Beam élasto-acoustics

Summary:

One presents an element of coupling élasto - acoustic right which applies to an element of structure of type beam of Timoshenko. This element makes it possible to realize, in vibro - acoustic, the modal analysis of a right piping containing of the fluid under pressure (water, vapor…). One can also carry out calculations of answer to fluid sources (flow masses, volume flow rate, pressure) by modal recombination. The boundary conditions applicable to the nodes of these elements are of Dirichlet type: displacement, pressure or potential can be imposed there.

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1 Notations

P	:	instantaneous total pressure in a point of the fluid
p	:	pressure realised on a cross-section
$\tilde{p}$	:	fluctuating pressure
$\mathbf{u}$	:	displacement of the structure
Φ	:	potential of displacements of the fluid
ρ_0	:	density of the acoustic fluid
ρ_s	:	density of the structure
ω, f	:	pulsation, frequency
c	:	speed of sound in the fluid
λ, k	:	number, wavelength of wave
s	:	tensor of the constraints of the structure
$\boldsymbol{\varepsilon}$	:	tensor of the structural deformations
dV	:	element of volume
dA	:	element of surface
Σ	:	surface of interaction enters piping and the fluid
S_f	:	cross-section of the fluid
S_s	:	cross-section of piping

2 Introduction

In order to be able to carry out calculations of dynamic response of structures filled of fluid to fluids, elements of coupling fluid-structure 3D were developed in *Code_Aster* (*cf* [bib2]).

These voluminal elements have the advantage of allowing a fine description of the structure into cubes particular places like, for example, connection between a principal piping and a pricking of instrumentation. On the other hand, their systematic use for the analysis of ramified and complex networks would lead to costs of modeling (realization of grid) and of calculation prohibitory.

For this reason, and in order to facilitate simplified studies of dynamic behavior of pipings, one developed an element of beam right élasto-acoustics allowing to realize, with lower costs of calculation and labour, of calculations of overall behavior of the right parts of pipings in low frequency.

One finds hereafter a presentation of the finite elements of pipings of type acoustic beam élasto -. The vibratory behavior of the networks of pipings is conditioned by the flow of the fluid which traverses them.

3 The model of beam élasto-acoustics

3.1 Assumptions

One studies low frequency the vibrations of a piping elastic, linear homogeneous and isotropic coupled to a compressible fluid.

The effects due to viscosity and the flow of the fluid are neglected.

Pipings are lengthened bodies. Indeed, their transverse dimensions are much lower than their length: $D \ll L$, and the thicknesses are such as one can neglect the modes of swelling and ovalization of the pipe. One can use a model of beam.

Low frequency the acoustic wavelengths associated with the studied problems are large compared to transverse and small dimensions compared to the longitudinal dimension of the circuit: $\omega . L/c > 1$ and $\omega . D/c \ll 1$. Compressibility acts indeed mainly on longitudinal displacements. Transversely, it is considered that the fluid moves like an indeformable solid, i.e. it acts like an added mass. The pressure in a cross-section of the pipe being then constant, one says that the acoustic wave is plane.

3.2 Functional calculus of the coupled problem

One can write the variational formulation of the problem of the pipings filled with fluid starting from and the behavior equilibrium equations of the fluid and the pipe as well as boundary conditions. From the general functional calculus of the three-dimensional coupled problem ([bib1], [bib2]), one can write the functional calculus applied to the typical case of the beams.

The variational formulation of the 3D problem amounts minimizing the functional calculus:

$$F(u, p, \Phi) = \frac{1}{2}\left\{\int_{\Omega_s}\left[\boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{u}) - \rho_s \omega^2 \mathbf{u}^2\right] dV + \int_{\Omega_f}\left[\frac{P^2}{\rho_0 c^2}\left(\frac{2\mathrm{P}\,\Phi}{\rho_0 c^2} - (\mathrm{grad}\,\Phi)^2\right)\right] dV\right\} - \rho_0 w^2 \int_S \Phi\, \mathbf{u} . \mathbf{n}\, dA$$

with:

- Ωs , the field of the structure
- Ωf , the field of the fluid
- Σ , the fluid surface of interaction - structure.

3.2.1 Contribution of piping

The model of beam used is that of Timoshenko with deformations of shearing action and inertia of rotation of the cross section. It corresponds to modeling `POU_D_T` it takes again elementary calculations. One does not take into account the effects of ovalization [bib3].

The terms associated with piping in the variational formulation are written then:

$$\int_L \left[\boldsymbol{\sigma}(u) : \boldsymbol{\varepsilon}(u) - \rho_s \omega^2 u^2\right] S_s\, ds$$

with: L , the average fibre of piping and S_s , the section of piping to the X-coordinate s (cf [Figure 3.2.1-a]).

