# Elements of Fourier for the axisymmetric structures

**Summary**

The elements of Fourier are intended to calculate the answer of structure for axisymmetric geometry solicited by nonaxisymmetric loadings broken up into Fourier series.

One exposes in this document a general theory of Analysis of Fourier with coupling of the symmetrical and antisymmetric modes in the anisotropic case. The case of isotropic, or orthotropic materials of axis $Oz$ , where the modes are uncoupled, is studied except for.

The elements of Fourier are usable in *Code_Aster* starting from modeling AXIS_FOURIER. The meshs supports of these elements are triangles and quadrangles of degree 1 and 2.

# Contents

# 1 Introduction

The analysis of Fourier is intended to calculate the answer of structures for axisymmetric geometries subjected to nonaxisymmetric loadings. In this case, it is necessary to develop the loadings in Fourier series. Generally convergence is reached for 4 or 5 harmonics, but the speed of this convergence depends on the nature of the loading: the more regular the loading is and the more quickly the corresponding series converges. The most unfavourable case is that of a concentrated force for which the practice shows that it is necessary to go to beyond (at least 7 harmonics).

In *Code_Aster*, the decomposition of the loading in Fourier series is supposed to be made as a preliminary by the user. *Code_Aster* allows to calculate the answers to this loading, harmonic by harmonic (modeling `AXIS_FOURIER`), and overall after recombination of the harmonics between them (operator `COMB_FOURIER`).

One will expose in a first chapter the framework general of the anisotropy, while insisting on the decoupling of the modes in the orthotropic case. The second chapter clarifies the calculation of the matrix of rigidity in the isotropic case.

For the use of the elements of Fourier in *Code_Aster*, one returns to the note of use of modeling Fourier [U2.07.01].

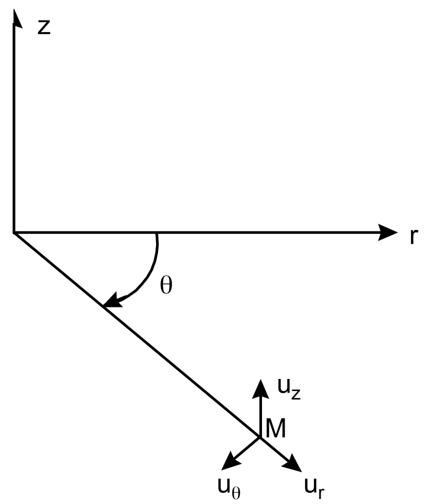
# 2 Anisotropic analysis of Fourier

## 2.1 General theory

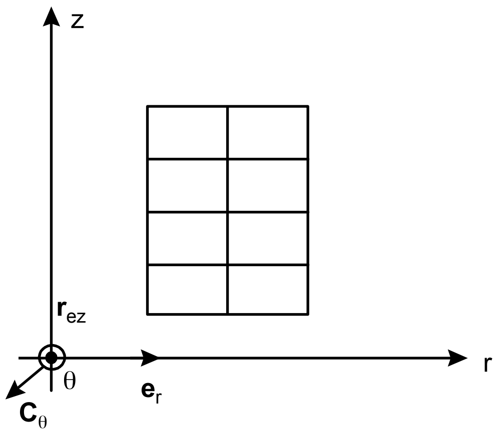
All the fields considered (forces, displacements, strains, stresses) are expressed in cylindrical coordinates with following convention on the order of the components:

1 radial component according to $r$
2 axial component according to $z$
3 tangential component according to $\theta$

Example: $(u_r, u_z, u_\theta), (f_r, f_z, f_\theta)$




The grid is localised in the plan $(r, z)$, the symmetry of revolution being done around the axis $Oz$.
The trihedron $(r, z, \theta)$ is directed in the direct direction.




Displacement is broken up $\mathbf{u}$ (or the loading $\mathbf{f}$ ) according to $\mathbf{u}=\mathbf{u}^s+\mathbf{u}^a$ where $\mathbf{u}^s$ (resp. $\mathbf{u}^a$ ) indicate the symmetrical part (resp. antisymmetric) of the development in Fourier series of $\mathbf{u}$ compared to the variable $\theta$ .

