Fluid coupling - Structure with free surface

Summary:

One has here the fluid coupling/structure if the fluid has a free surface. Elements of free surface were established in *Code_Aster* to calculate the modes of ballotement of a fluid coupled to an elastic structure for a three-dimensional problem.

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1 Notations

P	:	stationary pressure in the fluid
p	:	fluctuating pressure in the fluid,
$\mathbf{X_f}$	:	displacements in the fluid,
$\mathbf{X_s}$	:	the field of displacements in the structure,
g	:	gravity,
φ	:	potential of displacements of the fluid,
ρ_f , ρ_s	:	density of the fluid, the structure,
$\mathbf{T}$	:	the tensor of the constraints in the fluid,
$\boldsymbol{\sigma}$	:	the tensor of the constraints in the structure,
$\boldsymbol{\varepsilon}$	:	the tensor of the deformations in the structure,
$\mathbf{C}$	:	the tensor of elasticity,
c	:	the speed of sound in the fluid,
H	:	the height fluid (or average height),
$\mathbf{n}$	:	the normal external of the fluid.

2 Introduction

In order to study the behavior of structures filled of fluid, one can be resulted in taking into account the phenomena of shaking i.e. to add to the fluid system coupled - structure, the effect of gravity on the level of the free surface of the liquid. The structures concerned are, for example, the tanks of nuclear power plants of the fast reactor core, the swimming pools of fuel storage [bib4].

One thus supplemented the developments already carried out in coupling fluid-structure [bib3] by the introduction of new surface elements which take into account, in their formulation, the effect of gravity.

3 Theoretical formulation of the problem

The problem of interaction heavy structure-fluid amounts solving three problems simultaneously:

- the structure is subjected to a field of pressure P imposed by the fluid on the wall Σ ;
- the fluid is subjected to a field of displacement $\mathbf{X_s}$ imposed by the structure on Σ ;
- gravity acts on free surface where $p=\rho g z$.

It will be considered initially that the fluid is nonheavy before introducing gravity in the paragraph [§3.2].

3.1 Recalls on the coupling fluid-structure

In order to give an account of the interaction fluid-structure well, we will analyze the equations separately governing the behavior of the fluid and those which govern that of the structure, without considering in this chapter the boundary conditions relating to free surface.

3.1.1 Description of the fluid

It is considered that the studied system is subjected to small disturbances around its state of balance where the fluid and the structure are at rest: thus, $P=p_0+p$ and $\mathbf{X_s}=\mathbf{x_s}(x_0=0)$. What makes it possible to write [bib2]:

$\rho=-\rho_f \operatorname{div}(\mathbf{x_f})$ from where $p=-\rho_f c^2 \operatorname{div}(\mathbf{x_f})$.

With:

- p fluctuating pressure of the fluid,
- ρ the disturbance of density of the fluid, ρ_f density of the fluid at rest,
- $\mathbf{x_f}(\mathbf{r},t)$ the field of displacement of a particle of fluid.

The fluid is:

- perfect (i.e. nonviscous)
- barotrope:

$$p=\rho c^2 \quad \text{éq 3.1.1-1}$$

- and irrotational: there exists a potential of displacements φ, such as $p=\rho_f \dfrac{\partial^2 \varphi}{\partial t^2}$

The behavior of the volume of fluid eulérien is thus described by the following equations:

- law of behavior:

$$\mathbf{T}_{ij}=-p\,\boldsymbol{\delta}_{ij}$$

- conservation equation of the momentum in the fluid in the absence of source:

$$\operatorname{div}(\overline{\overline{\mathbf{T}}})=\rho_f \frac{\partial^2 \mathbf{x_f}}{\partial t^2} \quad \text{éq 3.1.1-2}$$

- conservation equation of the mass:

$$\frac{\partial \rho}{\partial t}+\rho_f \operatorname{div}\left(\frac{\partial \mathbf{x_s}}{\partial t}\right)=0 \quad \text{éq 3.1.1-3}$$

By combining the conservation equations of the momentum [éq 3.1.1-2] and of the mass [éq 3.1.1-3] written in harmonic mode with the pulsation w, one obtains, thanks to [éq 3.1.1-1], the equation of Helmholtz:

$$\Delta p+\frac{\omega^2}{c^2} p=0$$

3.1.2 Description of the structure

It is considered that the structure is elastic linear and that one remains in the field of the small disturbances. Taking into account these assumptions, one writes:

