Following pressure for the elements of hulls voluminal

Summary:

We present in this document, the model used to calculate the loading of type following pressure acting on the average surface of the finite elements of voluminal hulls corresponding to modeling COQUE_3D. Discretization of the loading led to a nodal vector of the external forces and to a nonsymmetrical contribution in the tangent matrix of rigidity. These finite elements objects are evaluated with each iteration of the algorithm of Newton of STAT_NON_LINE.

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1 Introduction

Our analysis leaves the weak formulation of balance under a loading of type following pressure activated by the keyword `TYPE_CHARGE: 'SUIV'` in the order `STAT_NON_LINE` [U4.32.01]. The difference compared to a classical geometrical linear analysis is that the pressure acts on the deformed geometry and either on the initial geometry. This new geometry is obtained starting from the transform of the initial average surface subjected to great displacements and great rotations [R3.07.05]. The notations are inspired by [R3.07.05].

This transform can be parametrized exactly as initial surface by using the reduced coordinates of the associated isoparametric element: the Co-variable or counter-variable reference marks are built in each point of deformed surface. The writing of the virtual work of the pressure with this parameterization is done in the configuration deformed by using the associated isoparametric elements. It results an independence from it from the field of integration with displacements which one uses to express the variation of the virtual work of the efforts external of pressure compared to known as displacements. That has an important advantage compared to the method applied for the pressure which follows the facets of the elements 3D [R3.03.04]. Indeed, this last method, based on a brought up to date Lagrangian formulation, led to nonlinear terms difficult to linearize, coming from the transformation jacobienne compared to the configuration of reference.

The finite elements objects obtained by linearization compared to incrémentaux displacements of the virtual work of the efforts external of pressure are to be reactualized with each iteration of the algorithm of Newton of `STAT_NON_LINE`. We underline the fact that the contribution of the following pressure to the tangent matrix of rigidity is nonsymmetrical, and we point out that the geometrical part of the tangent matrix is already nonsymmetrical [bib2].

2 Kinematics

For the elements of voluminal hull Ω a surface of reference is defined ω , or surfaces average, left (curvilinear coordinates $\xi_1 \xi_2$ for example) and a thickness $h(\xi_1 \xi_2)$ measured according to the normal on the average surface. The position of the points of the hull is given by the curvilinear coordinates (ξ_1, ξ_2) average surface ω and rise ξ_3 compared to this surface.

One points out the great transformation undergone by the hull:

> ω^φ (together of the points P^φ with $\xi_3=0$) is the transform of initial average surface ω (together of the points $P\,à\,\xi_3=0$).

The position of the point P^φ on the deformed configuration can be established according to the position of the initial point p as follows:

