Multifibre element of beam (right)

Summary :

This document presents the elements of multifibre beam of *Code_Aster* based on a resolution of a problem of beam for which each section of a beam is divided into several fibres. Each fibre behaves then like a beam of Euler. Several materials can be affected on only one support finite element (SEG2) what avoids having to duplicate the meshs (steel + concrete, for example).

The beams are right (element `POU_D_EM`). The section can be of an unspecified form, described by a “fibre grid”, to see `[U4.26.01]`.
The assumptions selected are the following ones:
- assumption of Euler: transverse shearing is neglected (this assumption is checked for strong twinges),
- the elements of multifibre beam take into account the effects of thermal dilation, drying and the hydration (terms of the second member) and in a simplified way torsion. The effort-normal coupling inflection is treated naturally, by integration in the section of the uniaxial answers of the models of behavior associated with each group with fibres. An enrichment of the axial deformation, solved by local condensation in the case of nonlinear behaviors, allows digital good performances, whatever the evolution in the section of the centre of gravity matériau of the section.

Contents

Notations

One gives the correspondence between the notations of this document and those of the documentation of use.

DX, DY, DZ and DRX, DRY, DRZ are in fact the names of the degrees of freedom associated with the components with displacement $u, v, w, \theta_x, \theta_y, \theta_z$.

E	Young modulus	E
ν	Poisson's ratio	NU
G	module of Coulomb = $\frac{E}{2.(1+\nu)}$	G
I_y, I_z	geometrical moments of inflection compared to the axes y, z	IY, IZ
J_x	constant of torsion	JX
K	matrix of rigidity	
M	matrix of mass	
M_x, M_y, M_z	moments around the axes x, y, z	MT, MFY, MFZ
N	normal effort with the section	N
S	surface of the section	A
u, v, w	translations on the axes x, y, z	DX, DY, DZ
V_y, V_z	efforts cutting-edges along the axes y, z	VY, VZ
ρ	density	ρ
$\theta_x, \theta_y, \theta_z$	rotations around the axes x, y, z	DRX, DRY, DRZ
q_x, q_y, q_z	external linear efforts	

1 Introduction

The analysis of the structures subjected to a dynamic loading requires models of behavior able to represent non-linearities of material.

Many analytical models were proposed. They can be classified according to two groups:
- detailed models founded on the mechanics of the solid and their description of the local behavior of the material (microscopic approach)
- models based on a total modeling of the behavior (macroscopic approach).

In the first type of models, we can find the models classical with the finite elements as well as "the fibre" models type (having an element of type beam how support).

While the "classical" models with the finite elements are powerful tools for the simulation of the nonlinear behavior of the complex parts of the structures (joined, assemblies,…), their application to the totality of a structure can prove not very practical because of a prohibitory computing time or size memory necessary to the realization of this calculation. On the other hand, a modeling of type multifibre beam [Figure 1-a], has the advantages of the simplifying assumptions of a kinematics of type beam of Euler - Bernoulli while offering a practical solution and effective for a nonlinear analysis complexes composite elements of structures such as those which one can meet for example out of reinforced concrete.

Moreover, this "intermediate" modeling is relatively robust and inexpensive in time calculation because of use of nonlinear models of behavior 1D.

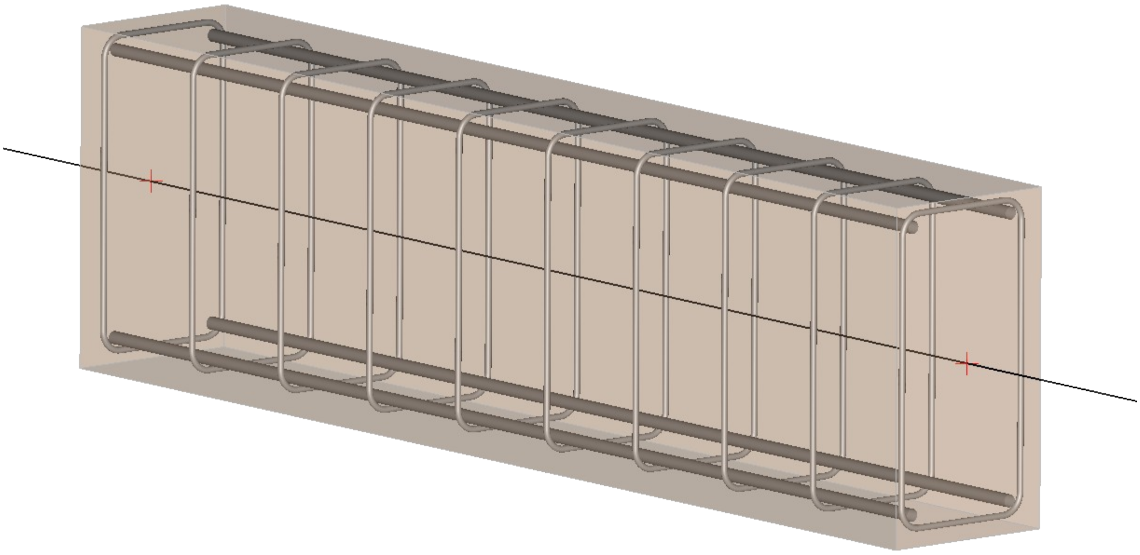

Figure 1-a : Beam reinforced concrete with frameworks and reinforcements.

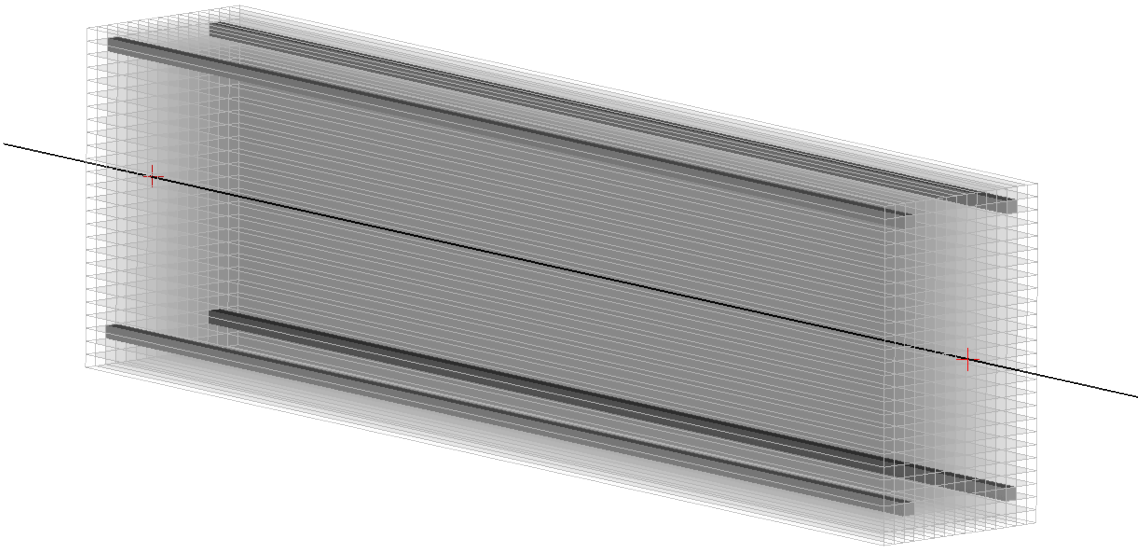

Figure 1-b : Modeling of a beam reinforced concrete by a multifibre beam.

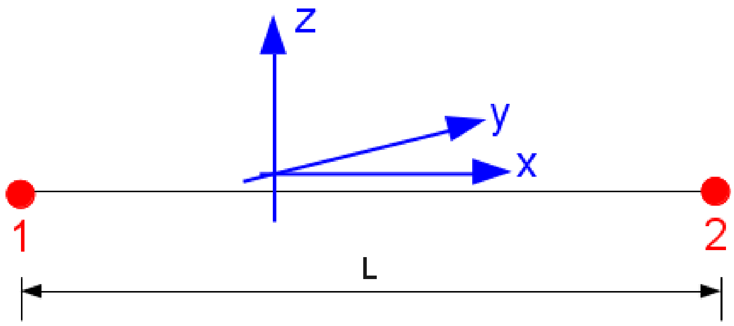

Figure 1-c : Support finite element of a modeling of type multifibre beam

2 Element of theory of the beams (recalls)

One takes again here the elements developed within the framework of the elements of beam of Euler, [feeding-bottle 4].

A beam is a solid generated by a surface of surface S of which the geometrical centre of inertia G followed a curve C called the average fibre or neutral fibre. The surface S is the cross-section (cross section) or profile, and it is supposed that if it is evolutionary, its evolutions (size, form) are continuous and progressive when G described the average line.

For the study of the beams in general, one makes the following assumptions:

- the cross-section of the beam is indeformable,
- transverse displacement is uniform on the cross-section.

