Element CABLE_GAINE

Summary:

The element CABLE_GAINE presented in this document is to model cables of prestressing being able to rub or slip into their sheaths and thus not following completely displacements of the concrete in which they are plunged. It thus comes in complement from modeling BAR allied with the operator DEFI_CABLE_BP who allows to model cables of prestressed completely adherent with the concrete (see [R7.01.02]).

This element is very strongly inspired by the element of interface describes in [R3.06.13]. It models the interface of the cable with its sheath and the cable itself.

Contents

1 Variational formulation

1.1 Potential energy and minimization

The potential energy, whose one will seek the local minimum, is written as the sum of the elastic deformation energy (in the cable), of cohesive energy (with the interface/cable sheaths) less the work of the external efforts:

$$E_{pot}(u)=E_{el}(u)+E_{gc}(u)-W_{ext}(u)$$

The expression of elastic energy is the following one:

$$E_{el}(u)=\int_{\Gamma}\Phi(\varepsilon(\boldsymbol{u}_{cable}(s)))ds$$

where Γ is the way (1D) cable, Φ is the density of energy elastic and $\boldsymbol{u}_{cable}$ is the displacement of the cable.

The displacement of the cable is defined as follows:

$$\boldsymbol{u}_{cable}(s)=\boldsymbol{u}_{gaine}(s)+g(s)\boldsymbol{T}(s)$$

where $\boldsymbol{T}$ represent the tangent with the cable, $\boldsymbol{u}_{gaine}$ the displacement of the sheath and g the relative displacement of the cable compared to the sheath.

There are then (mechanical cf Salençon "of the continuous mediums – Volume II: Elasticity – curvilinear Mediums"):

$$\varepsilon(\boldsymbol{u}_{cable}(s))=\frac{d(\boldsymbol{u}_{gaine}+g\boldsymbol{T})}{ds}\cdot\boldsymbol{T}=\frac{d(\boldsymbol{u}_{gaine})}{ds}\cdot\boldsymbol{T}+\frac{dg}{ds}$$

It is specified that the multiplication by the tangent vector with the cable $\boldsymbol{T}$ allows to pass from a vector "lengthening" according to the three directions of space to a scalar "lengthening" along the cable. In fact to obtain this expression one neglects $\dfrac{d\boldsymbol{T}}{ds}$.

The two following sizes are defined:

$$\varepsilon_{u_{gaine}}(\boldsymbol{u}_{gaine})=\frac{d(\boldsymbol{u}_{gaine})}{ds}\cdot\boldsymbol{T} \quad \text{and} \quad \varepsilon_g(g)=\frac{dg}{ds}$$

Thus the deformation of the cable is written finally:

$$\varepsilon(\boldsymbol{u}_{cable}(s))=\varepsilon_{u_{gaine}}(\boldsymbol{u}_{gaine})+\varepsilon_g(g)$$

Cohesive energy (sheath/cable) as for it is written:

$$E_{gc}(u)=\int_{\Gamma}\Pi(g(s))ds$$

where Π is the density of cohesive energy.

Taking into account nonthe derivability Π, one will use a method of decomposition-coordination to treat this minimization. That consists in introducing the slip δ, to pose $\delta(s)=g(s)$ and to be reduced to the problem of minimization under constraint according to:

$$\min_{\substack{u,\delta \\ g=\delta}} E(u,\delta)$$

with

$$E(u,\delta)=\int_\Gamma \Phi(\varepsilon(u_{cable}(s)))\,ds+\int_\Gamma \Pi(\delta(s))\,ds-W_{ext_{gaine}}(u_{gaine})-W_{ext_{cable}}(g)$$

That will make it possible to separately treat (decomposition) the minimization of cohesive energy at the local level (static condensation) whereas the minimization of elastic energy and work is treated at the total level.

1.2 Lagrangian increased

With the problem of minimization under constraints is dealt by dualisation: the Lagrangian one is introduced increased $\boldsymbol{L}$ and the field of multipliers λ (coordination):

$$\boldsymbol{L}(u,\delta,\lambda)=E(u,\delta)+\int_\Gamma \lambda(s)\cdot(g(s)-\delta(s))\,ds+\frac{r}{2}\int_\Gamma (g(s)-\delta(s))^2\,ds$$

with r coefficient of penalization.