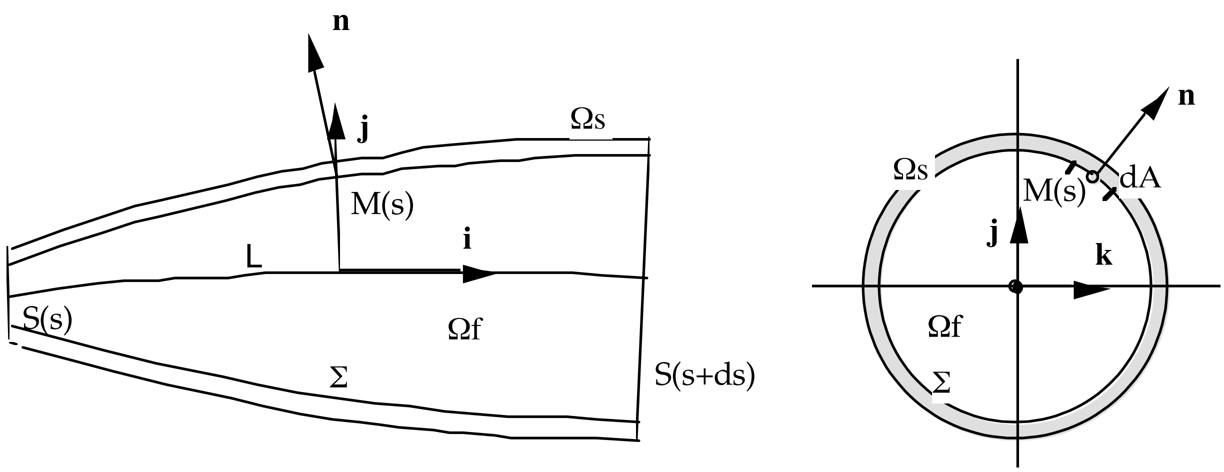


Figure 3.2.1-a: geometry of piping

3.2.2 Contribution of the fluid

In this paragraph, one is interested only in the fluid part of the functional calculus, i.e., term of coupling put except for, at the end which is written in 3D:

$$\int_{\Omega_f}\left[\frac{P^2}{\rho_0 c^2}-\rho_0\omega^2\left(\frac{2\mathrm{P}\Phi}{\rho_0 c^2}-\left(\mathrm{grad}\,\Phi\right)^2\right)\right]dV \qquad \text{éq 3.2.2-1}$$

It is supposed that the pressure breaks up into two terms:

$$P\left(M(s),t\right)=p(s,t)+\tilde{p}\left(M(s),t\right)$$

where p is the value realised on a cross-section of the pressure:

$$p(s,t)=\frac{1}{S_f(s)}\int_{S_f(s)}P\left(M(s),t\right)dM$$

and $\tilde{p}$ is a term of fluctuating pressure which corresponds to the contribution of the transverse modes.

According to the assumptions of the paragraph [§1], p check the equation 1-D of Helmholtz and $\tilde{p}$ the equation of Laplace (incompressible). The integral [éq 3.2.2-1] thus breaks up into two terms.

3.2.2.1 Term corresponding to the contribution of $\tilde{p}$

In the movements perpendicular to the axis of the pipe, one considers that the fluid intervenes only by its added mass [bib4], the term related to $\tilde{p}$ is thus a term of inertia:

$$\int_L \rho_0 \omega^2 (\mathbf{u_t})^2 S_f \, ds$$

$\mathbf{u_t}$ being transverse components of the vector displacement of the structure and S_f the section of the fluid to the X-coordinate s.

3.2.2.2 Term corresponding to the contribution of p

$$\int_L \left[\frac{P^2}{\rho_0 c^2} - \rho 0 \, \omega^2 \left(\frac{2\mathrm{P}\Phi}{\rho_0 c^2} - \left(\frac{\partial \Phi}{\partial s} \right)^2 \right) \right] S_f \, ds$$

3.2.3 Term of coupling

3.2.3.1 Current section

According to the references [bib4] and [bib5], one shows that the term of coupling C:

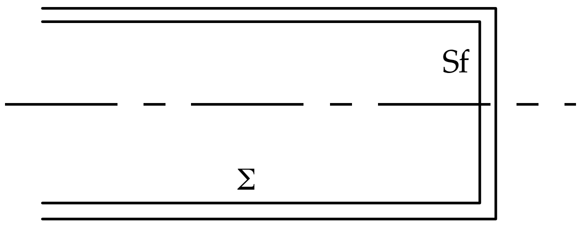
$$C = -\int_S \rho_0 \Phi \, \mathbf{u} \, . \, \mathbf{n} \, dA = \int_L -\frac{\rho_0 \Phi}{R} \mathbf{u} \, . \, \mathbf{j} S_f \, ds + \int_L \rho_0 \Phi \, \mathbf{u} \, . \, \mathbf{i} \frac{dS_f}{ds} ds$$

The equilibrium equations of the structure and the equation of propagation plane waves (Helmholtz) in the fluid are thus coupled with the level of the bent parts and the right parts where there is a change of section of piping. In the case of a pipe with constant cross-section:

$$R \to \infty \text{ and } \frac{dS_f}{ds} = 0 \text{ thus } C = 0$$

There is thus no coupling between the movements of beam of the structure and longitudinal displacements of the fluid in the right parts of the circuit. In this case, the fluid is characterized only by its added mass related to transverse displacements.

