Connection hull-beam

Summary:

One describes here the connection hull-beam, which make it possible to connect two parts of grid, one constituted by elements of beams (or of a discrete element), and the other with a grid one in elements of hulls (to represent phenomena except kinematics of beam). This development thus functions under assumptions translating that it is the same kinematics of beam which is transmitted between the two grids, on both sides of the connection. It results in 6 linear relations connecting displacements of the whole of the nodes of the edge of the hull with the 6 degrees of freedom of the node end of the beam.

Contents

1 Assumptions and applications

1.1 Assumptions and limitations

Here the connection hull-beam is described, which is used to connect two grids, one comprising of the elements of hulls (or plates), the other comprising of the elements of beams. This functionality makes it possible to model a slim structure in two parts: a part with a grid with classical elements of beams, representing a kinematics and a behavior of beams, and the other part with a grid in elements of hulls, to reveal other phenomena (ovalization, swelling, localised plasticity).

The following assumptions however are made:

- the transverse sectional surface of the end of the grid of hulls is identical to the right sectional surface of the element of beam which corresponds to him,
- the centres of gravity are identical,
- the sections are plane and coplanar,
- the normal with the section of hulls is confused with the axis of the beam.

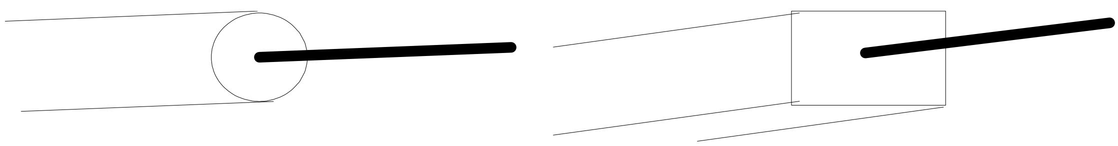
Limitations:

- one does not take account in the connection of the ovalization of the cross-sections,
- account of warping is not taken.

1.2 Applications concerned:

1.2.1 Modeling of pipings

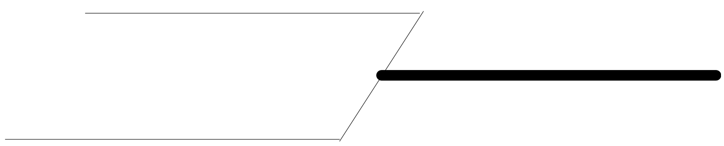
One of the major applications relates to pipings. The bent parts or prickings are then with a grid in hulls, which makes it possible to reveal an ovalization, a local elastoplastic behavior or a swelling in the event of internal pressure. This connection does not transmit the ovalization of the pipes since this one is not modelled in the elements of beams. With this intention, it is necessary to use the connection hull-pipe or to net a sufficient length of right piping in elements of hulls so that ovalization on the level of the connection is negligible.



Circular piping of section (or rectangular…) with a grid in hull then in beam.

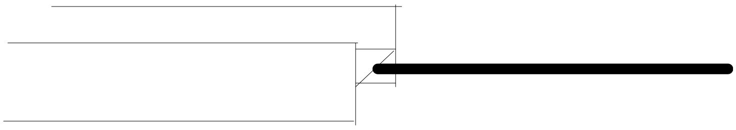
1.2.2 Connection plates beam

Connection plate-beam (mean rectangular section).



1.2.3 Beam with symmetrical profile

Beam with symmetrical profile with a grid partly in hulls.

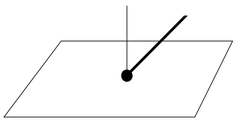


1.2.4 Application of a loading or boundary conditions of type “beam”

At the end of a slim structure with a grid in hulls, it is often useful to impose either a loading of type “beam” i.e. a torque of efforts, or of the boundary conditions (embedding) compatible with the kinematics of beam. One can then connect the transverse section of end of the grid hulls to a discrete element to which one will apply this torque or this embedding.

1.2.5 Application not considered:

This functionality does not make it possible to model ‘’ transverse prickings or orthogonaux' ‘D’ a beam on a plate or a hull:



2 Application of the method of the connection 3D-beam. Equations of connection

The approach is identical to that of the connection beam-3D [R3.03.03]: the connection results in 6 linear relations connecting displacements of the whole of the nodes hull of the section of connection (6 degrees of freedom per node, compared to 3 degrees of freedom per node in 3D) to the 6 degrees of freedom of the node of beam. The section of connection of hull is made up of elements of edge of hulls (segments). On the section connection crosses, one breaks up the field of displacement “hull” into a part “beam” and a “complementary” part. This leads us to define the conditions of kinematic connection between beam and hull like the equality of the displacement (torque distributor or kinematic torque) of beam and of the beam part of the field of hull displacement