- the law of behavior in linear elasticity:

$$\boldsymbol{\sigma}_{ij}=\mathbf{C}_{ijkl}\,\boldsymbol{\varepsilon}_{kl}$$

- the conservation equation of the momentum in the structure in the absence of voluminal forces others that inertial forces:

$$\operatorname{div}(\overline{\overline{\boldsymbol{\sigma}}})=\rho_s \frac{\partial^2 \mathbf{x_s}}{\partial t^2}$$

- the equation of compatibility on the tensor of deformation:

$$\boldsymbol{\varepsilon}_{kl}=\frac{1}{2}(\mathbf{u_{k,l}}+\mathbf{u_{l,k}})$$

3.1.3 Description of the interface fluid-structure

With the interface (Σ) between the fluid and the structure, like the fluid is not viscous, there is continuity of the normal constraints and normal speeds to the wall, and nullity of the tangential constraint (absence of viscous friction). These boundary conditions are written:

$$\begin{cases} \boldsymbol{\sigma}_{ij}\mathbf{n}_i = \mathbf{T}_{ij}\mathbf{n}_i = -p\,\boldsymbol{\delta}_{ij}\mathbf{n}_i \\ \dfrac{\partial \mathbf{x_f}}{\partial t}.\mathbf{n} = \dfrac{\partial \mathbf{x_s}}{\partial t}.\mathbf{n} \end{cases}$$

3.1.3.1 Formulation of the coupled problem

Finally, the equation of the problem coupled fluid-structure is written, while taking p like variable describing the field of pressure in the fluid and x_s the field of displacements in the structure:

$$\begin{cases} \mathbf{C}_{ijkl}\mathbf{x}_{\mathbf{s}_{k,lj}} + \omega^2 \rho_s \mathbf{x}_{\mathbf{s}_i} = 0 & \text{dans } V_s \\ \Delta p + \dfrac{\omega^2}{c^2} p = 0 & \text{dans } V_f \\ \boldsymbol{\sigma}_{ij}\mathbf{n_i} = \mathbf{C}_{ijkl}\mathbf{x}_{\mathbf{s}_{k,l}}\mathbf{n_i} = -p\,\boldsymbol{\delta}_{ij}\mathbf{n_i} & \text{sur } \Sigma \\ \dfrac{\partial p}{\partial n} = \rho_f \omega^2 \mathbf{x}_{\mathbf{f}_i}\mathbf{n_i} & \text{sur } \Sigma \end{cases}$$

Fields of displacements $\mathbf{x_s}$ for the structure and of pressure p for the fluid sought minimize the functional calculus:

$$\mathbf{L}(\mathbf{x_s}, p, z) = \frac{1}{2}\int_{V_s}\left[\boldsymbol{\sigma}_{ij}(\mathbf{x_s})\boldsymbol{\varepsilon}_{ij}(\mathbf{x_s}) - \rho_s\omega^2\mathbf{x_s^2}\right] - \int_{\Sigma} p\,\mathbf{x_s}\,\mathbf{n}\,d\Sigma + \frac{1}{2\rho_f}\int_{V_f}\left[\frac{1}{\omega^2}(\text{grad } p)^2 - \frac{p^2}{c^2}\right]dV$$

3.2 Action of gravity on free surface

3.2.1 Formulation of the problem

One points out the linearized dynamic equations here describing the small movements of a true fluid. One chooses a description eulérienne fluid:

$$\text{grad } P = \rho_f\left(\mathbf{g} - \frac{\partial^2 \mathbf{x_s}}{\partial t^2}\right) \text{ in } V_f$$

With balance the particle of fluid was in M_0 and thus: $\text{grad } P_0 = \rho_f \mathbf{g}$ in V_f .

One considers movements of low amplitude around the state of balance (it is the assumption of the small disturbances): then $\mathbf{M} = \mathbf{M_0} + \mathbf{x_f}(\mathbf{M_0}, \mathrm{t})$

Are p the fluctuation in pressure eulérienne and p_L the fluctuation in Lagrangian pressure, then:

$$p(\mathbf{M}, t) = P(\mathbf{M_0}, t) - P_0(\mathbf{M_0})$$
$$p_L = P(\mathbf{M}, t) - P_0(\mathbf{M_0})$$

Taking into account the assumption of small displacements:

$$p_L - p = \text{gradP}(\mathbf{M_o}, t)\mathbf{x_f}(\mathbf{M_o}, t) = -\rho_f g \mathbf{x_f}(\mathbf{M_o}, t) \quad \text{éq 3.2.1-1}$$

If one considers the case of a heavy fluid having a free surface in contact with a medium to constant pressure P_{atm}, one can write, by neglecting the effects of surface tension:

$P(\mathbf{M}, t) = P_{atm}$ on free surface SL i.e.: $p_L = 0$. Maybe, with [éq 3.2.1-1], $p = \rho_f g(\mathbf{x_f} . \mathbf{z})$
Taking into account the assumption of the small movements, the instantaneous slope of the tangent plan is a first order infinitely small. $\mathbf{x_f} . \mathbf{z}$ thus merges with the second order near with vertical rise h.