These assumptions make it possible to express displacements of an unspecified point of the section, according to displacements of the point corresponding located on the average line, and according to an increase in displacement due to the rotation of the section around the transverse axes.

The discretization in “exact” elements of beam is carried out on a linear element with two nodes and six degrees of freedom by nodes. These degrees of freedom are the three translations u, v, w and three rotations $\theta_x, \theta_y, \theta_z$ [Figure 2-a].

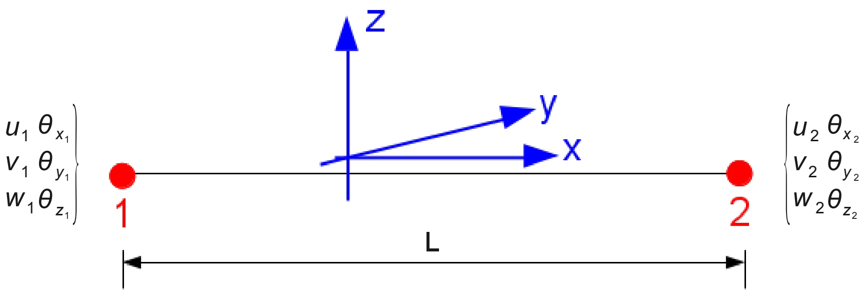

FigurE 2-a : Element beam.

Waited until the deformations are local, it is built in each top of the grid a local base depending on the element on which one works. The continuity of the fields of displacements is ensured by a basic change, bringing back the data in the total base.

In the case of the right beams, one traditionally places the average line on axis X of the local base, transverse displacements being thus carried out in the plan (y, z) .

Finally when we arrange sizes related to the degrees of freedom of an element in a vector or an elementary matrix (thus of dimension 12 or 12^2), one arranges initially the variables for the top 1 then those of the top 2 . For each node, one stores initially the sizes related to the three translations, then those related to three rotations. For example, a vector displacement will be structured in the following way:

$$\underbrace{u_1, v_1, w_1, \theta_{x_1}, \theta_{y_1}, \theta_{z_1}}_{\text{sommet 1}}\ ,\ \underbrace{u_2, v_2, w_2, \theta_{x_2}, \theta_{y_2}, \theta_{z_2}}_{\text{sommet 2}}$$

3 Equations of the movement of the beams

We will not include in this document all the equations of the movement of the beams. For more complements concerning this part one can refer to documentation concerning the elements POU_D_E and POU_D_T [feeding-bottle 4].

4 Element of right beam multifibre

One describes in this chapter obtaining the elementary matrices of rigidity and mass for the element of right beam multifibre, according to the model of Euler. The matrices of rigidity are calculated with the options 'RIGI_MECA' or 'RIGI_MECA_TANG', and matrices of mass with the option 'MASS_MECA' for the coherent matrix, and the option 'MASS_MECA_DIAG' for the matrix of diagonalized mass.

We present here a generalization [feeding-bottle 3] where the reference axis chosen for the beam is independent of any geometrical consideration, inertial or mechanical. The element functions for an unspecified section (heterogeneous is without symmetry) and is thus adapted to a nonlinear evolution of the behavior of fibres.

One also describes the calculation of the nodal forces for the nonlinear algorithms: 'FORC_NODA' and 'RAPH_MECA'.

4.1 Element beam of reference

[the Figure 4.1-a] the change of variable realized to pass from the real finite element [Figure shows us 2-a] with the finite element of reference.

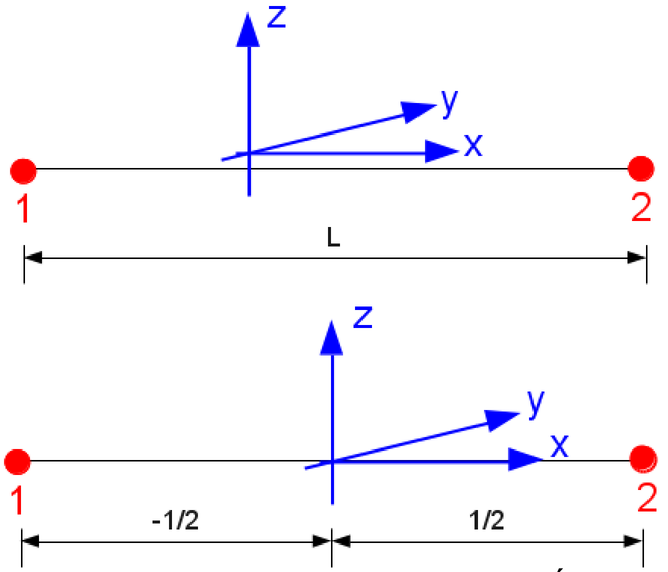

Figure 4.1-a : Element of reference vs real Élément.

One will then consider the continuous field of displacements in any point of the average line compared to the field of displacements discretized in the following way:

$$U_s=[N].\{U\} \quad \text{[éq 4.1-1]}$$

The index s indicate the quantities attached to average fibre.

By using the functions of form of the element of reference, the discretization of the variables $u_s(x), v_s(x), w_s(x), \theta_{sx}(x), \theta_{sy}(x), \theta_{sz}(x)$ becomes:

$$\begin{pmatrix} u_s(x) \\ v_s(x) \\ w_s(x) \\ \theta_{sx}(x) \\ \theta_{sy}(x) \\ \theta_{sz}(x) \end{pmatrix} = \begin{pmatrix} N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_3 & 0 & 0 & 0 & N_4 & 0 & N_5 & 0 & 0 & 0 & N_6 \\ 0 & 0 & N_3 & 0 & -N_4 & 0 & 0 & 0 & N_5 & 0 & -N_6 & 0 \\ 0 & 0 & 0 & N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 \\ 0 & 0 & -N_{3,x} & 0 & N_{4,x} & 0 & 0 & 0 & -N_{5,x} & 0 & N_{6,x} & 0 \\ 0 & N_{3,x} & 0 & 0 & 0 & N_{4,x} & 0 & N_{5,x} & 0 & 0 & 0 & N_{6,x} \end{pmatrix} . \begin{pmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{pmatrix}$$

[éq 4.1-2]

With the following functions of interpolation, and their derivative useful:

$$\begin{aligned}
N_1 &= 1-\frac{x}{L} \quad ; \quad N_{1,x} = -\frac{1}{L} \\
N_2 &= \frac{x}{L} \quad ; \quad N_{2,x} = \frac{1}{L} \\
N_3 &= 1-3\frac{x^2}{L^2}+2\frac{x^3}{L^3} \quad ; \quad N_{3,xx} = -\frac{6}{L^2}+12\frac{x}{L^3} \\
N_4 &= x-2\frac{x^2}{L}+\frac{x^3}{L^2} \quad ; \quad N_{4,xx} = -\frac{4}{L}+6\frac{x}{L^2} \\
N_5 &= 3\frac{x^2}{L^2}-2\frac{x^3}{L^3} \quad ; \quad N_{5,xx} = \frac{6}{L^2}-12\frac{x}{L^3} \\
N_6 &= -\frac{x^2}{L}+\frac{x^3}{L^2} \quad ; \quad N_{6,xx} = -\frac{2}{L}+6\frac{x}{L^2}
\end{aligned}$$

[éq 4.1-3]

4.2 Determination of the matrix of rigidity of the multifibre element

4.2.1 Case general (beam of Euler)

Let us consider a beam Euler, line, directed in the direction x, subjected to distributed efforts q_x, q_y, q_z [Figure 4.2.1-a].

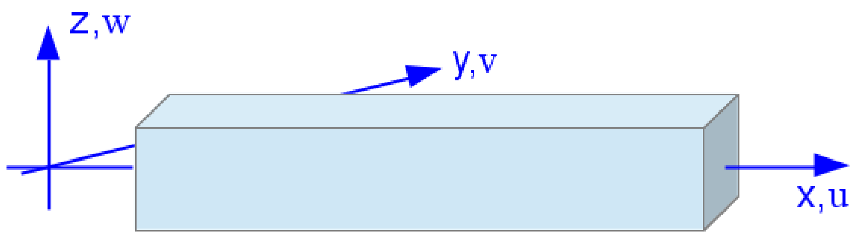

Figure 4.2.1-a : Beam of Euler 3D.

