Elements of absorbing border

Summary

This document describes the establishment in *Code_Aster* elements of absorbing border. These elements of the paraxial type, which one describes the theory here, are assigned to borders of elastic ranges or fluid to deal with problems 2D or 3D of interaction ground-structure or ground-fluid-structure. They make it possible to satisfy condition of Sommerfeld checking the assumption of anechoicity: the elimination of the elastic or acoustic plane waves diffracted and not physiques coming from the infinite one.

Contents

1 Introduction

1.1 Problems of a semi-infinite medium for the ISS

The standard problems of seismic answer and interaction ground-structure or ground-fluid-structure bring to consider infinite or supposed fields such. For example, in the case of stoppings subjected to the earthquake, one often deals with reserves of big size which enable us to make the assumption of anechoicity: the waves which leave towards the bottom reserve "do not return" not. The purpose of this is to reduce the size of the structure to be netted and to make it possible to pass from complex calculations with the current computer resources. One proposes on [Figure 1.1-a] below diagram which describes the type of situations considered.

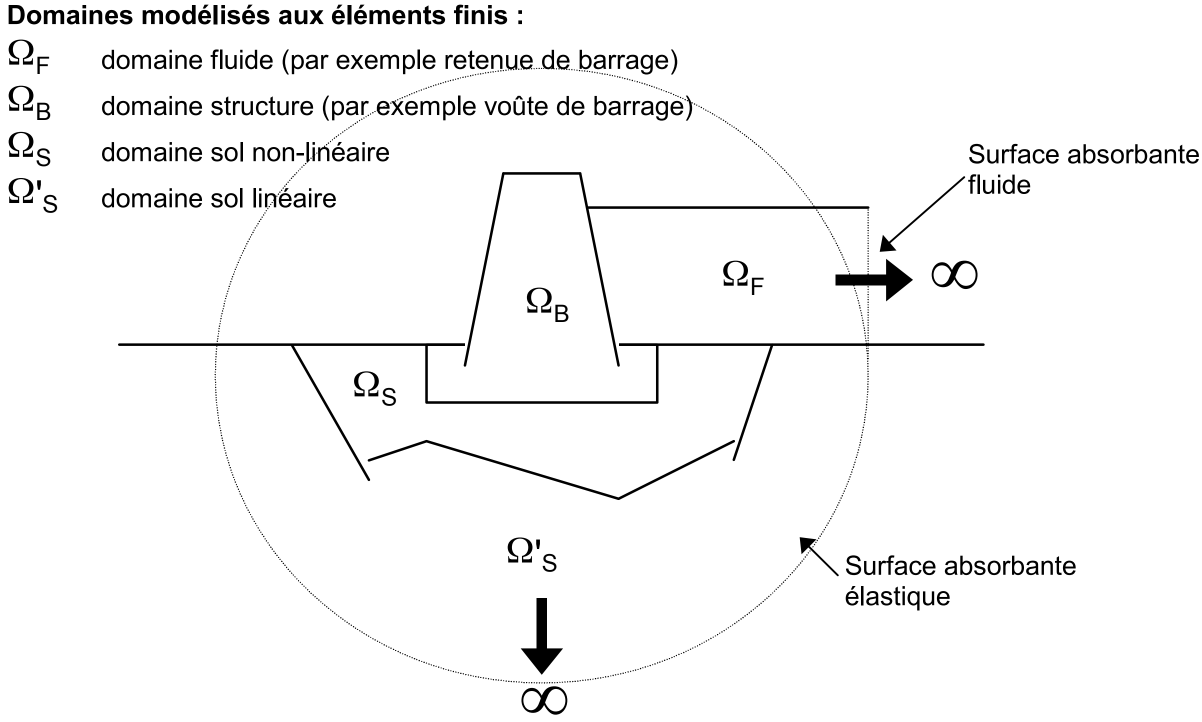


Figure 1.1-a: Field for the interaction ground-fluid-structure

In all the document, it is considered that the border of the grid finite elements of the ground is in a field with the elastic behavior.

The elliptic system theory ensures simply the existence and the unicity of the solution of the acoustic or elastoplastic problems in the limited fields, under the assumption of boundary conditions ensuring the closing of the problem. It goes from there differently for the infinite fields. One must resort to a condition particular, known as of Sommerfeld, formulated in the infinite directions of the problem. This condition in particular ensures, in the case of the diffraction of a plane wave (elastic or acoustic) by a structure, the elimination of the diffracted waves not physiques coming from infinite that the classical conditions on the edges of the field remotely finished are not enough to ensure.

1.2 State of the art of the digital approaches

The privileged method to treat infinite fields is that of the finite elements of border (or integral equations). The fundamental solution used checks the condition of Sommerfeld automatically. Only, the use of this method is conditioned by the knowledge of this fundamental solution, which is impossible in the case of a ground with complex geometry, for example, or when the ground or the structure is nonlinear. It is thus necessary then to resort to the finite elements. Consequently, conditions particular to the border of the grid finite elements are necessary to prohibit the reflection of the outgoing diffracted waves and thus artificially to reproduce the condition of Sommerfeld.

Several methods make it possible to identify boundary conditions answering our requirements. Some lead to an exact resolution of the problem: they are called “consistent borders”. They are founded on a precise taking into account of the wave propagation in the infinite field. For example, if this field can be presumedly elastic and with a simple stratigraphy far from the structure, one can consider a coupling finite elements - integral equations. One of the problems of this solution is that it is not local in space: it is necessary to make an assessment on all the border separating the field finished from the infinite field, which obligatorily leads us to a problem of under - structuring. This not-locality in space is characteristic of the consistent borders.

To lead in the local terms of border in space, one can use the theory of the infinite elements [bib1]. They are elements of infinite size whose basic functions reproduce the elastic or acoustic wave propagation as well as possible ad infinitum. These functions must be close to the solution because the classical mathematical theorems do not ensure any more convergence of the computation result towards the solution with such elements. In fact, one can find an analogy between the search for satisfactory basic functions and that of a fundamental solution for the integral equations. The geometrical constraints are rather close but especially, this research presents a disadvantage of size: it depends on the frequency. Consequently, such borders, local or not spaces some, can be used only in the field of Fourier, which prohibits a certain category of problems, with non-linearities of behavior or great displacements for example.
One thus arrives at having to find borders absorbing powerful who are local in space and time to treat with the finite elements of the transitory problems posed on infinite fields.

We will present in the continuation the theory of the paraxial elements which carry out the absorption sought with an effectiveness inversely proportional to their simplicity of implementation as well as the description of the constraints of implementation in *Code_Aster*. One presents the developments to deal 3D problems. Those for the cases 2D were carried out and their theory results simply from modeling 3D.

2 Theory of the paraxial elements

One presents in this part the principle of the paraxial approximation in the case of elastodynamic linear. Two theoretical approaches make it possible to determine the spirit and the practical application elastic paraxial elements: one owes the first in Cohen and Jennings [bib2] and the second with Modaressi [bib3]. The application of the theory of the paraxial elements to the fluid case will be made in the following part.
Subsequently, as presented on [Figure 1.1-a], one supposes that the border of the grid of the ground is located in a field at the elastic behavior.
Approach of Modaressi established in *Code_Aster* at the same time allows to build absorbing borders and to introduce the incidental seismic field.

