Parametric and not-parametric probabilistic models in dynamics

Summary:

This document describes two probabilistic approaches, one parametric and other the nonparametric one in order to take into account uncertainties of model and modeling for the dynamic systems in mechanics of the structures. The not-parametric approach is specific to the resolution on modal basis of the linear systems (in transient or harmonic) and to the resolution on modal basis of the systems with non- localised linearities (in transient). The parametric approach is a priori universal, but it is more particularly presented if it is combined with the nonparametric approach.

Contents

1 General information

Uncertainties, even when they are reduced, can significantly change the prediction of the vibratory behavior of the structure ([bib13], [bib16]). It is thus necessary to take them into account in a quantifiable and explicit way to increase the realism and the robustness of the forecasts. In this context, a probabilistic model of uncertainties contributes to the realism of the approach.

The probabilistic approach classical, known as parametric, makes it possible to incorporate in the mechanical analysis uncertainties on the data, that is to say for example parametric uncertainties on the geometry, the boundary conditions or the properties of materials. In this approach, each identified parameter as source of random uncertainties is modelled by a random variable. The parameters of entry of the model being thus characterized, the probabilistic digital methods seek to characterize in a probabilistic way the results quantities of the model. For complex structures, for which the vibratory behavior depends on a large number of parameters, this kind of probabilistic analysis is limited by the great amount of information necessary to characterize the parameters of entry and the difficulties of implementation of the propagation of variability.

A new approach, known as probabilistic approach nonparametric of random uncertainties in dynamics of the structures was recently proposed by C. Soize ([bib20] with [bib24]). This approach makes it possible to take into account uncertainties of model (uncertainties on the geometry for example) and uncertainties of modeling (uncertainties on the kinematics of beam or plate for example). It is based on the construction of random matrices of the linear dynamic systems, after projection on modal basis.

These two probabilistic approaches, one parametric and other the nonparametric one, are complementary. Thus a mixed, parametric and nonparametric approach, can be is developed (original method having caused publications ([bib6] and [bib11]). In particular, this mixed method is adapted well to the taking into account of uncertainties in the analysis of a nonlinear dynamic system composed of a structure linaire reduced on modal basis and of localised non-linearities. Indeed, uncertainties on the level of the linear structure can be treated naturally by the nonparametric approach and uncertainties on non-linearities can be treated naturally by the parametric approach.

The basic digital model of the nonlinear dynamic system is a model finite elements which will be called “model fine elements means”. That it is the or not parametric parametric approach, the laws of probability must be defined in an adequate way and most objectively possible starting from this average model. A Gaussian model of the matrices random is not adapted to dynamics in low frequency (negative Eigen frequencies). In order to build the law of corresponding probability, one uses the principle of the maximum of entropy of Jayne ([bib14], [bib15], [bib17]) as well as information available (model fine elements means, algebraic properties of the matrices, etc)

In this document, we present the nonparametric approach for resolutions transient or harmonic of the dynamic system. The parametric approach is more particularly presented if it combined with the non-parametric method.

The readers seeking the fundamental results of stochastic dynamics will be able to refer to [18] and the readers searching the theoretical details of the probabilistic approach presented in this document will be able to refer to [bib19]. Examples of uses of the approach are given in [bib5], and [bib7]. In [bib10], experimental tests made it possible to show the predictive character of the approach.

2 Modelings of the dynamic system

2.1 Average model finite elements

2.1.1 Transitory resolution in absolute coordinates

In the absolute reference mark, the mechanical system is modelled by the method with the finite elements. This basic model (in general that which would have been used in the deterministic study) is indicated under the name of “average model finite elements”. All the sizes relating to the average models are underlined.