One obtains:

$$\left.\begin{aligned} u_r^s &= \sum_{l=0}^{\infty} u_l^s(r,z)\cos l\theta \\ u_z^s &= \sum_{l=0}^{\infty} v_l^s(r,z)\cos l\theta \\ u_\theta^s &= \sum_{l=0}^{\infty} w_l^s(r,z)(-\sin l\theta) \end{aligned}\right\}\text{partie symétrique } u^s$$

$$\left.\begin{aligned} u_r^a &= \sum_{l=0}^{\infty} u_l^a(r,z)\sin l\theta \\ u_z^a &= \sum_{l=0}^{\infty} v_l^a(r,z)\sin l\theta \\ u_\theta^a &= \sum_{l=0}^{\infty} w_l^a(r,z)\cos l\theta \end{aligned}\right\}\text{partie antisymétrique } u^a$$

To note the choice of the sign $-$ for $u_\theta^s$ , which makes it possible to simplify later calculations. If one notes $\mathbf{U}_l^s=(u_l^s, v_l^s, w_l^s)$ $(\text{resp.}\, \mathbf{U}_l^a)$ $l-\text{ième}$ symmetrical component (resp. antisymmetric) of the development in Fourier series of $\mathbf{u}$ , one obtains:

$$\mathbf{u}=\sum_{l=0}^{\infty}\left[\begin{pmatrix}\cos l\theta & & 0\\ & \cos l\theta & \\ 0 & & -\sin l\theta\end{pmatrix}\mathbf{U}_l^s+\begin{pmatrix}\sin l\theta & & 0\\ & \sin l\theta & \\ 0 & & \cos l\theta\end{pmatrix}\mathbf{U}_l^a\right] \quad \text{éq 2.1-1}$$

If one indicates by $\boldsymbol{\varepsilon}$ the vector deformation linearized, one realizes that $\boldsymbol{\varepsilon}$ can be broken up into following Fourier series:

$$\boldsymbol{\varepsilon}=\sum_{l=0}^{\infty}\begin{pmatrix}\cos l\theta\, I_4 & 0_{4,2}\\ 0_{2,4} & -\sin l\theta\, I_2\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}\sin l\theta\, I_4 & 0_{4,2}\\ 0_{2,4} & \cos l\theta\, I_2\end{pmatrix}\boldsymbol{\theta}_l^a \quad \text{éq 2.1-2}$$

with $\boldsymbol{\varepsilon}=\left\{\varepsilon_r, \varepsilon_z, \varepsilon_q, \gamma_{rz}, \gamma_{rq}, \gamma_{zq}\right\}$

$$\boldsymbol{\varepsilon}_l^s=B_l^s\mathbf{U}_l^s \qquad \boldsymbol{\varepsilon}_l^a=B_l^a\mathbf{U}_l^a$$

with (see [bib1]):

$$B_l^s=\begin{bmatrix}\frac{\partial}{\partial r} & 0 & 0\\ 0 & \frac{\partial}{\partial z} & 0\\ \frac{1}{r} & 0 & -\frac{l}{r}\\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} & 0\\ \frac{l}{r} & 0 & \frac{\partial}{\partial r}-\frac{1}{r}\\ 0 & \frac{l}{r} & \frac{\partial}{\partial z}\end{bmatrix}$$

One has $B_l^a=B_l^s\ \forall l$ (this is due to the choice of the symmetrical development of $\mathbf{u}$ in $(\cos,\cos,-\sin)$ instead of $(\cos,\cos,\sin)$ ). One will omit starting from now the indices $a$ and $s$ and one will note $B_l$ the operator allowing to calculate the deformations corresponding to the harmonic $l$ .