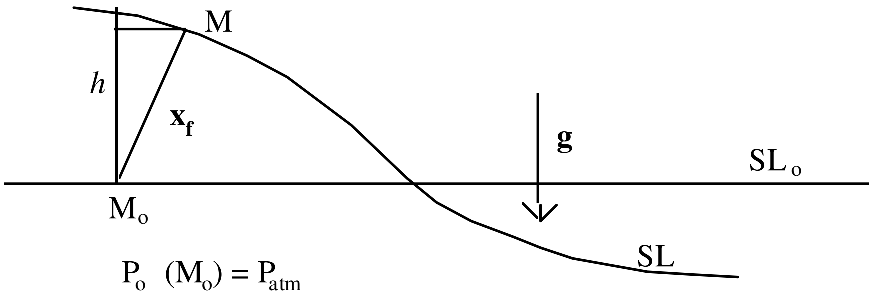

Figure 3.2.1-a: approximation on free surface

Thus, if one adds in the boundary conditions the condition of gravity on free surface, that amounts considering in $z = H$ the linearized condition:

$$p = \rho_f g z \quad \text{éq 3.2.1-2}$$

The equations of the total problem become:

$$\begin{cases} \mathbf{C}_{ijkl} \mathbf{x}_{\mathbf{s}_{k,lj}} + \omega^2 \rho_s \mathbf{x}_{\mathbf{s}_i} = 0 & \text{dans } V_s \\ \Delta p + \frac{\omega^2}{c^2} p = 0 & \text{dans } V_f \\ \boldsymbol{\sigma}_{ij} \mathbf{n_i} = \mathbf{C}_{ijkl} \mathbf{x}_{\mathbf{s}_{k,l}} \mathbf{n_i} = -p \boldsymbol{\delta}_{ij} \mathbf{n_i} & \text{sur } \Sigma \\ \frac{\partial p}{\partial n} = \rho_f \omega^2 \mathbf{x}_{\mathbf{f}_i} \mathbf{n_i} & \text{sur } \Sigma \text{ et sur } SL \\ p = \rho_f g z & \text{sur } SL \end{cases}$$

To express the functional calculus, one uses the law of behavior on free surface. By considering an acceptable field of displacement dz one obtains [bib2]:

$$\int_{SL} \rho g z \delta z \, ds = \int_{SL} p \delta z \, ds$$

Maybe, finally, the functional calculus of the total system fluid structure subjected to gravity:

$$\mathrm{L}(\mathbf{x_s}, p, z) = \frac{1}{2} \int_{V_s} \left[\boldsymbol{\sigma}_{ij}(\mathbf{x_s}) \boldsymbol{\varepsilon}_{ij}(\mathbf{x_s}) - \rho_s \omega^2 \mathbf{x_s^2} \right] - \int_{\Sigma} p \mathbf{x_s} . \mathbf{n} \, d\Sigma + \frac{1}{2} \rho_f ¿ \int_{V_f} \left[\frac{1}{\omega^2} (\text{grad } p)^2 - \frac{p^2}{c^2} \right] dV + \frac{1}{2} \int_{SL} \rho g z^2 dS - \int_{SL} p z \, dS$$

This taking into account of gravity implies two additional terms in the functional calculus describing the fluid:

- a term of potential energy related to free surface: $\frac{1}{2}\int_{SL} \rho g z^2 ds$
- a term due to the work of the hydrodynamic pressure in the displacement of free surface: $\int_{SL} p z ds$

However it should be noted that it is not the single effect of gravity since in any point of the wall Σ be exerted a permanent pressure $-\rho g z$ (where z is the altitude of the point M considered: it is supposed that $z=0$ on the level of free surface to balance). The point M is actuated by a movement X_s infinitesimal, the element of surface $d\Sigma$ thus vary and the effort due to the permanent pressure too. This effort is responsible for an additional term of rigidity being added to the rigidity of structure in the system. It could cause a buckling of the structure by cancelling structural rigidity. This effect is negligible on the vibratory characteristics ([bib2], [bib1]), one thus does not take it into account.

3.2.2 Discretization by finite elements

To obtain the discretized form of the functional calculus, one replaces each integral by a sum of integrals on each element i discretized system, then one uses an approximation by finite elements of the unknown functions of displacement and pressure on each element i [bib18].

