Finite elements in acoustics

Summary:

This document describes in low frequency stationary acoustics the equations used, the variational formulations which result from this as well as the translation corresponding in finite elements using a classical formulation to an unknown factor p (acoustic pressure). This document corresponds to PHENOMENE=' ACOUSTIQUE'.

Contents

1 Introduction

An option of modeling was developed in *Code_Aster*, allowing to study the low frequency stationary acoustic propagation, in closed medium, for fields of propagation to complex topology, i.e. to solve there under the quoted conditions the equation of Helmholtz.
To know the ways of propagation of energy in the fluid, the acoustics expert has the active acoustic intensity I :

$$I=\frac{1}{2}\mathrm{Re}\left[p\,v^*\right] \quad (1)$$

And of the reactive acoustic intensity J :

$$J=\frac{1}{2}\mathrm{Im}\left[p\,v^*\right] \quad (2)$$

Where v^* indicate the combined complex one vibratory speed. The knowledge of these sizes brings a very important further information in the resolution of problems of all kinds, for example the measurement of the powers radiated by the machines, the recognition and the localization of the sources.

2 Equations and boundary conditions of the problem

2.1 Equations and boundary conditions

The equation to be solved is the equation of Helmholtz [bib2]:

$$(\Delta+k^2)\,p=s \tag{3}$$

p is a complex size indicating the acoustic pressure and s , also complex, represents the sources terms of the problem. The parameter k indicate the number of wave of with the dealt problem; it can be complex or real, according to whether the propagation is carried out or not in a porous field [bib6]:

$$k=\frac{\omega}{c} \tag{4}$$

With c indicating the speed of sound, which can be complex in the case of a propagation in porous environment and ω is a reality in all the cases, which indicates the pulsation such as:

$$\omega=2\pi f \tag{5}$$

f is the work frequency of the harmonic problem. We represent on the figure 2.1-1 the unspecified confined field where the equation of Helmholtz applies (3) and conditions at the borders. Ω is open limited $\mathbb{R}^3$ of border $\partial\Omega$ regular, partitionnée in $\partial\Omega_v$ and $\partial\Omega_z$:

$$\partial\Omega=\partial\Omega_v\cup\partial\Omega_z \tag{6}$$

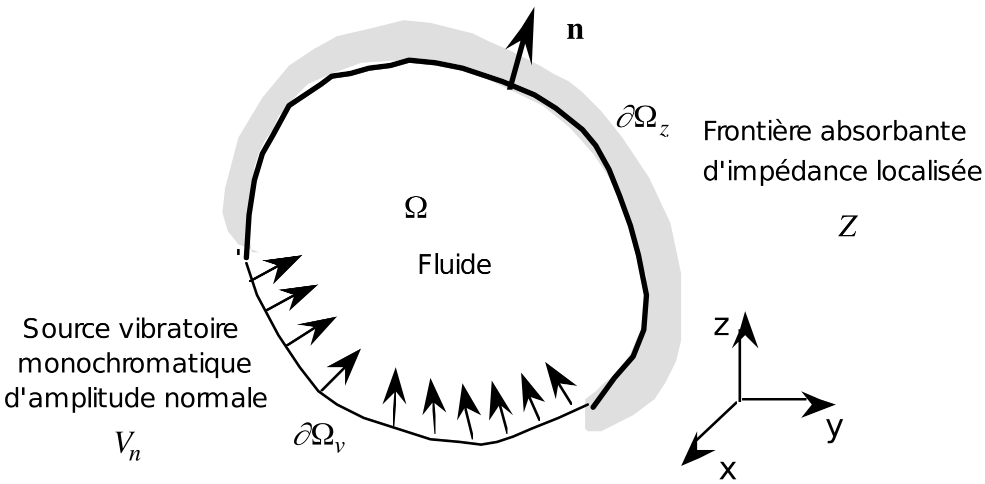

Figure 2.1-1: Configuration of the problem

The equation (3) is to be solved in a closed field Ω . Boundary conditions to take into account on the border $\partial\Omega$ field Ω express themselves in their most general form like:

$$\alpha\, p+\beta\frac{\partial p}{\partial \boldsymbol{n}}=\gamma \tag{7}$$

Classically, $\partial/\partial \boldsymbol{n}$ appoint the operator of normal derivative. α , β and γ are complex operators, who can be scalars, or integral operators according to whether the border of application of the boundary condition is with local reaction or nonlocal reaction (case of the interaction fluid-structure).

