Anisotropic elasticity

Summary

This document treats Lthermo hasanisotropic elasticity, used for modelings of continuous mediums 3D and 2D (C_PLAN, D_PLAN, AXIS), or layers of the composite hulls.

Medium thermorubber band can be anisotropic according to the 3 directions (one speaks oforthotropic elasticity), or in isotropic in two directions (one speaks about transverse isotropic elasticity), or it can have a cubic symmetry.

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Contents

1 Introduction

The objective of this document is to give the form of the matrices of flexibility and Hooke for materials thermorubber bands orthotropic, isotropic transverse and with cubic symmetry in the cases 3D, plane 2D-constraints, 2D - plane deformations and axisymetry.

In any rigour, for materials thermolinear rubber bands, the constraints are linear functions of the deformations and of the differential of temperature. One writes:

$$\sigma_{ij}=H_{ijkl}\left(\varepsilon_{kl}-\alpha_{kl}\left(T-T_{réf}\right)\right)$$

We speak about "matrices" of Hooke because, by preoccupation with a simplification, we did not adopt the notation of a tensor of order 4 for $\mathbf{H}$, with the profit of notation of W.Voigt, where tensors of order 2 are represented by vectors with 6 components and tensors of order 4 by matrices 6×6 .

The symmetrical nature of $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ and the adoption for these tensors of order 2 of a vectorial form makes it possible to write:

$$\{\boldsymbol{\sigma}\}=[\mathbf{H}]\left(\{\boldsymbol{\varepsilon}\}-\{\boldsymbol{\alpha}\}\left(T-T_{réf}\right)\right)$$

where $\{\boldsymbol{\sigma}\}$ and $\{\boldsymbol{\varepsilon}\}$ soNT the vectorial representation of the tensors of order 2 $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$, and where $[\mathbf{H}]$, matrices of Hooke, is a matrix 6×6 , necessarily symmetrical, that is to say a priori 21 independent coefficients, in the case Tridimensionnel:

$$[\mathbf{H}]=\begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ SYM & & & H_{44} & H_{45} & H_{46} \\ & & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{pmatrix}$$

L E v ector $\{\boldsymbol{\alpha}\}$ D ésigne the vectorial representation D U tensor of a nature 2 of thermal dilation, necessarily symmetrical, is a priori 6 independent coefficients, in the case T ridimensionnel .

2 Ttherepology of the matrices of Hooke

2.1 Orthotropism

It is about a situation where the elastic material displays two symmetries compared to two perpendicular plans (symmetry *orthorhombic*). LE tensor of elasticity have a priori 9 independent coefficients, consequence of the relations obtained with these two symmetries between the 21 coefficients.

LE tensor DE dilation thermal have a priori 3 independent coefficients, consequence of the relations obtained with these two symmetries.

In the axes of orthotropism:

$$[\mathbf{H}]=\begin{pmatrix} H_{11} & H_{12} & H_{13} & 0 & 0 & 0 \\ & H_{22} & H_{23} & 0 & 0 & 0 \\ & & H_{33} & 0 & 0 & 0 \\ SYM & & & H_{44} & 0 & 0 \\ & & & & H_{55} & 0 \\ & & & & & H_{66} \end{pmatrix} \quad (\boldsymbol{\alpha})=\begin{pmatrix} \alpha_{11} \\ \alpha_{22} \\ \alpha_{33} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2.2 Transverse isotropy

Transverse isotropy (or of revolution) ESt a restriction of the orthotropism in where one has the isotropy in one of the two orthogonal plans of elastic symmetry, following an invariance by rotation of $2\pi/3$ around the orthogonal axis with the transverse plan of isotropy for example $x_3=0$. LE tensor of elasticity have a priori 5 independent coefficients.

The matrix $[\mathbf{H}]$ the same form as for the orthotropism will have but with four additional relations between the components. Thus one will have thus for L' transverse isotropy in the plan $x_3=0$:

$$H_{11}=H_{22} \quad ; \quad H_{13}=H_{23} \quad ; \quad H_{44}=H_{55} \quad \text{and} \quad 2H_{44}=H_{11}-H_{12}$$ [éq 2.2-1]

LE tensor DE dilation thermal have a priori 2 independent coefficients:

$$\alpha_{11}=\alpha_{22}$$ éq 2.2-2]