That is to say $t \mapsto \underline{\mathbf{y}}(t)$ the transitory answer in the absolute reference mark of the “model finite elements average” definite on the interval of study $[0,T]$ and with value in $\mathbb{R}^k$ where k is the number of degrees of freedom. The matrices of mass, damping and rigidity are respectively noted $[\underline{M}]$, $[\underline{D}]$ and $[\underline{K}]$.
The transitory answer $\underline{\mathbf{y}}(t)$ “average model finite elements” checks the discretized nonlinear differential equation following:

$$[\underline{M}]\ddot{\underline{\mathbf{y}}}(t)+[\underline{D}]\dot{\underline{\mathbf{y}}}+\mathbf{f}_c(t,\dot{\underline{\mathbf{y}}}(t),\underline{\mathbf{y}}(t),\underline{\mathbf{w}})=\mathbf{f}(t) \quad , \quad T \square \text{ , éq 2.1.1-1}$$

with the initial conditions,

$$\underline{\mathbf{y}}(0)=\dot{\underline{\mathbf{y}}}(0)=0 \text{ , éq 2.1.1-2}$$

- $\mathbf{f}(t) \in \mathbb{R}^m$ represent the discretization by finite elements of the external forces.
- $\mathbf{f}_c(t,\underline{\mathbf{y}}(t),\dot{\underline{\mathbf{y}}}(t),\underline{\mathbf{w}}) \in \mathbb{R}^m$ corresponds to nonthe localised linearities (for example due to elastic thrusts of shock). Elements $\underline{w}_1,\ldots,\underline{w}_\nu$ vector $\underline{\boldsymbol{w}} \in \mathbb{R}^\nu$ represent a set of parameters defining these nonlinearities (for example game, stiffness of shock, damping of shock, etc).

2.1.2 Transitory resolution in relative coordinates (earthquake)

As in the transitory case in absolute coordinates, the mechanical system is modelled by a basic model, the “average model finite elements”.
One notes $t \to \underline{\mathbf{z}}(t)$ the transitory answer in absolute coordinates of this model on the interval of study $[0,T]$ with value in $\mathbb{R}^k$ (**caution:** notice the change of notation compared to the preceding paragraph.).

The transitory answer “average model finite elements” checks the discretized nonlinear differential equation following:

$$\begin{bmatrix}[\underline{M}] & [\underline{M}_{ls}]\\ [\underline{M}_{ls}]^T & [\underline{M}_s]\end{bmatrix}\begin{bmatrix}\ddot{\underline{\mathbf{z}}}(t)\\ \ddot{\underline{\mathbf{z}}}_s(t)\end{bmatrix}+\begin{bmatrix}[\underline{D}] & [\underline{D}_{ls}]\\ [\underline{D}_{ls}]^T & [\underline{D}_s]\end{bmatrix}\begin{bmatrix}\dot{\underline{\mathbf{z}}}(t)\\ \dot{\underline{\mathbf{z}}}_s(t)\end{bmatrix}+\begin{bmatrix}[\underline{K}] & [\underline{K}_{ls}]\\ [\underline{K}_{ls}]^T & [\underline{K}_s]\end{bmatrix}\begin{bmatrix}\underline{\mathbf{z}}(t)\\ \underline{\mathbf{z}}_s(t)\end{bmatrix}+\begin{bmatrix}F_c(t,\underline{\mathbf{z}}(t),\dot{\underline{\mathbf{z}}}(t);\underline{\mathbf{w}})\\ 0_d\end{bmatrix}=\begin{bmatrix}\mathbf{g}(t)\\ \mathbf{g}_s(t)\end{bmatrix}$$
$$t \in [\,0,T\,]. \quad \text{éq 2.1.2-1}$$

with the initial conditions,

$$\underline{\mathbf{z}}(0)=\dot{\underline{\mathbf{z}}}(0),\underline{\mathbf{z}}_s(0)=\dot{\underline{\mathbf{z}}}_s(0) \text{ éq 2.1.2-2}$$

- $\underline{g}(t) \in \mathbb{R}^m$ represent the discretization by finite elements of the external forces and $\underline{g_s}(t) \in \mathbb{R}^d$ corresponds to the discretization of the forces of reaction due to d conditions of Dirichlet.

- $F_c(t,\mathbf{z}(t),\dot{\mathbf{z}}(t),\underline{\mathbf{w}})\in\mathbb{R}^m$ corresponds to nonthe linearities located with like previously $\underline{\mathbf{w}}\in\mathbb{R}^v$ representing a set of parameters defining these nonlinearities.