## 2.2 Coupling and decoupling of the symmetrical and antisymmetric modes

By taking again the preceding notations, one a:

$$\mathbf{u}=\sum_l\begin{pmatrix}\cos l\,\theta\, I_2 & 0_{2,1}\\ 0_{1,2} & -\sin l\,\theta\end{pmatrix}\mathbf{u}_l^s+\sum_l\begin{pmatrix}\sin l\,\theta\, I_2 & 0_{2,1}\\ 0_{1,2} & \cos l\,\theta\end{pmatrix}\mathbf{u}_l^a$$

what is written, by introducing matrices $M_l^s$ et $M_l^a$ :

$$\mathbf{u}=\sum_l\left(M_l^s\mathbf{U}_l^s+M_l^a\mathbf{U}_l^a\right)$$
$$\mathbf{u}_l=M_l^s\mathbf{U}_l^s+M_l^a\mathbf{U}_l^a$$

One from of deduced that: $\boldsymbol{\varepsilon}_l=M'^s_l\boldsymbol{\varepsilon}_l^s+M'^a_l\boldsymbol{\varepsilon}_l^a$

avec $M'^{s}_{l}=\begin{pmatrix}\cos l\theta\, I_4 & 0_{4,2}\\ 0_{2,4} & -\sin l\theta\, I_2\end{pmatrix}$

$M'^{a}_{l}=\begin{pmatrix}\sin l\theta\, I_4 & 0_{4,2}\\ 0_{2,4} & \cos l\theta\, I_2\end{pmatrix}$

**Calculation of the deformation energy**

$$\begin{aligned}W_l &= \int_0^{2\pi}\int_s {}^t\varepsilon_l D\varepsilon_l\, ds\, d\theta \quad \text{avec } ds=rdrdz\\ &=\int_0^{2\pi} d\theta\int_s {}^t\varepsilon_l^s\, {}^tM'^{s}_{l} DM'^{s}_{l}\varepsilon_l^s ds+\int_0^{2\pi} d\theta\int_s {}^t\theta_l^a\, {}^tM'^{a}_{l} DM'^{a}_{l}\varepsilon_l^a ds\\ &+\int_0^{2\pi} d\theta\int_s {}^t\varepsilon_l^a\, {}^tM'^{a}_{l} DM'^{s}_{l}\varepsilon_l^s ds+\int_0^{2\pi} d\theta\int_s {}^t\varepsilon_l^s\, {}^tM'^{s}_{l} DM'^{a}_{l}\varepsilon_l^a ds\end{aligned}$$

Puisque $M'^{a}_{l}DM'^{s}_{l}=\begin{pmatrix}\sin l\theta\, I_4 & 0\\ 0 & \cos l\theta\, I_2\end{pmatrix}\begin{pmatrix}D_1 & D_3\\ {}^tD_3 & D_2\end{pmatrix}\begin{pmatrix}\cos l\theta\, I_4 & 0\\ 0 & -\sin l\theta\, I_2\end{pmatrix}$

$$M'^{a}_{l}DM'^{s}_{l}=\begin{pmatrix}D_1\sin l\theta\cos l\theta & -D_3(\sin l\theta)^2\\ {}^tD_3(\cos l\theta)^2 & -D_2\sin l\theta\cos l\theta\end{pmatrix}$$

and that $\int_0^{2\pi}\sin l\theta\cos l\theta\, d\theta=0$, si $D_3=0$ there is thus no term $({}^t\varepsilon_l^a,\varepsilon_l^s)$ ou $({}^t\varepsilon_l^s,\varepsilon_l^a)$ in $W$.
There is then no coupling $(U^a,U^s)$ ou $(U^s,U^a)$.

## 2.3 Calculation of the constraints

Just as $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$ can be broken up into following Fourier series:

$$\boldsymbol{\sigma}=\sum_l\left(M'^{s}_{l}\boldsymbol{\sigma}_l^s+M'^{a}_{l}\boldsymbol{\sigma}_l^a\right)$$

Law of Hooke $\boldsymbol{\sigma}=D\boldsymbol{\varepsilon}$, one deduces:

$$\mathbf{s}=\sum_l\begin{pmatrix}\cos l\theta D_1 & -\sin l\theta D_3\\ \cos l\theta D_3^t & -\sin l\theta D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}\sin l\theta D_1 & \cos l\theta D_3\\ \sin l\theta D_3^t & \cos l\theta D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^a$$