The fields of displacements and deformations take the following shape then when one writes the displacement of an unspecified point of the section according to displacement (U_s) and of rotation θ_s line of average:

$$u(x,y,z)=u_s(x)-y\theta_{sz}(x)+z\theta_{sy}(x) \quad \text{[éq 4.2.1-1]}$$
$$v(x,y,z)=v_s(x) \quad \text{[éq 4.2.1-2]}$$
$$w(x,y,z)=w_s(x) \quad \text{[éq 4.2.1-3]}$$
$$\varepsilon_{xx}=u_x^{'}(x)-y\theta_{sz}^{'}(x)+z\theta_{sy}^{'}(x) \quad \text{[éq 4.2.1-4]}$$
$$\varepsilon_{xy}=\varepsilon_{xz}=0 \quad \text{[éq 4.2.1-5]}$$

Note:

Torsion is treated overall by admitting an elastic assumption, except for, one does not calculate ε_{yz} *here.* $f'(x)$ *indicate the derivative of* $f(x)$ *compared to* x *.*

By introducing the equations [éq 4.2.1-4] and [éq 4.2.1-5] in the principle of virtual work one obtains:

$$\int_{V_0} \sigma_{xx}.\,\delta\varepsilon_{xx}\, dV_0 = \int_0^L \left(\delta u_s(x)\, q_x+\delta v_s(x)\, q_y+\delta w_s(x)\, q_z\right) dx \quad \text{[éq 4.2.1-6]}$$

q_x, q_y, q_z indicating the linear efforts applied. What gives by using the equation [éq 4.2.1-1]:

$$\begin{aligned}&\int_0^L \left(N\,\delta u_s^{'}(x)+M_x\,\delta\theta_{sx}^{'}(x)+M_y\,\delta\theta_{sy}^{'}(x)+M_z\,\delta\theta_{sz}^{'}(x)\right)dx\\ &\qquad=\int_0^L \left(q_x\,\delta u_s(x)+q_y\,\delta v_s(x)+q_z\,\delta w_s(x)\right)dx\end{aligned} \quad \text{[éq 4.2.1-7]}$$

with:

$$N=\int_S \sigma_{xx}\, dS \;\;;\;\; M_y=\int_S z\sigma_{xx}\, dS \;\;;\;\; M_z=\int_S -y\sigma_{xx}\, dS \quad \text{[éq 4.2.1-8]}$$

Note:

Torque M_x *is not calculated by integration but is not calculated directly starting from the stiffness in torsion (see [éq 4.2.2-4]).*

The theory of the beams associated with an elastic material gives: $\sigma_{xx}=E\,\varepsilon_{xx}$

4.2.2 Case of the multifibre beam

We suppose now that the section s is not homogeneous, materials with different mechanical characteristics.

Without adopting particular assumption on the intersection of the axis x with the section s or on the orientation of the axes Y,Z, the relation between the “generalized” constraints and deformations “generalized” $\mathbf{D}_s$ becomes [bib2]:

$$\mathbf{F}_s=\mathbf{K}_s\cdot\mathbf{D}_s \quad \text{[éq 4.2.2-1]}$$

with:

$$\begin{aligned}\mathbf{F}_s&=\left(N,\, M_y,\, M_z,\, M_x\right)^T\\ \mathbf{D}_s&=\left(u_s^{'}(x),\, \theta_{sy}^{'}(x),\, \theta_{sz}^{'}(x),\, \theta_{sx}^{'}(x)\right)^T\end{aligned} \quad \text{[éq 4.2.2-2]}$$

The matrix $\boldsymbol{K_s}$ can then put itself in the following form:

$$\mathbf{K}_s=\begin{vmatrix} K_{s11} & K_{s12} & K_{s13} & 0\\ & K_{s22} & K_{s23} & 0\\ & & K_{s33} & 0\\ sym & & & K_{s44}\end{vmatrix} \quad \text{[éq 4.2.2-3]}$$

with:

$$K_{s11}=\int_S EdS \;;\; K_{s12}=\int_S Ezds \;;\; K_{s13}=-\int_S Eyds$$
$$K_{s22}=\int_S Ez^2 dS \;;\; K_{s23}=-\int_S Eyzds \;;\; K_{s33}=\int_S Ey^2 ds \quad \text{[éq 4.2.2-4]}$$

where E can vary according to y and z. Indeed, it may be that in modeling section planes, several materials cohabit. For example, in a concrete section reinforced, there are at the same time concrete and reinforcements.
The discretization of the fibre section makes it possible to calculate the integrals of the equations [éq 4.2.2-4]. The calculation of the coefficients of the matrix $\mathbf{K}_s$ is detailed in the paragraph [§4.2.3] according to.

Note:

The term of torsion $K_{s44}=GJ_x$ is given by the user using the data of J_x, using the order `AFFE_CARA_ELEM`*.*

The introduction of the equations [éq 4.2.1-1] with [éq 4.2.2-4] in the principle of virtual work leads to:

$$\int_0^L \delta D_s^T . K_s . D_s \, dx - \int_0^L \left(\delta u_s(x)\, q_x + \delta v_s(x)\, q_y + \delta w_s(x)\, q_z\right) dx = 0 \quad \text{[éq 4.2.2-5]}$$

The generalized deformations are calculated by (D_S is given to the equation [éq 4.2.2-2]):

$$\mathbf{D}_s = \mathbf{B}\{\mathbf{U}\} \quad \text{[éq 4.2.2-6]}$$

With the matrix $\mathbf{B}$ following:

$$B=\begin{bmatrix} N_{1,x} & 0 & 0 & 0 & 0 & 0 & N_{2,x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -N_{3,xx} & 0 & N_{4,xx} & 0 & 0 & 0 & -N_{5,xx} & 0 & N_{6,xx} & 0 \\ 0 & N_{3,xx} & 0 & 0 & 0 & N_{4,xx} & 0 & N_{5,xx} & 0 & 0 & 0 & N_{6,xx} \\ 0 & 0 & 0 & N_{1,x} & 0 & 0 & 0 & 0 & 0 & N_{2,x} & 0 & 0 \end{bmatrix} \quad \text{[éq 4.2.2-7]}$$

Discretization of space $[0,L]$ with elements and the use of the equations [éq 4.2.2-5] the equation [éq returns 4.2.1-6] equivalent to the resolution of a classical linear system:

$$\mathbf{K}.\mathbf{U}=\mathbf{F} \quad \text{[éq 4.2.2-8]}$$

The matrix of rigidity of the element [Figure4.2.2-a] and the vector of the efforts results are finally given by:

$$\mathbf{K}_{elem}=\int_0^L \mathbf{B}^T.\mathbf{K}_s.\mathbf{B}\; dx$$
$$\mathbf{F}=\int_0^L \mathbf{N}^T.\boldsymbol{Q}\; dx \quad \text{[éq 4.2.2-9]}$$

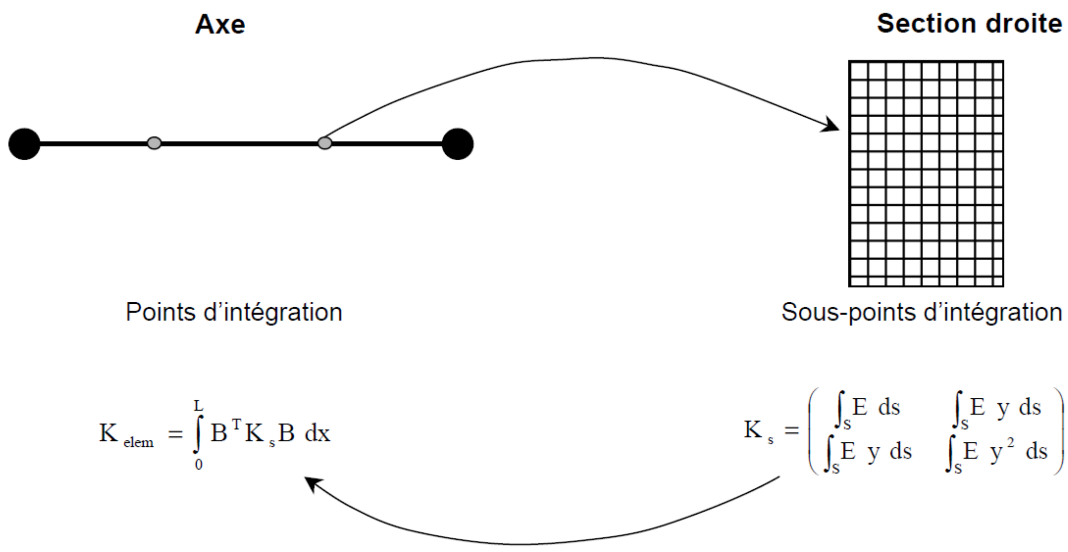

Figure 4.2.2-a : Multifibre beam – Calculation of $\mathbf{K}_{elem}$

With the vector Q who depends on the external loading: $\mathbf{Q}=\left(q_x \;\; q_y \;\; q_z \;\; 0 \;\; 0 \;\; 0\right)^T$.