$$x_P^\varphi(\xi_1, \xi_2) = x_P(\xi_1, \xi_2) + u_P(\xi_1, \xi_2) \ .$$

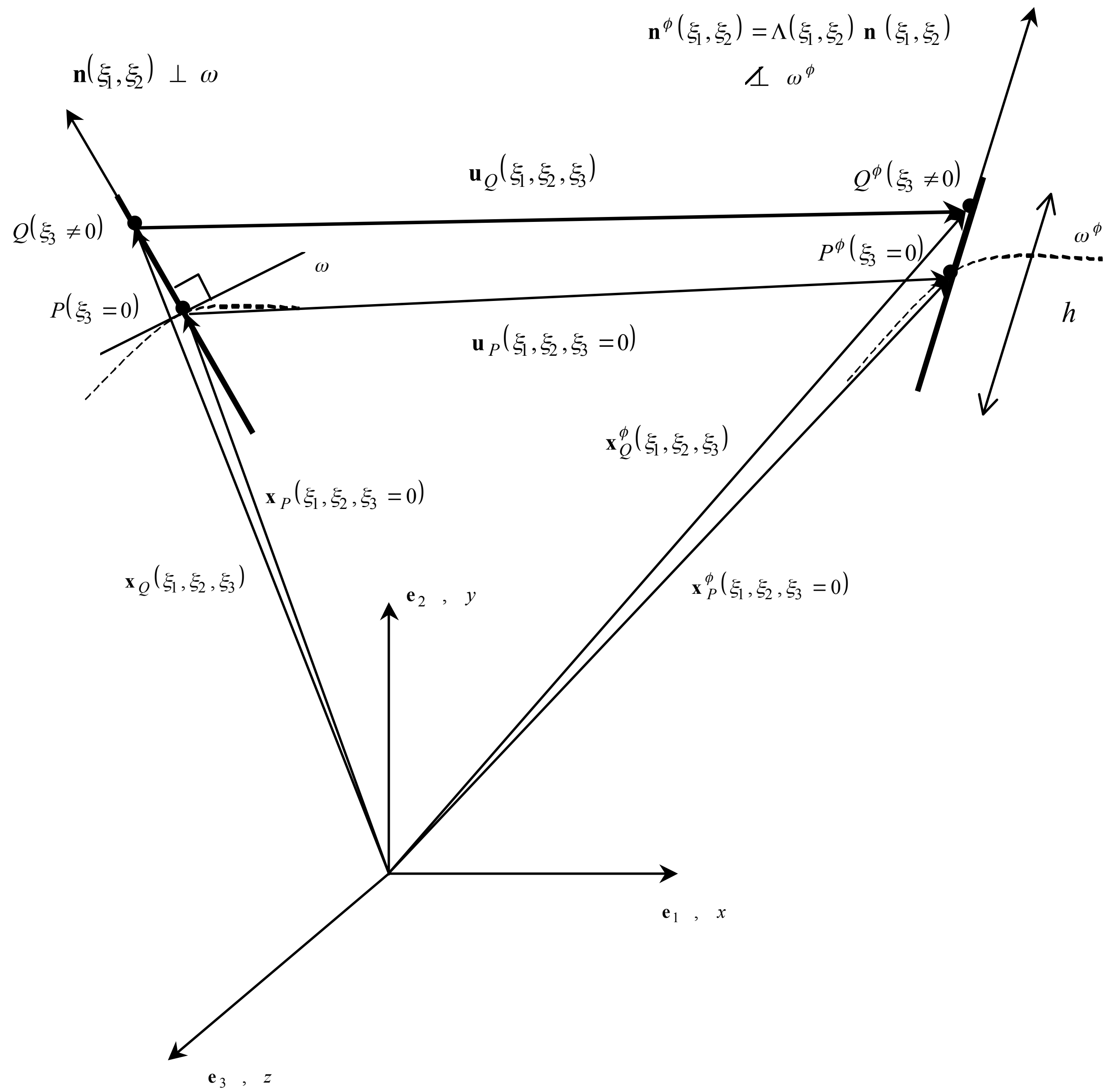

Figure 2-a: Voluminal hull.
Great transformations of an initially normal fibre on the average surface

2.1 Parameterization of the transform of average surface

The transform w^j can be parametrized in a way similar to parameterization of initial surface. Thus one can define the infinitesimal element of tangent vector in w^j :

$$\mathbf{dx}_{\boldsymbol{P}}^{\varphi}(\xi_1,\xi_2) = \frac{\partial x_P^{\varphi}}{\partial \xi_1} d\xi_1 + \frac{\partial x_P^{\varphi}}{\partial x_1} d\xi_2$$
$$\mathbf{dx}_{\boldsymbol{P}}^{\varphi}(\xi_1,\xi_2) = d\xi_1 a_1^{\varphi}(x_1,x_2) + d\xi_2 a_2^{\varphi}(\xi_1,\xi_2)$$

where $\left[a_1^{\varphi}\left(\xi_1,\xi_2\right);a_2^{\varphi}\left(\xi_1,\xi_2\right)\right]$ represent a nonorthogonal natural base $\left(a_1^{\varphi}.a_2^{\varphi}\neq 0\right)$ and not normalized $\left(\|a_1^{\varphi}\|\neq 1;\|a_1^{\varphi}\|\neq 1\right)$ tangent on the surface ω^{φ} . The two basic vectors can be related to displacements via the following formula:

$$a_1^{\varphi}\left(\xi_1,\xi_2\right)=\frac{\partial x_P^{\varphi}}{\partial \xi_1}=\frac{\partial\left(x_p+u_p\right)}{\partial \xi_1}$$
$$a_2^{\varphi}\left(\xi_1,\xi_2\right)=\frac{\partial x_P^{\varphi}}{\partial \xi_2}=\frac{\partial\left(x_p+u_p\right)}{\partial \xi_2}$$

what makes it possible to connect them to the vectors of the natural base related to initial surface ω by the relations:

$$a_1^{\varphi}\left(\xi_1,\xi_2\right)=a_1\left(\xi_1,\xi_2\right)+\frac{\partial u_p}{\partial \xi_1}$$
$$a_2^{\varphi}\left(\xi_1,\xi_2\right)=a_2\left(\xi_1,\xi_2\right)+\frac{\partial u_p}{\partial \xi_2}$$

It is important to note that these vectors are distinct from the vectors obtained by great rotation Λ vectors $a_1\left(\xi_1,\xi_2\right);a_2\left(\xi_1,\xi_2\right)$:

$$a_1^{\varphi}\left(\xi_1,\xi_2\right)\neq\Lambda\left(\xi_1,\xi_2\right)a_1\left(\xi_1,\xi_2\right)$$
$$a_2^{\varphi}\left(\xi_1,\xi_2\right)\neq\Lambda\left(\xi_1,\xi_2\right)a_2\left(\xi_1,\xi_2\right)$$

Indeed, because of deformation due to transverse shearing, the turned vectors are not tangent any more with ω^{φ} . The illustration of that is given by [Figure 3.1-a].

With this parameterization, the infinitesimal vector element of surface which is perpendicular to ω^{φ} can be written:

$$d\,\omega^{\varphi}\left(\xi_1,\xi_2\right)=a_1^{\varphi}\left(\xi_1,\xi_2\right)\times a_2^{\varphi}\left(\xi_1,\xi_2\right)d\xi_1\,d\xi_2$$