2.1 Spectral impedance of the border

To obtain the paraxial equation, we should initially determine the shape of the field of displacement diffracted in the vicinity of the border. For that, one leaves the equations of the elastodynamic 3D:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2}-\mathbf{E}_{11}c^2\frac{\partial^2 \mathbf{u}}{\partial x'^2}-\mathbf{E}_{12}c^2\frac{\partial^2 \mathbf{u}}{\partial x'\partial x_3}-\mathbf{E}_{22}c^2\frac{\partial^2 \mathbf{u}}{\partial x_3^2}=0$$

With: $\mathbf{u}=\begin{Bmatrix}u'\\u_3\end{Bmatrix}$ $\mathbf{E}_{11}=\frac{1}{c^2}\begin{bmatrix}c_P^2 & 0\\0 & c_S^2\end{bmatrix}$ $\mathbf{E}_{12}=\frac{1}{c^2}\left(c_P^2-c_S^2\right)\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
$\mathbf{E}_{22}=\frac{1}{c^2}\begin{bmatrix}c_S^2 & 0\\0 & c_P^2\end{bmatrix}$

The constant c , homogeneous at a speed, is introduced to make certain quantities adimensional. The equations and their solutions are of course independent of this constant.
One calls x' and u' directions and components of displacement in the tangent plan and x_3 and u_3 according to $\mathbf{e_3}$, normal direction at the border.
One proceeds to two transforms of Fourier, one compared to time, the other compared to the variables of space in the plan at the border. One limits oneself to the case of a plane border and without corner: The equations are written then:

$$\left(c_P^2-c_S^2\right)\left[-\boldsymbol{\xi}'.\hat{\mathbf{u}}'+i\frac{\partial \hat{\mathbf{u}}_3}{\partial x_3}\right]\boldsymbol{\xi}'+c_S^2\left[-|\boldsymbol{\xi}'|^2+\frac{\partial^2}{\partial x_3^2}\right]\hat{\mathbf{u}}'+\omega^2\hat{\mathbf{u}}'=0$$
$$\left(c_P^2-c_S^2\right)\left[-i\boldsymbol{\xi}'.\frac{\partial \hat{\mathbf{u}}'}{\partial x_3}+\frac{\partial^2 \hat{\mathbf{u}}_3}{\partial x_3^2}\right]+c_S^2\left[-|\boldsymbol{\xi}'|^2+\frac{\partial^2}{\partial x_3^2}\right]\hat{\mathbf{u}}_3+\omega^2\hat{\mathbf{u}}_3¿=0$$

where $\hat{\mathbf{u}}$ and $\hat{\mathbf{u}}_3$ the transforms of Fourier indicate and $\boldsymbol{\xi}'$ the vector of wave associated with x' .

It is about a differential connection in x_3 that one can solve by diagonalisant it. One from of deduced:

$$\frac{\left(\hat{\mathbf{u}}'\wedge\boldsymbol{\xi}'\right).\mathbf{e_3}}{|\boldsymbol{\xi}'|}=A\exp\left(-i\,\xi_S x_3\right)$$
$$\hat{\mathbf{u}}'.\boldsymbol{\xi}'=|\boldsymbol{\xi}'|\left[A_P\exp\left(-i\,\xi_P x_3\right)+A_S\exp\left(-i\,\xi_S x_3\right)\right]$$
$$|\boldsymbol{\xi}'|\hat{\mathbf{u}}_3=-A_P\xi_P\exp\left(-i\,\xi_P x_3\right)-A_S\xi_S\exp\left(-i\,\xi_S x_3\right)$$

With: $\xi_P = \sqrt{\frac{\omega^2}{c_P^2} - |\boldsymbol{\xi}'|^2}$ and $\xi_S = \sqrt{\frac{\omega^2}{c_S^2} - |\boldsymbol{\xi}'|^2}$

To determine the constants $A, A_S \text{ et } A_P$, one supposes known $\hat{\mathbf{u}}(\boldsymbol{\xi}', 0)$ on the border of the finite elements field. One expresses them according to $\hat{\mathbf{u}}'(\boldsymbol{\xi}', 0) = \hat{\mathbf{u}}_0' \text{ et } \hat{\mathbf{u}}_3(\boldsymbol{\xi}', 0) = \hat{\mathbf{u}}_{30}$.

One now will evaluate the vector forced on a facet of normal $\mathbf{e_3}$ in $x_3 = 0$, which will give us the impedance of the border. One subjects to $t(x', x_3)$ the same transform of Fourier in space as for the equations of elastodynamic, so that:

$$\hat{\mathbf{t}}(\boldsymbol{\xi}', x_3) = \left[i\lambda \hat{\mathbf{u}}' . \boldsymbol{\xi}' + (\lambda + 2\mu) \frac{\partial \hat{\mathbf{u}}_3}{\partial x_3} \right] \mathbf{e_3} + \mu \left(\frac{\partial \hat{\mathbf{u}}'}{\partial x_3} + i \hat{\mathbf{u}}_3 \mathbf{x}' \right)$$

One wishes to free oneself in $x_3 = 0$ terms containing of the derivative in x_3. The system obtained previously allows it to us according to $\hat{u}_0' \text{ et } \hat{u}_{30}$:

$$\frac{\partial \hat{\mathbf{u}}_0'}{\partial x_3} . \boldsymbol{\xi}' = i |\boldsymbol{\xi}'|^2 \hat{\mathbf{u}}_{30}$$
$$\left(\frac{\partial \hat{\mathbf{u}}_0'}{\partial x_3} \wedge \boldsymbol{\xi}' \right) . \mathbf{e_3} = -i \xi_S \left(\hat{\mathbf{u}}_0' \wedge \boldsymbol{\xi}' \right) . \mathbf{e_3}$$
$$\frac{\partial \hat{\mathbf{u}}_{30}}{\partial x_3} = i \left[-\xi_P \xi_S \frac{\hat{\mathbf{u}}_0' . \boldsymbol{\xi}'}{|\boldsymbol{\xi}'|^2} + (\xi_P + \xi_S) \hat{\mathbf{u}}_{30} \right]$$

One thus obtains the spectral impedance of the border:

$$\hat{\mathbf{t}}_0 = a^0 \mathbf{e_3} + b^0 \boldsymbol{\xi}' + c^0 \boldsymbol{\xi}' \wedge \mathbf{e_3}$$

where $a^0, \mathrm{b}^0 \text{ et } c^0$ are functions of $|\boldsymbol{\xi}'|$ and of ω who depend linearly on $\hat{\mathbf{u}}_0' \text{ et } \hat{\mathbf{u}}_{30}$

One can then write: $\hat{\mathbf{t}}_{\mathbf{0}} = A(|\boldsymbol{\xi}'|, \omega) \hat{\mathbf{u}}_0(\boldsymbol{\xi}', \omega)$
where A appoint the operator total spectral impedance. One returns to physical space by two transforms of Fourier opposite.

2.2 Paraxial approximation of the impedance

The spectral impedance calculated previously is local neither spaces some nor in time since it utilizes $\hat{\mathbf{u}}_{\mathbf{0}}(\boldsymbol{\xi}', \omega)$, transform of Fourier of $\mathbf{u}_{\mathbf{0}}(x', t)$ for all x' and all t .

The idea is then to develop ξ_P and ξ_S according to the powers of $\frac{|\boldsymbol{\xi}'|}{\omega}$. This approximation will be good either high frequency, or for $|\boldsymbol{\xi}'|$ small.