As in [R3.03.03], one introduces space $\mathbf{T}$ fields associated with a kinematic torque (defined by two vectors):

$$\mathbf{T}=\{v\in V/\exists(T,\Omega)\ \text{tel que}\ v(M)=\mathbf{T}+\mathbf{\Omega}\wedge\mathbf{GM}\}$$ éq 2-1

Here, $\mathbf{G}$ represent the centre of gravity of the section of connection (in front of being identical to that of the beam). For the fields of displacement of $\mathbf{T}$, $\mathbf{T}$ is the translation of the section (or point $\mathbf{G}$), $\mathbf{\Omega}$ infinitesimal rotation and fields $\mathbf{v}$ are displacements of the space of acceptable displacements V preserving the section S plane and not deformed there (One uses still the Assumptions of NAVIER-BERNOULLI).

The vectorial subspace $\mathbf{T}$ being of finished size (equal to 6) has additional orthogonal for the definite scalar product on V :

$$\mathbf{T}^{\perp}=\left\{\mathbf{v}\in V/\int_s \mathbf{v}(M).\mathbf{w}(M)\,dS=0 \quad \forall\,\mathbf{w}\in T\right\}$$ éq 2-2

Maybe, in a more explicit way:

$$\mathbf{T}^{\perp}=\left\{\mathbf{v}\in V/\int_s \mathbf{v}(M)\,dS=0 \text{ et } \int_s \mathbf{GM}\wedge\mathbf{v}\,dS=0\right\}$$ éq 2-3

Any field of V all in all breaks up in a single way of an element of $\mathbf{T}$ and of an element of $\mathbf{T}^{\perp}$.

$$\mathbf{u}=\mathbf{u}^p+\mathbf{u}^s \quad \mathbf{u}^p\in\mathbf{T} \quad , \quad \mathbf{u}^s\in\mathbf{T}^{\perp}$$ éq 2-4

One has moreover the following property:

For any couple of field hull $(\mathbf{w},\mathbf{v})$ defined on S ,

$$\begin{array}{l}\mathbf{w}=\mathbf{w}^p+\mathbf{w}^s\\ \mathbf{v}=\mathbf{v}^p+\mathbf{v}^s\end{array} \Rightarrow \int_s \mathbf{v}.\mathbf{w}\,dS=\int_s \mathbf{v}^p.\mathbf{w}^p\,dS+\int_s \mathbf{v}^s.\mathbf{w}^s\,dS$$ éq 2-5

Definition:

> *One calls component of displacement of beam of a field of hull* $\mathbf{u}$ *defined on the section the component* $\mathbf{u}^p$ de $\mathbf{u}$ *on the subspace* $\mathbf{T}$.

The characterization immediately is obtained:

$$\mathbf{T}_{\mathbf{u}}=\frac{1}{|S|}\int_S \mathbf{u}\,dS, \qquad \mathbf{\Omega}_{\mathbf{u}}=\mathbf{I}^{-1}\left(\int_s \mathbf{GM}\wedge\mathbf{u}\,dS\right)$$ éq 2-7

where $|S|$ represent the surface of the section S et $\mathbf{I}$ the geometrical tensor of inertia of surface S , expressed in $\mathbf{G}$.

In other words, one can as say as the calculation of the beam part of a field hull $\mathbf{u}$ take place by using the property of orthogonal projection since $\mathbf{T}$ et $\mathbf{T}^{\perp}$ are orthogonal by definition.

If one notes $\mathbf{u}^p=\mathbf{T}_{\mathbf{u}}+\mathbf{\Omega}_{\mathbf{u}}\wedge\mathbf{GM}$, then:

$$\left(\mathbf{T}_{\mathbf{u}},\mathbf{\Omega}_{\mathbf{u}}\right)=\underset{(\mathbf{T},\Omega)}{\text{Argmin}}\int_S\left(\mathbf{u}-\mathbf{T}-\mathbf{\Omega}\wedge\mathbf{GM}\right)^2$$ éq 2-6

The component beam of $\mathbf{u}$ can thus be interpreted like the field of displacement of beam nearest to $\mathbf{u}$ within the meaning of least squares.