After static raising, the matric equations [éq 2.1.2-1] and [éq 2.1.2-2] in the absolute reference mark are rewritten in "relative" coordinates:

$$[\underline{\mathbf{M}}]\ddot{\underline{\mathbf{y}}}(t)+[\underline{D}]\dot{\underline{\mathbf{y}}}+[\underline{K}]\underline{\mathbf{y}}(t)+\mathbf{f}_c(t,\dot{\underline{\mathbf{y}}}(t),\underline{\mathbf{y}}(t),\underline{\mathbf{w}})=\mathbf{f}(t)\ ,\ t\in[0,T]\ .$$ éq 2.1.2-3

$$\underline{\mathbf{y}}(0)=\dot{\underline{\mathbf{y}}}(0)=0\ ,$$ éq 2.1.2-4

- $\underline{\mathbf{y}}(t)\in\mathbb{R}^m$ is the vector of the free degrees of freedom in the "relative" frame of reference such as

$\underline{\mathbf{z}}(t)=\underline{\mathbf{y}}(t)+[\underline{R}]\underline{\mathbf{z}}_s(t)$ with $[\underline{R}]=-[\underline{K}]^{-1}[\underline{K_{ls}}]$

- The function $t\to\underline{\mathbf{f}}(t)$ defined on $[0,T]$ and with value in $\mathbb{R}^m$ and the nonlinear application $(\mathbf{x},\mathbf{y})\mapsto\mathbf{f}_c(t,\mathbf{x},\mathbf{y}\ ;\ \underline{\mathbf{w}})$ of $\mathbb{R}^m\times\mathbb{R}^m$ in $\mathbb{R}^m$ are such as:

$$\mathbf{f}(t)=\mathbf{g}(t)-([\underline{M}][\underline{R}]+[\underline{M_{ls}}])\ddot{\underline{\mathbf{z}}}(t)-([\underline{D}][\underline{R}]+[\underline{D_{ls}}])\dot{\underline{\mathbf{z}}}(t)$$ éq 2.1.2-5

$$\mathbf{f}_c(t,\mathbf{x},\mathbf{y}\ ;\ \underline{\mathbf{w}})=F_c(t,\mathbf{x}+[\underline{R}]\underline{\mathbf{z}}_s(t),\mathbf{y}+[\underline{R}]\dot{\underline{\mathbf{z}}}_s(t)\ ;\ \underline{\mathbf{w}}).$$ éq 2.1.2-6

Note:

- *In the continuation, according to whether or not a static raising were carried out, $\underline{\mathbf{y}}(t)$ corresponds either to the transitory answer in absolute coordinates defined by [the § 2.1.1], or the transitory answer in "relative" coordinates defined by [§ 2.1.2].*
- *It is supposed that if them d conditions of Dirichlet were homogeneous no movement of rigid body could not occur. Consequently, $[\underline{K}]$ is symmetrical definite positive and its reverse $[\underline{K}]^{-1}$ is defined, which makes it possible to introduce the real matrix $[\underline{R}]=-[\underline{K}]^{-1}[\underline{K_{ls}}]$ of dimension $(m\times d)$.*
- *In Code_Aster the term of damping in [éq 2.1.2-5] is neglected.*

2.1.3 Harmonic resolution

As in the transitory case, the mechanical system is modelled by a basic model, the "average model finite elements". On a frequential tape $[\omega_1,\omega_2]$, the harmonic answer $\mathbf{q}(\omega)$ linear "model finite elements average" checks the following equation:

$$(-\omega^2[\underline{M}]+i\omega[\underline{D}]+[\underline{K}])q(\omega)=F(\omega)\quad,\quad\omega\in[\omega_1,\omega_2]$$ éq 2.1.3-1

with $\mathbf{f}(\omega)$ representing the discretization by finite elements of the external forces.

2.2 Average reduced matric model

It is supposed that the energy of vibration of the dynamic response is mainly localised in the field of the low frequencies. One can thus build the average matric model reduced while projecting $\underline{\mathbf{y}}(t)$ or $\underline{y}(\omega)$ on under clean space generated by N first modes of the system dynamic linear (infinite games) conservative homogeneous (blocked supports) associated which is written,

$$[\underline{K}]\underline{\varphi}=\lambda[\underline{M}]\varphi\ .$$ éq 2.2-1

Matrices $[\underline{M}]$ and $[\underline{K}]$ being definite positive (for $[\underline{K}]$ cf notices 2 [§2.1.2]), the eigenvalues are real and positive,

$$0 \leq \underline{\lambda_1} \leq \underline{\lambda_2} \leq \ldots \leq \lambda_n \qquad \text{éq 2.2-2}$$