Let us examine the dependence in x_3, for example of $\hat{\mathbf{u}}_{\mathbf{3}}$: one will have, for $\mathbf{u}_{\mathbf{3}}(x', x_3, t)$ terms of the form: $\exp\left[i \left(\xi' x' + \omega t - \xi_\mathrm{P} x_3 \right) \right]$

With the development of ξ_P : $\xi_P = \frac{\omega}{c_P} \left[1 - \left(\frac{c_P |\boldsymbol{\xi}'|}{\omega} \right)^2 + \ldots \right]$

One shows that for $|\boldsymbol{\xi}'|$ small, there will be waves being propagated according to directions close to the normal $\mathbf{e_3}$ at the border, because the exponential one is written:

$$\exp\left\{i\omega\left[\left(t-\frac{x_3}{c_P}\right)+i\,o\left(\frac{|\xi'|}{\omega}\right)\right]\right\}$$

Consequently, with an asymptotic development of ξ_P and ξ_S , while multiplying by a suitable power of w to remove this quantity with the denominator, one obtains:

$$A_0(\boldsymbol{\xi}',\omega)\,\hat{\mathbf{t}}_0=A_1(\boldsymbol{\xi}',\omega)\,\hat{\mathbf{u}}_0$$

where A_0 and A_1 are polynomial functions in $\boldsymbol{\xi}'$ and ω .

Maybe, after the two transforms of Fourier opposite:

$$A_0\left(\frac{\partial}{\partial x'},\frac{\partial}{\partial t}\right)\mathbf{t_0}=A_1\left(\frac{\partial}{\partial x'},\frac{\partial}{\partial t}\right)\mathbf{u_0}$$

One thus obtains the final form of the approximate transitory local impedance according to the last term in $\frac{|\xi'|}{\omega}$ retained. One can find the calculation detailed of A_i in [bib5].
For example, for order 0:

$$\mathbf{t_0}=coeam\left(\rho\,c_P\frac{\partial u_3}{\partial t}\mathbf{e_3}+\rho\,c_S\frac{\partial \mathbf{u}'}{\partial t}\right)$$

This corresponds to viscous shock absorbers distributed along the border of the finite elements field, with the corrective coefficient $coeam$ informed behind the keyword `COEF_AMOR` affected by `DEFI_MATERIAU` with material of the elements of absorbing border.
By analogy, for order 0, one can add a term in displacement corresponding to the rigidities distributed along the border of the finite elements field:

$$\mathbf{t_1}=\frac{\lambda+2\mu}{L}u_3\,\mathbf{e_3}+\frac{\mu}{L}\mathbf{u}'$$

This additional formulation understands the coefficients of Lamé λ and μ , as well as a characteristic dimension L .

With order 1:

$$\frac{\partial \mathbf{t_0}}{\partial t}=\rho\,c_P\frac{\partial^2\mathbf{u_3}}{\partial t^2}\mathbf{e_3}+\rho\,c_S\frac{\partial^2\mathbf{u}'}{\partial t^2}+\rho\,c_S\left[(2c_S-c_P)\frac{\partial^2\mathbf{u_3}}{\partial x'\partial t}\mathbf{e_3}+(c_P-2c_S)\frac{\partial^2\mathbf{u}'}{\partial x'\partial t}\right]$$
$$+\rho\,c_P^2\left(c_S-\frac{c_P}{2}\right)\frac{\partial^2\mathbf{u_3}.\mathbf{e_3}}{\partial x'^2}+\rho\,c_S^2\left(c_P-\frac{c_S}{2}\right)\frac{\partial^2\mathbf{u}'}{\partial x'^2}$$

One sees appearing the derivative compared to the time of the vector forced. In the digital processing, it will be necessary to resort to an integration of this term on the elements of the border.
To conclude, it will be retained that the paraxial approximation led to a transitory local impedance utilizing primarily only derivative in time and in the tangent plan at the border, like possibly an additional term without derived in time.

In way symbolic system, one writes:

$$\mathbf{t_0}=A_0\left(\frac{\partial \mathbf{u}}{\partial t}\right)+A_1(\mathbf{u}) \qquad \text{l'ordre 0}$$
$$\frac{\partial \mathbf{t_0}}{\partial t}=A_1\left(\frac{\partial^2 \mathbf{u}}{\partial t^2},\frac{\partial \mathbf{u}}{\partial t},\mathbf{u}\right) \qquad \text{à l'ordre 1}$$

2.3 Taking into account of the incidental seismic field

It is pointed out that the behavior of the ground is supposed to be elastic at least in the vicinity of the border. Ad infinitum, the total field u must be equal to the incidental field $\mathbf{u_i}$ (one of the consequences of the condition of radiation of Sommerfeld). The diffracted field is thus introduced $\mathbf{u_r}$ such as:

$$\mathbf{u}=\mathbf{u_i}+\mathbf{u_r}$$
$$\lim_{x\to+\infty}\mathbf{u_r}=0$$

At the border of the grid finite elements, one writes the condition of absorption for the diffracted field:

$$\mathbf{t_0}(\mathbf{u_r})=A_0\left(\frac{\partial \mathbf{u_r}}{\partial t}\right) \quad \text{with order 0}$$
$$\frac{\partial \mathbf{t_0}}{\partial t}(\mathbf{u_r})=A_1\left(\frac{\partial^2 \mathbf{u_r}}{\partial t^2},\frac{\partial \mathbf{u_r}}{\partial t},\mathbf{u_r}\right) \quad \text{with order 1}$$

One from of deduced the total vector forced on the border from the grid finite elements:

$$\mathbf{t_0}(\mathbf{u})=\mathbf{t_0}(\mathbf{u_i})+\mathbf{t_0}(\mathbf{u_r})=\mathbf{t_0}(\mathbf{u_i})+A_0\left(\frac{\partial \mathbf{u}}{\partial t}\right)-A_0\left(\frac{\partial \mathbf{u_i}}{\partial t}\right) \quad \text{with order 0}$$

One thus obtains the variational formulation of the problem in the vicinity of the border for order 0:

$$\rho\int_\Omega \frac{\partial^2 \mathbf{u}}{\partial t^2}\mathbf{v}+\int_\Omega \boldsymbol{\sigma}(\mathbf{u}):\boldsymbol{\varepsilon}(\mathbf{v})-\int_S A_0\left(\frac{\partial \mathbf{u}}{\partial t}\right)\mathbf{v}=\int_S\left[\mathbf{t}(\mathbf{u_i})-A_0\left(\frac{\partial \mathbf{u_i}}{\partial t}\right)\right]\mathbf{v}$$

For any field $\mathbf{v}$ kinematically acceptable

For order 1, one preserves the classical formulation:

$$\rho\int_\Omega \frac{\partial^2 \mathbf{u}}{\partial t^2}v+\int_\Omega \boldsymbol{\sigma}(u):\boldsymbol{\varepsilon}(v)-\int_S t(\mathbf{u})\,v=0$$

where $\mathbf{t}(\mathbf{u})$ the law of following evolution follows:

$$\frac{\partial \mathbf{t}(\mathbf{u})}{\partial t}=\frac{\partial \mathbf{t}(\mathbf{u_i})}{\partial t}+A_1\left(\frac{\partial^2 \mathbf{u}}{\partial t^2},\frac{\partial \mathbf{u}}{\partial t},\mathbf{u}\right)-A_1\left(\frac{\partial^2 \mathbf{u_i}}{\partial t^2},\frac{\partial \mathbf{u_i}}{\partial t},\mathbf{u_i}\right)$$

The request due to the incidental field appears explicitly in the case of order 0, but it is contained in the law of evolution of $\mathbf{t}(\mathbf{u})$ for order 1.