Maybe, while revealing the matrices $M'^{s}_{l}$ et $M'^{a}_{l}$ :

$$\begin{aligned}\boldsymbol{\sigma}= &\sum_l M'^{s}_{l}\left[\begin{pmatrix}D_1 & 0_{4,2}\\ 0_{2,4} & D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}0_{4,4} & D_3\\ -D_3^t & 0_{2,2}\end{pmatrix}\boldsymbol{\varepsilon}_l^a\right]\\ &+M'^{a}_{l}\left[\begin{pmatrix}0_{4,4} & -D_3\\ D_3^t & 0_{2,2}\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}D_1 & 0_{4,2}\\ 0_{2,4} & D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^a\right]\end{aligned}$$

While posing $D^s\begin{pmatrix} D_1 & 0_{4,2} \\ 0_{2,4} & D_2 \end{pmatrix}$ and $D^a=\begin{pmatrix} 0_{4,4} & D_3 \\ -D_3^t & 0_{2,2} \end{pmatrix}$, one from of deduced the parts symmetrical and antisymmetric of the constraint relating to the harmonic $l$ :

$$\begin{cases} \boldsymbol{\sigma}_l^s = D^s\,\boldsymbol{\varepsilon}_l^s + D^a\,\boldsymbol{\varepsilon}_l^a = D^s\,B_l\,\mathbf{u}_l^s + D^a\,B_l\,\mathbf{u}_l^a \\ \boldsymbol{\sigma}_l^a = -D^a\,\boldsymbol{\varepsilon}_l^s + D^s\,\boldsymbol{\varepsilon}_l^a = -D^a\,B_l\,\mathbf{u}_l^s + D^s\,B_l\,\mathbf{u}_l^a \end{cases} \quad \text{éq 2.3-1}$$

**Note:**

*In the case of the orthotropism compared to $Oz$, one has $D^a=0$ and [éq 2.1-1] is reduced to:*

$$\begin{cases} \boldsymbol{\sigma}_l^s = D^s\,B_l\,\mathbf{u}_l^s \\ \boldsymbol{\sigma}_l^a = D^s\,B_l\,\mathbf{u}_l^a \end{cases}$$

*I.e. if displacements are symmetrical (or antisymmetric), the constraints are it too.*

# 3 Calculation of the matrix of rigidity

## 3.1 Case general

Are $\mathbf{u}$ and $\boldsymbol{\varepsilon}$ two unspecified fields kinematically acceptable. By applying the principle of virtual work to the element of volume $v$, one can write:

$$\int_v \left({}^t\,\delta\,\boldsymbol{\varepsilon}\,.\,\mathbf{s}\right) dv = \int_v \left({}^t\,\delta\,\mathbf{u}\,.\,f\right) dv$$

After decomposition in Fourier series and integration compared to $\theta$, one obtains, for fields $\varepsilon_l^s, \varepsilon_l^a, u_l^s, u_l^a$ C.A. unspecified and for any harmonic $l$ :

$$\int_{s_l} \left({}^t\,\delta\,\varepsilon_l^s\,.\,\sigma_l^s + {}^t\,\delta\,\varepsilon_l^a\,.\,\sigma_l^a\right) ds_l = \int_{s_l} \left({}^t\,\delta\,u_l^s\,.\,f_l^s + {}^t\,\delta\,u_l^a\,.\,f_l^a\right) ds_l$$

Maybe, while using [éq 2.3-1] and while posing:

$$K_l^s = \int_{s_l} {}^t\,B_l\,D^s\,B_l\,ds_l$$
$$K_l^a = \int_{s_l} {}^t\,B_l\,D^s\,B_l\,ds_l = K_l^s = K_l$$
$$K_l^{as} = \int_{s_l} {}^t\,B_l\,D^a\,B_l\,ds_l$$