Clean modes of vibration associated $\{\Phi_1, \Phi_2, \ldots\}$ the properties of orthogonality check,

$$<[\underline{M}]\varphi_\alpha, \varphi_\beta> = \underline{\mu_\alpha}\,\delta_{\alpha\beta} \qquad \text{éq 2.2-3}$$

$$<[\underline{K}]\varphi_\alpha, \varphi_\beta> = \underline{\mu_\alpha}\,\underline{\omega_\alpha^2}\,\delta_{\alpha\beta} \qquad \text{éq 2.2-4}$$

with

$$\omega_\alpha = \sqrt{(\lambda_\alpha)} \qquad \text{éq 2.2-5}$$

One notes the matrix of generalized mass respectively, the matrix of generalized stiffness and damping stamps it generalized by:

$$[\underline{M_n}] = [\underline{\Phi_n}]^T [\underline{M}] [\underline{\Phi_n}] \qquad \text{éq 2.2-6}$$

$$[\underline{K_n}] = [\underline{\Phi_n}]^T [\underline{K}] [\underline{\Phi_n}] \qquad \text{éq 2.2-7}$$

$$[\underline{D_n}] = [\underline{\Phi_n}]^T [\underline{D}] [\underline{\Phi_n}] \qquad \text{éq 2.2-8}$$

2.2.1 Resolution in transient

Projection $\underline{y^n}(t)$ of $\underline{y}(t)$ on under space generated by N first modes of the associated homogeneous conservative system dynamic linear is written:

$$\underline{\mathbf{y^n}}(t) = [\underline{\Phi_n}]\underline{\mathbf{q^n}}(t) = \sum_{\alpha=1}^{n} \underline{q_\alpha^n}(t)\,\underline{\Phi_\alpha}\,, \qquad \text{éq 2.2.1-1}$$

Generalized displacements are solutions of the average reduced matric model (dynamic system not linaire),

$$[M_n]\ddot{q}^n(t) + [D_n]\dot{q}^n(t) + [K_n]\,q^n(t) + F_c^n(t, q^n(t), \dot{q}^n(t)\ ;\ W) = F^n(t)\,, \qquad \text{éq 2.2.1-2}$$

$$\underline{\dot{q}^n}(0) = \underline{q^n}(0) = 0\,, \qquad \text{éq 2.2.1-3}$$

with

$$\mathbf{F^n}(t) = [\underline{\Phi_n}]^T \mathbf{f}(t)\,, \qquad \text{éq 2.2.1-4}$$

$$\mathbf{F_c^n}(t, \mathbf{q}, \mathbf{p}\ ;\ \underline{\mathbf{w}}) = [\Phi_n]^T \mathbf{f_c}(t, [\Phi_n]\,\mathbf{q}, [\Phi_n]\,\mathbf{p}, \underline{\mathbf{w}}) \qquad \text{éq 2.2.1-5}$$

2.2.2 Resolution in harmonic

Projection $\underline{y^n}(\omega)$ of $\underline{y}(\omega)$ on under space generated by N first modes of the associated homogeneous conservative system dynamic linear is written

$$\mathbf{q^n}(\omega) = [\Phi_n]^T \mathbf{f}(\omega)\,, \qquad \text{éq 2.2.2-1}$$

Generalized displacements $\mathbf{q^n}(\omega)$ are solutions of the average reduced matric model

$$(-\omega^2[\underline{M_n}] + i\omega[\underline{D_n}] + [\underline{K_n}])\,\mathbf{q^n}(\omega) = \mathbf{F^n}(\omega) \qquad \text{éq 2.2.2-2}$$

with

$$\mathbf{F^n}(\omega) = [\underline{\Phi_n}]^T \mathbf{f}(\omega)\,, \qquad \text{éq 2.2.2-3}$$

3 Probabilistic model

3.1 Introduction of the probabilistic model into the problem dynamics

In order to take into account uncertainties of modeling and uncertainties on the data, a mixed probabilistic formulation nonparametric – parametric is used. For that, the vector of n generalized degrees of freedom $\mathbf{q}^{\mathbf{n}}(t)$ (resp. $\mathbf{q}^{\mathbf{n}}(\omega)$) is replaced by a random variable $Q^n(t)$ (resp. $Q^n(\omega)$).