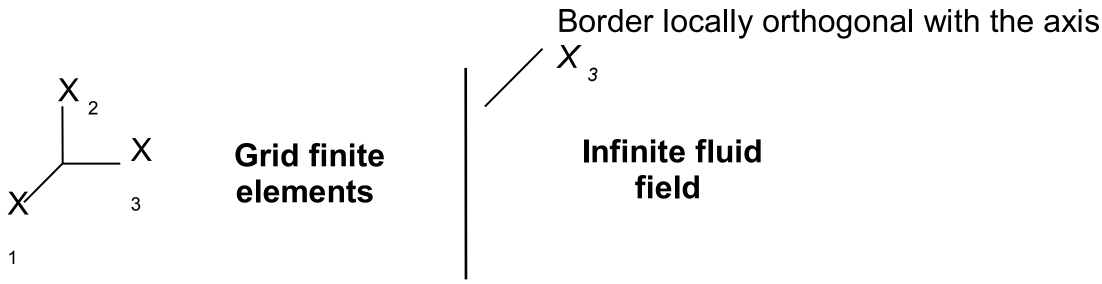
3 Anechoic fluid elements in transient

This part presents the main part of the general constraints of implementation of anechoic fluid elements of border absorbents with the paraxial approximation of order 0 in *Code_Aster*. For reasons of simplicity related to the handling of scalar sizes such as the pressure or the potential of displacement, in opposition to the vector quantities like displacement, one is interested initially in the fluid elements.

3.1 Standard formulation

One takes again here the reasoning of Modaressi by adapting it to an acoustic fluid field. Initially, one is interested in the only data of the size pressure in this fluid. One will then reconsider this modeling to adapt to the constraints of *Code_Aster*, by underlining the adjustments to be made.

That is to say thus following configuration, by taking again conventions of the preceding part in the vicinity of the border:



The definition of a local reference mark on the level of the element makes it possible to bring back for us systematically in such a situation.

3.1.1 Finite elements formulation

Pressure p check the equation of Helmholtz in all the field Ω modelled with the finite elements, which gives, for any virtual field of pressure q :

$$-\int_{\Omega} \nabla p . \nabla q - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_{\Omega} pq + \int_{\Sigma} \frac{\partial p}{\partial n} q = 0$$

Σ represent the border of the field Ω.

Size to be estimated on Σ thanks to the paraxial approximation is here $\frac{\partial p}{\partial n}$.

3.1.2 Paraxial approximation

In the configuration suggested, the term $\frac{\partial p}{\partial n}$ corresponds to $\frac{\partial p}{\partial x_3}$.

Let us consider a wave consequently planes harmonic being propagated in the fluid:

$$p = A \exp\left[i \left(k_1 x_1 + k_2 x_2 + k_3 x_3 - \omega t \right) \right]$$

While replacing in the equation of Helmholtz, one obtains:

$$k_3 = \frac{\omega}{c} \sqrt{1 - \frac{c^2}{\omega^2} \left(k_1^2 + k_2^2 \right)}$$

One obtains the following development then, for high frequencies (ω large) or in the vicinity of the border (k_1 and k_2 small):

$$k_3=\frac{\omega}{c}\left(1-\frac{c^2}{2\omega^2}\left(k_1^2+k_2^2\right)\right)$$

Maybe, while multiplying by ω to make disappear this quantity with the denominator and after a transform from Fourier reverses in space and time:

$$\frac{\partial^2 p}{\partial x_3 \partial t}=-\frac{1}{c}\frac{\partial^2 p}{\partial t^2}+\frac{1}{2}c\left(\frac{\partial^2 p}{\partial x_1^2}+\frac{\partial^2 p}{\partial x_2^2}\right)$$

As had presented it Modaressi, this equation utilizes the derivative compared to the time of the surface term. Within the framework of this part, one is interested only at the end of order 0, that is to say, after an integration in time, which makes disappear the awkward derivative:

$$\frac{\partial p}{\partial x_3}=-\frac{1}{c}\frac{\partial p}{\partial t} \quad \text{or more generally:} \quad \frac{\partial p}{\partial n}=-\frac{1}{c}\frac{\partial p}{\partial t}$$

It is this relation of impedance which we will discretize on the border of the finite elements field.

Note:

Taking into account the disappearance of the term of order 1 in the development of the square root, the minimal order of approximation for the paraxial fluids is in fact 1 and not 0. We will preserve the name of elements of order 0 for coherence with the solid. However, one speaks about fluid elements of order 2 at the time to consider elements of a strictly positive nature.

3.2 Impedance of the vibroacoustic elements in *Code_Aster*

Code_Aster have vibroacoustic elements. One recalls in this paragraph the choices of formulation made at the time of their implementation. One is inspired to present existing it of the reference material of *Code_Aster* [bib6].

3.2.1 Limits of the formulation out of p

In the framework of the interaction fluid-structure in harmonic, the formulation in pressure only of the acoustic fluid led to nonsymmetrical matrices. Indeed, the total system is expressed, in variational form, in the following way:

$$\int_{\Omega_s} \mathbf{C}_{ijkl}\cdot\mathbf{u}_{k,l}\,\mathbf{v}_{i,j}-\omega^2\int_{\Omega_s}\rho_s\,\mathbf{u}_i\,\mathbf{v}_i-\int_{\Sigma} p\,\mathbf{v}_i\,.\,\mathbf{n}_i=0 \quad \text{for the structure}$$

$$\frac{1}{\rho_f\,\omega^2}\int_{\Omega_f}\nabla p\,.\,\nabla q-k^2\int_{\Omega_f} pq-\int_{\Sigma}\mathbf{u}_i\,.\,\mathbf{n}_i\,q=0 \quad \text{for the fluid}$$

with $k=\frac{\omega}{c}$, number of wave for the fluid, v and q two virtual fields in the structure and the fluid respectively.

After discretization by finite elements, one obtains the following matric system:

$$\begin{bmatrix}\mathbf{K} & -\mathbf{C}\\ 0 & \mathbf{H}\end{bmatrix}\begin{bmatrix}u\\ p\end{bmatrix}-\omega^2\begin{bmatrix}\mathbf{M} & 0\\ \rho_f\,\mathbf{C}^{\mathrm{T}} & \dfrac{\mathbf{Q}}{c^2}\end{bmatrix}\begin{bmatrix}u\\ p\end{bmatrix}=0$$

where : $\mathbf{K}$ and $\mathbf{M}$ are the matrices of rigidity and mass of the structure

$\mathbf{H}$ and $\mathbf{Q}$ are the fluid matrices obtained respectively starting from the bilinear forms: $\int_{\Omega_f} \nabla p . \nabla q$ and $\int_{\Omega_f} pq$

$\mathbf{C}$ is the matrix of coupling obtained starting from the bilinear form: $\int_{\Sigma} p \mathbf{u_i} . \mathbf{n_i}$

The nonsymmetrical character of this system does not make it possible to use the algorithm of classics resolution of *Code_Aster*. This justifies the introduction of an additional variable into the description of the fluid.