The kinematic condition of connection sought between the field hull on S and elements of the torque of displacement of the beam in $\mathbf{G}$ is given by:

$$|S|\mathbf{T}-\int_S \mathbf{u}\, dS=0 \qquad \mathbf{I}(\boldsymbol{\Omega})-\int_S \mathbf{GM}\wedge\mathbf{u}\, dS=0 \qquad \text{éq 2-8}$$

The equation [éq 2-8] shows that the situation is identical to the case 3D-beam. The linear relations will have the same form. The only difference comes from the integrals on S *(which represents a curve here corresponding to the section of the hull, modelled by elements of edge of hull). Moreover, the field of displacement of hull utilizes degrees of freedom of rotation.*

To translate the equation [éq 2-8] into linear relations, the two integrals should be calculated:

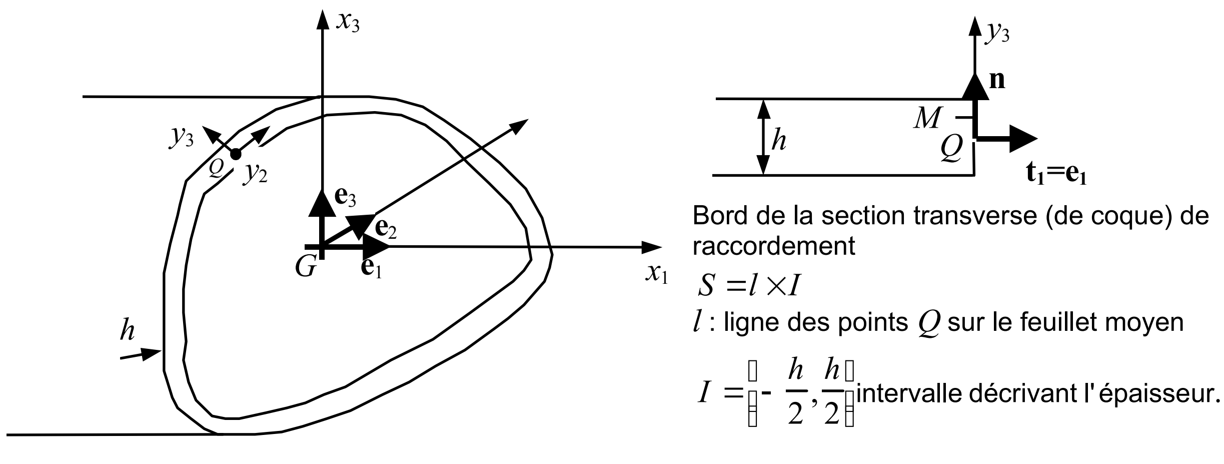
- average displacement: $\int_S \mathbf{u}\, dS$
- average rotation: $\int_S \mathbf{GM}\wedge\mathbf{u}\, dS$

3 Integrals to be calculated. Kinematics of hull.

For each node, the program calculates the coefficients of the 6 linear relations [éq 2-8] which connect:

- 6 degrees of freedom of the node of beam P (geometrically confused with the centre of gravity $\mathbf{G}$ transverse section of the grid hulls)
- with the degrees of freedom of **all** nodes of the list of the meshs of the edge of hull.

These linear relations are dualisées, like all the linear relations resulting, for example, of the keyword `LIAISON_DDL` of `AFFE_CHAR_MECA`. They are built as for the connection 3D-beam starting from the assembly of elementary terms.



Kinematics of hull or linear plate in the thickness:

$$\mathbf{u}(M)=\mathbf{u}(Q)+\left(\boldsymbol{\theta}(Q)\wedge\mathbf{n}\right).\,y_3$$

- $\mathbf{u}$ is the vector displacement of average surface in Q ,
- $\mathbf{n}$ is the normal vector on the average surface of the hull in Q ,
- $\boldsymbol{\theta}$ is the vector rotation in Q normal according to the directions $\mathbf{t}_1$ et $\mathbf{t}_2$ tangent plan
- y_3 is the coordinate in the thickness ($y_3 \in \left]-\frac{h}{2},\frac{h}{2}\right[$).

3.1 Calculation of average displacement on the section S

It is a question of calculating the integral $\int_S \mathbf{u}\,dS$, where $\mathbf{u}$ is the displacement of hull (comprising 6 ddl by node), S is the edge of hull of the transverse section of connection.

Average displacement on the section S is written:

$$\int_s \mathbf{u}(M)\,ds = h\int_l \mathbf{u}(Q)\,ds + \int_l \left(\boldsymbol{\theta}(Q)\wedge\mathbf{n}\right)\left(\int_{-h/2}^{h/2} y_3\,dy_3\right)ds$$

that is to say $\int_s \mathbf{u}(M)\,ds = h\int_l \mathbf{u}(Q)\,ds$

One neglects in this expression the variations of metric in the thickness of the hull.