1.3 Characterization of the point saddles

The conditions of optimality of order 1 make it possible to write:

$$\forall \delta u_{gaine}:\int_\Gamma \sigma:\varepsilon_{u_{gaine}}(\delta u_{gaine}(s))\,ds=W_{ext_{gaine}}(\delta u_{gaine}) \quad \text{with} \quad \sigma=\frac{\partial\Phi}{\partial\varepsilon}(\varepsilon) \qquad \text{[éq. 1.1]}$$

$$\forall \delta g:\int_\Gamma \sigma:\varepsilon_g(\delta g(s))\,ds+\int_\Gamma [\lambda(s)+r(g(s)-\delta(s))]\cdot\delta g(s)\,ds=W_{ext_{cable}}(\delta g) \qquad \text{[éq. 1.2]}$$

$$\forall \delta\lambda:\int_\Gamma [g(s)-\delta(s)]\cdot\delta\lambda(s)\,ds=0 \qquad \text{[éq. 1.3]}$$

$$\forall \delta\delta:\int_\Gamma [t(s)-\lambda(s)-r(g(s)-\delta(s))]\cdot\delta\delta(s)\,ds=0 \quad \text{with} \quad t\in\partial\Pi(\delta) \qquad \text{[éq. 1.4]}$$

2 Discretization of the finite element

Notice preliminary:
The finite element considered models the interface of the cable with its sheath and the cable itself. It would have been possible to model separately the interface cable-sheath (with a finite element of interface) and even cables it him (with a finite element of cable), nevertheless, as it thereafter will be seen the rubbing law of behavior requires the knowledge of the tension in the cable, it is thus advantageous to have in the same element of the interface and cable. Moreover, as it below will be seen, modeling suggested is pressed on quadratic elements, it is thus not possible to re-use the element `BAR` existing.

The finite element considered is linear (geometrical support in the form of segments), contrary to the elements of interface which are surface or voluminal.
It is considered to represent a cable plunged in a volume (of concrete) $3D$. Consequently, the degrees of freedom are:

- displacements of the sheath (3 degrees of freedom): u_{gaine}
- the relative longitudinal displacement of the cable in the sheath g
- and the multiplier of Lagrange λ .

In a way similar to the elements of interface, relative slip δ is discretized at the points of Gauss then eliminated by static condensation. The adopted interpolation is P2 for displacements (sheath and cable) and P1 for the multiplier of Lagrange. The geometric standards of support will be thus SEG3. The constraint $g=\delta$ is imposed on the weak direction.

The points of selected Gauss are the same ones as for the element of interface under-integrated, that is to say 2 points of integration (contrary to the elements of interface presented in [R3.06.13], the fact that the cable does not have that the longitudinal displacements controlled by the rigidity of the cable does not impose the use of an integration at 3 points of Gauss. This point was checked at the time as of tests). The following figure makes it possible to visualize all this information.

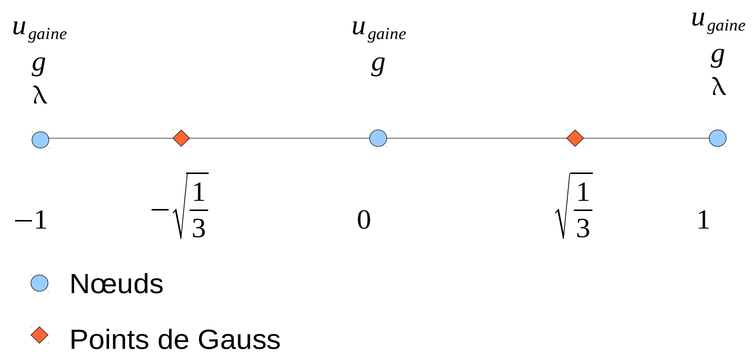

Figure 2-1: discretization of the element CABLE_GAINE

3 Static condensation

The field δ disappears from the total formulation thanks to static condensation. In each point of collocation s_g , one has (according to éq.1.4):

$$t_g=\lambda_g+r(g_g-\delta_g)\in\partial\Pi(\delta_g)$$

with $t_g=t(s_g)$, $\lambda_g=\lambda(s_g)$, $g_g=g(s_g)$ and $\delta_g=\delta(s_g)$.

Once the discretized problem, g_g and λ_g will be obtained by interpolations with the functions of forms appropriate of the discretized values to the nodes of g and λ that one notes $\{G\}$ and $\{\Lambda\}$.