3.2.2 Symmetrical formulation out of p and phi

The new introduced size is the potential of displacements ϕ , such as $x=\nabla\phi$. According to [bib6], one obtains the new variational form of the system coupled fluid-structure:

$$\int_{\Omega_s} \mathbf{C}_{ijkl}\cdot\mathbf{u}_{\mathbf{k,l}}\mathbf{v}_{\mathbf{i,j}}-\omega^2\int_{\Omega_s}\rho_s\mathbf{u}_{\mathbf{i}}\mathbf{v}_{\mathbf{i}}-\rho_f\omega^2\int_{\Sigma}\phi\, p\,\mathbf{v}_{\mathbf{i}}\cdot\mathbf{n}_{\mathbf{i}}=0 \quad \text{for the structure}$$

$$\frac{1}{\rho_f c^2}\int_{\Omega_f} pq-\rho_f\omega^2\left[\frac{1}{\rho_f c^2}\int_{\Omega_f}(\phi q+p\psi)-\int_{\Omega_f}\nabla\phi.\nabla\psi+\int_{S}\psi\,\mathbf{u}_{\mathbf{i}}\mathbf{n}_{\mathbf{i}}\right]=0 \quad \text{for the fluid}$$

With: $p=\rho_f\omega^2 f$ in the fluid and ψ a field of potential of virtual displacement

This leads us to the symmetrical matric system:

$$\begin{bmatrix}\mathbf{K} & 0 & 0\\ 0 & \dfrac{\mathbf{M_f}}{\rho_f c^2} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}u\\ p\\ \phi\end{bmatrix}-\omega^2\begin{bmatrix}\mathbf{M} & 0 & \rho_f\mathbf{M}_{\Sigma}\\ 0 & 0 & \dfrac{\mathbf{M}_{fl}}{c^2}\\ \rho_f\mathbf{M}_{\Sigma}^{\mathbf{T}} & \dfrac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f H\end{bmatrix}\begin{bmatrix}u\\ p\\ \phi\end{bmatrix}=0$$

where: $\mathbf{K}$ and $\mathbf{M}$ are the matrices of rigidity and mass of the structure

$\mathbf{M}_{\Sigma}$ is the matrix of coupling obtained starting from the bilinear form $\int_{\Sigma}\phi\,\mathbf{u}_{\mathbf{i}}\mathbf{n}_{\mathbf{i}}$

$\mathbf{M_f},\mathbf{M}_{fl}$ and $\mathbf{H}$ are the fluid matrices obtained starting from the bilinear forms: $\int_{\Omega_f} pq$, $\int_{\Omega_f} pq$ (or $\int_{\Omega_f}\phi q$) and $\int_{\Omega_f}\nabla\phi.\nabla\psi$

3.2.3 Imposition of an impedance with the formulation out of p and phi

Generally, a relation of impedance at the border of the fluid is expressed as follows:

$$p=Z\,\mathbf{v}.\mathbf{n}$$

where: Z is the imposed impedance

$\mathbf{v}.\mathbf{n}$ is the outgoing normal speed of the fluid particles

One from of deduced, according to the law of behavior of the fluid, which connects the pressure to the displacement of the fluid particles for an acoustic fluid $\nabla p-\rho_f\dfrac{\partial^2 u}{\partial t^2}=0$:

$$\frac{\rho_f}{Z}\frac{\partial p}{\partial t}=\frac{\partial p}{\partial n}$$

The discretization of such an equation leads to a nonsymmetrical term in a formulation in p and ϕ . One prefers to formulate the condition compared to the potential of displacement, that is to say:

$$\nabla\phi+\frac{\rho_f}{Z}\frac{\partial\phi}{\partial t}=0$$

One obtains then like expression for the term of edge associated with the relation with impedance:

$$\rho_f \frac{\partial^2}{\partial t^2} \int_\Sigma \phi \frac{\partial \psi}{\partial n} = \frac{\partial^3}{\partial t^3} \int_\Sigma \frac{\rho_f^2}{Z} \phi \psi$$

One then notes the appearance (somewhat artificial) of a term in derived third compared to time. In harmonic, which is the privileged scope of application of the vibroacoustic elements in *Code_Aster*, that does not pose a problem. One treats a term in ω^3 without difficulty. For transitory calculation, rather than to introduce an approximation of a derivative third into the diagram of Newmark implemented in the operators of direct integration in dynamics in *Code_Aster* `DYNA_LINE_TRAN` [U4.53.02] and `DYNA_NON_LINE` [U4.53.01], one prefers to operate a simple correction of the second member, which returns in fact to consider the impedance explicitly. The stability conditions of the diagram of Newmark are not rigorously any more the same ones, but the experiment showed us that it is simple to arrive at convergence starting from the old conditions.

This choice of an explicit correction of the second member will be also justified at the time of the implementation of paraxial elements of order 1, qu ' it makes easier definitely.

3.2.4 Detailed formulation

One proposes here the precise formulation for an acoustic fluid modelled on a field Ω with an anechoic condition on a part Σ_a border Σ field. Apart from that, one breaks up the border into a free surface and a part in contact with a rigid solid. The introduction of requests external or the presence of an elastic structure is modelled easily by the current methods. The elements of volume and surface are formulated in p and ϕ .
The equations in the fluid are:

$\rho_f \Delta \phi + \frac{1}{c^2} p = 0$ in volume Ω éq 3.2.4-1

$p = \rho_f \frac{\partial^2 \phi}{\partial t^2}$ in volume Ω éq 3.2.4-2

$p = 0$ on free surface éq 3.2.4-3

$\frac{\partial \phi}{\partial n} = 0$ on the rigid wall éq 3.2.4-4

$\frac{\partial p}{\partial n} = -\frac{1}{c} \frac{\partial p}{\partial t}$ on the part of the border with anechoic condition éq 3.2.4-5

One multiplies the equation [éq 3.2.4-1] by a field of virtual potential y and one integrates in Ω :

$$\int_{\Omega_f} \left[\frac{1}{c^2} p \psi + \rho_f \frac{\partial^2}{\partial t^2} (\nabla \phi . \nabla \psi) \right] + \int_\Sigma \psi \rho_f \frac{\partial^2}{\partial t^2} \left(\frac{\partial \phi}{\partial n} \right) = 0$$ according to the formula of Green

Maybe, with the boundary conditions on Σ and the equation [éq 3.2.4-2]:

$$\int_{\Omega_f} \left[\frac{1}{c^2} p \psi + \rho_f \frac{\partial^2}{\partial t^2} (\nabla \phi . \nabla \psi) \right] + \int_\Sigma \psi \rho_f \frac{\partial p}{\partial n} = 0$$

One can consequently apply the condition of impedance formulated in pressure:

$$\int_{\Sigma_a} \psi \rho_f \frac{\partial p}{\partial n} = -\frac{1}{c} \int_{\Sigma_a} \psi \rho_f \frac{\partial p}{\partial t}$$

Moreover, to arrive to a symmetrical formulation of the terms of volume, one multiplies the equation [éq 3.2.4-2] by a virtual field of pressure q and one integrates in Ω :

$$\int_{\Omega_f} \frac{pq}{\rho_f c^2} - \frac{\partial^2}{\partial t^2} \int_{\Omega_f} \frac{\phi q}{c^2} = 0$$

By summoning the two variational equations, one obtains:

$$\frac{1}{\rho_f c^2} \int_{\Omega_f} pq + \rho_f \frac{\partial^2}{\partial t^2} \left[\frac{1}{\rho_f c^2} \int_{\Omega_f} (\phi q + p \psi) - \int_{\Omega_f} \nabla \phi . \nabla \psi \right] - \frac{1}{c} \int_{\Sigma_a} \psi \rho_f \frac{\partial p}{\partial t} = 0$$

Matriciellement:

$$\begin{bmatrix} \mathbf{M_f} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ \varphi \end{bmatrix} - \frac{1}{c} \begin{bmatrix} 0 & 0 \\ \mathbf{A} & 0 \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\mathbf{M}_{fl}}{c^2} \\ \frac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f \mathbf{H} \end{bmatrix} \begin{bmatrix} \ddot{p} \\ \ddot{\varphi} \end{bmatrix} = 0$$

where submatrices $\mathbf{M_f}, \mathbf{M}_{fl}$ and $\mathbf{H}$ the same bilinear forms discretize as previously.

The submatrix $\mathbf{A}$ discretize the term $\int_{\Sigma_a} \psi \rho_f \frac{\partial p}{\partial t}$. The matrix of damping obtained is not symmetrical, as one had predicted higher. This is why one rejects this term with the second member.