Developments currently carried out in *Code_Aster* concern only boundary conditions with local reaction, for which α , β and γ are scalars; the cases spécifiables are the following:

- The case $\alpha=0$, $\beta\neq 0$ and $\gamma\neq 0$ indicate a border of the field at imposed vibratory speed. Indeed, there exists a relation connecting the acoustic gradient of pressure complexes at the complex particulate vibratory speed. I.e.:

$$\frac{\partial p}{\partial \boldsymbol{n}}=-j\omega\rho_0 V_n \quad (8)$$

Where ρ_0 indicate the density of the fluid considered, and one imposes V_n , normal vibratory speed with the wall ($V_n=\boldsymbol{v}\cdot\boldsymbol{n}$ where $\boldsymbol{n}$ indicate the unit vector of the normal external with the border $\partial\Omega$).

- The case $\alpha\neq 0$, $\beta\neq 0$ and $\gamma=0$ relate to a border with acoustic impedance Z imposed. Acoustic impedance Z is defined like the report of the pressure at the particulate vibratory speed in the vicinity of the wall with imposed impedance:

$$Z=\frac{p}{V_n} \quad (9)$$

- The case $\alpha\neq 0$, $\beta=0$ and $\gamma\neq 0$ represent the case where the acoustic pressure is imposed p at a border (generally $\gamma=0$, corresponding to $p=0$).

3 Formulation in pressure

3.1 Mathematical expression of the problem

One uses the standard procedure aiming at posing the problem with the classical finite elements. One supposes the sufficiently regular solution of the problem, $p \in H^2(\Omega)$. One multiplies the equation of Helmoltz (3) without source term s by a function test φ . One integrates on Ω and the formula of Green is used. According to (6), the border $\partial\Omega$ field Ω , subdivides itself in two zones, a zone at imposed vibratory speed, $\partial\Omega_v$ and a zone with imposed acoustic impedance, $\partial\Omega_Z$. The equation obtained can be rewritten in the form:

$$\int_{\Omega} \boldsymbol{grad}(p) \cdot \boldsymbol{grad}(\varphi) dV - \int_{\Omega} \frac{\omega^2}{c^2} p \varphi dV + j \int_{\partial\Omega_Z} \frac{\rho_0 \omega}{Z} p \varphi dS + j \int_{\partial\Omega_v} \rho_0 \omega V_n \varphi dS = 0 \quad (10)$$

Where dV represent an element of differential volume in Ω and dS represent an element of surface on $\partial\Omega$. Particulate vibratory speed is then determined by:

$$\boldsymbol{v} = \frac{j}{\rho_0 \omega} \boldsymbol{grad}(p) \quad (11)$$

3.2 Discretization by finite elements

In the case of the classical finite elements, the elementary integrals are $\boldsymbol{K}^e$, $\boldsymbol{M}^e$, $\boldsymbol{C}^e$ and $\boldsymbol{S}^e$ according to the decomposition indicated by (14) ($\boldsymbol{K}^e$ is the matrix of rigidity, $\boldsymbol{M}^e$ the matrix of mass, $\boldsymbol{C}^e$ the matrix of damping and $\boldsymbol{S}^e$ the vector source). Two of them come from voluminal integrals, the two others are the result of integrals respectively on a vibrating surface and a surface with imposed impedance. It will be supposed that the total coordinates of an element can be written thanks to the data of m elementary functions of form H_i :

$$\boldsymbol{OM}^e = \sum_{i=1}^{m} N_i \boldsymbol{OM}_i^e \quad (12)$$

One is given moreover, of the basic functions N_i , to describe the elementary pressure. The pressure inside an element will be able to be written:

$$p^e(x, y, z) = \sum_{i=1}^{m} N_i p_i^e \quad (13)$$

Where p_i^e is the pressure with the node i element e . In the case of isoparametric finite elements, basic functions N_i are equal to the functions of form H_i . On each element of the field, the problem with the finite elements in pressure is written:

$$\left(\boldsymbol{K}^e - \omega^2 \boldsymbol{M}^e + j \omega \boldsymbol{C}^e\right) \boldsymbol{p}^e = -j \omega \boldsymbol{S}^e \quad (14)$$

Where $\boldsymbol{p}^e$ is the vector-column of the nodal values of the pressure on the element.