In transient, the stochastic process $t \to \mathbf{Q}^{\mathbf{n}}(t)$ indexed by $[0,T]$ and with value in R^n is solution of the nonlinear dynamic system,

$$[M_n]\ddot{Q}^n(t)+[D_n]\dot{Q}^n(t)+[K_n]Q^n(t)+F_c^n(t,Q^n(t),\dot{Q}^n(t)\ ;\ W)=F^n(t), \qquad \text{éq 3.1-1}$$

$$\dot{Q}^n(0)=Q^n(0)=0, \qquad \text{éq 3.1-2}$$

and in harmonic, the stochastic process $t \mapsto Q^n(\omega)$ indexed on $[\omega_1,\omega_2]$ and with value in $\mathbb{R}^n$ is solution of the system:

$$(-\omega^2[\underline{M}_n]+i\omega[\underline{D}_n]+[\underline{K}_n])\,Q^n(\omega)=F^n(\omega) \qquad \text{éq 3.1-3}$$

where, in the two transitory and harmonic cases, $[\underline{M}_n]$, $[\underline{D}_n]$, and $[\underline{K}_n]$ are random full positive matrices real symmetrical definite and where W is a random variable with value in $\mathbb{R}^\nu$. The introduction of random matrices into the equations [éq 3.1-1] and [éq 3.1-3] makes it possible to model random uncertainties associated with the linear part of the dynamic system. The random variable W vectorial value introduced into the equation [éq 3.1-1] allows to model random uncertainties concerning the parameters of nonthe linearities of shock.

The parametric probabilistic approach and the nonparametric probabilistic approach introduce random matrices (, and) and a random variable W whose laws of probability are a priori nonknown. The choice of a probabilistic model rather than another must rest only on information available (algebraic properties of the generalized matrices, median values of the parameters and the generalized matrices, etc). In order to objectively build the laws of probability of the probabilistic model of uncertainties, ([bib20] with [bib24]), the principle of the maximum of entropy ([bib14], [bib15], [bib17]) is used with a system of constraints defined by this information available. Information available and the probabilistic model which results from this are presented in the next paragraph.

3.2 Probabilistic model for the matrices of the dynamic system (nonparametric uncertainties)

3.2.1 Information available on the matrices of the dynamic system

The nonparametric probabilistic model is built in substituent the matrices $[\underline{M}_n]$, $[\underline{D}_n]$, and $[\underline{K}_n]$ by respectively noted random matrices $[\mathbf{M}_n]$, $[\mathbf{D}_n]$, and $[\mathbf{K}_n]$. So that the probabilistic dynamic system thus built either mechanically and statistically correct, the construction of the random matrices $[\mathbf{M}_n]$, $[\mathbf{K}_n]$ and $[\mathbf{D}_n]$ must be such as:

1) $[\mathbf{M}_n]$, $[\mathbf{K}_n]$ and $[\mathbf{D}_n]$ are of the random variables of the second order with values in the

whole of the positive definite real matrices symmetrical and dimension $(n \times n)$.

$$\left[\mathbf{M}_n\right] , \left[\mathbf{K}_n\right] \text{ and } \left[\mathbf{D}_n\right] \in M_n^+ \text{ P.S. (almost surely),} \qquad \text{éq 3.2.1-1}$$

where M_n^+ is the whole of the positive definite symmetrical matrices real of dimension $(n \times n)$

.

This algebraic property is absolutely required to have a random model of equation which corresponds to that of a dynamic system of the second deadened order.

2) Median values of the random matrices $\left[\mathbf{M}_n\right]$, $\left[\mathbf{K}_n\right]$ and $\left[\mathbf{D}_n\right]$ are respectively $\left[\underline{M}_n\right]$ $\left[\underline{K}_n\right]$ $\left[\underline{D}_n\right]$:

$$E\left(\left[\mathbf{M_n}\right]\right)=\left[\underline{M}_n\right] , \quad E\left(\left[\mathbf{K_n}\right]\right)=\left[\underline{K}_n\right] \quad \text{and} \quad E\left(\left[\mathbf{D_n}\right]\right)=\left[\underline{D}_n\right] , \qquad \text{éq 3.2.1-2}$$

where E indicate the expectation.