3.2.5 Direct temporal integration

In our case, because of nonthe symmetry of the matrix of impedance, one chooses to consider the anechoic term explicitly as we evoked before. That amounts calculating it at the moment t and to place it among the requests at the time of the expression of dynamic balance at the moment $t+\Delta t$. One solves:

$$\begin{bmatrix} \mathbf{M_f} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_{t+\Delta t} \\ \varphi_{t+\Delta t} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\mathbf{M}_{fl}}{c^2} \\ \frac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f \mathbf{H} \end{bmatrix} \begin{bmatrix} \ddot{p}_{t+\Delta t} \\ \ddot{\varphi}_{t+\Delta t} \end{bmatrix} = \frac{1}{c} \begin{bmatrix} 0 & 0 \\ \mathbf{A} & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_t \\ \dot{\varphi}_t \end{bmatrix} \quad \text{éq 3.2.5-1}$$

Instead of:

$$\begin{bmatrix} \mathbf{M_f} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_{t+\Delta t} \\ \varphi_{t+\Delta t} \end{bmatrix} - \frac{1}{c} \begin{bmatrix} 0 & 0 \\ \mathbf{A} & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{t+\Delta t} \\ \dot{\varphi}_{t+\Delta t} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\mathbf{M}_{fl}}{c^2} \\ \frac{\mathbf{M}_{fl}^{\mathbf{T}}}{c^2} & \rho_f \mathbf{H} \end{bmatrix} \begin{bmatrix} \ddot{p}_{t+\Delta t} \\ \ddot{\varphi}_{t+\Delta t} \end{bmatrix} = 0$$

Thus, there is not a nonsymmetrical matrix to treat in the system giving $\mathbf{X}$ at the moment $t+\Delta t$.

Note:

In a nonlinear calculation, one reactualizes the second member with each internal iteration. Calculation can thus prove more exact and more stable in this case.

3.3 Use in *Code_Aster*

The taking into account of anechoic fluid elements and the calculation of their impedance require a specific modeling on the absorbing borders:

- in 2D with modeling '2D_FLUI_ABSO' on the finite elements of type MEFASEn ($n=2,3$) on the edges absorbing with n nodes.
- in 3D with modeling '3D_FLUI_ABSO' on the finite elements of type MEFA_FACEn ($n=3,4,6,8,9$) on the faces absorbing with n nodes.

In harmonic analysis with the operator DYNA_LINE_HARM [U4.53.11], one calculates as a preliminary a mechanical impedance by the option IMPE_MECA of the operator CALC_MATR_ELEM [U4.61.01] and one informs it in DYNA_LINE_HARM (keyword MATR_IMPE_PHI).

The calculation of the option IMPE_MECA require a specific charge definite by the keyword IMPE_FACE of the operator AFFE_CHAR_MECA ; one informs there the group of fluid meshs of absorbents as well as the value of impedance that one affects to it behind the operand IMPE : this value is worth $\rho_f c$. It should be noted that if one wants to represent a rate of reflection r , for example in the case of the presence of alluvia in bottom of reserve, it is then necessary to correct the value of celerity c by a new value c' such as $c'=c\frac{1+r}{1-r}$.

In transitory analysis, the taking into account of the correct force due under the terms of impedance is automatic with modelings of elements absorbents in the operators DYNA_LINE_TRAN [U4.53.02] and DYNA_NON_LINE [U4.53.01]. For its explicit formulation, the use of this kind of absorbing element can condition the digital stability of the solution of the total problem, even whenever a diagram of integration in time of the family of Newmark is used. In this case, the choice of a step of sufficiently fine time, even near to the condition of Current, is consequently advised for the transitory resolution.

4 Elastic elements absorbents in *Code_Aster*

This part presents the main part of the general constraints of implementation of elastic elements of border absorbents with the paraxial approximation of order 0 in *Code_Aster*. One points out the relation of paraxial impedance of order 0 such as it was established by Modaressi for a linear elastic range:

$$\mathbf{t}(\mathbf{u})=\rho\left(c_p\frac{\partial\mathbf{u}_\perp}{\partial t}+c_s\frac{\partial\mathbf{u}_{//}}{\partial t}\right)$$

$\mathbf{u_T}$ becomes $\mathbf{u}_3$ and $\mathbf{u}_{//}$ becomes $\mathbf{u}'$

4.1 Adaptation of the seismic loading to the paraxial elements

One presented in the first part the principle of taking into account of the incidental field thanks to the paraxial elements. It is advisable here to present the methods of modeling of the seismic loading in *Code_Aster* to be able to adapt the data to the requirements of the paraxial elements.

The fundamental equation of dynamics associated with an unspecified model 2D or 3D discretized in finite elements with continuous medium or structure and in the absence of external loading is written in the absolute reference mark:

$$\mathbf{M}\ddot{\mathbf{X}}_\mathbf{a}+\mathbf{C}\dot{\mathbf{X}}_\mathbf{a}+\mathbf{K}\mathbf{X}_\mathbf{a}=0$$

One breaks up the movement of the structures into a movement of training $\mathbf{X_e}$ and a relative movement $\mathbf{X_r}$.