The unknown factors are $\mathbf{X_s}(u, v, w), p, z$, one has then while posing Ni functions of forms (or nodal functions of interpolation on the element i) :

$$\begin{cases} \boldsymbol{\sigma}=\mathbf{D}\,\boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon}=\mathbf{B}\,\mathbf{u} \end{cases} \quad \begin{cases} \mathbf{x_s}=\mathbf{N_i}\,\mathbf{u} \\ \mathbf{p}(x, y, z)=\mathbf{N_i}\,\mathbf{p} \\ \nabla\,\mathbf{p}=\overline{\mathbf{N_i}}\,\mathbf{p} \end{cases} \quad \begin{cases} \mathbf{x_s}.\mathbf{n}=\mathbf{N}_{\Sigma i}\,\mathbf{u} \\ \mathbf{z}=\mathbf{N}_{Si}\,\mathbf{z} \end{cases}$$

where δ, p , are the unknown factors with the nodes structures and the fluid nodes, and z unknown factors at free surface.

From where the discretized expression of the functional calculus associated with the problem:

$$\mathrm{L}=\mathbf{u^t}(\mathbf{K}-\omega^2\mathbf{M})\mathbf{u}+\mathbf{p^t}\left(\frac{\mathbf{H}}{\rho_f\omega^2}-\frac{\mathbf{Q}}{\rho_f c^2}\right)\mathbf{p}+\mathbf{z^t}\rho_f g\,\mathbf{K_z}\mathbf{z}-2\,\mathbf{p^t}\mathbf{M_z}\mathbf{z}-2\,\mathbf{p^t}\mathbf{C}\,\mathbf{u}$$

with

$$\mathbf{K}=\sum_i \int_{Vi} \mathbf{N_i^t}\mathbf{B_i^t}\mathbf{D_i}\mathbf{B_i}\mathbf{N_i}\,dV_i$$ matrix stiffness of the structure

$$\mathbf{M}=\sum_i \int_{Si} \mathbf{N_i^t}\rho_f\mathbf{N_i}\,dV_i$$ matrix masses structure

and

$$\mathbf{Q}=\sum_i \int_{Vi_{fl}} \mathbf{N_i^t N_i}\, dV_i \qquad \mathbf{M_z}=\sum_i \int_{Si} \mathbf{N_{\Sigma i}^t N_{\Sigma i}}\, dS_i$$

$$\mathbf{K_z}=\sum_i \int_{Si} \mathbf{N_{Si}^t N_{Si}}\, dS_i \qquad \mathbf{M_z}=\sum_i \int_{Si} \mathbf{N_i^t N_{Si}}\, dS_i$$

$$\mathbf{H}=\sum_i \int_{Vi_{fl}} \overline{\mathbf{N_i^t}}\, \overline{\mathbf{N_i}}\, dV_i$$

where c is the speed of sound in the fluid, ρ_f density of the fluid and where $\mathbf{K_f}$ corresponds to the potential energy of the fluid, $\mathbf{K_z}$ with the potential energy of free surface, $\mathbf{H}$ with the kinetic energy of the fluid, $\mathbf{M}$ with the coupling fluid-solid and $\mathbf{M_z}$ with the coupling $p-z$ on free surface.

The approximation by finished parts of the complete problem leads then to the following matric system:

$$\begin{bmatrix} \mathbf{K} & -\mathbf{C} & 0 \\ 0 & \mathbf{H} & 0 \\ 0 & -\mathbf{M_z} & \mathbf{K_z} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \\ \mathbf{z} \end{Bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M} & 0 & 0 \\ \rho_f \mathbf{C} & \frac{\mathbf{Q}}{c^2} & \rho_f \mathbf{M_z} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \\ \mathbf{z} \end{Bmatrix} = 0$$

The first equation corresponds to the movement of the structure subjected to the forces of pressure, the second with that of the movement of the fluid coupled with the structure and at free surface, the third is the equation of free surface.

However the written problem of the kind has matrices masses and rigidity nonsymmetrical what prevents the use of the classical algorithms of resolution of *Code_Aster*.

3.2.3 Introduction of an additional variable

To make the problem symmetrical and to be able to use the classical methods of resolution, an additional variable is introduced: potential of displacements in the fluid j [bib2].

$$\mathbf{X_f}=\mathrm{grad}\,\varphi \text{ i.e. } \rho\,\omega^2 \varphi = p$$

This additional unknown factor is related to the unknown factors of the problem, which leads to a matrix of singular rigidity.