3 Variational formulation

3.1 Virtual work

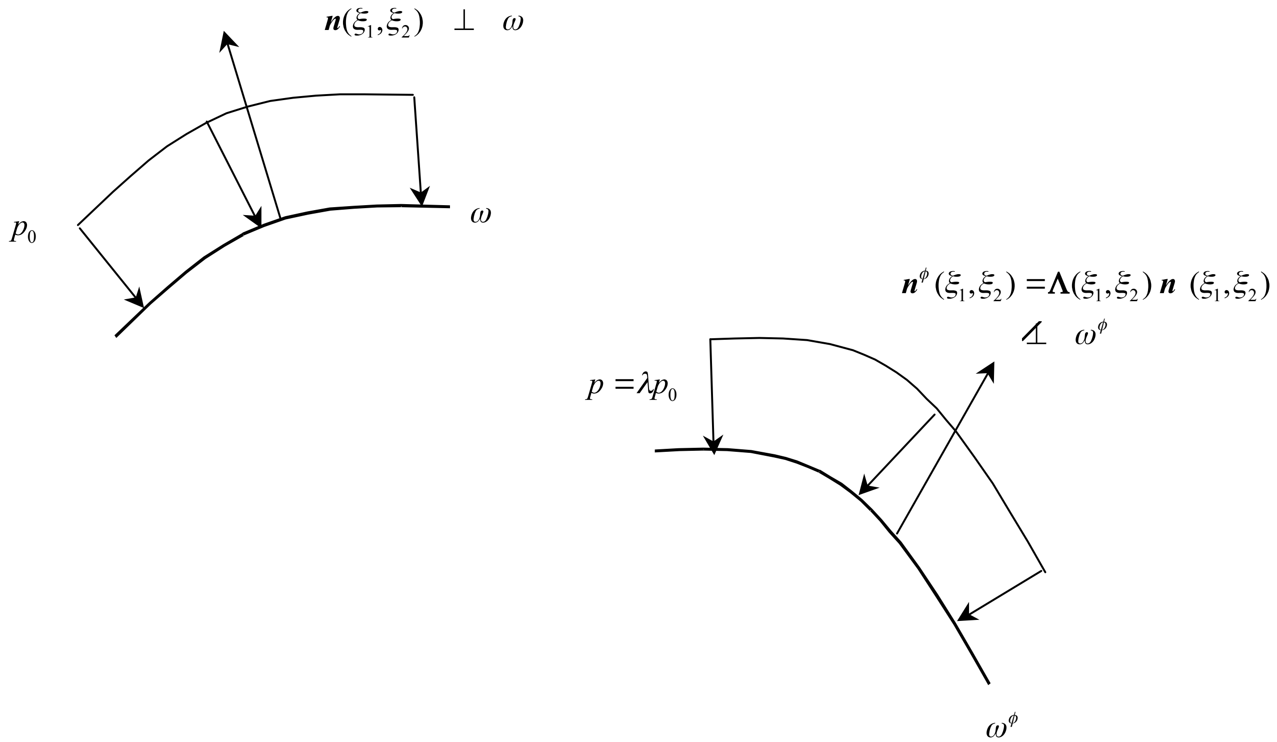

Figure 3.1-a: Voluminal hull.
Following pressure on initial average surface and its transform

The virtual work of a pressure **following** p (i.e. acting on transformed average surface and moving with) can be expressed in the form:

$$\delta\pi_{\substack{pression\\suiveuse}} = -\int_{\omega^\varphi} \delta u_p \,.\, p\, d\omega^\varphi$$

If one uses the isoparametric element of surface corresponding to our modeling of voluminal hull, surface $d\omega^\varphi$ express yourself directly according to the isoparametric coordinates $d\xi_1 d\xi_2$ and one obtains the following simple form of the equation above:

$$\delta\pi_{\substack{pression\\suiveuse}} = -\int_{[-1,+1]\times[-1,+1]} \delta u_p \,.\, p(\xi_1,\xi_2)\, a_1^\varphi(\xi_1,\xi_2)\times a_2^\varphi(\xi_1,\xi_2)\, d\xi_1 d\xi_2$$

3.2 Tangent operator

As the virtual work of the following pressure depends on the current configuration, its linear variation Δ is not worthless and must be taken into account. The tangent operator associated with this virtual work is written with the iteration $(i+1)$ in the form:

$$L\left[\Delta\delta\pi^{(i+1)}_{\substack{pression\\suiveuse}}\right]=\delta\pi^{(i)}_{\substack{pression\\suiveuse}}+\Delta\delta\pi^{(i)}_{\substack{pression\\suiveuse}}$$

where $\Delta\delta\pi^{(i)}_{\substack{pression\\suiveuse}}$ is the increment between two iterations of the virtual work of the following pressure. If the pressure is given in the form:

$$p=\lambda\, p_0$$

λ being the level of load which is fixed lasting the iterations (piloting in load $\Delta\lambda=0$), one can write:

$$\Delta\delta\pi_{\substack{pression\\suiveuse}}=-\int_{[-1,+1]\times[-1,+1]}\delta u_p\,.\,p\left(a_1^{\varphi}\times\Delta a_2^{\varphi}-a_2^{\varphi}\times\Delta a_1^{\varphi}\right)d\xi_1\,d\xi_2$$

The incremental variations of the vectors of the tangent local base to the transform of average surface are given by:

$$\Delta\boldsymbol{a}_1^{\varphi}=\frac{\partial}{\partial\xi_1}\Delta\boldsymbol{u}_P$$
$$\Delta\boldsymbol{a}_2^{\varphi}=\frac{\partial}{\partial\xi_2}\Delta\boldsymbol{u}_P$$

since initial surface average “does not move” not during the iterations what involves $\Delta\boldsymbol{x}_P=0$.