The integration of the constitutive relation (cf below) makes it possible to calculate δ_g according to $\{G\}$ and $\{\Lambda\}$ that one notes δ :

$$t_g=\lambda_g+r(g_g-\delta_g)\in\partial\Pi(\delta_g)\Leftrightarrow\delta_g=\hat{\delta}(g_g,\lambda_g)=\delta(\{G\},\{\Lambda\})$$

4 Integration

The laws of friction considered are perfect adherence, the slip without friction and a friction of Coulomb with a threshold making it possible to find the profiles of tensions given by the BPEL (or by the ETCC since the 2 codes use equivalent formulas). These three cases are usable with the law of behavior `CABLE_GAINE_FROT`.

The addition of new laws of friction adapted to other regulation is possible.

Of a point of Gauss given, the integration of the laws consists in determining δ_g . For that, one seeks the point of intersection between the line representative of the equation $t_g=\lambda_g+r(g_g-\delta_g)$ (green curve on the figures Figure 4-1, Figure 4-2, Figure 4-3) and the curve representative of the derivative of $\Pi(\delta_g)$ (blue curve on the figures).

Note: in the paragraphs which follow, one adopts a notation on displacements not to rather weigh down the notations than on the increments of displacements as that is really the case.

4.1 Perfect adherence

In the first case, corresponding to perfect adherence, one a:

$$\Pi(\delta_g)=I_{R^+}(\delta_g)+I_{R^-}(\delta_g)$$

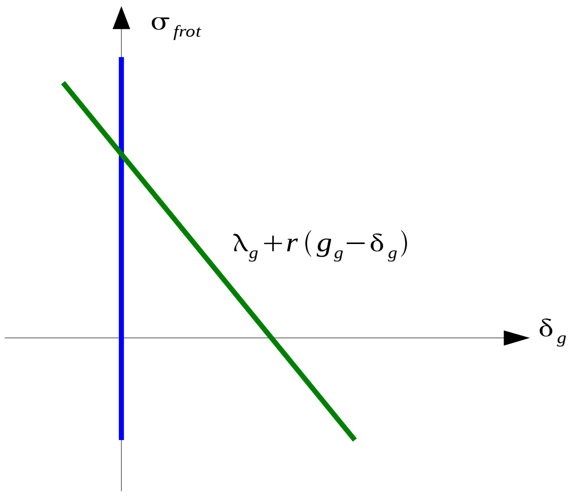

Figure 4-1: Graphic interpretation of the integration of the law of perfect adherence

The solution is: $\delta_g=0$

4.2 Slip without friction

In the second case, slip without friction, one a:

$$\Pi(\delta_g)=0$$

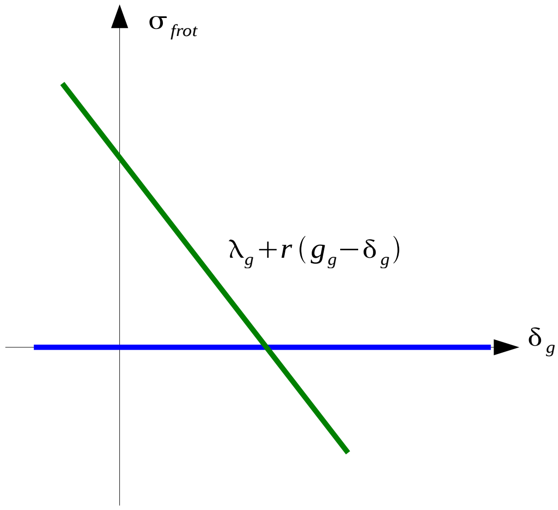

Figure 4-2: Graphic interpretation of the integration of the perfect law of slip

The solution checks:

$$0=\lambda_g+r(g_g-\delta_g)$$

from where

$$\delta_g=g_g+\frac{\lambda_g}{r}$$

4.2.1 Slip with friction (BPEL)

This law makes it possible to take into account rectilinear friction and curved friction. The tensions imposed by the BPEL are found by choosing the threshold of the law of Coulomb σ_c as follows:

$$\sigma_c=-(\varphi+f\frac{\partial\alpha}{\partial s})N$$

with the notations of [R7.01.02] §2.2.2 which one points out:

- f the coefficient of friction of the cable on the partly curved concrete, in rad^{-1} ,
- φ the coefficient of friction per unit of length, in m^{-1} ,
- α the cumulated angular deviation.

and with N normal effort.