3.2.1 The matrix of rigidity

The matrix of rigidity $\boldsymbol{K}^e$ Correspond with the calculation of the first term of (10), that is to say:

$$\int_{\Omega^e} \left(\boldsymbol{grad}(p) \cdot \boldsymbol{grad}(\varphi)\right) dV \quad (15)$$

She admits like term general:

$$K_{ij}^e = \int_{\Omega^e} \left(\nabla N_i \nabla N_j\right) . dV \quad (16)$$

3.2.2 The matrix of mass

The matrix of rigidity $\boldsymbol{M}^e$ corresponds to the calculation of the second term of (10), that is to say:

$$M^{e}_{ij}=\int_{\Omega^e}\frac{1}{c^2}\,p\,\varphi\,dV \tag{17}$$

She admits like term general:

$$M^{e}_{ij}=\int_{\Omega^e}\frac{1}{c^2}N_i N_j.\,dV \tag{18}$$

3.2.3 The matrix of damping

The matrix of damping $\boldsymbol{C}^e$ Correspond with the calculation of the third term of (10), that is to say:

$$C^{e}_{ij}=\int_{\partial\Omega^e_Z}\frac{\rho_0}{Z}\,p\,\varphi\,dS \tag{19}$$

She admits like term general:

$$C^{e}_{ij}=\int_{\partial\Omega^e_Z}\frac{\rho_0}{Z}N_i N_j\,dS \tag{20}$$

3.2.4 The vector source

The vector source $\boldsymbol{S}^e$ Correspond with the calculation of the last term of (10), that is to say:

$$S^{e}_{i}=\int_{\partial\Omega_v}\rho_0 V_n\varphi\,dS=0 \tag{21}$$

He admits like term general:

$$S^{e}_{i}=\int_{\partial\Omega^e_v}\rho_0 V_n N_i\,dS \tag{22}$$

4 Orders specific to acoustic modeling

At the time of a study by modeling in acoustic finite elements with *Code_Aster* one uses general orders and orders which are specific to acoustics, or whose keywords and options are particular with this discipline; we present the list below of it.

4.1 Definition of the characteristics of the propagation mediums

It is necessary to give the density (actual value) and the celerity of propagation (complex value); one uses for that the order `DEFI_MATERIAU` with the following keywords:

keyword factor:	`FLUID`	
keywords:	`RHO`	(density ρ_0)
	`CELE_C`	(celerity c)

Example:

```
air = DEFI_MATERIAU (FLUIDE=_F (RHO= 1.3, CELE_C = ('IH', 343.0, 0. ,))
```

In this case: $\rho_0 = 343$ is real (the imaginary part is worthless)

4.2 Boundary conditions

One must assign values normal vibratory speed per face (or edge into two-dimensional) to the meshs defining the borders sources, and also values of acoustic impedance per face (edge into two-dimensional) with the meshs defining the borders in imposed impedance. One uses the order specific to acoustics `AFFE_CHAR_ACOU` with the following keywords:

Keyword:	`MODEL`	
Keyword factor:	`VITE_FACE`	
Keyword:	`MESH`	
	`GROUP_MA`	
	`VNOR`	(normal vibratory speed V_n)
Keyword factor:	`IMPE_FACE`	
Keyword:	`MESH`	
	`GROUP_MA`	
	`IMPE`	(acoustic impedance Z)
Keyword factor:	`PRES_IMPO`	
	`NODE`	
	`GROUP_NO`	
	`NEAR`	(pressure p imposed on the nodes)

4.3 Calculation of the elementary matrices

The various elementary matrices (rigidity, mass and damping) are calculated by specific options. The order is employed `CALC_MATR_ELEM` with the keyword `OPTION` for which one specifies the possible values of assignment:

Keywords:	`OPTION`	`'RIGI_ACOU'`
		`'MASS_ACOU'`
		`'AMOR_ACOU'`

Note:

The assembled matrices can be obtained directly with the macro order `ASSEMBLY` and same options.