If we consider that the distributed efforts q_x, q_y, q_z are constant, we obtain the vector nodal forces according to:

$$\mathbf{F}=\left(\frac{Lq_x}{2} \;\; \frac{Lq_y}{2} \;\; \frac{Lq_z}{2} \;\; 0 \;\; -\frac{L^2 q_z}{12} \;\; \frac{L^2 q_y}{12} \;\; \frac{Lq_x}{2} \;\; \frac{Lq_y}{2} \;\; \frac{Lq_z}{2} \;\; 0 \;\; \frac{L^2 q_z}{12} \;\; \frac{L^2 q_y}{12}\right)^T \quad \text{[éq 4.2.2-10]}$$

4.2.3 Discretization of the fibre section – Calculation of K_s

The discretization of the fibre section makes it possible to calculate the various integrals which intervene in the matrix of rigidity, and the other terms necessary.

Geometry of fibres gathered in groups of fibres, via the operator `DEFI_GEOM_FIBRE` [U4.26.01]) contains in particular the characteristics (`Y, Z, SURFACE`) for each fibre. One can envisage with more the 10 groups of maximum fibres by element beam.

Thus, if we have a section which comprises n fibres we will have the following approximations of the integrals:

$$K_{s11}=\sum_{i=1}^{n} E_i S_i \;\; ; \;\; K_{s12}=\sum_{i=1}^{n} E_i z_i S_i \;\; ; \;\; K_{s13}=\sum_{i=1}^{n} E_i y_i S_i$$
$$K_{s22}=\sum_{i=1}^{n} E_i z_i^2 S_i \;\; ; \;\; K_{s23}=-\sum_{i=1}^{n} E_i y_i z_i S_i \;\; ; \;\; K_{s33}=\sum_{i=1}^{n} E_i y_i^2 S_i \quad \text{[éq 4.2.3-1]}$$

with E_i the initial or tangent module and S_i the section of each fibre. The state of stress is constant by fibre.
Each fibre is also located using y_i and z_i coordinates of the centre of gravity of fibre compared to the axis of the section defined by the keyword '`COOR_AXE_POUTRE`' (see the order `DEFI_GEOM_FIBRE` [U4.26.01]).
The classification of fibres depends on the choice of the keyword '`FIBRE`' or '`SECTION`' (see the order `DEFI_GEOM_FIBRE` [U4.26.01]).

4.2.4 Integration in the linear elastic case (`RIGI_MECA`)

When the behavior of material is linear, if the element beam is homogeneous in its length, the integration of the equation [éq 4.2.2-9] can be made analytically.
One obtains the matrix of following rigidity then:

$$\mathbf{K}_{elem}=\begin{vmatrix}
\frac{K_{s11}}{L} & 0 & 0 & 0 & \frac{K_{s12}}{L} & \frac{K_{s13}}{L} & \frac{-K_{s11}}{L} & 0 & 0 & 0 & \frac{-K_{s12}}{L} & \frac{-K_{s13}}{L} \\
 & \frac{12K_{s33}}{L^3} & \frac{-12K_{s23}}{L^3} & 0 & \frac{6K_{s23}}{L^2} & \frac{6K_{s33}}{L^2} & 0 & \frac{-12K_{s33}}{L^3} & \frac{12K_{s23}}{L^3} & 0 & \frac{6K_{s23}}{L^2} & \frac{6K_{s33}}{L^2} \\
 & & \frac{12K_{s22}}{L^3} & 0 & \frac{-6K_{s22}}{L^2} & \frac{-6K_{s23}}{L^2} & 0 & \frac{12K_{s23}}{L^3} & \frac{-12K_{s22}}{L^3} & 0 & \frac{-6K_{s22}}{L^2} & \frac{-6K_{s23}}{L^2} \\
 & & & \frac{K_{s44}}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{K_{s44}}{L} & 0 & 0 \\
 & & & & \frac{4K_{s22}}{L} & \frac{4K_{s23}}{L} & \frac{-K_{s12}}{L} & \frac{-6K_{s23}}{L^2} & \frac{6K_{s22}}{L^2} & 0 & \frac{2K_{s22}}{L} & \frac{2K_{s23}}{L} \\
 & & & & & \frac{4K_{s33}}{L} & \frac{-K_{s13}}{L} & \frac{-6K_{s33}}{L^2} & \frac{6K_{s23}}{L^2} & 0 & \frac{2K_{s23}}{L} & \frac{2K_{s33}}{L} \\
 & & & & & & \frac{K_{s11}}{L} & 0 & 0 & 0 & \frac{K_{s12}}{L} & \frac{K_{s13}}{L} \\
 & & & & & & & \frac{12K_{s33}}{L^3} & \frac{-12K_{s23}}{L^3} & 0 & \frac{-6K_{s23}}{L^2} & \frac{-6K_{s33}}{L^2} \\
 & & & SYM & & & & & \frac{12K_{s22}}{L^3} & 0 & \frac{6K_{s22}}{L^2} & \frac{6K_{s23}}{L^2} \\
 & & & & & & & & & \frac{K_{s44}}{L} & 0 & 0 \\
 & & & & & & & & & & \frac{4K_{s22}}{L} & \frac{4K_{s23}}{L} \\
 & & & & & & & & & & & \frac{4K_{s33}}{L}
\end{vmatrix}$$

[éq 4.2.4-1]

with the following terms $K_{s11}, K_{s12}, K_{s13}, K_{s22}, K_{s33}, K_{s23}, K_{s44}$ given to the equation [éq 4.2.2-4].

Note:

The matrix of rigidity presented above does not take into account a possible eccentricity of the reference axis compared to the elastic center, not to weigh down the presentation. However the additional terms are well taken into account in the programming (see §4.5.3).

4.2.5 Integration in the non-linear case (`RIGI_MECA_TANG`)

When the behavior of material is nonlinear, to allow a correct integration of the internal efforts (see paragraph [§4.4]), it is necessary to have at least two points of integration along the beam. We chose to use two points of Gauss.

The integral of $\mathbf{K}_{elem}$ [éq 4.2.2-9] is calculated under digital form:

$$\mathbf{K}_{elem}=\int_0^L \mathbf{B}^T.K_s.\mathbf{B}\ dx=j\sum_{i=1}^{2} w_i \mathbf{B}(x_i)^T.\mathbf{K}_s(x_i)\ .\ \mathbf{B}(x_i) \quad \text{[éq 4.2.5-1]}$$

- where x_i is the position of the point of Gauss i in an element of reference length 1, i.e.: $(1\pm 0{,}57735026918963)/2$;
- w_i is the weight of the point of Gauss i . One takes here $w_i=0{,}5$ for each of the 2 points; j is Jacobien. One takes here $j=L$, the real element having a length L and the function of form to pass to the element of reference being $\frac{x}{L}$.

$\mathbf{K}_s$ is calculated using the equations [éq 4.2.1-4], [éq4.2.2-4] (see paragraph [§4.2.3] for the digital integration of these equations).

The analytical calculation of $\mathbf{B}(x_i)^T . \mathbf{K}_s(x_i) . \mathbf{B}(x_i)$ give:

$$\begin{pmatrix}
B_1^2K_{s11} & -B_1B_2K_{s13} & B_1B_2K_{s12} & 0 & -B_1B_3K_{s12} & -B_1B_3K_{s13} & -B_1^2K_{s11} & B_1B_2K_{s13} & -B_1B_2K_{s12} & 0 & -B_1B_4K_{s12} & -B_1B_4K_{s13} \\
 & B_2^2K_{s33} & B_2^2K_{s23} & 0 & B_2B_3K_{s23} & B_2B_3K_{s33} & B_1B_2K_{s13} & -B_2^2K_{s33} & B_2^2K_{s23} & 0 & B_2B_4K_{s23} & B_2B_4K_{s33} \\
 & & B_2^2K_{s22} & 0 & -B_2B_3K_{s22} & -B_2B_3K_{s23} & -B_1B_2K_{s12} & B_2^2K_{s23} & -B_2^2K_{s22} & 0 & -B_2B_4K_{s22} & -B_2B_4K_{s23} \\
 & & & B_1^2K_{s44} & 0 & 0 & 0 & 0 & 0 & -B_1^2K_{s44} & 0 & 0 \\
 & & & & B_3^2K_{s22} & B_3^2K_{s23} & B_1B_3K_{s12} & -B_2B_3K_{s23} & B_2B_3K_{s22} & 0 & B_3B_4K_{s22} & B_3B_4K_{s23} \\
 & & & & & B_3^2K_{s33} & B_1B_3K_{s13} & -B_2B_3K_{s33} & B_2B_3K_{s23} & 0 & B_3B_4K_{s23} & B_3B_4K_{s33} \\
 & & & & & & B_1^2K_{s11} & -B_1B_2K_{s13} & B_1B_2K_{s12} & 0 & B_1B_4K_{s12} & B_1B_4K_{s13} \\
 & & & & & & & B_2^2K_{s33} & -B_2^2K_{s23} & 0 & -B_2B_4K_{s23} & -B_2B_4K_{s33} \\
 & & & & & & & & B_2^2K_{s22} & 0 & B_2B_4K_{s22} & B_2B_4K_{s23} \\
 & & & & & & & & & B_1^2K_{s44} & 0 & 0 \\
 & & & & & & & & & & B_4^2K_{s22} & B_4^2K_{s23} \\
 & & & & & & & & & & & B_4^2K_{s33}
\end{pmatrix} \qquad \text{[éq 4.2.5-2]}$$

where them B_i are calculated with the X-coordinate x_i element of reference with:

$$\begin{aligned}
B_1 &= -N_{1,x} = N_{2,x} = \frac{1}{L} \\
B_2 &= -N_{3,xx} = N_{5,xx} = -\frac{6}{L^2} + \frac{12x_i}{L^2} \\
B_3 &= N_{4,xx} = -\frac{4}{L} + \frac{6x_i}{L} \\
B_4 &= N_{6,xx} = -\frac{2}{L} + \frac{6x_i}{L}
\end{aligned} \qquad \text{[éq 4.2.5-3]}$$

