

Connection Harlequin 3D – Beam

Summary:

This document explains the method Harlequin developed in *Code_Aster* to connect a modeling continuous medium 3D and a modeling beam.

In 3D, this connection results in linear relations connecting displacements of the whole of nodes 3D (3 degrees of freedom per node) dependent with the whole of the nodes of beam (6 degrees of freedom per node).

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1 Presentation

The order employed to treat the connections of the type Harlequin between elements of beam and elements 3D is `AFPE_CHAR_MECA [U4.25.01]`, keywords `LIAISON_ELEM` and `OPTION = '3D_POU_ARLEQUIN'`.

2 The connection Harlequin 3D-beam

2.1 Objectives and assumptions

Contrary to the classical adaptive methods, the framework Harlequin makes it possible to superimpose adapted local models, rather than to correct the existing total models. The physical solution is then reconstituted in the zone of superposition like a partition of the fields solutions.

In what follows, the qualifications “end” or “room” and “coarse” or “total” are used here by anticipation of heterogeneities of modelings used (beam 1D and voluminal 3D).

The method Harlequin is a method of coupling with covering based on the combination of models of smoothnesses and/or different modelings. It allows the mixture as well as the connection “in volume” of the formulations of behaviors heterogeneous, and this without forcing a priori constraints on the grids to be restuck¹. It is founded on two principal ideas:

- The connection of under fields via a weak formulation: the introduction of the multipliers of Lagrange into the zone of joining guarantees the coupling of the models, the continuity of the kinematics quantities, as well as the control of the variations of the constraints and the deformations between the coupled zones;
- Distribution of energy between fields and models: with an aim of not twice counting the energy of the total system in the zone of covering, the virtual work associated with the two models is distributed between the under-fields coupled through the zone of joining by the means of weight functions $(\gamma, 1-\gamma)$ who form a partition of the unit (the sum of the two functions is equal to 1) on the whole of the field of study. One notes that for two under-fields Ω_{1D} and Ω_{3D} , the zone of covering is defined by their intersection.

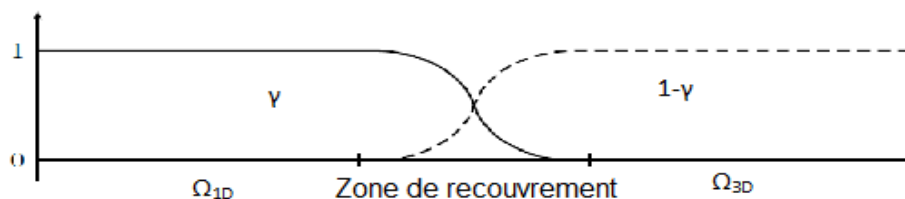


Figure 2.1-a weight functions within the framework Harlequin

This weighting makes it possible to avoid taking into account energy several times in the same zone and thus authorizes a certain freedom with the user for the choice of the prevalent model. Indeed, it makes it possible to put the weight on the model which one wishes to make express.

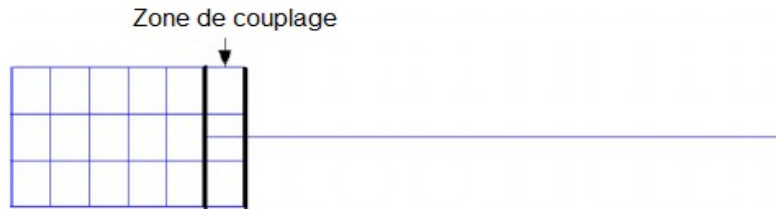
Within the framework Harlequin, as soon as heterogeneous modelings are coupled, it is recommended to discretize the multipliers of Lagrange on the coarse model in space. This makes it possible to avoid the digital phenomenon of locking and, consequently, to authorize more freedom with each model while making it possible the fine model to express its wealth. It results from it that space finite elements of these multipliers of Lagrange is built on the coarse model (model of beam for this case).

¹ In practice, one will ask the user of Code_Aster to provide hierarchical grids.

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The connection 3D-Beam Harlequin, as developed in Code_Aster, must meet a requirement highly important. Indeed, Lbe grids 1D and 3D must be “hierarchically compatible”, with the direction where all the elements 3D are included in the cylindrical space of the element 1D in opposite (not of elements 3D to horse between two elements beam). The only constraint that one imposes to the user thus consists in making an effort during the development of the grid, which is not a bad thing because the final model will be controlled better.



Schematic figure 2.1-b representation of a connection of a mixed model 1D-3D of hierarchical beam cantilever with “grids”

The method Harlequin is used here to connect two models 1D and 3D (operation of junction). Consequently, the zone of coupling corresponds to the zone of joining. The management of weightings to ensure the partition of the unit in the free zone (of the conditions Harlequin) is managed by the user.