3) So that the solution of the probabilistic dynamic system is also a variable of the second order, one imposes at the times of the second order standards of Frobenius of the matrices opposite $\left[\mathbf{M}_n\right]^{-1}$, $\left[\mathbf{K}_n\right]^{-1}$ and $\left[\mathbf{D}_n\right]^{-1}$ to be finished:

$$E\{\left\|\left[\boldsymbol{M_n}\right]^{-1}\right\|_F^2\}<+\infty \qquad E\{\left\|\left[\boldsymbol{K_n}\right]^{-1}\right\|_F^2\}<+\infty \qquad E\{\left\|\left[\boldsymbol{D_n}\right]^{-1}\right\|_F^2\}<+\infty \qquad \text{éq 3.2.1-3}$$

with $\|[A]\|_F=\left(\mathrm{tr}\left([A][A]^T\right)\right)^{1/2}$.

Note:

The only property of positivity of the matrices is not enough and it should be made sure that their opposite are of the second order, from where (3 (a random variable of the second order almost surely invertible does not have in the case general a random variable reverses of the second order). For more details to see [bib19].

3.2.2 Construction of the probabilistic model by the principle of the maximum of entropy

The entropy “measures” the level of uncertainty of a law of probability. Thus, if $p_{[\mathbf{A}]}$ is the function of density of probability corresponding to a random matrix $[\mathbf{A}]$ (representing the matrices $\left[\mathbf{M}_n\right]$, $\left[\mathbf{K}_n\right]$ or $\left[\mathbf{D}_n\right]$) of law given, then entropy (or probabilistic uncertainty) $S(p_{[\mathbf{A}]})$ of $p_{[\mathbf{A}]}$ is defined by:

$$S(p_{[\mathbf{A}]})=-\int_{M_n^+} p_{[A]}([A])\ln(p_{[A]}([A]))d\tilde{A} \qquad \text{éq 3.2.2-1}$$

The principle of the maximum of entropy of Jayne consists in building the function of density of probability which maximizes the probabilistic entropy while checking a system of constraints. In this case, the system of constraints is defined by information available corresponding to the equations [éq 3.2.1-1] with [éq 3.2.1-3]. For the random matrix, this system of constraints is written

$$[\mathbf{A}]\in M_n^+ \text{ p.s. , } E\{[\mathbf{A}]\}=[\underline{A}] \qquad E\{\left\|\left[\boldsymbol{A}\right]^{-1}\right\|_F^2\}<+\infty . \qquad \text{éq 3.2.2-2}$$

It is shown whereas the random matrix $[\mathbf{A}]$ is such as (see [bib20] with [bib24])

$$[\mathbf{A}]=[\underline{L_A}]^T[\boldsymbol{G_A}][\underline{L_A}] \qquad \text{éq 3.2.2-3}$$

where $[\underline{L_A}]$ is the lower triangular matrix resulting from the factorization of Cholesky of the average matrix $[\mathbf{A}]$ and where the function of density of probability of the random matrix $[\boldsymbol{G_A}]$ is defined on the unit M_n^+ compared to measurement $\tilde{d}A$ such as:

$$\tilde{d}G = 2^{n(n-1)/4} \prod_{1 \le i \le j \le n} dG_{ij} \qquad \text{éq 3.2.2-4}$$

$$p_{[G_A]}([G]) = \mathbf{1}_{\mathbf{M}_n^+}([G]) \times C_{G_A} \times (\det[G])^{(1-\delta_A^2)(2\delta_A^2)^{-1}(n+1)} \times e^{-(n+1)(2\delta_A^2)^{-1} tr[G]} \qquad \text{éq 3.2.2-5}$$

with

$$C_{G_A} = \frac{(2\pi)^{-n(n-1)/4} \left(\frac{n+1}{2\delta_A^2} \right)^{n(n+1)(2\delta_A^2)^{-1}}}{\prod_{j=1}^{n} \Gamma\left(\frac{n+1}{2\delta_A^2} + \frac{1-j}{2} \right)} \qquad \text{éq 3.2.2-6}$$

where $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt$, éq 3.2.2-7

and where $\mathbf{1}_{\mathbf{M}_n^+}$ is the indicating function of M_n^+, and where the parameter δ_A controlling the dispersion of the random matrix $[\mathbf{A}]$ is defined by:

$$\delta_A = \left\{ \frac{E\left\{ \| [G_A] - [\underline{G}_A] \|_F^2 \right\}}{\| [\underline{G}_A] \|_F^2} \right\}^{1/2} , \qquad \text{éq 3.2.2-8}$$

The theoretical construction of the model provides an acceptable terminal for the level of uncertainty introduced. must be selected so that

$$0 < \delta_A < \sqrt{\frac{n_0+1}{n_0+5}} , \qquad \text{éq 3.2.2-9}$$

where $n_0 \in \mathbb{N}$ is a constant of the probabilistic model selected so that.