4.3 Determination of the matrix of mass of the multifibre element

4.3.1 Determination of $\mathbf{M}_{elem}$

In the same way, the virtual work of the efforts of inertia becomes [bib2]:

$$\begin{aligned}
W_{inert} &= \int_0^L \int_S \rho \left(\delta u(x,y) \frac{d^2 u(x,y)}{dt^2} + \delta v(x,y) \frac{d^2 v(x,y)}{dt^2} + \delta w(x,y) \frac{d^2 w(x,y)}{dt^2} \right) dS\ dx \\
&= \int_0^L \delta \mathbf{U}_s . \mathbf{M}_s . \frac{d^2 \mathbf{U}_s}{dt^2}\ dx
\end{aligned} \qquad \text{[éq 4.3.1-1]}$$

with $\mathbf{U}_s$ the vector of "generalized" displacements.
What gives for the matrix of mass:

$$\mathbf{M}_s = \begin{pmatrix}
M_{s11} & 0 & 0 & 0 & M_{s12} & M_{s13} \\
 & M_{s11} & 0 & -M_{s12} & 0 & 0 \\
 & & M_{s11} & -M_{s13} & 0 & 0 \\
 & & & M_{s22}+M_{s33} & 0 & 0 \\
 & & & & M_{s22} & M_{s23} \\
sym & & & & & M_{s33}
\end{pmatrix} \qquad \text{[éq 4.3.1-2]}$$

with:

$$M_{s11}=\int_S \rho\, ds \;;\; M_{s12}=\int_S \rho\, z ds \;;\; M_{s13}=-\int_S \rho\, y ds$$
$$M_{s22}=\int_S \rho\, z^2 ds \;;\; M_{s23}=-\int_S \rho\, yz ds \;;\; M_{s33}=\int_S \rho\, y^2 ds \qquad \text{[éq 4.3.1-3]}$$

with ρ who can vary according to y and z.

As for the matrix of rigidity, we take into account the generalized deformations and the discretization of space $[0,L]$. What gives finally for the elementary matrix of mass of dimension 12x12 :

$$M^1_{elem}=\left[\frac{LM_{s11}}{3}\quad\frac{-M_{s13}}{2}\quad\frac{M_{s12}}{2}\quad 0\quad\frac{LM_{s12}}{12}\quad\frac{LM_{s13}}{12}\quad\frac{LM_{s11}}{6}\quad\frac{M_{s13}}{2}\quad\frac{-M_{s12}}{2}\quad 0\quad\frac{-LM_{s12}}{12}\quad\frac{-LM_{s13}}{12}\right]$$

$$M^2_{elem}=\left[sym\quad\frac{13LM_{s11}}{35}+\frac{6M_{s33}}{5L}\quad\frac{-6M_{s23}}{5L}\quad\frac{-7LM_{s12}}{20}\quad\frac{M_{s23}}{10}\quad\frac{11L^2M_{s11}}{210}+\frac{M_{s33}}{10}\quad\frac{-M_{s13}}{2}\quad\frac{9LM_{s11}}{70}-\frac{6M_{s33}}{5L}\quad\frac{6M_{s23}}{5L}\quad\frac{-3LM_{s12}}{20}\quad\frac{M_{s23}}{10}\quad\frac{-13L^2M_{s11}}{420}+\frac{M_{s33}}{10}\right]$$

$$M^3_{elem}=\left[sym\; sym\quad\frac{13LM_{s11}}{35}+\frac{6M_{s22}}{5L}\quad\frac{-7LM_{s13}}{20}\quad\frac{-11L^2M_{s11}}{210}-\frac{M_{s22}}{10}\quad\frac{-M_{s23}}{10}\quad\frac{M_{s12}}{2}\quad\frac{6M_{s23}}{5L}\quad\frac{9LM_{s11}}{70}-\frac{6M_{s22}}{5L}\quad\frac{-3LM_{s13}}{20}\quad\frac{13L^2M_{s11}}{420}-\frac{M_{s22}}{10}\quad\frac{-M_{s23}}{10}\right]$$

$$M^4_{elem}=\left[sym\; sym\; sym\quad\frac{LM_{s22}+LM_{s33}}{3}\quad\frac{L^2M_{s13}}{20}\quad\frac{-L^2M_{s12}}{20}\quad 0\quad\frac{-3LM_{s12}}{20}\quad\frac{-3LM_{s13}}{20}\quad\frac{LM_{s22}+LM_{s33}}{6}\quad\frac{-L^2M_{s13}}{30}\quad\frac{L^2M_{s12}}{30}\right]$$

$$M^5_{elem}=\left[sym\; sym\; sym\; sym\quad\frac{L^3M_{s11}}{105}+\frac{2LM_{s22}}{15}\quad\frac{2LM_{s23}}{15}\quad\frac{-LM_{s12}}{12}\quad\frac{-M_{s23}}{10}\quad\frac{-13L^2M_{s11}}{420}+\frac{M_{s22}}{10}\quad\frac{L^2M_{s13}}{30}\quad\frac{-L^3M_{s11}}{140}-\frac{LM_{s22}}{30}\quad\frac{-LM_{s23}}{30}\right]$$

$$M^6_{elem}=\left[sym\; sym\; sym\; sym\; sym\quad\frac{L^3M_{s11}}{105}+\frac{2LM_{s33}}{15}\quad\frac{-LM_{s13}}{12}\quad\frac{13L^2M_{s11}}{420}-\frac{M_{s33}}{10}\quad\frac{M_{s23}}{10}\quad\frac{-L^2M_{s12}}{30}\quad\frac{-LM_{s23}}{30}\quad\frac{-L^3M_{s11}}{140}-\frac{LM_{s33}}{30}\right]$$

$$M^7_{elem}=\left[sym\; sym\; sym\; sym\; sym\; sym\quad\frac{LM_{s11}}{3}\quad\frac{M_{s13}}{2}\quad\frac{-M_{s12}}{2}\quad 0\quad\frac{LM_{s12}}{12}\quad\frac{LM_{s13}}{12}\right]$$

$$M^8_{elem}=\left[sym\; sym\; sym\; sym\; sym\; sym\; sym\quad\frac{13LM_{s11}}{35}+\frac{6M_{s33}}{5L}\quad\frac{-6M_{s23}}{5L}\quad\frac{-7LM_{s12}}{20}\quad\frac{-M_{s23}}{10}\quad\frac{-11L^2M_{s11}}{210}-\frac{M_{s33}}{10}\right]$$

$$M^9_{elem}=\left[sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\quad\frac{13LM_{s11}}{35}+\frac{6M_{s22}}{5L}\quad\frac{-7LM_{s13}}{20}\quad\frac{11L^2M_{s11}}{210}+\frac{M_{s22}}{10}\quad\frac{M_{s23}}{10}\right]$$

$$M^{10}_{elem}=\left[sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\quad\frac{LM_{s22}+LM_{s33}}{3}\quad\frac{-L^2M_{s13}}{20}\quad\frac{L^2M_{s12}}{20}\right]$$

$$M^{11}_{elem}=\left[sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\quad\frac{L^3M_{s11}}{105}+\frac{2LM_{s22}}{15}\quad\frac{2LM_{s23}}{15}\right]$$

$$M^{12}_{elem}=\left[sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\; sym\quad\frac{L^3M_{s11}}{105}+\frac{2LM_{s33}}{15}\right]$$

with the following terms: $M_{s11}, M_{s12}, M_{s13}, M_{s22}, M_{s33}, M_{s23}$ who are given to the equation [éq 4.3.1-3].

Note:
The matrix of diagonal mass is reduced by the technique of the concentrated masses ([bib4]) . This matrix of diagonal mass is obtained by the option 'MASS_MECA_DIAG' of the operator CALC_MATR_ELEM [U4.61.01].
The additional terms in the event of eccentricity of the axes are not presented here but are well taken into account (see §4.5.3).