2.3 Isotropy

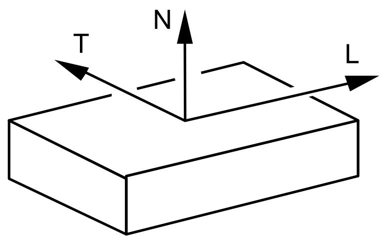
The material is isotropic if $[\mathbf{H}]$ remain invariant in any change of reference mark. LE tensor of elasticity have a priori 2 independent coefficients. There is only one thermal dilation coefficient

3 Matrix of Hooke and matrix of flexibility

3.1 Notations

Instead of using indices 1.2 and 3 to locate the axes Cartesian reference mark, one will use the corresponding indices L , T and N :

1. L for longitudinal
2. T for transverse
3. N for normal



The coefficients which intervene are the following:

Keyword	Notation	significance
E_L	E_L	Module DE Longitudinal Young
E_T	E_T	Module DE Transverse Young
E_N	E_N	Module DE Normal Young
G_LT	G_{LT}	Modulus of rigidity in the plan (L,T)
G_TN	G_{TN}	Modulus of rigidity in the plan (T,N)
G_LN	G_{LN}	Modulus of rigidity in the plan (L,N)
NU_LT	ν_{LT}	Poisson's ratio in the plan (L,T)
NU_TN	ν_{TN}	Poisson's ratio in the plan (T,N)
NU_LN	ν_{LN}	Poisson's ratio in the plan (L,N)
ALPHA_L	α_L	Coefficient of thermal dilation longitudinal means
ALPHA_T	α_T	Coefficient of thermal dilation means transverse
ALPHA_N R	α_N	Coefficient of thermal dilation means normal

Notice very important:

> ν_{LT} *is different from* ν_{TL} *:*
>
> *If one applies a traction along the axis* L *:*
>
> $\varepsilon_{LL}=\dfrac{\sigma_{LL}}{E_L}$ *(law of Hooke following a direction).*

This traction is accompanied, proportionally, of a contraction $-\nu_{LT}\cdot\dfrac{\sigma_{LL}}{E_L}$ along the axis T, and of a contraction $-\nu_{LN}\cdot\dfrac{\sigma_{LL}}{E_L}$ along the axis N.

The first index indicates the axis where the effect of the loading is exerted and the second index indicates the direction of the loading.

Then one exerts a traction along the axis T, then a traction according to N, one obtains:

$$\left.\begin{aligned}\varepsilon_{LL}&=\frac{\sigma_{LL}}{E_L}-\nu_{TL}\frac{\sigma_{TT}}{E_T}-\nu_{NL}\frac{\sigma_{NN}}{E_N}\\ \varepsilon_{TT}&=-\nu_{LT}\frac{\sigma_{LL}}{E_L}+\frac{\sigma_{TT}}{E_T}-\nu_{NT}\frac{\sigma_{NN}}{E_N}\\ \varepsilon_{NN}&=-\nu_{LN}\frac{\sigma_{LL}}{E_L}-\nu_{TN}\frac{\sigma_{TT}}{E_T}+\frac{\sigma_{NN}}{E_N}\end{aligned}\right\} \quad \text{[éq 3.1-1]}$$

The matrix of flexibility $[\mathbf{H}]^{-1}$ being symmetrical; one from of deduced:

$$\frac{\nu_{LT}}{E_L}=\frac{\nu_{TL}}{E_T} \quad ; \quad \frac{\nu_{LN}}{E_L}=\frac{\nu_{NL}}{E_N} \quad ; \quad \frac{\nu_{TN}}{E_T}=\frac{\nu_{NT}}{E_N}$$

3.2 Case 3D

3.2.1 Orthotropie

3.2.1.1 Matrix of flexibility

$$\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}=\begin{bmatrix}\frac{1}{E_L} & \frac{-\nu_{TL}}{E_T} & \frac{-\nu_{NL}}{E_N} & 0 & 0 & 0\\ \frac{-\nu_{LT}}{E_L} & \frac{1}{E_T} & \frac{-\nu_{NT}}{E_N} & 0 & 0 & 0\\ \frac{-\nu_{LN}}{E_L} & \frac{-\nu_{TN}}{E_T} & \frac{1}{E_N} & 0 & 0 & 0\\ & & & \frac{1}{G_{LT}} & 0 & 0\\ & SYM & & & \frac{1}{G_{LN}} & 0\\ & & & & & \frac{1}{G_{TN}}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}$$