3.2.3.2 Fund of pipe



In the case of a bottom of pipe, one notes $\Sigma t = \Sigma + S_f$, the entire surface of intEraction enters the fluid and piping.
In the case of a right piping with closed constant section, the term of coupling C is worth then:

$$C = -\int_{S+S_f} \rho_0 \Phi \, \mathbf{u} \, . \, \mathbf{n} \, dA = -\int_{S_f} \rho_0 \Phi \, \mathbf{u} \, . \, \mathbf{n} \, dA$$

It is the basic effect.

This term is added at the end of coupling of a current section. Thus, a free node which carries out a condition of null flow through the section [bib6] carries out an acoustic basic condition. Indeed, an incidental plane wave is completely considered on the bottom: the acoustic pressure in the conduit obeys the equation of Helmoltz with normal gradient of pressure no one (fluid displacements and solid being worthless).

$$\begin{cases} \dfrac{\partial^2 p}{\partial x^2}+k^2 p=0 \\ (\dfrac{\partial p}{\partial x})_{S_f}=0 \end{cases}$$

One seeks the solution in the form: $p=A\cos(\omega t-kx)+B\cos(\omega t+kx)$
i.e. in the form of a linear combination of an incidental plane acoustic wave and a considered wave. The condition of null gradient on the bottom, checked for every moment, imposes:

$$A=B$$

The considered wave is thus “equal” to the incidental wave (coefficient of reflection equal to the unit).

3.2.4 Functional calculus of the system coupled in the case of pipings

In the typical case that we treat of a not bent piping, with constant section, the functional calculus of the coupled problem is thus written in the following form:

$$F(u,p,\Phi)= \frac{1}{2}\left\{\int_L \boldsymbol{\sigma}(\mathbf{u}):\boldsymbol{\varepsilon}(\mathbf{u})\,S_s\,ds-\omega^2\int_L\left[\rho_s S_s u^2+\rho_0 S_f(\mathbf{u_t})^2\right]ds \right.$$
$$\left. +\int_L\left[\frac{P^2}{\rho_0 c^2}-\rho_0\omega^2\left(\frac{2\mathrm{P}\Phi}{\rho_0 c^2}-\left(\frac{\partial\Phi}{\partial\boldsymbol{\sigma}}\right)^2\right)\right]S_f\,ds\right\}-\omega^2\int_{S_f}\rho_0\Phi\,\mathbf{u}.\mathbf{n}\,dA$$

3.3 Discretization by finite elements

The solution (u,p,Φ) sought the functional calculus minimizes F . The approximation by finished parts of the complete problem leads then to the symmetrical system:

$$\begin{bmatrix}\mathbf{K} & 0 & 0\\ 0 & \dfrac{\mathbf{K_f}}{\rho_0.c^2} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ p\\ \Phi\end{bmatrix}-\omega^2\begin{bmatrix}\mathbf{M}+\mathbf{M_f} & 0 & \mathbf{M_\Sigma}\\ 0 & 0 & \dfrac{\mathbf{M_{fl}}}{c^2}\\ \mathbf{M_\Sigma^T} & \dfrac{\mathbf{M_{fl}^T}}{c^2} & \rho_0.\mathbf{H}\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ p\\ F\end{bmatrix}=0$$

$\mathbf{K}$ and $\mathbf{M}$ being respectively matrices of rigidity and mass of the structure,

$\mathbf{K_f}$, $\mathbf{M_{fl}}$, $\mathbf{H}$ being fluid matrices, respectively obtained starting from the quadratic forms:

$$\int_L p^2 S_f\,ds,\int_L p\Phi S_f\,ds,\int_L\left(\frac{\partial\Phi}{\partial s}\right)^2 S_f\,ds$$

$\mathbf{M_f}$ being the fluid matrix obtained starting from the quadratic form: $\int_L p_0 S_f(\mathbf{u_t})^2\,ds$

$\mathbf{M_\Sigma}$ being the matrix of coupling obtained starting from the bilinear form: $\int_{S_f} p_0\Phi\,\mathbf{u}\,\mathbf{n}\,dA$.