4.3.2 Discretization of the fibre section - Calculation of M_s

The discretization of the fibre section makes it possible to calculate the various integrals which intervene in the matrix of mass. Thus, if we have a section which comprises n fibres we will have the following approximations of the integrals:

$$M_{s11}=\sum_{i=1}^{n}\rho_i S_i \;;\; M_{s12}=\sum_{i=1}^{n}\rho_i z_i S_i \;;\; M_{s13}=-\sum_{i=1}^{n}\rho_i y_i S_i$$
$$M_{s22}=\sum_{i=1}^{n}\rho_i z_i^2 S_i \;;\; M_{s23}=-\sum_{i=1}^{n}\rho_i y_i z_i S_i \;;\; M_{s33}=\sum_{i=1}^{n}\rho_i y_i^2 S_i \qquad \text{[éq 4.3.2-1]}$$

with ρ_i and S_i density and the section of each fibre. y_i and z_i are the coordinates of the centre of gravity of fibre defined as previously.

4.4 Calculation of the internal forces

The calculation of the nodal forces $\mathbf{F}_{\mathrm{int}}$ had in a state of internal stresses given is done by the integral:

$$\boldsymbol{F}_{\mathbf{int}}=\int_0^L \boldsymbol{B}^T . \boldsymbol{F}_s \, dx \qquad \text{[éq 4.4-1]}$$

where $\boldsymbol{B}$ is the matrix giving the generalized deformations according to nodal displacements [éq 4.2.2-6] and where $\mathbf{F}_s$ is the vector of the generalized constraints given to the equation [éq 4.2.2-2],

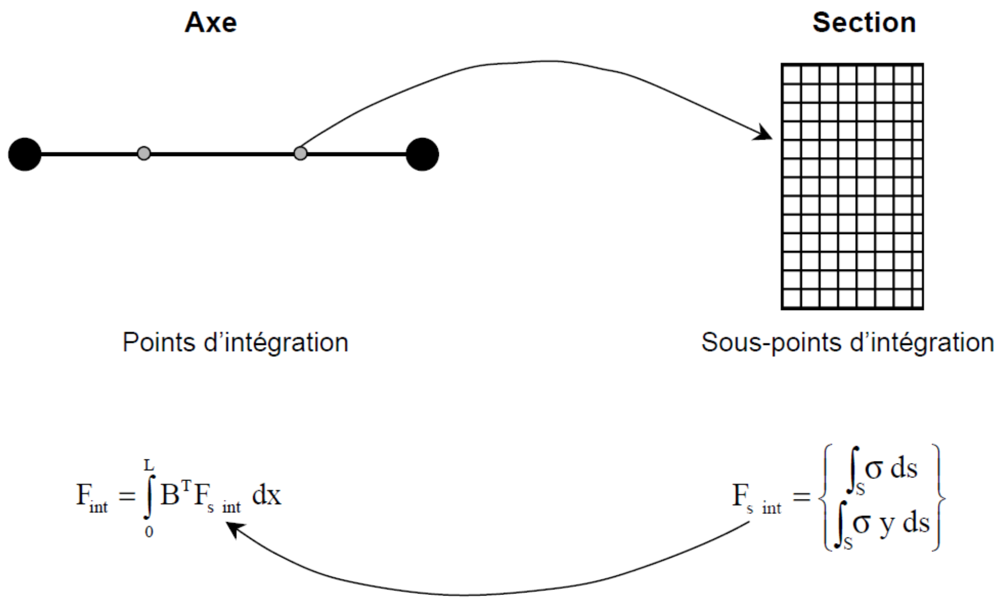

Figure 4.4-a : Multifibre beam – Calculation of $\mathbf{F}_{\mathrm{int}}$

$$\mathbf{F}_s^T = \left(N \quad M_y \quad M_z \quad M_x \right) \qquad \text{[éq 4.4-2]}$$

Normal effort NR and bending moments M_y and M_z are calculated by integration of the constraints on the section [éq 4.2.1-8].

Behaviour in torsion being supposed to remain linear, the torque is calculated with nodal axial rotations:

$$M_x = GJ_x \frac{\theta_{x2} - \theta_{x1}}{L} \qquad \text{[éq 4.4-3]}$$

The equation [éq 4.1-1] is integrated numerically:

$$\mathbf{F}_i = \int_0^L \mathbf{B}^T . \mathbf{F}_s \, dx = j \sum_{i=1}^{2} w_i \mathbf{B}(x_i)^T . \mathbf{F}_s(x_i) \qquad \text{[éq 4.4-4]}$$

The positions and weights of the points of Gauss as well as Jacobien are given in the paragraph [§4.2.5].

The analytical calculation of $B(x_i)^T . F_s(x_i)$ give:

$$\left[B(x_i)^T . F_s(x_i) \right]^T = \begin{bmatrix} -B_1 N & B_2 M_z & -B_2 M_y & 0 & B_3 M_y & B_3 M_z \\ B_1 N & -B_2 M_z & B_2 M_y & 0 & B_4 M_y & B_4 M_z \end{bmatrix} \qquad \text{[éq 4.4-5]}$$

where them B_i are given to the equation [éq 4.2.4-1].

4.5 Formulation enriched in deformation

With the interpolations of displacements of the equation [éq 4.1-1], the axial generalized deformation is constant and the curves are linear (see equations [éq 4.2.2-6], [éq 4.2.2-7] and [éq 4.2.5-3]):

$$\begin{cases} \varepsilon_s(x)=\dfrac{u_2-u_1}{L} \\ \chi_{ys}(x)=-\left(-\dfrac{6}{L^2}+\dfrac{12x}{L^3}\right)w_1+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{y1}-\left(-\dfrac{12x}{L^3}+\dfrac{6}{L^2}\right)w_2+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{y2} \\ \chi_{zs}(x)=\left(-\dfrac{6}{L^2}+\dfrac{12x}{L^3}\right)v_1+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{z1}+\left(-\dfrac{12x}{L^3}+\dfrac{6}{L^2}\right)v_2+\left(\dfrac{6x}{L^2}-\dfrac{4}{L}\right)\theta_{z2} \end{cases} \quad \text{[éq 4.5-1]}$$

If it there has not coupling between these two deformations (elastic case, with the average line of reference which passes by the barycentre of the section), that does not pose problems. But in the nonlinear case general, there is a shift of the neutral axis, and the terms K_{s12} and K_{s13} of $\mathbf{K}_s$ (equations [éq 4.2.1-4] and [éq 4.2.2-4]) are not worthless, there is coupling between the moments and the normal effort. There is then an incompatibility in the approximation of the axial deformations of a fibre:

$$\varepsilon=\varepsilon_s(x)-y\chi_{zs}(x)+z\chi_{ys}(x) \quad \text{[éq 4.5-2]}$$

A means of eliminating this incompatibility is to enrich the field by axial deformation:

$$\varepsilon_s(x) \mapsto \varepsilon_s(x)+\tilde{\varepsilon}_s(x) \ ; \ \tilde{\varepsilon}_s(x)=\alpha . G(x) \ ; \ G(x)=\frac{4}{L}-\frac{8x}{L^2} \quad \text{[éq 4.5-3]}$$

for $x\in\left[-\frac{L}{2},\frac{L}{2}\right]$

where $G(x)$ is an enriched deformation which derives from a function "bubble" in displacement and α the degree of freedom of enrichment. The variational base of such an enrichment is provided by the principle of Hu-Washizu [bib5] which can be presented same manner as the method of the incompatible modes [bib6].

4.5.1 Method of the incompatible modes

The regular field of generalized displacements $\mathbf{U}_s$ is defined by the equation [éq 4.1-1]. Generalized deformations $\mathbf{D}_s$ and generalized constraints $\mathbf{F}_s$ by the equation [éq 4.2.2-2].