$[\mathbf{H}]^{-1}$ – Orthotropism

3.2.1.2 Matrix of Hooke

$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{LL}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}=\begin{bmatrix}\frac{(1-\nu_{TN}\nu_{NT})}{\Delta\cdot E_T E_N} & \frac{(\nu_{TL}+\nu_{NL}\nu_{TN})}{\Delta\cdot E_T.E_N} & \frac{(\nu_{NL}+\nu_{TL}\nu_{NT})}{\Delta\cdot E_T.E_N} & 0 & 0 & 0\\ \frac{(\nu_{LT}+\nu_{LN}\nu_{NT})}{\Delta\cdot E_L E_N} & \frac{(1-\nu_{NL}\nu_{LN})}{\Delta\cdot E_L.E_N} & \frac{(\nu_{NT}+\nu_{NL}.\nu_{LT})}{\Delta\cdot E_L.E_N} & 0 & 0 & 0\\ \frac{(\nu_{LN}+\nu_{LT}.\nu_{TN})}{\Delta\cdot E_L.E_T} & \frac{(\nu_{TN}+\nu_{TL}.\nu_{LN})}{\Delta\cdot E_L.E_T} & \frac{(1-\nu_{LT}.\nu_{TL})}{\Delta\cdot E_L.E_T} & 0 & 0 & 0\\ & & & G_{LT} & 0 & 0\\ & SYM & & & G_{LN} & 0\\ & & & & & G_{TN}\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}$$

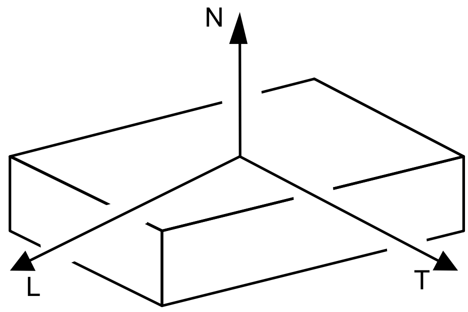
$[\mathbf{H}]$ – Orthotropism

with: $$\frac{\nu_{TL}}{E_T}=\frac{\nu_{LT}}{E_L} \quad ; \quad \frac{\nu_{NL}}{E_N}=\frac{\nu_{LN}}{E_L} \quad ; \quad \frac{\nu_{NT}}{E_N}=\frac{\nu_{TN}}{E_T}$$

with: $$\frac{1}{\Delta}=\frac{E_L E_T E_N}{1-\nu_{TN}\nu_{NT}-\nu_{NL}\nu_{LN}-\nu_{LT}\nu_{TL}-2\nu_{TN}\nu_{NL}\nu_{LT}}$$

3.2.2 Transverse isotropy

The transverse isotropy is here defined in the plan (L, T), and the direction of orthotropism is thus N. One can draw the attention of the reader to the fact that this convention differs from a usual convention which indicates by “longitudinal direction” the direction of orthotropism of isotropic transverse materials.



It is noted that the dilation coefficients check: $\alpha_T = \alpha_L$.

3.2.2.1 Matrix of flexibility

The matrix $[\mathbf{H}]^{-1}$ can be deduced directly from the matrix $[\mathbf{H}]^{-1}$ - Orthotropism by using the properties of the transverse isotropy.

In the plan (L, T), cf [éq 2.2-1] :

$$E_L = E_T$$
$$\nu_{TL} = \nu_{LT}$$
$$G_{LT} = \frac{E_L}{2(1+\nu_{LT})}$$

In the plans (L, N) and (T, N) :

$$\nu_{NT} = \nu_{NL}$$
$$\nu_{LN} = \nu_{TN}$$
$$G_{TN} = G_{LN}$$
$$\frac{\nu_{NT}}{E_N} = \frac{\nu_{LN}}{E_L}$$

$$\begin{bmatrix}\varepsilon_{LL}\\\varepsilon_{TT}\\\varepsilon_{NN}\\2\varepsilon_{LT}\\2\varepsilon_{LN}\\2\varepsilon_{TN}\end{bmatrix}=\begin{bmatrix}\frac{1}{E_L} & \frac{-\nu_{LT}}{E_L} & \frac{-\nu_{NL}}{E_N} & 0 & 0 & 0\\ \frac{-\nu_{TL}}{E_L} & \frac{1}{E_L} & \frac{-\nu_{NT}}{E_N} & 0 & 0 & 0\\ \frac{-\nu_{LN}}{E_L} & \frac{-\nu_{TN}}{E_L} & \frac{1}{E_N} & 0 & 0 & 0\\ & & & \frac{2(1+\nu_{LT})}{E_L} & 0 & 0\\ & SYM & & & \frac{1}{G_{LN}} & 0\\ & & & & & \frac{1}{G_{TN}}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\\sigma_{TT}\\\sigma_{NN}\\\sigma_{LT}\\\sigma_{LN}\\\sigma_{TN}\end{bmatrix}$$