Indeed, by considering that the cable slips over all its length at the time of the setting in tension and noting s the curvilinear X-coordinate along the cable, the balance of the cable is written:

$$\frac{dN}{ds}=-(\varphi+f\frac{\partial\alpha}{\partial s})N$$

from where by integration:

$$N=N_0\exp(-(\varphi s+f\alpha(s)))$$

One then recognizes the expression of the tension of the cable in the presence of rectilinear friction of the BPEL ([R7.01.02] §2.2.2).

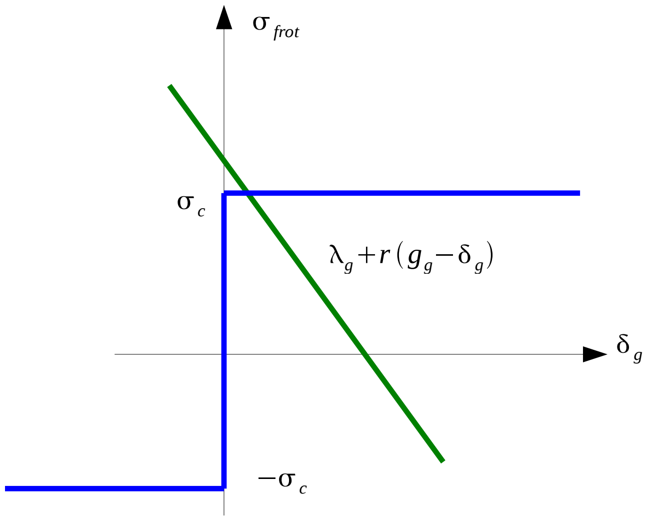

Figure 4-3: Graphic interpretation of the integration of the law of friction BPEL

To write the solution, there are 3 cases:

- if $|(\lambda_g+rg_g)|\leq\sigma_c$: $\delta_g=0$
- if $\lambda_g+rg_g>\sigma_c$: $\delta_g=g_g+\dfrac{\lambda_g-\sigma_c}{r}$
- if $\lambda_g+rg_g<-\sigma_c$: $\delta_g=g_g+\dfrac{\lambda_g+\sigma_c}{r}$

The case where normal effort N is negative does not have normally to occur for the problems of cables of prestressings. However it can sometimes happen to have values very slightly negative. One makes the choice to adopt the behavior slipping into such a case.

Notice 1 : by introducing such a dependence of the slip to the tension into the cable, the contribution of the variation of this tension to the slip $\dfrac{\partial\delta_g}{\partial N}$ induced a nonsymmetrical matrix.

Notice 2: So that friction curves is taken into account at the elementary level, it is imperative that the curve of the cable is retranscribed in the elements the component, i.e. the three nodes of meshs SEG3 should not be aligned.

5 Discretized writing

5.1 Notations

It is pointed out that the elements CABLE_GAINE have 3 nodes and that only the nodes ends have degrees of freedom of Lagrange (cf. Figure 2-1).

- The functions of form of displacements are noted n_m with $m\in\{1,2,3\}$.
- The functions of form of the multipliers of Lagrange are noted l_p with $p\in\{1,2\}$.

One notes (the quantities are evaluated in each point of Gauss):

- s_g the curvilinear X-coordinate
- ω_g the weight of the points of Gauss
- σ_g the tensor of the constraints
- N_g^3 matrix of the values of the functions of form at the point of Gauss g discretizing displacements of the sheath (3 components)

$$N_g^3=\begin{pmatrix} n_1(s_g) & 0 & 0 & n_2(s_g) & 0 & 0 & n_3(s_g) & 0 & 0 \\ 0 & n_1(s_g) & 0 & 0 & n_2(s_g) & 0 & 0 & n_3(s_g) & 0 \\ 0 & 0 & n_1(s_g) & 0 & 0 & n_2(s_g) & 0 & 0 & n_3(s_g) \end{pmatrix}$$