As it is about a coupling of the L2 type, it is enough to impose the coefficients de weighting in the definition of materials relative to the various zones free and of joining.

Note:

A general procedure to manage the incompatible grids would be very heavy and expensive to program, it would require a complicated management of the weight functions in the zone of joining as well as an intersection or a cutting in subelements of the grids 1D and 3D to manage the integration of the terms of coupling.

2.2 Notations

One will note:

- Ω the zone of coupling
- \mathbf{u}_{1D} and \mathbf{u}_{3D} fields of displacements corresponding to the models 1D beam and 3D
- λ the multiplier of Lagrange associated with the 1D field
- M the classical matrix of stiffness beam

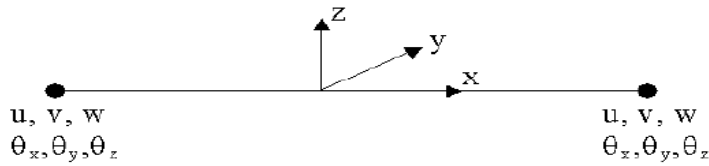
2.3 Expression of the condition of connection Harlequin

The equation of coupling L_2 in continuous medium is written:

$$\int_{\Omega} \lambda : (\mathbf{u}_{1D} - \mathbf{u}_{3D}) d\Omega = \int_{\Omega} \lambda : \mathbf{u}_{1D} d\Omega - \int_{\Omega} \lambda : \mathbf{u}_{3D} d\Omega \quad \text{éq 2.4-1}$$

λ and \mathbf{u}_{1D} have exactly the same type of degrees of freedom and the same type of functions of form because they are defined in the same space of the fields kinematically acceptable. Indeed, for an element 1D of the multiplier of Lagrange an element 1D beam corresponds.

It is supposed, to simplify, that the axis of the beam is according to X.



Schematic figure 2.3-a representation of an element of beam to 6 degrees of freedom

Then, the displacement of type beam is written classically as follows:

$$\begin{cases} u = u(x) + z\theta_y(x) - y\theta_z(x) + 0 \\ v = 0 + 0 + v(x) - (z - z_c)\theta_x(x) \\ w = 0 + w(x) + 0 + (y - y_c)\theta_x(x) \end{cases}$$

The multiplier of Lagrange will be written then according to a kinematics of beam as follows:

$$\begin{cases} \lambda_u = \lambda_u(x) + z\lambda_{\theta_y}(x) - y\lambda_{\theta_z}(x) + 0 \\ \lambda_v = 0 + 0 + \lambda_v(x) - (z - z_c)\lambda_{\theta_x}(x) \\ \lambda_w = 0 + \lambda_w(x) + 0 + (y - y_c)\lambda_{\theta_x}(x) \end{cases}$$

On an element 1D noted E_{1D} the first term of coupling 1D-1D is written in same space 1D:

$$\int_{e_{1D}} \boldsymbol{\lambda} : \mathbf{u}_{1D} d\Omega = [\boldsymbol{\lambda}]^t C_{1D-1D} [\mathbf{u}_{1D}] = [\boldsymbol{\lambda}]^t \mathbf{M} [\mathbf{u}_{1D}] \quad \text{éq 2.4-2}$$

where $[\boldsymbol{\lambda}]$ is the vector of the 12 degrees of freedom of the multiplier of Lagrange and $[\mathbf{u}_{1D}]$ is the vector of the 12 degrees of freedom of the elements beam:

$$\begin{aligned} [\mathbf{u}_{1D}] &= (u^1, v^1, w^1, \theta_x^1, \theta_y^1, \theta_z^1, u^2, v^2, w^2, \theta_x^2, \theta_y^2, \theta_z^2) \\ [\boldsymbol{\lambda}] &= (\lambda_u^1, \lambda_v^1, \lambda_w^1, \lambda_{\theta_x}^1, \lambda_{\theta_y}^1, \lambda_{\theta_z}^1, \lambda_u^2, \lambda_v^2, \lambda_w^2, \lambda_{\theta_x}^2, \lambda_{\theta_y}^2, \lambda_{\theta_z}^2) \end{aligned}$$

In the typical case of the coupling L_2 , the matrix C_{1D-3D} is equal to the classical matrix of mass of an element of beam.