The principle of Hu-Washizu consists in writing the weak form of the equilibrium equations, but also of the calculation of the deformations and the law of behavior, in projection on the three virtual fields (generalized displacements $\mathbf{U}_s^*$, generalized deformations $\mathbf{D}_s^*$ and generalized constraints $\mathbf{F}_s^*$):

$$\begin{cases} \displaystyle\int_0^L \frac{dU_s^*}{dx}\cdot F_s\,dx-W_{\text{ext}}=0 \\ \displaystyle\int_0^L \boldsymbol{F}_s^*\cdot\left(\frac{d\boldsymbol{U}_s}{dx}-\boldsymbol{D}_s\right)dx=0 \\ \displaystyle\int_0^L \boldsymbol{D}_s^*\cdot\left(\boldsymbol{F}_s-\boldsymbol{K}_s\cdot\boldsymbol{D}_s\right)dx=0 \end{cases} \quad \text{[éq 4.5.1-1]}$$

One introduces the enrichment of the real deformations, and one chooses to break up the virtual field of deformations into a "regular" part exit of the virtual field of displacements and an enriched part:

$$D_s=\frac{dU_s}{dx}+\bar{D}_s \quad D_s^*=\frac{dU_s^*}{dx}+\bar{D}_s^* \quad \text{[éq 4.5.1-2]}$$

One defers [éq 4.5.1-2has] in [éq 4.5.1-1B], which justifies "enrichment" by orthogonality:

$$\int_0^L \mathbf{F}_s^* . \mathbf{D}_s \, dx = 0$$ [éq 4.5.1-3]

The equation [éq 4.5.1-1C] breaks up into two since one has two independent virtual fields in [éq 4.5.1-2B]:

$$\int_0^L \frac{dU_s^*}{dx} . \left(F_s - K_s . D_s \right) dx = 0 \quad \int_0^L \bar{\mathbf{D}}_s^* \cdot \left(\mathbf{F}_s - \mathbf{K}_s \cdot \mathbf{D}_s \right) dx = 0$$ [éq 4.5.1-4]

Lastly, the method of the incompatible modes consists in choosing the orthogonal space of the constraints to the space of the enriched deformations, so that [éq 4.5.1-3] is automatically checked and [éq 4.5.1-4B] thus gives simply:

$$\int_0^L \bar{\mathbf{D}}_s^* . \mathbf{K}_s . \mathbf{D}_s \, dx = 0$$ [éq 4.5.1-5]

If one returns to the strong formulation of the law of behavior in [éq 4.5.1-4has] and [éq 4.5.1-5], the system [éq 4.5.1-1] becomes:

$$\int_0^L \frac{dU_s^*}{dx} . F_s \, dx - W_{ext} = 0 \quad \int_0^L \bar{\mathbf{D}}_s^* \cdot \mathbf{F}_s \, dx = 0 \quad \mathbf{F}_s = \mathbf{K}_s \cdot \mathbf{D}_s$$ [éq 4.5.1-6]

Note:

Here one enriches only the axial deformation by an element of beam of Euler-Bernoulli, with a continuous function, therefore $\bar{\mathbf{D}} = \left(\bar{\varepsilon}_s \quad 0 \quad 0 \quad 0 \right)^T$.

4.5.2 Digital establishment

From the finite elements point of view, one can write displacements and the deformations in matric form, with the enriched part:

$$\mathbf{B}_s = \mathbf{N} . (\mathbf{U}) + \mathbf{Q} . (\alpha) \quad \mathbf{D} = \mathbf{B} . (\mathbf{U}) + \mathbf{G} . (\alpha)$$ [éq 4.5.2-1]

where $\mathbf{N}$ and $\mathbf{B}$ are the classical matrices of the functions of interpolation and their derivative (see [éq 4.1-1] and [éq 4.2.2-7]) and:

$$\mathbf{Q} = \left(\frac{4x}{L} - \frac{4x^2}{L^2} \quad 0 \quad 0 \quad 0 \right)^T \text{ and } \mathbf{G} = \left(\frac{4}{L} - \frac{8x}{L^2} \quad 0 \quad 0 \quad 0 \right)^T$$ [éq 4.5.2-2]

Note:

$\mathbf{G}$ *was selected so that the element always passes the "patch test" (worthless deformation energy for a movement of solid)* : $\int_0^L \mathbf{G}(x) dx = \mathbf{0}$ [éq 4.5.2-3]

After classical handling of passage of continuous to discrete, the system of equations [éq 4.5.1-6], written for the whole of the structure, approximates itself by:

$$\begin{cases} A_{e=1}^{N_{elem}} \left(F_{\text{int}} - F_{ext} \right) = 0 \\ h_e = 0 \quad \forall e \in [1, N_{elem}] \end{cases}$$ [éq 4.5.2-4]

with:

$$\begin{cases} \mathbf{F}_{\text{int}} = \int_0^L \mathbf{B}^T . \mathbf{F}_s \, dx = \int_0^L \mathbf{B}^T . \mathbf{K}_s . \left(\mathbf{B} . \mathbf{U}_s + \mathbf{G} . \alpha \right) dx \\ \mathbf{F}_{ext} = \int_0^L \mathbf{N}^T . \mathbf{f} \, dx \\ h_e = \int_0^L \mathbf{G}^T . \mathbf{F}_s \, dx \end{cases}$$ [éq 4.5.2-5]

$A_{e=1}^{N_{elem}}$ indicate the assembly on all the elements of the grid; $\mathbf{f}$ is the axial loading distributed on the element beam. The system of equations [éq 4.5.2-4] is nonlinear, it is solved in an iterative way (see `STAT_NON_LINE`).

With the iteration $(i+1)$, with $\Delta\mathbf{U}^{(i)}=\mathbf{U}^{(i+1)}-\mathbf{U}^{(i)}$ and $\Delta\alpha^{(i)}=\alpha^{(i+1)}-\alpha^{(i)}$, the linearization of the system gives (iterations of correction of Newton):

$$\begin{cases} A_{e=1}^{N_{elem}}\left(\left(\mathbf{F}_{\text{int}}^{(i+1)}-\mathbf{F}_{ext}^{(i+1)}\right)+\mathbf{K}_e^{(i)}.\,\Delta\,\mathbf{U}^{(i)}+\mathbf{X}_e^{(i)}\Delta\,\alpha^{(i)}\right)=0 \\ h_e^{(i+1)}+\mathbf{X}_e^{(i)\,T}.\,\Delta\,\mathbf{U}^{(i)}+H_e^{(i)}\Delta\,\alpha^{(i)}=0 \qquad \forall\, e\in[1,\ldots,N_{elem}] \end{cases} \quad \text{[éq 4.5.2-6]}$$

with:

$$\begin{cases} \mathbf{K}_e^{(i)}=\int_L \mathbf{B}^T.\mathbf{K}_s^{(i)}.\mathbf{B}\;dx \\ \mathbf{X}_e^{(i)}=\int_L \mathbf{B}^T.\mathbf{K}_s^{(i)}.\mathbf{G}\;dx \\ H_e^{(i)}=\int_L \mathbf{G}^T.\mathbf{K}_s^{(i)}.\mathbf{G}\;dx \end{cases} \quad \text{[éq 4.5.2-7]}$$

The second equation of the system [éq 4.5.2-6] is local. It makes it possible to calculate the degree of freedom of enrichment α independently on each element. One calculates it by a local iterative method (iterations (j) for a displacement $d^{(i)}=\Delta\mathbf{U}^{(i)}$ fixed):

$$\alpha_{(j+1)}^{(i)}=\alpha_{(j)}^{(i)}-\left(H_{e(j)}^{(i)}\right)^{-1}h_{e(j)}^{(i)} \quad \text{[éq 4.5.2-8]}$$

Thus, when one converged at the local level, one a:

$$h_e(d^{(i)},\alpha^{(i)})=0 \quad \text{[éq 4.5.2-9]}$$

And one can operate a static condensation to eliminate α at the total level.

$$\mathbf{K}_e^{(i)}=\mathbf{K}_e^{(i)}-\mathbf{X}_e^{(i)}\left(H_e^{(i)}\right)^{-1}\mathbf{X}_e^{(i)\,T} \quad \text{[éq 4.5.2-10]}$$

From a practical point of view, this technique makes it possible to treat enrichment at the elementary level without disturbing the number of total degrees of freedom. It is established with the level of the elementary routine charged to calculate the options `FULL_MECA`, `RAPH_MECA` and `RIGI_MECA_TANG`.

Note:

- in the typical case exposed here, $H_e^{(i)}$ is a reality, therefore very easy to reverse.
- in the same way, h_e and α are also realities.
- The calculation of $K_e^{(i)}$ is explained in the paragraph §4.2.5, the other sizes of the equation [éq 4.5.2-8] are calculated according to the same technique.
- In the same way the calculation of $\mathbf{F}_{\text{int}}$ is explained in the paragraph § 4.4, h_e in the equation [éq 4.5.2-5] is calculated according to the same technique.

4.5.3 Taking into account of offsetting

For an elastic behavior, the enrichment of the axial deformations makes it possible to take account correctly coupling between the normal effort and the bending moments, and to return the answer of the beam independent of the position chosen for the reference axis (see keyword `COOR_AXE_POUTRE` in the operator `DEFI_GEOM_FIBRE`, [U4.26.01]). It thus makes it possible to treat the case of the offset beams.

The digital establishment of the enrichment of the axial deformations presented in paragraph 4.5.2 relates to nonlinear calculations with `STAT_NON_LINE` or `DYNA_NON_LINE`, for the options `FULL_MECA`, `RAPH_MECA` and `RIGI_MECA_TANG`. Indeed, determination of α is done by local iterations at the elementary level [éq 4.5.2-8].