3.2 Calculation of the average rotation of the section S

$$\begin{aligned}\int_s \mathbf{GM}\wedge\mathbf{u}(M)\,ds &= \int_l \int_{-h/2}^{h/2}\left(\mathbf{GQ}+y_3\,\mathbf{n}(Q)\right)\wedge\left(\mathbf{u}(Q)+\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q)\,.\,y_3\right)ds\,dy_3\\ &= h\int_l \mathbf{GQ}\wedge\mathbf{u}(Q)\,ds + \int_l \mathbf{GQ}\wedge\left(\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q)\right)ds\int_{-h/2}^{h/2} y_3\,dy_3\\ &+\int_l \mathbf{n}(Q)\wedge\mathbf{u}(Q)\left(\int_{-h/2}^{h/2} y_3\,dy_3\right)ds + \int_l \mathbf{n}(Q)\wedge\left(\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q)\right)\int_{-\frac{h}{2}}^{\frac{h}{2}} y_3^2\,dy_3\,.\,ds\end{aligned}$$

that is to say $\int_s \mathbf{GM}\wedge\mathbf{u}(M)\,ds = h\int_l \mathbf{GQ}\wedge\mathbf{u}(Q)\,ds + \frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge\left(\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q)\right)ds$.

3.3 Calculation of the tensor of inertia

The tensor of inertia is defined by [R3.03.03]:

$$\mathbf{I}(\boldsymbol{\Omega}) = \int_s \mathbf{GM}\wedge\left(\boldsymbol{\Omega}\wedge\mathbf{GM}\right)ds$$

while posing: $\mathbf{GM}=\mathbf{GQ}+\mathbf{n}(Q)\,.\,y_3$.

One obtains: $\mathbf{I}(\boldsymbol{\Omega}) = h\int_l \mathbf{GQ}\wedge\left(\boldsymbol{\Omega}\wedge\mathbf{GQ}\right)ds + \frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge\left(\boldsymbol{\Omega}\wedge\mathbf{n}(Q)\right)ds$

3.4 Establishment of the method

The calculation of the coefficients of the linear relations is done in two times:

- calculation of elementary quantities on the elements of the list of the meshs of edges of hulls (mesh of type SEG2 or SEG3):
  - the 9 terms are calculated:
  - $\int_{elt} ds\,;\int_{elt} xds\,;\int_{elt} yds\,;\int_{elt} x^2\,ds\,;\int_{elt} y^2\,ds\,;\int_{elt} z^2\,ds\,;\int_{elt} xyds\,;\int_{elt} xzds\,;\int_{elt} yzds$

    as well as terms resulting from $\mathbf{I}(\boldsymbol{\Omega})$: $\frac{h^3}{12}\int_l \mathbf{n}\wedge\left(\boldsymbol{\Omega}\wedge\mathbf{n}\right)ds$

    what makes it possible to calculate: $\frac{h^3}{12}\int_l\left(n_y^2+n_z^2\right)ds\,,\frac{h^3}{12}\int_l n_x n_y\,ds$, etc…
  - summation of these quantities on (S) from where the calculation of:

    - $A=|S|$
    - position of $\mathbf{G}$
    - tensor of inertia $\mathbf{I}$

- knowing $\mathbf{G}$, elementary calculation on the elements of the list of the meshs of edges of hulls of:

$$\int_{elt} N_i\,ds\,;\int_{elt} xN_i\,ds\,;\int_{elt} yN_i\,ds\,;\int_{elt} zN_i\,ds \quad \text{where} \quad \begin{array}{l}\mathbf{GM}=(x\,,\,y\,,\,z)\\ N_i=\text{fonctions de forme de l'élément}\end{array}$$

(It should simply be noticed that in this case, the integrals on the elements of edge are to be multiplied by the thickness of the hull: $\int_{elt} N_i\,ds=h\int_l N_i\,dl$ where l represent the curvilinear X-coordinate of average fibre of the element of edge of hull).

Moreover, one adds the terms additional coming from: $\frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge(\mathbf{\Omega}\wedge\mathbf{n}(Q))\,ds$

While noting $\mathbf{n}=\begin{vmatrix}n_x\\ n_y\\ n_z\end{vmatrix}$ and $\boldsymbol{\theta}=\begin{vmatrix}\theta_x\\ \theta_y\\ \theta_z\end{vmatrix}$ in the total reference mark one obtains:

$$\mathbf{n}(Q)\wedge(\boldsymbol{\theta}\wedge\mathbf{n}(Q))=\begin{vmatrix}(n_y^2+n_z^2)\theta_x-n_xn_y\theta_y-n_xn_z\theta_z\\ -n_xn_y\theta_x+(n_x^2+n_z^2)\theta_y-n_yn_z\theta_z\\ -n_xn_z\theta_x-n_yn_z\theta_y+(n_x^2+n_y^2)\theta_z\end{vmatrix}=\mathbf{A}\,\boldsymbol{\theta}$$

then:

$$\frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge(\mathbf{\Omega}\wedge\mathbf{n}(Q))\,ds=\frac{h^3}{12}\sum_{el}\left(\int_{el}\mathbf{A}(s)N_j(s)\,ds\right)\theta_j$$

- “assembly” of the terms calculated above to obtain of each node of the meshs of edge, coefficients of the terms of the linear relations.

