Linear relations kinematics of type RBE3

Summary:

This document describes the way in which the linear relations kinematics of type RBE3 are calculated.

Contents

1 Introduction

1.1 Principle general

A linear kinematic relation of type RBE3 implies a node says main and several node known as slaves. The relation causes to distribute the torque of efforts seen by the main node on the nodes slaves. The assumptions of construction of the linear constraints impose, for each node slave:

- a distribution of the torques balanced by the distance enters the main node and the node slave running,
- a complementary relative weighting using coefficients indicated by the user.

By informing one coefficient (not no one) for the whole of the nodes, all the nodes slaves are considered with the same weight, and only weighting with the distance will be active.

The distribution is made so that the torques on the implied nodes are with balance.

1.2 Notations

X_M	Coordinates of the main node
X_i	Coordinates of i - ème node slave ($1 \leq i \leq n$)
$\xi_i = X_i - X_M = \begin{bmatrix} \xi_{ix} \\ \xi_{iy} \\ \xi_{iz} \end{bmatrix}$	Relative coordinates of i - ème node slave
ω_i	Weighting coefficient of i - ème node slave
$F_i = \begin{bmatrix} F_{ix} \\ F_{iy} \\ F_{iz} \end{bmatrix}$	Efforts applied to the node i
$M_i = \begin{bmatrix} M_{ix} \\ M_{iy} \\ M_{iz} \end{bmatrix}$	Moments applied to the node i
$u_i = \begin{bmatrix} u_{ix} \\ u_{iy} \\ u_{iz} \end{bmatrix}$	Displacements of the node i
$\theta_i = \begin{bmatrix} \theta_{ix} \\ \theta_{iy} \\ \theta_{iz} \end{bmatrix}$	Rotations of the node
$d_i = \begin{bmatrix} u_i \\ \theta_i \end{bmatrix}$	Degrees of freedom of the movement, potentially carried by the node i
$T_i = \begin{bmatrix} F_i \\ M_i \end{bmatrix}$	Torque of effort to the node i

One will note M^T transposed of the matrix M.

In addition, it is considered that the nodes carry by default the components of displacements and rotation, that is to say six degrees of freedom per node.

2 Definitions

The construction of the relations kinematics require the introduction of the matrices which make it possible to clarify the formulas of change of point of reduction for the various torques. It is pointed out that the torque of efforts of a node slave i transferred onto the main node M is written:

$$T_M = \begin{bmatrix} F_M \\ M_M \end{bmatrix} = \begin{bmatrix} F_i \\ M_i + \xi_i \wedge F_i \end{bmatrix} \quad (1)$$

The matrix thus is introduced S_i :

$$S_i = \begin{bmatrix} 1 & 0 & 0 & 0 & \xi_{iz} & -\xi_{iy} \\ 0 & 1 & 0 & -\xi_{iz} & 0 & \xi_{ix} \\ 0 & 0 & 1 & \xi_{iy} & -\xi_{ix} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

So that one can write:

$$T_M = S_i^T T_i \quad (3)$$

The calculation of the relations requires a setting at the level of components of rotation, so that the relations created are not modified during a scaling of the problem. For that, the following characteristic length is defined:

$$L_c = \frac{\sum_{i=1}^{n} \|\xi_i\|}{n} \quad (4)$$

and matrices (diagonal), associated with the weighting coefficient for i - ème node slave:

$$W_i = \omega_i \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_c^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_c^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_c^2 \end{bmatrix} \quad (5)$$

3 Expression of the relations kinematics

3.1 Obtaining the relations kinematics

The reasoning is carried out on the transmission of the efforts between the main node and the nodes slaves. Initially, it is considered that all the components of the main node and all the components of the nodes slaves are implied in the relation (the section §4 approach the restriction of the relations on certain components).

That is to say T_M the torque the effort of the main node which one seeks to divide into T_i torques of efforts on n nodes slaves. The distribution is done by using the following assumptions:

- torques T_i are resulting from the same linear combination $X\,T_M$ torque with the main node, where X is the matrix associated with the linear combination,
- this linear combination $X\,T_M$ is distributed on the nodes slaves by balancing transport by the distance between the main node and the node slave on the one hand, and a weighting coefficient fixed by the user on the other hand.