One obtains the system of equations according to:

$$\begin{cases} K_l\,u_l^s + K_l^{as}\,u_l^a = f_l^s \\ {}^t\,K_l^{as}\,u_l^s + K_l\,u_l^a = f_l^a \end{cases} \quad \text{éq 3-1}$$

where ${}^t\,K_l^{as} = -K_l^{as}$ it is seen that if $D_a \neq 0$, the decoupling of the modes in symmetrical and antisymmetric harmonics is not possible any more. On the other hand, if $D_a=0$ (orthotropism compared to $Oz$ ) then $K_l^{as}=0$ and [éq 3-1] is reduced to:

$$\begin{cases} K_l\,u_l^s = f_l^s \\ K_l\,u_l^a = f_l^a \end{cases}$$

While taking for vectors displacement (resp. force) corresponding to the harmonic $l$ vectors:

$$\mathbf{u}_l=\left\{u_r^s, u_z^s, u_\theta^s, u_r^a, u_z^a, u_\theta^a\right\}_l$$
$$\mathbf{f}_l=\left\{f_r^s, f_z^s, f_\theta^s, f_r^a, f_z^a, f_\theta^a\right\}_l$$

One has then:

$$K_l^g \mathbf{u}_l=\mathbf{f}_l \text{ avec } K_l^g=\begin{pmatrix} K_l & K_l^{as} \\ {}^t K_l^{as} & K_l \end{pmatrix}$$

## 3.2 Calculation of $K_l^g$ in the isotropic case

In this case one thus has $K_l^{as}=0$. One details in the continuation the calculation of $K_l=\int_{s_l} {}^t B_l D^s B_l ds_l$

In the isotropic case, one a:

$$D=D^s=\begin{bmatrix} D1 & D2 & D2 & 0 & 0 & 0 \\ D2 & D1 & D2 & 0 & 0 & 0 \\ D2 & D2 & D1 & 0 & 0 & 0 \\ 0 & 0 & 0 & D3 & 0 & 0 \\ 0 & 0 & 0 & 0 & D3 & 0 \\ 0 & 0 & 0 & 0 & 0 & D3 \end{bmatrix}$$

$$\text{avec} \quad D1=\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$
$$D2=\frac{E\nu}{(1+\nu)(1-2u)}$$
$$D3=\frac{E}{2(1+\nu)}$$

One can write:

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \\ \gamma_{r\theta} \\ \gamma_{z\theta} \end{Bmatrix}=B_l\begin{Bmatrix} u_r^s \\ u_z^s \\ u_\theta^s \end{Bmatrix}=B_l^{'}{}^t\left\{\frac{u_r}{r}, \frac{u_z}{r}, \frac{u_\theta}{r}, \frac{\partial u_r}{\partial r}, \frac{\partial u_z}{\partial r}, \frac{\partial u_\theta}{\partial r}, \frac{\partial u_r}{\partial z}, \frac{\partial u_z}{\partial z}, \frac{\partial u_\theta}{\partial z}\right\}$$

← fcts of form → ← derived from the fcts of form →

$$\text{avec } B'_l=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -l & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ l & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & l & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

While indicating by $\{W_J\}_{J=1 \text{ à } n}$ functions of form of the element considered, one a:

$$U=\begin{bmatrix} \frac{u_r}{r} \\ \frac{u_z}{r} \\ \frac{u_\theta}{r} \\ \frac{\partial u_r}{\partial r} \\ \frac{\partial u_z}{\partial r} \\ \frac{\partial u_\theta}{\partial r} \\ \frac{\partial u_r}{\partial z} \\ \frac{\partial u_z}{\partial z} \\ \frac{\partial u_\theta}{\partial z} \end{bmatrix} = \begin{bmatrix} \cdots & \frac{W_J}{r} & \overbrace{0}^{\text{noeud } J} & 0 & \cdots \\ \cdots & 0 & \frac{W_J}{r} & 0 & \cdots \\ \cdots & 0 & 0 & \frac{W_J}{r} & \cdots \\ \cdots & \frac{\partial W_J}{\partial r} & 0 & 0 & \cdots \\ \cdots & 0 & \frac{\partial W_J}{\partial r} & 0 & \cdots \\ \cdots & 0 & 0 & \frac{\partial W_J}{\partial r} & \cdots \\ \cdots & \frac{\partial W_J}{\partial z} & 0 & 0 & \cdots \\ \cdots & 0 & \frac{\partial W_J}{\partial z} & 0 & \cdots \\ \cdots & 0 & \underbrace{0}_{\text{bloc} P_J} & \frac{\partial W_J}{\partial z} & \cdots \end{bmatrix} \begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ u_r(J) \\ u_z(J) \\ u_\theta(J) \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}$$

One notes $(P)=(P_1, \ldots, P_N)$ where $N$ is the number of nodes of the element.