- N_g^1 matrix of the values of the functions of form at the point of Gauss g discretizing relative displacements of the cable (1 component):

$$N_g^1=\begin{pmatrix} n_1(s_g) & n_2(s_g) & n_3(s_g) \end{pmatrix}$$

- T_g the tangent vector at the point of Gauss g
- ∇N_g^1 the derivative of the functions of form discretizing relative displacements of the cable
- ∇N_g^3 the derivative of the functions of form discretizing displacements of the sheath
- One also poses to simplify the notations: $B_g=T_g^T\nabla N_g^3$
- L_g matrix of the values of the functions of form discretizing the multiplier of Lagrange at the point of Gauss $L_g=\begin{pmatrix} l_1(s_g) & l_2(s_g) \end{pmatrix}$

- $\{U\}$ nodal displacements (sheath + cable)
- $\{U_{ga}\}$ nodal displacements of the sheath
- $\{G_c\}$ the nodal relative displacement of the cable
- $\{\Lambda\}$ the multiplier of nodal Lagrange
- $\{F_{ext}^{ga}\}$ the vector forces external nodal, dual displacements of the sheath
- $\{F_{ext}^{c}\}$ the vector forces external nodal, dual nodal relative displacement of cable
- A the section of the cable

5.2 Discretization of the conditions of optimality

Equations 1.1,1.2 and 1.3 (the éq. 1.4 being treated with the level of the law of behavior) which characterizes the point saddle are discretized in the form:

$$\begin{aligned}
&\sum \omega_g(B_g^T:\sigma_g)=\{F_{ext}^{ga}\}\\
&\sum \omega_g[[\nabla N_g^1]^T:\sigma_g+[N_g^1]^T\cdot[[L_g]\{\Lambda\}+r[N_g^1]\{G_c\}-r\delta(\{G_c\},\{\Lambda\})]]=\{F_{ext}^{c}\}\\
&\sum \omega_g[[L_g]^T\cdot[[N_g^1]\{G_c\}-\delta(\{G_c\},\{\Lambda\})]]=\{0\}
\end{aligned}$$

5.3 Internal forces

The notations are introduced:

$$(f_u^g)=(B_g^T:\sigma_g)$$

$$(f^g_{g_c})=[\nabla N^1_g]^T:\sigma_g+[N^1_g]^T\cdot[[L_g]\{\Lambda\}+r[N^1_g]\{G_c\}-r\,\delta(\{G_c\},\{\Lambda\})]$$
$$(f^g_\lambda)=[L_g]^T\cdot[[N^1_g]\{G_c\}-\delta(\{G_c\},\{\Lambda\})]$$

Contributions to the forces, for $i\in\{1,2,3\}$, $m\in\{1,2,3\}$ and $p\in\{1,2\}$, are written then:

$$(f_u)_{i,m}=\sum_g \omega_g (f^g_{u_{gaine}})_{i,m}$$
$$(f_{g_c})_m=\sum_g \omega_g (f^g_{u_{cable}})_m$$
$$(f_\lambda)_p=\sum_g \omega_g (f^g_\lambda)_p$$

where

$(f^g_u)_{i,m}$ is the component $(3(m-1)+i)$ of (f^g_u) ,

$(f^g_{g_c})_m$ is the component m of $(f^g_{g_c})$ and

$(f^g_\lambda)_p$ is the component p of (f^g_λ)

5.4 Matrix of rigidity

One points out the notations in a point of Gauss:

- the relative displacement of the cable: $g_g=N^1_g\{G_c\}$
- deformation of the cable: $\varepsilon_g=B_g\{U_{ga}\}+(\nabla N^1_g)\{G_c\}$
- the multiplier of Lagrange: $\lambda_g=(L_g)\{\Lambda\}$

While posing $\tau_g=\lambda_g+rg_g$, the two paragraphs which follow give the contributions to the tangent matrix for the cases adherent and slipping and the rubbing case.

5.4.1 Case adherent and slipping

In the cases adherent and slipping, the tangent matrix is symmetrical (minimization of a point saddles):

$$K_{uu}=\sum_g \omega_g B^T_g \frac{d\sigma}{d\varepsilon} B_g$$
$$K_{ug_c}=(K_{g_c u})^T=\sum_g \omega_g B^T_g \frac{d\sigma}{d\varepsilon}(\nabla N^1_g)$$
$$K_{u\lambda}=(K_{\lambda u})^T=0$$
$$K_{g_c g_c}=\sum_g \omega_g[(\nabla N^1_g)^T \frac{d\sigma}{d\varepsilon}(\nabla N^1_g)+(N^1_g)^T r(N^1_g)-(N^1_g)^T r^2 \frac{d\delta_g}{d\tau_g}(N^1_g)]$$
$$K_{g_c\lambda}=(K_{\lambda g_c})^T=\sum_g \omega_g[(N^1_g)^T(L_g)+(N^1_g)^T r \frac{d\delta_g}{d\tau_g}(L_g)]$$
$$K_{\lambda\lambda}=\sum_g \omega_g[-(L_g)^T \frac{d\delta_g}{d\tau_g}(L_g)]$$

It is specified that $\frac{d\delta_g}{d\tau_g}$ is obtained starting from the law of behavior of friction.