Always on an element 1D noted E_{1D} , the second term of coupling 1D-3D is written in same space 1D:

$$-\int_{e_{1D}} \boldsymbol{\lambda} : \mathbf{u}_{3D} d\Omega = -[\boldsymbol{\lambda}]^t C_{1D-3D} [\mathbf{u}_{3D}] \quad \text{éq 2.4-3}$$

and $[\mathbf{u}_{3D}]$ is the vector of $(3 \times N_{no3D})$ degrees of freedom of the elements 3D (N_{no3D} being the number of nodes of the element 3D):

$$[\mathbf{u}_{3D}] = (u^1, v^1, w^1, \dots, u^{N_{no3D}}, v^{N_{no3D}}, w^{N_{no3D}})$$

The approach of construction of the matrix C_{1D-3D} on the model fine 3D is more delicate than that of the coarse model of Beam. The complexity of this coupling lies in the coexistence of two fields belonging to different spaces. It is obtained by considering all the elements 3D contained in the cylindrical space of the element 1D.

$$C_{1D-3D} = \sum_{k=1}^{N_{el3D}} C_{1D-3D}^k$$

One is reminded that, by construction even grid, all the elements 3D must be included in this space. These elements are determined by a procedure of pairing (graph of relationship) which consists in projecting their barycentres in the geometrical support of the associated element 1D (projection of a point on a segment).

For each element 3D number K (K going of 1 to NR_{el3D}), the matrix is calculated C_{1D-3D}^k in the following way:

$$C_{1D-3D}^k = \sum_{g=1}^{N_{bgauss}} \mathbf{W}^g \mathbf{B}_{1D}^{gt}(\xi_{1D}) \mathbf{G}^g \mathbf{B}_{3D}^g(\xi_{3D}, \eta_{3D}, \zeta_{3D}) \det(\mathbf{J}^g)$$

with G the matrix of passage Element of reference – Element physical, containing of the zeros and the elements of the reverse of the jacobian calculated at the point of gauss G.

The matrix \mathbf{B}_{1D}^g of dimension 3 X are (3 X many nodes of the element 1D). Its terms are calculated at the points of Gauss of the elements 1D.

$$\mathbf{B}_{1D}^g = \begin{bmatrix} N_1^p & y \xi_5 & z \xi_5 & 0 & z \xi_6 & -y \xi_6 & N_2^p & y \xi_7 & z \xi_7 & 0 & z \xi_8 & -y \xi_8 \\ 0 & \xi_1 & 0 & -z N_1^p & 0 & -\xi_2 & 0 & \xi_3 & 0 & -z N_2^p & 0 & -\xi_4 \\ 0 & 0 & \xi_1 & y N_1^p & \xi_2 & 0 & 0 & 0 & \xi_3 & y N_2^p & \xi_4 & 0 \end{bmatrix}$$

where X, there and Z are the local variables element 1D, N_1^p and N_2^p functions of form for traction-compression-torsion, $\xi_{1...8}$ are the functions of the shape of inflection of the beam (cf document [r3.08.01] of Code-Aster).

The matrix \mathbf{B}_{3D}^g of dimension 3 X are (3 X many nodes of the element 3D). For example, the matrix \mathbf{B}_{3D}^g associated with the voluminal element linear tetrahedron (TETRA4) is written:

$$\mathbf{B}_{3D}^g = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix}$$

Variables X, there, Z as well as the functions of form and their derivative are calculated at the point of Gauss G of the element 3D. Thus, for example, one a:

$$\begin{aligned} x &= \langle N \rangle \{ x_n \}, u_x = \langle N \rangle \{ u_n \} \\ y &= \langle N \rangle \{ y_n \}, u_y = \langle N \rangle \{ v_n \} \\ z &= \langle N \rangle \{ z_n \}, u_z = \langle N \rangle \{ w_n \} \end{aligned}$$

where:

- X_N , there N and Z_N are the coordinates of the nodes of the element 3D,
- U_N , v_N and W_N are the degrees of freedom associated with the nodes with the element 3D,
- $\langle N \rangle$ is the vector containing all the functions of form of the element 3D following the order of the nodes 3D (cf document r3.01.01 of Code-Aster).

For each point of Gauss of coordinates $(\xi_{3D}, \eta_{3D}, \zeta_{3D})$ into the element of reference 3D, we introduce his physical representation in the form of the interpolated coordinates $(\bar{X}, \bar{Y}, \bar{Z})$ following:

$$\begin{aligned}\bar{X} &= \sum_{i=1}^{N_{el3D}} N^i x^i(\xi_{3D}, \eta_{3D}, \zeta_{3D}) \\ \bar{Y} &= \sum_{i=1}^{N_{el3D}} N^i y^i(\xi_{3D}, \eta_{3D}, \zeta_{3D}) \\ \bar{Z} &= \sum_{i=1}^{N_{el3D}} N^i z^i(\xi_{3D}, \eta_{3D}, \zeta_{3D})\end{aligned}$$