Then $K_l=\int_{s_l} {}^t P \, {}^t B_l^{'} D B_l^{'} P \, ds_l$

$K_l$ symmetrical and is formed by blocks $(K_l)^{I,J}$ $3\times3$ :

$$(K_l)^{I,J}=\int_{s_l} {}^t P_I \, {}^t B_l^{'} D B_l^{'} P_J \, ds_l$$

The calculation of the blocks $(K_l)^{I,J}$ is clarified below:

$$
tB_l^{'}DB_l^{'}=\begin{bmatrix}
D1+1^2D3 & 0 & -l(D1+D3) & D2 & 0 & lD3 & 0 & D2 & 0\\
0 & l^2D3 & 0 & 0 & 0 & 0 & 0 & 0 & lD3\\
-l(D1+D3) & 0 & l^2D1+D3 & -lD2 & 0 & -D3 & 0 & -lD2 & 0\\
D2 & 0 & -lD2 & D1 & 0 & 0 & 0 & D2 & 0\\
0 & 0 & 0 & 0 & D3 & 0 & D3 & 0 & 0\\
lD3 & 0 & -D3 & 0 & 0 & D3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & D3 & 0 & D3 & 0 & 0\\
D2 & 0 & -lD2 & D2 & 0 & 0 & 0 & D1 & 0\\
0 & lD3 & 0 & 0 & 0 & 0 & 0 & 0 & D3
\end{bmatrix}
$$

$$
{}^tP_I\,{}^tB_l^{'}DB_l^{'}P_J=\left(K_{ij}^l\right)_{3\times 3}^{I,J}=\begin{bmatrix}
K_{11}^{I,J} & K_{12}^{I,J} & K_{13}^{I,J}\\
K_{21}^{I,J} & K_{22}^{I,J} & K_{23}^{I,J}\\
K_{31}^{I,J} & K_{32}^{I,J} & K_{33}^{I,J}
\end{bmatrix} \quad \text{avec}
$$

$$
\begin{cases}
K_{11}^{I,J}=\left(\dfrac{D1+l^2D3}{r^2}\right)W_IW_J+D1\dfrac{\partial W_I}{\partial r}\dfrac{\partial W_J}{\partial r}+D3\dfrac{\partial W_I}{\partial z}\dfrac{\partial W_J}{\partial z}+\dfrac{D2}{r}\left(W_I\dfrac{\partial W_J}{\partial r}+W_J\dfrac{\partial W_I}{\partial r}\right)\\
K_{22}^{I,J}=\left(\dfrac{l^2D3}{r^2}\right)W_IW_J+D3\dfrac{\partial W_I}{\partial r}\dfrac{\partial W_J}{\partial r}+D1\dfrac{\partial W_I}{\partial z}\dfrac{\partial W_J}{\partial z}\\
K_{33}^{I,J}=\left(\dfrac{l^2D1+D3}{r^2}\right)W_IW_J+D3\left(\dfrac{\partial W_I}{\partial r}\dfrac{\partial W_J}{\partial r}+\dfrac{\partial W_I}{\partial z}\dfrac{\partial W_J}{\partial z}\right)-\dfrac{D3}{r}\left(W_I\dfrac{\partial W_J}{\partial r}+\dfrac{\partial W_I}{\partial r}W_J\right)\\
K_{12}^{I,J}=D2\dfrac{\partial W_I}{\partial r}\dfrac{\partial W_J}{\partial z}+D3\dfrac{\partial W_I}{\partial z}\dfrac{\partial W_J}{\partial r}+\dfrac{D2}{r}W_I\dfrac{\partial W_J}{\partial z}\\
K_{21}^{I,J}=D3\dfrac{\partial W_I}{\partial r}\dfrac{\partial W_J}{\partial z}+D2\dfrac{\partial W_I}{\partial z}\dfrac{\partial W_J}{\partial r}+\dfrac{D2}{r}W_J\dfrac{\partial W_I}{\partial z}\\
K_{13}^{I,J}=-\dfrac{l}{r^2}(D1+D3)W_IW_J-\dfrac{l}{r}D2W_J\dfrac{\partial W_I}{\partial r}+\dfrac{l}{r}D3W_I\dfrac{\partial W_J}{\partial r}\\
K_{31}^{I,J}=-\dfrac{l}{r^2}(D1+D3)W_IW_J-\dfrac{l}{r}D2W_I\dfrac{\partial W_J}{\partial r}+\dfrac{l}{r}D3W_J\dfrac{\partial W_I}{\partial r}\\
K_{23}^{I,J}=-\dfrac{l}{r}D2\dfrac{\partial W_I}{\partial z}W_J+\dfrac{l}{r}D3W_I\dfrac{\partial W_J}{\partial z}\\
K_{32}^{I,J}=-\dfrac{l}{r}D2W_I\dfrac{\partial W_J}{\partial z}+\dfrac{l}{r}D3\dfrac{\partial W_I}{\partial z}W_J
\end{cases}
$$