In the case of calculations with the option RIGI_MECA (MECA_STATIQUE, calculation of modes, calculations nonlinear with prediction 'ELASTIC'), one can determine α explicitly according to offsettings of the elastic center compared to the reference axis:

$$e_y=\frac{\int_S Ey\,dS}{\int_S E\,dS}=\frac{K_{S13}}{K_{S11}} \text{ and } e_z=\frac{\int_S Ez\,dS}{\int_S E\,dS}=\frac{K_{S12}}{K_{S11}}$$

One can then treat static condensation to eliminate α on the level of the elementary matrix of rigidity. These calculations were done analytically and it matrix is established explicitly in *Code_Aster* (modification of about twenty terms if e_y and/or e_z are not worthless).

Since displacement is enriched, the matrix of mass (see §4.3) is modified. As for the matrix of rigidity, the terms modified if e_y and/or e_z are not worthless, were calculated analytically and programmed explicitly.

The enrichment of the deformations also modifies the calculation of the options DEGE_ELNO [éq 4.5.2-1] and EPSI_ELGA [éq 4.5-2].

4.6 Nonlinear models of behavior usable

The supported models are on the one hand the relations of behavior 1D of type VMIS_ISOT_LINE, VMIS_CINE_LINE, VMIS_ISOT_TRAC, CORR_ACIER and PINTO_MENEGOTTO [R5.03.09] for steels, in addition the model MAZARS_GC [R7.01.08] dedicated to the uniaxial behavior of the concrete into cyclic. One can thus have several materials by multifibre element of beam.

In addition, if the behavior used is not available in 1D, one can use the other laws 3D using the method of R.De Borst [R5.03.09]). For example, one can treat: GRAN_IRRA_LOG, VISC_IRRA_LOG. However in this case, one can treat one material by multifibre element of beam.

Note:

The internal, constant variables by fibre, are stored in the under-points attached to the point of integration considered.
The access to the postprocessing of the sizes defined in the under-points is done via format MED3.0, of Salomé.

5 Element multipoutre

The element multipoutre is a generalization of the multifibre element of beam presented to the preceding section. This element consists of an element having kinetics of beam, but where batches of fibres are gathered in bolsters. These bolsters themselves multifibre, like are shown with the figure 5-a. This element is only accessible within the framework from kinematics of beams of Euler.

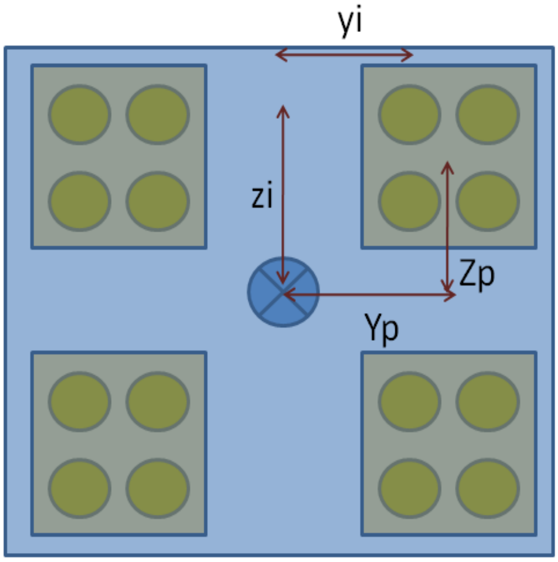

Figure 5-a : Example of a section multipoutre made up by 4 bolsters described by a rectangular section. In the example the bolsters consist of 4 fibres.

5.1 Element multipoutre of reference

The kinematics of the element is the same one as for the multifibre beams. Nevertheless, since the fibres are assembled in the form of bolsters, a field of local displacement (u_p, v_p, w_p) bound to the bolster p is calculated:

$$u_p(x,y,z)=u_0(x)-Y^p\theta_{sz}(x)+Z^p\theta_{sy}(x)$$
$$v_p(x,y,z)=v_0(x)-Z^p\theta_{sx}(x)+Y^p\theta_{sz}(x)$$
$$w_p(x,y,z)=w_0(x)+Y^p\theta_{sx}(x)-Z^p\theta_{sy}(x)$$

where the vector (u_0, v_0, w_0) is the field of displacement in the center of the section of the element and the couples (Y^p, Z^p) are the positions of the bolsters p in the section of the element. For all the bolsters, it is supposed that rotations θ_{sx} , θ_{sy} and θ_{sz} are constant in the section.

The field of local displacement to the bolster p is then employed in order to determine a local field of deformation for each bolster p as for the classical multifibre beams.

5.2 Internal stress analysis of the element

For all the bolsters p, a field of the generalized constraints is in the same way given that for the multifibre classics. One thus has, for a bolster p, the field of generalized efforts $\mathbf{F}_s^p$.

Moreover, for the classical multifibre beams, the torsion of the bolsters is considered linear and calculated independently

$$M_x^p=GJ_x^p\frac{\theta_{x2}-\theta_{x1}}{L} \quad \text{[éq 5.2-1]}$$

Effort with the nodes of the element associated with the bolster p also describes itself as for the multifibre beams:

$$\mathbf{F}_i^p=(F_x^p, V_y^p, V_z^p, M_x^p, M_y^p, M_z^p)^T=\int_0^L \mathbf{B}^T.\mathbf{F}_s^p\,dx=j\sum_{i=1}^{2} w_i\,\mathbf{B}(x_i)^T.\mathbf{F}_s^p(x_i)$$

The efforts in the bolsters are then assembled in order to describe the total effort in the element. The normal effort and the efforts cutting-edges are obtained by summoning the individual efforts on N_p bolsters:

$$\mathbf{F}_x=\sum_{p=1}^{N_p} F_x^p;\ \ \mathbf{V}_y=\sum_{p=1}^{N_p} V_y^p;\ \ \mathbf{V}_z=\sum_{p=1}^{N_p} V_z^p$$

For the moment, those are given according to the individual moments on all the bolsters as well as offsetting of the bolsters compared to the element. This offsetting leads to a coupling between torsion and shearings and also between the inflection and the normal efforts of the various bolsters p :

$$\mathbf{M}_x=\sum_{p=1}^{N_p} M_x^p+\sum_{p=1}^{N_p} V_z^p Y^p-\sum_{p=1}^{N_p} V_y^p Z^p;\ \ \mathbf{M}_y=\sum_{p=1}^{N_p} M_y^p+\sum_{p=1}^{N_p} F_x^p Z^p;\ \ \mathbf{M}_z=\sum_{p=1}^{N_p} M_z^p-\sum_{p=1}^{N_p} F_x^p Y^p$$

5.3 Determination of the matrix of mass of the multifibre element and formulation enriched in deformation

Calculation is done exactly as for the multifibre beams.

6 Case of application

One will be able usefully to consult the cases following tests:

- ssll111a: Static response of a reinforced concrete beam (section in T) to linear behavior thermoelastic, [V3.01.111].
- ssls143a: Beam cantilever with offset heart, [V3.03.143].
- sdll130b: Seismic response of a reinforced concrete beam (rectangular section) to linear behavior, [V2.02.130].
- sdll132a : Clean modes of a frame in multifibre beams; [V2.02.132].
- sdll150a: clean modes of a beam with offset heart, [V2.02.150].
- ssnl119a, ssnl119b: Static response of a reinforced concrete beam (rectangular section) to nonlinear behavior, [V6.02.119].
- sdnl130a: Seismic response of a reinforced concrete beam (rectangular section) to nonlinear behavior, [V5.02.130].
- ssll102j: Fixed beam subjected to unit efforts, [V3.01.102].
- ssnl106g, ssnl106h: Elastoplastic beam in traction and pure inflection, [V6.02.106].
- ssnl122a: Beam cantilever multifibre subjected to an effort [V6.02.122].
- ssnl504a: Beam of multifibre beams [V6.02.504].
- ssnl123a: Buckling of a beam multifibre [V6.02.123].

7 Bibliography

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8 Description of the versions of the document

Index document	Version Aster	Author (S) Organization (S)	Description of the modifications
With	6.4	**S.Moulin** (EDF-R&D/AMA), L.Davenne (ENSC/LMT),	Initial version
B	9.5	L.Davenne (ENSC/LMT), F.Voldoire (EDF-R&D/AMA)	Enrichment of the axial deformation by function bubble and static condensation into nonlinear, taken into account of torsion into linear, adaptation to the new structure of data `GROUP_FIBRE`, cf drives REX 9141. List of the cases of applications.
C	10	F.Voldoire (EDF-R&D/AMA)	Corrections of working Openoffice.
D	12	J-L.Fléjou	Addition incompatible Modes
E		D. Geoffroy	Addition of the multipoutre formulation