One shows moreover than, under the only constraints of the equations [éq 3.2.1-1] to [éq 3.2.1-3], the principle of the maximum of entropy leads so that the random matrices, or are statistically independent as a whole.

This probabilistic model for the positive definite symmetrical matrices random real differs from the more classical models of the random matrices based on the Gaussian Units and the Circular units (references in [bib19]). The orthogonal Gaussian unit used in addition for fields high frequencies would lead in the low frequencies field (in which one places oneself) to negative eigenvalues, which one cannot admit for the systems considered. Moreover, one matrix of the orthogonal unit Gaussian does not have in the case general an opposite matrix of the second order, which would lead to a solution of the dynamic system of infinite variance, which one cannot admit either.

3.3 Probabilistic model for the real variables (parametric uncertainties)

3.3.1 Information available on the real variables

In the mixed probabilistic approach, parametric probabilistic modeling consists in substituting the parameter $\underline{w}$ non-linearities in the nonlinear dynamic systems given by [éq 2.1.1-1] or [éq 2.1.2-3] by a noted random variable $W = (W_1, \ldots, W_\nu)$. In a purely parametric approach (i.e without making random the matrices of the system dynamic), parametric probabilistic modeling consists in substituting certain parameters $\underline{w}$ matrices $[\underline{M}_n(\underline{w})]$, $[\underline{K}_n(\underline{w})]$ and $[\underline{D}_n(\underline{w})]$ average dynamic system reduced by a random variable W . These parameters can be for example parameters of material.

It is supposed that the components of W are real random variables independent between them and independent of the random matrices of the dynamic system. In the continuation, to reduce the writing, one notes W an unspecified coordinate W_j . The construction of the probabilistic model requires to

define the information available, which constitutes a system of constraint under which entropy of the density of probability of the random variable W *is* maximized.

Information available is the following one:

1) The support of the random variable W is an interval D of $\mathbb{R}^{\nu}$
$$W \in D, \ p.s. \ . \qquad \text{éq 3.3.1-1}$$
2) The median value of the random variable W is $\underline{w}$:
$$E[W] = \underline{w} \ . \qquad \text{éq 3.3.1-2}$$
3) Possibly, according to information indeed available, moment of the second order of the random variable $|W|^{-1}$ is finished:
$$E\left[|W|^{-2}\right] < +\infty \ . \qquad \text{éq 3.3.1-3}$$

3.3.2 Construction of the probabilistic model by the principle of the maximum of entropy

If p_W is the function of density of probability corresponding to the random variable W then the entropy (probabilistic uncertainty) $S(p_W)$ of p_W is defined by:
$$S(p_W) = -\int_{-\infty}^{+\infty} p_W(w) \ln(p_W(w)) dw \ , \qquad \text{éq 3.3.2-1}$$

By using the principle of the maximum of entropy, one obtains three densities of probability according to nature of the support D and according to whether the constraint corresponding to the equation [éq 3.2.2-6] is considered or not.

3.3.3 Closed support limited without information on the reverse

If there exist two realities a and b such as $D = [a, b]$ and if the information available is given by the equations [éq 3.3.1-1] and [éq 3.2.2-5], then the random variable W follows a truncated exponential law whose function of density of probability is:
$$p_W(w) = 1_{[a,+\infty]}(w) \frac{k}{\alpha(k)} \exp(-kw) \qquad \text{éq 3.3.3-1}$$

where $1_{[a,b]}$ is the indicating function of and where $\alpha(k)$ and k are such as:
$$(\underline{w_k} - 1)\alpha(k) - k\beta(k) = 0 \qquad \text{éq 3.3.3-2}$$