One reformulates the problem coupled heavy structure-fluid:

$$\begin{cases} \mathbf{C}_{ijkl}\, \mathbf{x}_{\mathbf{s}_{k,lj}} + \omega^2 \rho_s \mathbf{x}_{\mathbf{s}_i} = 0 & \text{dans } V_s \\ \Delta \varphi + \dfrac{\omega^2}{\rho_f c^2} p = 0 & \text{dans } V_f \\ p = \rho_f \omega^2 \varphi & \text{dans } V_f \\ \boldsymbol{\sigma}_{ij} \mathbf{n_i} = \mathbf{C}_{ijkl}\, \mathbf{x}_{\mathbf{s}_{k,l}} \mathbf{n_i} = -\rho_f \omega^2 \varphi\, \boldsymbol{\delta}_{ij} \mathbf{n_i} & \text{sur } S \\ \dfrac{\partial \varphi}{\partial n} = \mathbf{x}_{\mathbf{f}_i} \mathbf{n_i} & \text{sur } S \\ p = \rho_f g z & \text{sur } SL \end{cases}$$

What leads to the functional calculus of the coupled system:

$$L(\mathbf{x_s}, p, j, z)=\frac{1}{2}\int_{V_s}\left[\boldsymbol{\sigma}_{ij}(\mathbf{x_s})\boldsymbol{\varepsilon}_{ij}(\mathbf{x_s})-\rho_s\omega^2\mathbf{x_s^2}\right]+\frac{1}{2}\int_{V_f}\frac{p^2}{c^2}dV+\frac{1}{2}\int_{SL}\rho g z^2 ds$$
$$-\omega^2\left[\int_{\Sigma}\rho_f\varphi\,\mathbf{x_s}\mathbf{n}\,d\Sigma+\int_{SL}\rho_f\varphi z\,ds+\int_{V_f}\left[\frac{\rho_f}{2}(\text{grad}\,\varphi)^2+\frac{p\varphi}{c^2}\right]dV\right]$$

Maybe while discretizing:

$$\mathrm{L}=\boldsymbol{\delta}^{\mathbf{t}}(\mathbf{K}-\omega^2\mathbf{M})\boldsymbol{\delta}+\frac{1}{\rho_f c^2}\mathbf{p^t}\mathbf{Q}\,\mathbf{p}+\rho_f g\,\mathbf{z^t}\mathbf{K_z}\mathbf{z}$$
$$-2\omega^2\left[\frac{\rho_f}{2}\boldsymbol{\varphi}^{\mathbf{t}}\mathbf{H}\boldsymbol{\varphi}+\rho_f\boldsymbol{\varphi}^{\mathbf{t}}\mathbf{C}\boldsymbol{\delta}+\rho_f\boldsymbol{\varphi}^{\mathbf{t}}\mathbf{M_z}\mathbf{z}+\frac{1}{c^2}\mathbf{p^t}\mathbf{Q}\boldsymbol{\varphi}\right]$$

What is written, in matric form:

$$\begin{bmatrix}\mathbf{K} & 0 & 0 & 0\\ 0 & \frac{\mathbf{Q}}{\rho_f c^2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \rho_f g\mathbf{K_z}\end{bmatrix}\begin{bmatrix}\boldsymbol{\delta}\\ \mathbf{p}\\ \boldsymbol{\varphi}\\ \mathbf{z}\end{bmatrix}-\omega^2\begin{bmatrix}\mathbf{M} & 0 & \rho_f\mathbf{C} & 0\\ 0 & 0 & \frac{\mathbf{Q}}{c^2} & 0\\ \rho_f\mathbf{C^t} & \frac{\mathbf{Q^t}}{c^2} & \rho_f\mathbf{H} & \rho_f\mathbf{M_z}\\ 0 & 0 & \rho_f\mathbf{M_z^t} & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{\delta}\\ \mathbf{p}\\ \boldsymbol{\varphi}\\ \mathbf{z}\end{bmatrix}=0$$

4 Establishment in *Code_Aster*

The library of the finite elements of *Code_Aster* was enriched by five isoparametric surface elements having like degrees of freedom the deflection of free surface and the potential of displacements of the fluid at free surface. They are compatible with the elements 3D which deal with the problem of coupling fluid/structure [bib3]

One names:

MEFP_FACE3 and MEFP_FACE6 respectively triangles with 3 or 6 nodes,
MEFP_FACE4, MEFP_FACE8 and MEFP_FACE9 respectively quadrangles with 4.8 or 9 nodes.

These elements belong to modeling 2D_FLUI_PESA phenomenon MECHANICS.

5 Bibliography

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4) C. LEPOUTERE, F. WAECKEL: Effect of gravity on the free surface of a fluid coupled to a structure, Notes HP intern - 61/93.139

6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
3	*G. ROUSSEAU, Fe WAECKEL (EDF/EP/AMV)*	Initial text