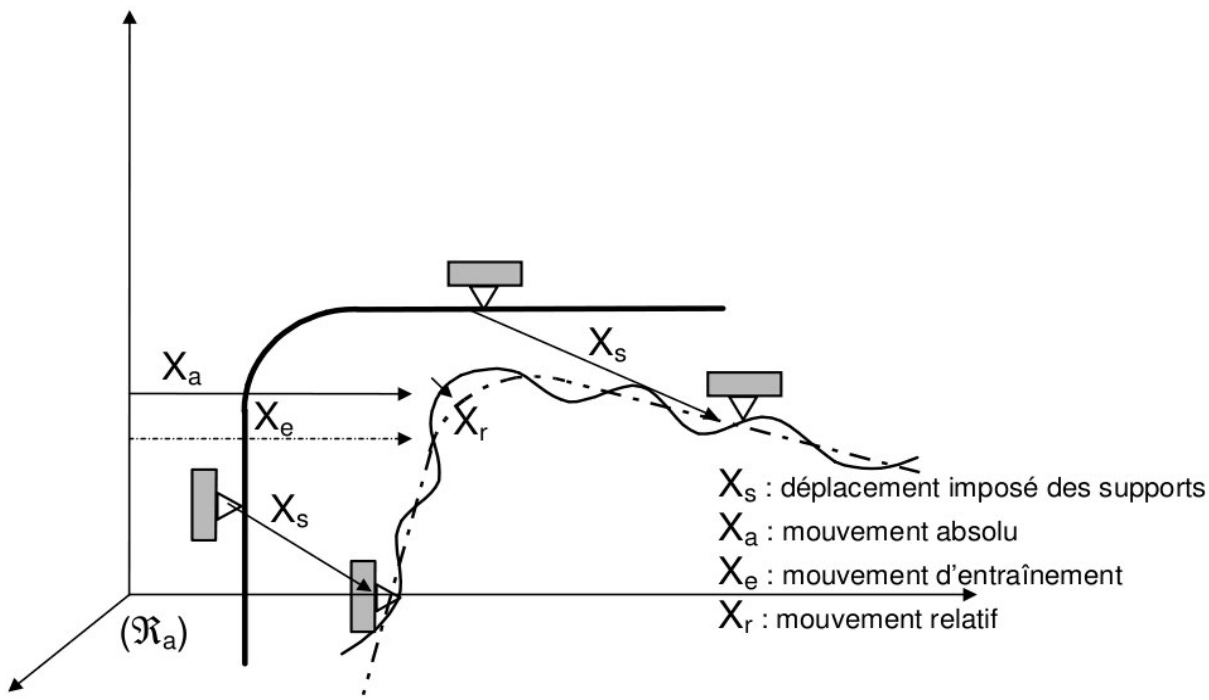


Figure 4.1-a: Decomposition of the movement of the structures

Thus, $\mathbf{X_a}=\mathbf{X_r}+\mathbf{X_e}$

- $\mathbf{X_a}$ is the vector of displacements in the absolute reference mark,
- $\mathbf{X_r}$ is the vector of relative displacements, i.e. the vector of displacements of the structure compared to the deformation which it would have under the static action of the displacements imposed on the level of the supports $\mathbf{X_S}$. $\mathbf{X_r}$ is thus null at the points of anchoring,
- $\mathbf{X_e}$ is the vector of displacements of training of the structure produces statically by the imposed displacement of the supports $\mathbf{X_S}$: $\mathbf{X_e}=\mathbf{\Psi}\,\mathbf{X_s}$,
- $\mathbf{\Psi}$ is the matrix of the static modes. The static modes represent the response of the structure to a unit displacement imposed on each degree of freedom of connection (others being blocked), in the absence of external forces. Thus, $\mathbf{K}\mathbf{\Psi}=0$, i.e., $\mathbf{K}\mathbf{X}_e=0$.
  In the case of the mono-support (all the supports undergo the same imposed movement), $\mathbf{\Psi}$ is a rigid mode of body.

Assumption in *Code_Aster* :

> *It is supposed that the damping dissipated by the structure is of viscous type i.e. the force of damping is proportional to the relative speed of the structure. Thus,* $\mathbf{C}\dot{\mathbf{X}}_e=0$.

The fundamental equation of dynamics in the relative reference mark is written then:

$$\mathbf{M}\ddot{\mathbf{X}}_\mathbf{r}+\mathbf{C}\dot{\mathbf{X}}_\mathbf{r}+\mathbf{K}\mathbf{X}_\mathbf{r}=-\mathbf{M}\mathbf{\Psi}\ddot{\mathbf{X}}_\mathbf{S}$$

The operator CALC_CHAR_SEISME [U4.63.01] the term calculates $-\mathbf{M}\mathbf{\Psi}$, or more exactly, $-\mathbf{M}\mathbf{\Psi}\,\mathbf{d}$ where $\mathbf{d}$ is an unit vector such as $\mathbf{X_s}=\mathbf{d}.f(t)$ with f a scalar function of time.

One distinguishes two types of seismic loadings introduced into *Code_Aster* thanks to the operator CALC_CHAR_SEISME :

1) The loading of the type MONO_APPUI, for which $\mathbf{\Psi}$ is the matrix identity (the static modes are modes of rigid body),
2) The loading of the type MULTI_APPUI, for which $\mathbf{\Psi}$ is unspecified.

According to the method of taking into account of the incidental field with the paraxial elements presented in the first part, it is necessary for us to know on the border displacement and the constraints due to the incidental field. For the loading of the type MULTI_APPUI, only displacement is directly accessible at any moment. It thus seems difficult to allow the use of such a load pattern with paraxial elements in the ground. Moreover, if such a loading models imposed displacements of the supports, it does not require a modeling of the ground since all the influence is taken into account by these displacements.

The case MONO_APPUI can be perceived differently. It represents an overall acceleration applied to the model. Consequently, the wave propagation in the ground can play a role to play in the behavior of the structure, since the movements of the interface ground-structure are not imposed. Moreover, the paraxial elements are usable with this kind of loading because it does not create constraints at the border of the grid (a rigid mode of body does not create deformations). Consequently, one has all the data necessary to calculation of the impedance absorbing on the border.

Notice 1:

In the case of a seismic request `MONO_APPUI` *, dynamic calculation is done in the relative reference mark. If one amounts on the term discretizing on the paraxial elements (see first part), one notices that* u_i *corresponds exactly to the displacement of training* $\mathbf{X_e}$ *presented higher. Thus,* $\mathbf{u}-\mathbf{u_i}$ *corresponds to the relative displacement calculated during calculation. Consequently, the relation to be taken into account on the paraxial elements in such a configuration is simply:*

$$\mathbf{t}(\mathbf{u})=A_0\left(\frac{\partial \mathbf{u}}{\partial t}\right)$$

Notice 2:

In the case of a calculation of interaction ground-fluid-structure with infinite fluid, the pressure to be taken into account for the calculation of the anechoic impedance in the fluid is well the absolute pressure, if there is not an incidental field in the fluid (what is often the case). The correction which one could exempt to make for the ground must then be made for the fluid paraxial elements.

4.2 Implementation of the elements in transient and harmonic

4.2.1 Implementation in transient

The mode of implementation of the elastic paraxial elements in transient is very close to that presented for the fluid elements. The difference comes primarily from the need for breaking up displacement into a component according to the normal with the element, corresponding to a wave P , and a component in the plan of the element, corresponding to a wave S. One is then capable to discretize the relation of impedance introduced into the first part:

$$\mathbf{t}(\mathbf{u})=\rho C_p\frac{\partial \mathbf{u_3}}{\partial t}+\rho C_s\frac{\partial \mathbf{u}^/}{\partial t}$$

One does not reconsider the diagram of temporal integration which one already described in the preceding part, knowing that one considers the relation of impedance by an operator of damping added to the first member.
For the taking into account of the additional term:

$$\mathbf{t_1}(\mathbf{u})=\frac{\lambda+2\mu}{L}u_3\mathbf{e_3}+\frac{\mu}{L}\mathbf{u}'$$

one uses this time an operator of rigidity added to the first member.

4.2.2 Implementation in harmonic

Fluid acoustic elements of *Code_Aster* propose already the possibility of taking into account an impedance imposed on the border of the grid in harmonic. That corresponds to the treatment of a term in ω^3 in the equations, as referred to above. It is trying to introduce the possibility of imposing an impedance absorbing for an elastic problem in harmonic.

For a harmonic calculation of answer of an infinite structure, the taking into account of the absorbing impedance as a correction of the second member is obviously not applicable. However, the relation of impedance to order 0 expresses the surface terms according to the speed of the nodes of the element. One can thus build a pseudo-matrix of viscous damping translating the presence of the infinite field.
In the same way, one builds (cf 4.1.1) a pseudo-matrix of rigidity supplementing the role of the matrix of damping defined previously.

Decomposition of the relation of impedance according to the components normal or tangential of displacement on the constrained element us to build the matrix of impedance in a local reference mark on the element. One defines this local reference mark in the elementary routine as well as the matrix of passage which allows the return to the total base.

4.3 Seismic load pattern per plane wave

In complement of the methods of taking into account of the seismic loading already available and because of the inadequacy of the mode `MULTI_APPUI` with the paraxial elements, it seems interesting to introduce a principle of loading per plane wave. That corresponds to the loadings classically met during calculations of interaction ground-structure by the integral equations.