While discretizing linearly p and Φ , one thus has:

$p=p_1\frac{L-x}{L}+p_2\frac{x}{L}\text{et}\,\Phi=\Phi_1\frac{L-x}{L}+\Phi_2\frac{x}{L}$, L being the length of the element considered.

In this case, the elementary matrix of stiffness of the fluid is written:

$$\mathbf{K_f}=\frac{S_f L}{3}\{p_1\,p_2\}\begin{bmatrix}1 & 1/2\\ 1/2 & 1\end{bmatrix}\begin{Bmatrix}p_1\\ p_2\end{Bmatrix}$$

The elementary matrix of coupling is written:

$$\mathbf{M_\Sigma}=r_0 S_f\{\Phi_1\Phi_2\}\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix}$$

The various matrices of fluid mass elementary are written:

$$\mathbf{M_\Sigma}=r_0 S_f\{\Phi_1\Phi_2\}\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{Bmatrix}p_1\\ p_2\end{Bmatrix}$$
$$\mathbf{M_\Sigma}=-\frac{S_f}{L}\{\Phi_1\Phi_2\}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}\Phi_1\\ \Phi_2\end{Bmatrix}$$

3.4 Establishment in *Code_Aster*

On the principles which we have just described, an acoustic element of beam vibro -, Timoshenko for the part piping, right to constant section or variable (in this case, only the circular sections are authorized), was established in *Code_Aster*. It belongs to modeling 'FLUI_STRU' phenomenon 'MECHANICAL'.

This element has 8 degrees of freedom per node: displacements and rotations of piping, pressure and potential of displacement of the fluid (cf [Figure 3.4-a]). The formulation is written for displacements **buildings** in the local reference mark with the element made up of neutral fibre (axis X) and of the main axes of inertia (axis Y , axis Z) section. Two scalars p and Φ (pressure and potential of fluid displacements) are invariants by change of reference mark.

On each node of this element, one can impose boundary conditions of the Dirichlet type in pressure, potential of fluid displacements and displacements (translation or rotation).

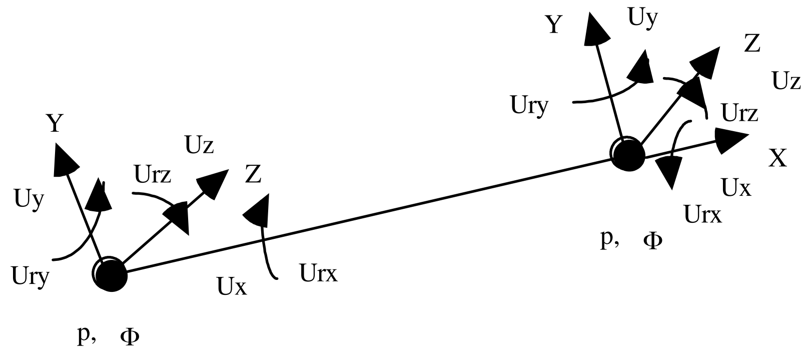


Figure 3.4-a: element of beam filled with fluid

This element to date makes it possible only to calculate the clean modes of a right piping filled with fluid and to do harmonic calculation of answer. The effects of curve or abrupt widening of section are not taken into account for the moment, but these effects fluid - structure, when one deals with not very dense fluids like the vapor of a piping of admission, do not seem to have a determining importance on the calculation of the first modes: the correct mechanical representation of the elbow (coefficient of flexibility) seems sufficient to calculate these frequencies [bib7].

In modal analysis, one can quote the case of a right piping filled with fluid with loose lead:

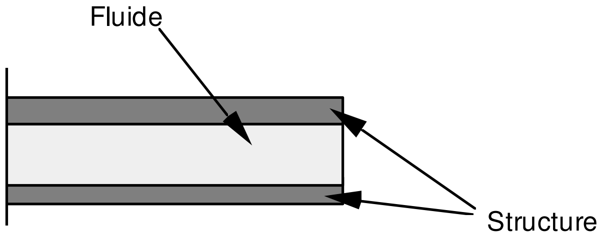


Figure 3.4-b: beam filled with fluid embedded - free

the Eigen frequency of the mode of traction and compression of this fluid coupled system/structure is given by the relation:

$$\mathrm{tg}\left(\frac{\omega L}{c}\right)=\sqrt{\frac{S_s}{S_f}\frac{E}{\rho_0 c^2}}$$

One indicates by:

- E : Young modulus of solid material
- S_S : section of the solid
- S_f : section of the fluid

It is supposed here that the speed of speed of sound in the fluid is equal to the speed of sound in the solid $c_S=\sqrt{\frac{E}{\rho_S}}$ [bib7].

The transitory calculation of answer for this kind of finite element $(\mathbf{u}, p, \varphi)$ is not yet available in *Code_Aster*.

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5 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
3	*G. ROUSSEAU, Fe WAECKEL (EDF/EP/AMV)*	Initial text