `NOEUD_1` or `GROUP_NO_1`) carrying the 3 classical degrees of freedom of beam: DX , DY , DRZ

Note:

- *the node P can be a node of element of beam or discrete element,*
- *the list of the meshs lma must represent the section of the beam.*

For each node, the program calculates the coefficients of the 3 linear relations [éq 2.3-8] which connect:

- 3 degrees of freedom of the node P ,
- with the degrees of freedom of **all** nodes of lma .

These linear relations will be dualisées, like all the linear relations resulting for example from the keyword `LIAISON_DDL` of `AFFE_CHAR_MECA`.

The calculation of the coefficients of the linear relations is done in several stages:

- calculation of elementary quantities on the elements of lma : (`OPTION: CARA_SECT_POUT3`)
  - $\int_{elt} 1 ; \int_{elt} x ; \int_{elt} y ; \int_{elt} x^2 ; \int_{elt} y^2$
- summation of these quantities on the edge (S) from where the calculation of:
  - $A=|S|$
  - position of G
  - tensor of inertia $\mathbf{\Omega}$
- knowing G , elementary calculation on the elements of lma of: (`OPTION: CARA_SECT_POUT4`)

$$\int_{elt} Ni ; \int_{elt} xNi ; \int_{elt} yNi ; \quad \text{où :} \quad \begin{array}{l} \mathbf{GM}=\{x , y\} \\ Ni=\text{fonctions de forme de l'élément} \end{array}$$

- "assembly" of the terms calculated above to obtain of each node of lma , coefficients of the terms of the linear relations.

4 Which uses can one make this modeling?

In addition to the two aimed uses to [§2] [Figure 2.1-a] and [Figure 2.1-b], this connection can also be used for:

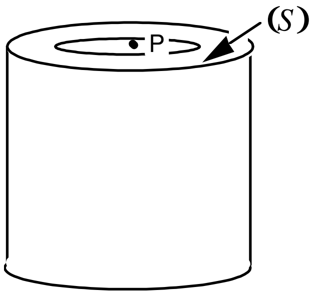
- to apply a torque of efforts to a known surface of a modeling 3D:

  For that, the user defines the surface of load application (lma), it "connects it" with a node (P) of discrete element (`DIS_TR_N`) **without rigidity** then it applies the torque wanted to this node (`FORCE_NODALE`).

  In this way, the torque is applied in "softness", without generating secondary stresses to surface.

- "to retain" a structure without too much the encaster:

  For example, if there is with a grid in 3D a pipe and that one wants to prevent his movements of solid body



one connects (S) with P then one blocks the 6 degrees of freedom of P .

The structure being then retained, without (S) that is to say embedded. In particular, the section (S) can ovalize itself.

5 Bibliography

1) S. ANDRIEUX: "Connections Beam 3D, Hull 3D and generalizations". Note EDF/DER/IMA/MMN HI-70/97/001/0, 1997.

2) Doc. R3.03.06. Connection hull-beam.

6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
3	*J. PELLET (EDF/IMA/MMN)*	Initial text