Blocks $K^{I,J}$ are not symmetrical except for $I=J$ (on the diagonal of $K$ ). One notices in fact that the blocks $K^{I,J}$ can be written for any harmonic ( $l=0$ understood).

$$\left|\begin{array}{l}
K_{11}^{I,J}=K0_{11}^{I,J}+l^2\frac{D3}{r^2}W_I W_J\\
K_{22}^{I,J}=K0_{22}^{I,J}+l^2\frac{D3}{r^2}W_I W_J\\
K_{33}^{I,J}=K0_{33}^{I,J}+l^2\frac{D1}{r^2}W_I W_J\\
K_{12}^{I,J}=K0_{12}^{I,J}\\
K_{21}^{I,J}=K0_{21}^{I,J}\\
K_{13}^{I,J}=-lK0_{13}^{I,J}\\
K_{31}^{I,J}=-lK0_{31}^{I,J}\\
K_{23}^{I,J}=-lK0_{23}^{I,J}\\
K_{32}^{I,J}=-lK0_{32}^{I,J}
\end{array}\right.$$

where blocks $K0^{I,J}$ are independent of the harmonic $l$ .

# 4 Loadings

It is supposed that the loading was broken up according to the same base which displacements, that is to say:

$$\mathbf{f}=\sum_{l=0}^{\infty}\left[\begin{pmatrix}\cos l\theta & & 0\\ & \cos l\theta & \\ 0 & & -\sin l\theta\end{pmatrix}\mathbf{F}_l^s+\begin{pmatrix}\sin l\theta & & 0\\ & \sin l\theta & \\ 0 & & \cos l\theta\end{pmatrix}\mathbf{F}_l^a\right]$$

There is not coupling for the same harmonic between the parts symmetrical and antisymmetric of the loading because of orthogonality of the goniometrical functions $\sin l\theta$ and $\cos l\theta$ , this for all the types of loading. This wants to say in particular that the equivalent nodal forces are the same ones for the harmonics symmetrical and antisymmetric if the amplitudes $F_l^s$ et $F_l^a$ are the same ones.

For the nature of the acceptable loadings with modeling Fourier, one returns to the note of use [U2.01.07].

# 5 Conclusion and Outlines

Currently, it is supposed that the decomposition of the loading was made as a preliminary by the user, i.e. $\left\{F_l^s, F_l^a\right\}_{l\geq 0}$ is known. This decomposition could be realized by an operator of *Code_Aster* who would project the loading on the modes of Fourier.

For the moment, only the nonanisotropic case is established, i.e. there is never coupling of the modes. The extension to the anisotropy can constitute a later development.

# 6 Bibliography

1 DUVAUT G.: “Mechanical of the continuous mediums” p282

2 ASKA HS.: “Axisymmetric structures in Fourier series”, May 1982, ISD

# 7 Description of the versions of the document

| **Version Aster** | **Author (S) Organization (S)** | **Description of the modifications** |
|---|---|---|
| 5 | X.Desroches EDF-R&D/AMA | Initial text |