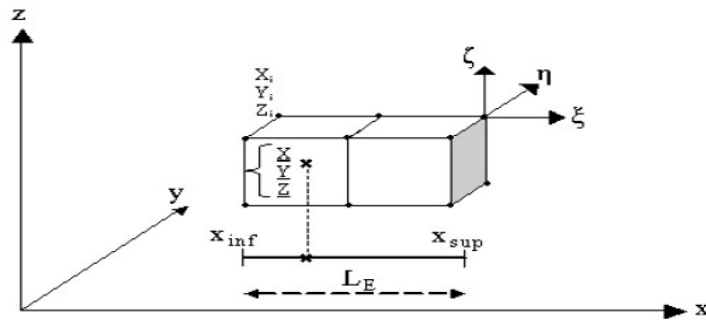


Figure 2.3-b operation of localization of the points of Gauss of an element 3D in the single element of beam which is paired to him

To simplify the illustration, we suppose that the axis of the beam is according to X. For each point of Gauss of the element 3D, we introduce his projection on the single real element 1D which is paired to him as follows:

$$\bar{\xi}_{1D} = \frac{\bar{X} - X_{inf}}{L_e}$$

The coordinates of projection in the two other directions thickness of the beam are given by $y = \bar{Y}$ and $z = \bar{Z}$.

This technique makes it possible to discretize the multiplier of Lagrange on the elements of beam and to calculate the integrals in the forms of the matrices of coupling C_{1D-3D}^k . Thus, for each element 3D one calculates the matrix C_{1D-3D}^k as follows:

$$C_{1D-3D}^k = \sum_{g=1}^{N_{bgauss}} \mathbf{W}^g \mathbf{B}_{1D}^{gT}(\bar{\xi}_{1D}) \mathbf{G}^g \mathbf{B}_{3D}^g(\xi_{3D}, \eta_{3D}, \zeta_{3D}) \det(\mathbf{J}^g)$$

where the functions of the shape of type beam are interpolated with the parametric coordinates of the projected points obtained.

2.4 Establishment of the method of connection

For each connection Harlequin, the user must define the following operands under the option 3D_POU_ARLEQUIN keyword LIAISON_ELEM :

The group of meshes 3D of the zone of joining (keyword GROUP_MA_1)

The group of meshes 1D of the zone of joining (keyword GROUP_MA_2)

The concept `CHAM_MATER` defining materials (being used to ensure the partition of the unit)
The concept `CARA_ELEM` defining the elementary characteristics being used with calculation of the matrices as coupling C_{1D-1D} and C_{1D-3D}

For each element beam, Code_aster calculates the linear relations resulting from the writing of the assembled matrices of coupling [éq 2.4-1]. These relations kinematics connect:

- 12 degrees of freedom of the two nodes of the element beam,
- with the degrees of freedom of the nodes of **all** elements paired with the element beam.

Lastly, the resulting linear relations will be dualisées, like all the linear relations resulting for example from the keyword `LIAISON_DDL` of `AFFE_CHAR_MECA`.

3 Which uses can one make this modeling?

Currently, it is possible to consider three types of use of the method Harlequin:

- **The zoom** : This kind of application aims at the detection and the study of a local phenomenon (contact, crack,...) on the level of a zone specifies pertaining to a total field. A fine patch is then superimposed on the existing coarse model, with an aim of refining the solution in the vicinities of the zones of interest. In this case, case the zone of joining is equal to one of the under-fields.
- **The junction** : The principal goal aimed by this application consists in connecting two models which can be different natures and/or smoothnesses. In this configuration, the zone of joining is strictly equal to the zone of covering of the two fields.
- **Substitution** : This operation consists in locally substituting the existing model by another finer. This technique proves very useful to introduce with a great flexibility, a defect in a healthy model such as cracking. Contrary to the two preceding cases, by applying substitution, the zone of joining constitutes a under-field of the zone of covering of the two models.

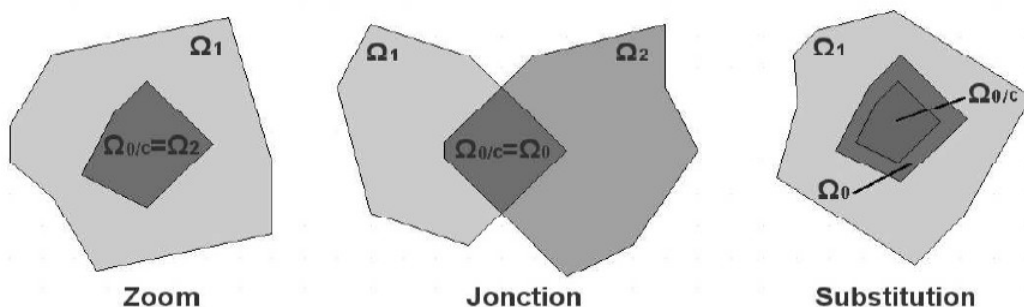


Figure 3-a various uses of connection 1D-3D within the framework Harlequin

4 Bibliography

- [1] A. Ghanem, “Contribution to the advanced modeling of the revolving machines in transitory dynamics within the framework Harlequin”, thesis of INSA de Lyon.

5 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
12	MR. TORKHANI	Initial text

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Code_Aster

Version
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