4.3.1 Characterization of a wave planes in transient

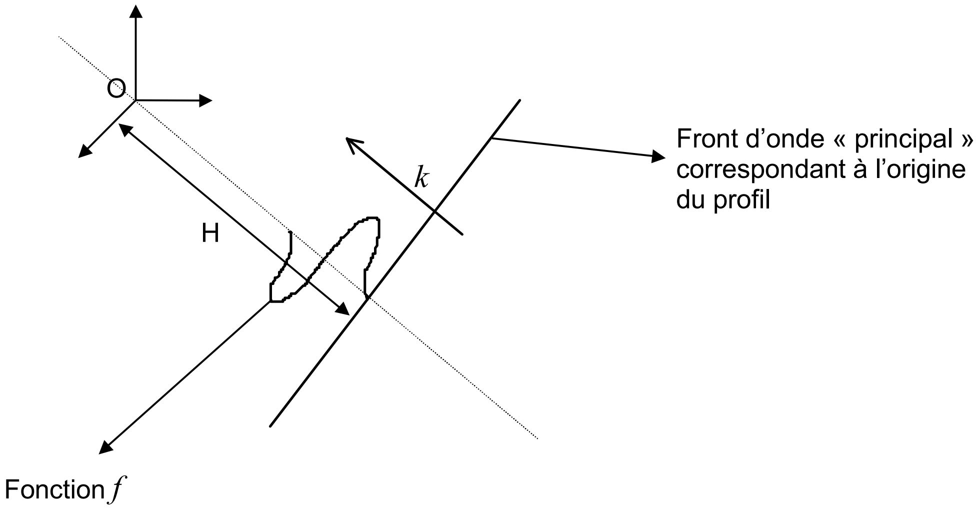
In harmonic, a wave planes elastic is characterized by its direction, its pulsation and its type (wave P for the compression waves, waves SV or SH for the waves of shearing). In transient, the data of the pulsation, corresponding to a standing wave in time, must be replaced by the data of a profile of displacement which one will take into account the propagation in the course of time in the direction of the wave.
Directions of the waves P , SV and SH are given starting from the vector V informed by the parameter `DIRECTION`. Namely:

– P is colinéaire with V and normalized to 1,
– SH is the intersection of the horizontal plane and normal plan with V , and normalized to 1,
– SV is the vector product of SH and of P . There exists a case of indetermination with this rule when the horizontal plane and the normal plan are confused. In this case, if $V=Z$ purely vertical, one imposes $SH=Y$, and $SV=X$.

More precisely, one will consider a plane wave in the form:

$$\mathbf{u}(\mathbf{x},t)=f(\mathbf{k}.\mathbf{x}-C_p t)\mathbf{k} \quad \text{for a wave } P \text{ (with } \mathbf{k} \text{ unit)}$$
$$\mathbf{u}(\mathbf{x},t)=f(\mathbf{k}.\mathbf{x}-C_S t)\wedge\mathbf{k} \quad \text{for a wave } S \text{ (with always } \mathbf{k} \text{ unit)}$$

f then represent the profile of the wave given according to the direction $\mathbf{k}$.



H is the distance from the origin to the principal wave front.

4.3.2 User data for the loading by plane wave

In accordance with the theory exposed in first part, it is necessary for us to calculate the constraint at the border of the grid due to the incidental wave and the term of impedance corresponding to incidental displacement, is:

$$\mathbf{t}(\mathbf{u_i}) \quad \text{and} \quad A_0\left(\frac{\partial \mathbf{u_i}}{\partial t}\right)+A_1(\mathbf{u_i})$$

To express the constraints, it is necessary for us to have the deformations due to the incidental wave, the law of behavior of material enabling us to pass from the ones to the others.
On the elements of border, one can express the tensor of the deformations linearized in each node by the classical formula:

$$\boldsymbol{\varepsilon}(x,t)=\frac{1}{2}\left[\nabla\mathbf{u}(x,t)+{}^{t}\nabla\mathbf{u}(x,t)\right]$$

Finally, to consider the constraints due to the incidental field, we thus should determine the derivative $\frac{\partial(\mathbf{u_i})_j}{\partial x_k}$ for j and k traversing the three directions of space. One obtains these quantities starting from the definition of the incidental plane wave:

$$\frac{\partial(\mathbf{u_i})_j}{\partial x_k}=k_k f'(\mathbf{k}.\mathbf{x}-C_m t)k_j \text{ with } m=S \text{ or } P$$

With regard to the term of impedance, it is necessary for us $\frac{\partial\mathbf{u_i}}{\partial t}=-C_m\dot{f}(\mathbf{k}.\mathbf{x}-C_m t)\mathbf{k}$, always with $m=S$ or P.

It is seen whereas the important function for a loading by wave planes with paraxial elements of order 0 is not the profile of the wave f, but its derivative, is f' that is to say $\dot{f}$. The wave being plane, the wave front is characterized by the plans $\mathbf{k}.\mathbf{x}-C_m t=cte$, from where the relation: $\mathbf{k}.d\mathbf{x}=C_m dt$. There is thus following equivalence between the two derivative of f : $f'=\frac{1}{C_m}\dot{f}$.
One chooses to ask for the function $\dot{f}$ with the user like data of calculation.
However, in the optional case where one adds the rigidities distributed along the border of the finite elements field, the user must moreover provide the profile of the wave f .
In addition, for calculating well dephasing in time due on the way of the wave, it is also necessary to indicate the entrance point of the loading by plane wave in the structure given by the scalar product of the vector of wave and the position of the point source of entry $\mathbf{k}.\mathbf{x_0}$ provided by the parameter `DIST`.
For the taking into account of the reflected wave, this one is activated if one gives the position of the exit point $\mathbf{k}.\mathbf{x_1}$ provided by the parameter `DIST_REFLECHI`. In this case, the expression of the profile of the wave by taking account of the space dephasing dependent on the way of the wave $\dot{f}(\mathbf{k}.(\mathbf{x}-\mathbf{x_0})-C_m t)$ becomes then $\dot{f}(\mathbf{k}.(\mathbf{x}-\mathbf{x_0})-C_m t)+\dot{f}(\mathbf{k}.(\mathbf{2x_1}-\mathbf{x_0}-\mathbf{x})-C_m t)$ with $m=S$ or P.
One can consequently recapitulate the parameters to enter for the definition of a loading per plane wave in transient:

Type of the wave	: P, SV or SH
Direction of the wave	: k_x, k_y, k_z
Derived from the profile of the wave	: $\dot{f}(t)$ for $t\in[0,+\infty[$
Profile of the wave (optional)	: $f(t)$ for $t\in[0,+\infty[$
Space source of the loading	: $\mathbf{k}.\mathbf{x_0}$
Space exit of the loading	: $\mathbf{k}.\mathbf{x_1}$

4.4 Use in *Code_Aster*

The taking into account of elastic elements absorbents and the calculation of their impedance requires a specific modeling on the absorbing borders:

- in 2D with modeling 'D_PLAN_ABSO' on the absorbing edges.
- in 3D with modeling '3D_ABSO' on the absorbing faces.

The formulation of these elements being enough rudimentary to precisely be able that they are compared to discrete shock absorbers and thus to be used in harmonic analyses, consequently, on the one hand, one should not block them during dynamic analyses, and on the other hand, a counterpart is that the quality of their use depends on quality of the form on the border. A good test of this quality takes as a starting point the cases tests SDLV120 and SDLV121 and can be based on more or less total absorption on the level of this border of a passage of wave of displacement imposed at the top of structure. As for these tests, one can make sure that the wave does not return while looking at a degree of freedom speed on Nœud near the border.

In harmonic analysis with the operator DYNA_LINE_HARM [U4.53.11], one calculates as a preliminary a mechanical cushioning by the option AMOR_MECA of the operator CALC_MATR_ELEM [U4.61.01] and one informs it in DYNA_LINE_HARM (keyword MATR_AMOR).

In transitory analysis, the calculation of the mechanical cushioning is automatic with modelings of elements absorbents in the operator DYNA_NON_LINE [U4.53.01]. With the operator DYNA_LINE_TRAN [U4.53.02], one calculates this mechanical cushioning as a preliminary in an explicit way by the option AMOR_MECA of the operator CALC_MATR_ELEM [U4.61.01] and one informs it by the keyword MATR_AMOR.

For these two analyses, the calculation of added mechanical rigidity is automatic with modelings of elements absorbents when the option systematically is calculated RIGI_MECA whatever the operator of calculation.

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