4 Use

4.1 Modeling

For each connection, the user must define under the keyword factor LIAISON_ELEM of AFFE_CHAR_MECA:

S : the trace of the cross-section of the beam on the hull: it does it by the keywords MAILLE_1 and/or GROUP_MA_1 i.e. it gives the list of the linear meshs (affected of elements "edge" of modeling hull) which represent this section geometrically.

P : a node (keyword NOEUD_1 or GROUP_NO_1) carrying the 6 classical degrees of freedom of beam: DX, DY, DZ, DRX, DRY MARTINI, DRZ

$\mathbf{V}$: the vector defining the axis of the beam, directed hull towards the beam, and defined by its coordinates in the assistance keyword AXE_POUTRE: (v1, v2, v3)

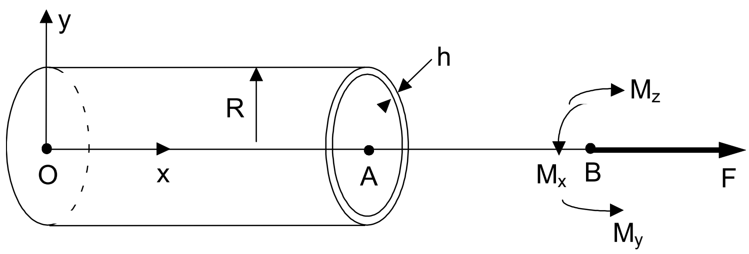
Note:

- *the node P can be a node of element of beam or discrete element,*
- *the list of the meshs of edge of hull, defined by MESH or GROUP_MA must represent **exactly** the cross-section of the beam. It is an important constraint for the grid.*

4.2 Examples and tests

4.2.1 Test SSLX101

It is about a subjected right beam has unit efforts in B (traction, moments bending and torsion). One takes a section of thin tube thickness $\mathrm{h} \ll \mathrm{R}$.

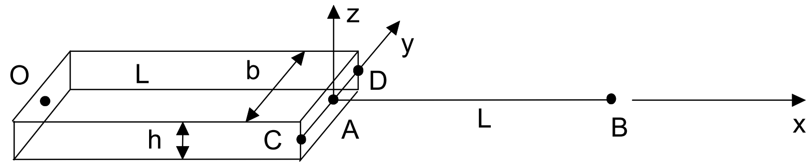


Embedding in O is realized using a connection between the edge of the hull and a specific element located in O . This element is embedded (worthless translations and rotations).

This makes it possible to obtain in the hull a state of stresses very close to a solution "beam": there is no disturbance of the stress field. The solution differs from the analytical solution (solution RDM) of 3%, this being only due to the smoothness of the grid in elements of hulls.

4.2.2 Inflection of a plate

Let us consider a sufficiently long thin section, length $2L$, of width b , thickness h , modelled by an element of hull OA and an element of beam on AB :



- The 1era condition of connection is written:

$$b\,h\,\mathbf{U}(A)=h\int_{CD}\mathbf{U}(y)\,dy$$

the displacement of the point A (pertaining to the beam) is the average of displacements of the edge CD plate.

- 2ème condition of connection is written:

$$\mathbf{I}(\mathbf{\Omega})=h\int_{CD}\mathbf{AQ}\wedge\mathbf{U}(Q)\,ds+\frac{h^3}{12}\int_{CD}\mathbf{\theta}(Q)\,ds$$

In the case of an inflection around y , the only term not no one is: $\frac{h^3}{12}\int_{-\frac{b}{2}}^{b/2}\mathbf{\theta}(y)\,dy$

Indeed, $h\int_{CD}\mathbf{AQ}\wedge\mathbf{U}(Q)\,ds=h\left(\int_{-\frac{b}{2}}^{\frac{b}{2}}U_z\,ydy\right).\mathbf{x}=0$

For an inflection around y , the connection is thus written:

$$I_y\theta_y(A)=\frac{b\,h^3}{12}\theta_y \text{ because } \theta_y \text{ is constant on CD.}$$

This application is put in work in test SSLX100B: 3D_coque_poutre mixture.

5 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6	J.M.PROIX-R&D/AMA	