5.4.2 Rubbing case

In the rubbing case, there is a dependence of the slip to the tension in the cable. following expressions are some modified:

$$K_{g_c u}=\sum_g \omega_g[(\nabla N_g^1)^T \frac{d\sigma}{d\varepsilon} B_g - r(N_g^1)^T \frac{\partial \delta_g}{\partial N} A \frac{\partial \sigma}{\partial \varepsilon} B_g]$$

$$K_{\lambda u}=\sum_g \omega_g[-(L_g)^T \frac{\partial \delta_g}{\partial N} A \frac{\partial \sigma}{\partial \varepsilon} B_g]$$

$$K_{g_c g_c}=\sum_g \omega_g[(\nabla N_g^1)^T \frac{d\sigma}{d\varepsilon}(\nabla N_g^1)+(N_g^1)^T r(N_g^1)-(N_g^1)^T r^2 \frac{d\delta_g}{d\tau_g}(N_g^1)-(N_g^1)^T r \frac{\partial \delta_g}{\partial N} A \frac{\partial \sigma}{\partial \varepsilon}(\nabla N_g^1)]$$

$$K_{\lambda g_c}=\sum_g \omega_g[(L_g)^T (N_g^1)+(L_g)^T r \frac{d\delta_g}{d\tau_g}(N_g^1)-(L_g)^T \frac{\partial \delta_g}{\partial N} A \frac{\partial \sigma}{\partial \varepsilon}(\nabla N_g^1)]$$

(The other expressions are unchanged compared to 5.4.1)
It is noted that the matrix is not symmetrical any more.

6 Convergence of calculation: adjustment of the parameters

The convergence of a calculation with the elements `CABLE_GAINE` is sometimes difficult to obtain taking into account as of different order of magnitude involved. This is why it is necessary well to choose the coefficient of penalization r and to prefer convergence criteria by values of reference (keyword `RESI_REFE_RELA`). The two following paragraphs give advices of use on these points.

6.1 Coefficient of penalization

At the time of the writing of Lagrangian increased, the coefficient r known as coefficient of penalization was introduced. To choose the value of r (`PENA_LAGR` in the cohesive law `CABLE_GAINE_FROT`), which one points out that the converged solution does not depend, or to clarify the construction of the force of reference used in the criterion `RESI_REFE_RELA` (see § according to), it is necessary to carry out a rapid analyzes dimensional.

For the choice of r, one sees in the preceding equations which it is judicious that it is of about size of $\frac{\lambda}{u}$.
u is the order of magnitude of expected displacements.
λ is the force of friction per i.e, unit of length the stress shear (friction) integrated on the perimeter of the cable. If a constraint is taken σ^{ref} typical waited in the concrete in the vicinity of the cable, λ will be of the order $2\pi r_{cable}\sigma^{ref}$.

To obtain best possible convergence, it is necessary to choose r of about size of $2\pi r_{cable}\frac{\sigma^{ref}}{u^{ref}}$.

6.2 Convergence criteria by values of reference

The use of convergence criteria per value is often necessary to go at the end of calculation.
For that it is necessary to activate the keyword `RESI_REFE_RELA` in the keyword factor `CONVERGENCE` of `STAT_NON_LINE` or `CALC_PRECONT`. In the case of the element `CABLE_GAINE`, this keyword must be accompanied by three others:

- a force of reference (keyword `EFFORT_REFE`): tension expected in the cable,
- a displacement of reference (keyword `DEPL_REFE`): a typical displacement of the structure,
- a constraint of reference (keyword `SIGM_REFE`): the order of magnitude of the constraints expected in the vicinity of the cable (the typical constraint of the concrete except typical case), this

constraint makes it possible to build a reference for λ by using the section A cable in the form: $\lambda^{ref}=\sigma^{ref}*\sqrt{A}$