4.4 Calculation of the elementary vector source

The elementary vector is calculated by a specific option; the loading should obligatorily be indicated. The order is employed `CALC_VECT_ELEM` with the keyword `OPTION` for which one specifies the only value of possible assignment:

Keywords:	`OPTION`	`'CHAR_ACOU'`
Keywords:	`LOAD`	

4.5 Calculation of the solution

After assembly of the elementary matrices and vector the harmonic solution can be calculated directly with the order `DYNA_VIBRATED`.

4.6 Postprocessings

Starting from the result of the resolution, orders of postprocessing make it possible to obtain the nodal fields of following acoustic sizes:

- Level L_p of acoustic pressure p in dB : $L_p = 20\log_{10}\left[\frac{|p|}{2{,}0\times 10^{-5}}\right]$
- real part of the acoustic pressure
- imaginary part of the acoustic pressure
- active acoustic intensity $I = \frac{1}{2}\text{Re}\left[p\, v^*\right]$
- reactive acoustic intensity $J = \frac{1}{2}\text{Im}\left[p\, v^*\right]$

These fields are calculated by the use of the ordering of postprocessing `CALC_CHAMP` (the concept of the result is of the type `'ACOU_HARMO'` or 'MODE_ACOU') with the keywords `RESULT` and `OPTION` for which one specifies the possible values of assignment:

Keyword:	`RESULT`		
Keyword:	`ACOUSTICS`	`'PRAC_ELNO'`	(pressure level in decibels) (real part of the pressure) (imaginary part of the pressure)
		`'INTE_ELNO'`	(intensity activates) (intensity reactivates)

5 Conclusion

A modeling was thus added in *Code_Aster*, allowing to do calculations of interior acoustics in low frequencies for complex geometries by finite elements acoustic classics. The formulation was validated by comparison with an analytical solution; cases tests are presented in the handbook of V7 validation under coding AHLV100.

6 Bibliography

1) A. BOUIZI: 'Put in work of a computer code of finite elements for treatment of the equation of Helmholtz in closed space' - Work of end of studies, E.C.L. 1986.

2) A. BOUIZI: 'Spectral analysis of the equation of Helmholtz.' - Report of DEA, Central School of Lyon, 1986.

3) A. BOUIZI: 'Mixed finite elements in stationary linear acoustics: Development of code AIRMEF' - Acoustic Department. DER - EDF. HE - 24/88.02. 1988.

4) A. BOUIZI, MR. COURTADE, D. JEANDEL, E. LUZZATO, A. MIGNOT, C. SURRY. : 'Conditions of compatibility of Brezzi-Babuska for methods finite elements mixed in

conformity in Mechanics and Acoustique' AUM, Acts of the 8th French Congress of Mechanics, Nantes, 1987.

5) A. BOUIZI, MR. COURTADE, D. JEANDEL, E. LUZZATO, C. SURRY: 'Treatment of the equation of Helmholtz by a code of mixed finite elements in closed space' GAMI, Conference Vibrations Shocks, 1988, ECL, 1988.

6) A. BOUIZI: 'Resolution of the equations of linear Acoustics by a method finite elements mixed'. Thesis presented in front of the Central School of Lyon - Speciality: Mechanics -. Supported the 3/2/89.

7) C. HABASQUE: 'Experimental validation of the code computer of acoustics internal, Low frequency'. Report of internship of DEA. ECL 1986 (+ Project of end of studies).

8) A.ADOBES: 'Digital and experimental study of the fields of standing waves' Ratio DER/EDF - HE-2287.22

9) F. STIFKENS, A.ADOBES: 'Assessment of the integration of the classical finite elements in Aster' - Ratio DER/EDF - HP-64/91.149

10) F. STIFKENS: 'Integration of the mixed acoustic finite elements in *Aster*' Ratio DER/EDF – HP-61/92.081

7 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
3	F.STIFKENS EDF-R&D/AMV	Initial text
13.2	M.Abbas EDF R & D – AMA	Suppression of the mixed formulation