These calculations finally make it possible to establish the expression of the increment of the virtual work of following pressure in the form:

$$\Delta\delta\pi_{\substack{pression\\suiveuse}}=-\int_{[-1,+1]\times[-1,+1]}\delta u_P\cdot P\left(\left[a_1^{\varphi}\times\right]\frac{\partial}{\partial\xi_2}\Delta u_P-\left[a_2^{\varphi}\times\right]\frac{\partial}{\partial\xi_2}\Delta u_P\right)d\xi_1\,d\xi_2$$

where $\left[\boldsymbol{a}_1^{\varphi}\times\right] et \left[\boldsymbol{a}_2^{\varphi}\times\right]$ are respectively the antisymmetric matrices of the tangent vectors $\boldsymbol{a}_1^{\varphi}\, et\, \boldsymbol{a}_2^{\varphi}$ respectively.

Note:

In the reference [bib2], an integration by part is undertaken on the expression above. It is shown that the tangent matrix can be broken up into a symmetrical part resulting from an integration on the field and an antisymmetric part resulting from integration on contour. Il is as shown as the assembly of the antisymmetric parts of the elementary tangent matrices leads to a worthless matrix when the pressure is continuous of a finite element with another, because of existence of a potential associated with work with the pressure in this case there.

4 Discretization

At the points P average surface, the interpolation of virtual displacement is written:

$$\boldsymbol{\delta u}(\xi_{1,}\xi_2)=\sum_{I=1}^{NB1} N_I^{(1)}(\xi_{1,}\xi_2)\begin{pmatrix}\delta u\\ \delta v\\ \delta w\end{pmatrix}_I$$

and the interpolation of incremental displacement between two iterations is written:

$$\boldsymbol{\delta u}(\xi_{1,}\xi_2)=\sum_{I=1}^{NB1} N_I^{(1)}(\xi_{1,}\xi_2)\begin{pmatrix}\Delta u\\ \Delta v\\ \Delta w\end{pmatrix}_I$$

We rewrite the two preceding equations in the matric form:

$$\delta \boldsymbol{u}(\xi_1,\xi_2)=[\boldsymbol{N}]\{\delta \boldsymbol{u}\}^e$$
$$\Delta \boldsymbol{u}(\xi_1,\xi_2)=[\boldsymbol{N}]\{\Delta \boldsymbol{u}\}^e$$

where $[N]$ is the matrix of the functions of form of translation on the average surface, whose expression is:

$$[N]=\left[\ldots\left[N_I^{(1)}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right]_{I=1,\,NB1}\ldots\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}_{NB2}\right]$$

Functions of form $N_I^{(1)}\mathrm{et}\,N_I^{(2)}$ (used thereafter are given in appendix of [R3.07.04]. Nodes $I=1,NB1$ are the nodes tops and the mediums on the sides (for the quadrangle and the triangle). The node $NB2$ is with the barycentre of the element.

The vector $\{\delta \boldsymbol{u}\}^e$ is the nodal vector of virtual displacements given by:

$$\{\delta \boldsymbol{u}\}^e = \begin{pmatrix} . \\ . \\ . \\ \begin{pmatrix} \delta w \\ \delta \theta_x \\ \delta \theta_y \\ \delta \theta_z \end{pmatrix}_I \\ . \\ . \\ . \\ I=1, NB1 \\ \begin{pmatrix} \delta \theta_x \\ \delta \theta_x \\ \delta \theta_x \end{pmatrix}_{NB2} \end{pmatrix}$$

The vector $\{\Delta \boldsymbol{u}\}^e$ is the nodal vector of displacements incremental between two iterations.

$$\{\Delta \boldsymbol{u}\}^e = \begin{pmatrix} . \\ . \\ . \\ \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta \theta_x \\ \Delta \theta_y \\ \Delta \theta_z \end{pmatrix}_I \\ . \\ . \\ . \\ I = 1, NB1 \\ \begin{pmatrix} \Delta \theta_x \\ \Delta \theta_x \\ \Delta \theta_x \end{pmatrix}_{NB2} \end{pmatrix}$$