$[\mathbf{H}]^{-1}$ - Transverse isotropy

3.2.2.2 Matrix of Hooke

The matrix $[\mathbf{H}]$ have same symmetries as $[\mathbf{H}]^{-1}$.

$$\begin{bmatrix}\sigma_{LL}\\\sigma_{TT}\\\sigma_{NN}\\\sigma_{LT}\\\sigma_{LN}\\\sigma_{TN}\end{bmatrix}=\begin{bmatrix}\frac{1-\nu_{NL}.\nu_{LN}}{\Delta'\cdot E_L.E_N} & \frac{\nu_{LT}+\nu_{NL}\nu_{LN}}{\Delta'\cdot E_L.E_N} & \frac{\nu_{NL}+\nu_{LT}\nu_{NL}}{\Delta'\cdot E_L.E_N} & 0 & 0 & 0\\ \frac{\nu_{TL}+\nu_{NL}\nu_{LN}}{\Delta'\cdot E_L.E_N} & \frac{1-\nu_{NL}.\nu_{LN}}{\Delta'\cdot E_L.E_N} & \frac{\nu_{LN}+\nu_{LT}\nu_{LN}}{\Delta'\cdot E_L.E_N} & 0 & 0 & 0\\ \frac{\nu_{LN}+\nu_{LT}.\nu_{LN}}{\Delta'\cdot E_L^2} & \frac{\nu_{TN}+\nu_{LT}.\nu_{TN}}{\Delta'\cdot E_L^2} & \frac{1-\nu_{LT}^2}{\Delta'\cdot E_L^2} & 0 & 0 & 0\\ & & & \frac{E_L}{2(1+\nu_{LT})} & & \\ & & & & G_{LN} & \\ & & & & & G_{LN}\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\\varepsilon_{TT}\\\varepsilon_{NN}\\2\varepsilon_{LT}\\2\varepsilon_{LN}\\2\varepsilon_{TN}\end{bmatrix}$$

$[\mathbf{H}]$ – Transverse isotropy

hasvec: $\frac{1}{\Delta'}=\frac{E_L^2.E_N}{1-2\nu_{NL}\cdot\nu_{LN}-\nu_{LT}^2-2\nu_{NL}\cdot\nu_{LN}\cdot\nu_{LT}}$

3.2.3 Elasticity with symmetry cubic

Elasticity with cubic symmetry occurs when in addition to the three Cartesian plans of symmetry, six plans turned with 45° are also of symmetry . There are then 3 coefficients élasti ques independent. That corresponds to a matrix of elasticity of the form:

$$\begin{bmatrix} H_{1111} & H_{1122} & H_{1122} & & & \\ H_{1122} & H_{1111} & H_{1122} & & & \\ H_{1122} & H_{1122} & H_{1111} & & & \\ & & & H_{1212} & & \\ & & & & H_{1212} & \\ & & & & & H_{1212} \end{bmatrix}$$

Being given cubic symmetry, it remains to determine 3 coefficients:

$$E_L = E_N = E_T = E, \quad G_{LT} = G_{LN} = G_{TN} = G, \quad \nu_{LN} = \nu_{LT} = \nu_{LN} = \nu$$

It is noted that the dilation coefficients check: $\alpha_T = \alpha_L = \alpha_N$.

To reproduce elasticity with symmetry cubic with ELAS_ORTH, it is enough to calculate the coefficients of the orthotropism such that the matrix of elasticity obtained is form above:

$$H_{1111} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$
$$H_{1122} = \frac{\nu E}{(1+\nu)(1-2\nu)}$$
$$H_{1212} = G_{LT} = G_{LN} = G_{TN}$$

therefore, as long as $(1+\nu)(1-2\nu) \neq 0$ (i.e. ν different from 0.5).