Each torque T_i is written then:

$$T_i = W_i S_i X T_M \tag{6}$$

The matrix X linear coefficients is given by ensuring that the torque applied to the main node is identical to the whole of the torques T_i applied to n nodes slaves. X must thus check:

$$T_M = \sum_{i=1}^{n} S_i^T T_i = \sum_{i=1}^{n} S_i^T W_i S_i X T_M \tag{7}$$

While reformulating the problem in a matric way, it comes that X is solution of the following system:

$$\begin{bmatrix} S_1^T & \cdots & S_n^T \end{bmatrix} \begin{bmatrix} W_1 & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & W_n \end{bmatrix} \begin{bmatrix} S_1 \\ \vdots \\ S_n \end{bmatrix} [X] = [I_d] \tag{8}$$

Where I_d is the matrix identity, that is to say:

$$X = (S^T W S)^{-1} \tag{9}$$

While noting S is the assembly of S_i and W is the assembly of W_i .

It is noted that the matrix $S^T W S$ must be invertible. This is ensured by a relevant choice of the degrees of freedom Master and slave by the user. For example, if one seeks to transmit a complete torque of a three-dimensional model to a model beam, it will have to be made sure that the nodes slaves associated with the three-dimensional model are not aligned. If it is the case, the nodes of a three-dimensional model not carrying the degrees of freedom of rotation, one creates a mechanism such as rotation around the axis formed by the alignment of nodes slaves is not blocked.
One notes now $B_i = W_i S_i X$. Relation $T_i = B_i T_M$, one uses the equality of virtual work to generate a constraint on the degrees of freedom starting from the relation on the torques. The work of the torque T_M on the degrees of freedom of the main node d_M must be equal to that of the various torques T_i on each degree of freedom d_i nodes slaves, is:

$$T_M^T d_M = \sum_{i=1}^{n} T_i^T d_i = T_M^T \left(\sum_{i=1}^{n} B_i^T d_i \right) \tag{10}$$

The kinematic relation applied between the main node and the nodes slave is thus written:

$$d_M = \sum_{i=1}^{n} B_i^T d_i \tag{11}$$

Interpretation of kinematics:
By taking again the expression (11), it is noted that one “reversed” the initial problem, since if the initial problem consisted in distributing the torque of effort to the main node on the nodes slaves, in kinematics, that amounts expressing the degrees of freedom d_M main node like linear combination of the degrees of freedom d_i nodes slaves.
By analyzing more closely kinematics associated with the matrix S , it is noted that each column corresponds to a mode of rigid body (overall movement) of the whole of the nodes slaves. The first three columns are the translatory movements (displacement unit in a direction, no one in the two others), and the three last columns are the rotation movements around the main node.
Let us consider the simple case of a uniform weighting $w_i = 1 \quad \forall i$, with a factor of correction L_c presumedly unit. Under these conditions, W is a matrix identity, the relation (11) becomes:

$$d_M = (S^T S)^{-1} S^T \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} \tag{12}$$

The relation (12) is the solution with least square of a problem of minimization which is written:

$$d_M = \underset{d_m}{ArgMin} \left\| S\, d_m - \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} \right\|^2 \quad (13)$$

In other words, the relation (13) seek to determine the best possible linear combination d_M rigid modes of body passing by all the degrees of freedom slaves d_i. When weighting is uniform, the linear constraint to build by the relation (11) thus consist in imposing the movement of average rigid body of the whole of the nodes slaves on the main node. The addition of a weighting w_i on a node led to amplify or decrease the relative influence of this node in the movement of the main node.

The principal interest of standard connection “RBE3” is to avoid on-forcing the model. Contrary to the case of the solid connection, the nodes slaves of a standard relation RBE3 can continue to have relative movements.

3.2 Dimension of the matrices

One notes $NDDLES$ the full number of the degrees of freedom slaves. The matrices which one handles have following dimensions:

- Matrix W (assembly of W_i) : $NDDLES$ lines, $NDDLES$ columns;
- Matrix S (assembly of S_i) : $NDDLES$ lines, six columns;
- Matrix $S^T W S$: six lines, six columns;
- Matrix X : six lines, six columns;
- Matrix B : $NDDLES$ lines, six columns.

4 Restriction of the relations on certain components

4.1 Components on the nodes slaves

To restrict the components on the nodes slaves with the wanted components, only the lines of the matrices S_i corresponding to the desired components are preserved (these matrices become consequently of the rectangular and either square matrices).
Matrices W_i remain square because one removes the lines and the columns corresponding to the degrees of freedom slaves not implied in the linear relation.

4.2 Components on the main node

To restrict the components on the main node with the wanted components, only the lines of the matrices B_i corresponding to the desired components are preserved.