This discretization also enables us to establish the expression of the derivative of the incremental displacement of average surface compared to the surface isoparametric coordinates in the form:

$$\frac{\partial}{\partial \xi_1} \Delta \boldsymbol{u}(\xi_1, \xi_2) = \left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right] \{\Delta \boldsymbol{u}\}^e$$
$$\frac{\partial}{\partial \xi_2} \Delta \boldsymbol{u}(\xi_1, \xi_2) = \left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right] \{\Delta \boldsymbol{u}\}^e$$

where $\left[\frac{\partial}{\partial \xi_1} \boldsymbol{N}\right] \text{et} \left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right]$ are the matrices derived from the functions of forms of translation on the average surface, whose expressions are:

$$\left[\frac{\partial}{\partial \xi_1} \boldsymbol{N}\right] = \left[\dots \left[\frac{\partial N_I^{(1)}}{\partial \xi_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right]_{I=1, NB1} \dots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{NB2} \right]$$
$$\left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right] = \left[\dots \left[\frac{\partial N_I^{(1)}}{\partial \xi_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right]_{I=1, NB1} \dots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{NB2} \right]$$

Thus one can express the virtual work of the following pressure in the following matric form:

$$\delta \pi_{\substack{pression \\ suiveuse}} = \{\delta \boldsymbol{u}^e\} . \left\{ \boldsymbol{f}^e_{\substack{pression \\ suiveuse}} \right\}$$

with $\left\{ \boldsymbol{f}^e_{\substack{pression \\ suiveuse}} \right\}$ the nodal vector of the external forces which can be expressed in the following way:

$$\left\{ \boldsymbol{f}^e_{\substack{pression \\ suiveuse}} \right\} = \int_{[-1,+1] \times [-1,+1]} [\boldsymbol{N}]^T \left(\boldsymbol{a}_1^{\varphi} \times \boldsymbol{a}_1^{\varphi} \right) d\xi_1 \, d\xi_2$$

It is important to note that with our parameterization of the transform of average surface, the jacobien $\det\left(\left[\boldsymbol{J}(\xi_3=0)\right]\right)$ of this surface is not implied in the calculation of the finite elements objects.

It will be also noted that the pressure is discretized with an isoparametric interpolation of the values with the NB2 nodes:

$$p(\xi_1, \xi_2) = \sum_{I=1}^{NB2} N_I^{(2)}(\xi_1, \xi_2) \, p_I$$

One can also express the increment between two iterations of the virtual work of the following pressure in the matric form:

$$\Delta\,\delta\,\pi_{\substack{pression\\suiveuse}} = -\{\delta\,\boldsymbol{u}^e\}\{\boldsymbol{K}^e_{\boldsymbol{T}\,\substack{pression\\suiveuse}}\}\{\Delta\,\boldsymbol{u}^e\}$$

where $\{\boldsymbol{K}^e_{\boldsymbol{T}\,\substack{pression\\suiveuse}}\}$ is the contribution in the tangent matrix of rigidity of the external forces which can be expressed in the form:

$$\{\boldsymbol{K}^e_{\boldsymbol{T}\,\substack{pression\\suiveuse}}\} = \int_{[-1,+1]\times[-1,+1]} [\boldsymbol{N}]\, p \left[\boldsymbol{a}_1^{\varphi}\times\right]\left[\frac{\partial}{\partial\,\xi_2}\boldsymbol{N}\right] d\,\xi_1\, d\,\xi_2 - \int_{[-1,+1]\times[-1,+1]} [\boldsymbol{N}]\, p \left[\boldsymbol{a}_2^{\varphi}\times\right]\left[\frac{\partial}{\partial\,\xi_1}\boldsymbol{N}\right] d\,\xi_1\, d\,\xi_2$$

Note:

It is noted that the finite elements formulations resulting from this approach do not utilize the degrees of freedom of rotations. The treatment is thus also valid for the facets of the finite elements of three-dimensional elasticity.

5 Bibliography

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9. P. MASSIN and A. LAULUSA: “Digital modeling of the Voluminal Hulls”, handbook of Reference of *Code_Aster* [R3.07.04]

6 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
01/05/00	P.MASSIN EDF: R & D /MMN *AL MIKDAD (SAMTECH)*	Initial text