$\frac{H_{1122}}{H_{1111}} = \frac{\nu}{1-\nu}$ what provides $\nu = \frac{1}{1+\frac{H_{1111}}{H_{1122}}}$ then $E = H_{1111} \frac{(1+\nu)(1-2\nu)}{(1-\nu)}$

3.2.4 Isotropy

The law of Hooke takes the following shape with the coefficients of Lamé λ and $\mu = G$:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2 \mu \varepsilon_{ij}$$

3.2.4.1 Matrix of flexibility according to E and ν

$$\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}=\begin{bmatrix}\frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0\\ & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0\\ & & \frac{1}{E} & 0 & 0 & 0\\ & & & \frac{1}{G}=\frac{2(1+\nu)}{E} & 0 & 0\\ & SYM & & & \frac{1}{G}=\frac{2(1+\nu)}{E} & 0\\ & & & & & \frac{1}{G}=\frac{2(1+\nu)}{E}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}$$

$[\mathbf{H}]^{-1}$ – Complete isotropy

3.2.4.2 Matrix of Hooke according to E and ν

$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ & 1-\nu & \nu & 0 & 0 & 0\\ & & 1-\nu & 0 & 0 & 0\\ & SYM & & \frac{1-2\nu}{2} & 0 & 0\\ & & & & \frac{1-2\nu}{2} & 0\\ & & & & & \frac{1-2\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}$$

$[\mathbf{H}]$ – Complete isotropy

3.2.4.3 Matrix of flexibility according to the coefficients of Lamé λ and μ

$$\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}=\begin{bmatrix}\frac{\lambda+\mu}{\mu(3\lambda+2\mu)} & \frac{-\lambda}{2\mu(3\lambda+2\mu)} & \frac{-\lambda}{2\mu(3\lambda+2\mu)} & 0 & 0 & 0\\ & \frac{\lambda+\mu}{\mu(3\lambda+2\mu)} & \frac{-\lambda}{2\mu(3\lambda+2\mu)} & 0 & 0 & 0\\ & & \frac{\lambda+\mu}{\mu(3\lambda+2\mu)} & 0 & 0 & 0\\ & & & \frac{1}{\mu} & 0 & 0\\ & SYM & & & \frac{1}{\mu} & 0\\ & & & & & \frac{1}{\mu}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}$$

$[\mathbf{H}]^{-1}$ – Complete isotropy

3.2.4.4 Matrix of Hooke according to the coefficients of Lamé λ and μ

$$\begin{bmatrix}\sigma_{LL}\\\sigma_{TT}\\\sigma_{NN}\\\sigma_{LN}\\\sigma_{LT}\\\sigma_{TN}\end{bmatrix}=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ & \lambda+2\mu & \lambda & 0 & 0 & 0\\ & & \lambda+2\mu & 0 & 0 & 0\\ & SYM & & \mu & 0 & 0\\ & & & & \mu & 0\\ & & & & & \mu\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\\varepsilon_{TT}\\\varepsilon_{NN}\\2\varepsilon_{LN}\\2\varepsilon_{LT}\\2\varepsilon_{TN}\end{bmatrix}$$

$[\mathbf{H}]$ – Isotropy supplements with the coefficients of Lamé

3.3 Orthotropic in plane deformations and axisymmetric case 2D

3.3.1 Matrix of flexibility

$$\begin{bmatrix}\varepsilon_{LL}\\\varepsilon_{TT}\\0\\2\varepsilon_{LT}\end{bmatrix}=\begin{bmatrix}\frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & -\frac{\nu_{NL}}{E_N} & 0\\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & -\frac{\nu_{NL}}{E_N} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{LT}}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\\sigma_{TT}\\\sigma_{NN}\\\sigma_{LT}\end{bmatrix}$$

$[\mathbf{H}]^{-1}$ – Orthotropism planes in plane deformations and axisymetry

3.3.2 Matrix of Hooke

$$\begin{bmatrix}\sigma_{LL}\\\sigma_{TT}\\\sigma_{NN}\\\sigma_{LT}\end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix}\frac{(1-\nu_{TN}\nu_{NT})}{E_T E_N} & \frac{(\nu_{TL}+\nu_{NL}\nu_{TN})}{E_T.E_N} & \frac{(\nu_{NL}+\nu_{TL}\nu_{NT})}{E_T.E_N} & 0\\ \frac{(\nu_{LT}+\nu_{LN}\nu_{NT})}{E_L E_N} & \frac{(1-\nu_{NL}\nu_{LN})}{E_L.E_N} & \frac{(\nu_{NT}+\nu_{NL}.\nu_{LT})}{E_L.E_N} & 0\\ \frac{(\nu_{LN}+\nu_{LT}.\nu_{TN})}{E_L.E_T} & \frac{(\nu_{TN}+\nu_{TL}.\nu_{LN})}{E_L.E_T} & \frac{(1-\nu_{LT}.\nu_{TL})}{E_L.E_T} & 0\\ 0 & 0 & 0 & G_{LT}\cdot\Delta\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\\varepsilon_{TT}\\0\\2\varepsilon_{LT}\end{bmatrix}$$

$[\mathbf{H}]$ – Orthotropism planes in plane deformations and axisymetry

with: $$\frac{1}{\Delta}=\frac{E_L E_T E_N}{1-\nu_{TN}\nu_{NT}-\nu_{NL}\nu_{LN}-\nu_{LT}\nu_{TL}-2\nu_{TN}\nu_{NL}\nu_{LT}}$$

3.4 Orthotropic case 2D in plane constraints

3.4.1 Matrix of flexibility

$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\,\varepsilon_{LT} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \end{bmatrix}$$

$[\mathbf{H}]^{-1}$ – Orthotropism planes in plane constraints

3.4.2 Matrix of Hooke

By using the system of equations [éq 3.1-1], one obtains:

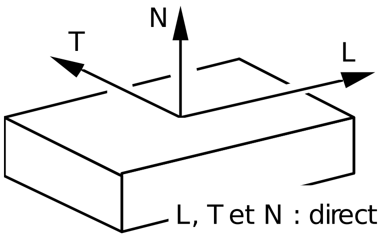
$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ 0 \\ \sigma_{LT} \end{bmatrix} = \frac{1}{1-\nu_{LT}.\nu_{TL}} \begin{bmatrix} E_L & \nu_{TL} E_T & 0 & 0 \\ \nu_{LT} E_L & E_T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{LT} \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\,\varepsilon_{LT} \end{bmatrix}$$

$[\mathbf{H}]$ – Orthotropism in plane constraints

4 Use in *Code_Aster*

In Code_*Aster*, the definition of the constant orthotropic elastic characteristics or functions of the temperature is carried out by the order `DEFI_MATERIAU`, keywords `ELAS_ORTH`, `ELAS_ISTR`, `ELAS_ISTR_FO` or `ELAS_ORTH_FO` for the elements of hull and the solid elements isoparametric or the layers constitutive of a composite (see the order `DEFI_COMPOSITE`).

To define the reference mark of orthotropism (L, T, N) bound to the elements, one can refer to documentations [U4.42.03] `DEFI_COMPOSITE` and [U4.42.01] `AFFE_CARA_ELEM`.



/ `ELAS_ORTH = _F (`♦ `E_L = ygl` Module DE Longitudinal Young.
♦ `E_T = ygt` Module DE Transverse Young.
◊ `E_N = ygn` Module DE Normal Young.
♦ `GL_T = glt` Modulus of rigidity in the plan LT.
◊ `G_TN = gtn` Modulus of rigidity in the plan TN.
◊ `G_LN = gln` Modulus of rigidity in the plan LN.
♦ `NU_LT = nult` Coefficient of Fish in the plan LT.
◊ `NU_TN = nutn` Coefficient of Fish in the plan TN.
◊ `NU_LN = nuln` Coefficient of Fish in the plan LN.
◊ `ALPHA_L = dilN` Coefficient of thermal dilation longitudinal means.
◊ `ALPHA_T = known as` Coefficient of thermal dilation means transversal.
`ALPHA_ NR = di N` Coefficient of thermal dilation means N orm al.

Notice important:

The talk of this note of reference is based on the convention of the books of J.L.Batoz and D.Gay. The documentation of `DEFI_MATERIAU` [U4.43.01] described these choices, and the coefficient `NU_LT` be interpreted in the following way in Code_Aster:

if one exerts a traction according to the axis L *causing a deformation according to this axis equalizes with* $\varepsilon_L = \frac{\sigma_L}{\mathrm{ygl}}$ *, there is a deformation according to the axis* T *equalize with:* $\varepsilon_T = -\nu_{\mathrm{LT}} \cdot \frac{\sigma_L}{\mathrm{ygl}}$.

5 Bibliography

1) J.C. MASSON: Matrix of Hooke for orthotropic materials, Report interns Applications in Mechanics, n°79-018, CiSi, 1979.

2) D. GAY: Composite materials, Hermes Edition, 1987.

3) J.L. BATOZ, G. DHATT: Modeling of the structures by finite elements, Volume 1, Hermes Edition.

6 Description of the versions of the document

Version Code_Aster	Author (S) Organization (S)	Description of the modifications
6.4	A. ASSIRE, EDF-R&D/AMA	Initial text
8.4	A. ASSIRE, X. DESROCHES, J.M. PROIX, EDF-R&D/AMA	Tiny corrections
15	F.VOLDOIRE, EDF-R&D/ERMES	Corrections (for a better comprehension)