with
$$\alpha(k) = e^{-ak} - e^{-bk} , \qquad \text{éq 3.3.3-3}$$

and
$$\beta(k) = a e^{-ak} - b e^{-bk} \qquad \text{éq 3.3.3-4}$$

3.3.4 Closed semi support not limited without information on the reverse

If there exists a reality a such as $D = [a, +\infty[$ and if the information available is given by the equations [éq 3.3.1-1] and [éq 3.3.1-2] then the random variable W follows an exponential law whose function of density of probability is:
$$p_W(w) = 1_{[a,+\infty]}(w) \frac{1}{\underline{w} - a} \exp\left(-\frac{w-a}{\underline{w}-a}\right) , \qquad \text{éq 3.3.4-1}$$

where $1_{[a,+\infty]}$ is the indicating function of $[a, +\infty[$.

3.3.5 Closed semi support not limited with information on the reverse

If there exists a reality a such as $D=[a,+\infty[$ and if the information available is given by the equations. [éq 3.3.1-1], [éq 3.3.1-2] and [éq 3.3.1-3], then the random variable W a law gamma follows whose function of density of probability is,

$$p_W(w)=1_{[a,+\infty]}(w)\frac{(\underline{w}\,\delta^2-a\,\delta^2)^{-1/\delta^2}}{\Gamma(1/\delta^2)}(w-a)^{(1-\delta^2)/\delta^2}\exp\left\{-\frac{w-a}{(\underline{w}-a)\,\delta^2}\right\}, \qquad \text{éq 3.3.5-1}$$

where δ is a parameter controlling the level of uncertainty of the random variable which is written W (in a way similar to the nonparametric case [éq 3.2.2-8]) *:*

$$\delta=\left\{\frac{E\left\{(w-\underline{w})^2\right\}}{\underline{w}^2}\right\}^{1/2}, \qquad \text{éq 3.3.5-2}$$

3.4 Construction of the stochastic answer and the associated statistics

3.4.1 Transitory case

3.4.1.1 Stochastic transitory answer

The excitations of the dynamic system are supposed to be deterministic, but in the paragraph [§3.1], of the matrices and the random parameters were introduced into the reduced matric model. Therefore, the transitory answer $t\to Q_n(t)$ is a nonstationary stochastic process indexed by $[0,T]$ with value in $\mathbb{R}^n$ (by using some additional assumptions of existence, unicity and regularity of the deterministic solution, cf [bib19]).

Consequently, with the vector of m free degrees of freedom $\underline{\mathbf{y}}^{\mathbf{n}}(t)$ corresponds the stochastic process $Y^n(t)$ indexed by $[0,T]$ and with value in $\mathbb{R}^m$ such as

$$Y^n(t)=[\underline{\Phi}_n]Q^n(t), \qquad \text{éq 3.4.1.1 - 1}$$

In the case of the passage in relative coordinates, with the stochastic process $t\to\mathbf{Y}_{\mathbf{n}}(t)$ indexed by $[0,T]$ and with value in $\mathbb{R}^m$ defined by the equation [éq 3.4.1.1 - 1] corresponds the stochastic process $t\to\mathbf{Z}_{\mathbf{n}}(t)$ free d.d.l of the structure in absolute coordinates indexed by and with value in $\mathbb{R}^m$ such as

$$\mathbf{Z}_{\mathbf{n}}(t)=\mathbf{Y}_{\mathbf{n}}(t)+[R]\mathbf{z}_{\mathbf{s}}(t). \qquad \text{éq 3.4.1.1 - 2}$$

3.4.1.2 Elastic spectrum of answer

One notes $Z_j^n(t)$ $j^{\text{ème}}$ component of the vector $\mathbf{Z}_{\mathbf{n}}(t)$ correspondent with a random realization of the stochastic answer of $j^{\text{ème}}$ free degree of freedom of the structure. $Z_j^n(t)$ perhaps characterized by its spectrum of elastic answer (also called spectrum of oscillator in the documentation of *Code_Aster*) that one notes $S_j(\xi,\omega)$ where ξ and ω are respectively the associated rate of depreciation and the pulsation.
With reasonable assumptions, in particular on the regularity of the nonlinear application $(t,\mathbf{q},\mathbf{p}\ ;\ \mathbf{w})\mapsto\mathbf{F}_{\mathbf{c}}^{\mathbf{n}}(t,\mathbf{q},\mathbf{p}\ ;\ \mathbf{w})$, one can show that is a process of the second order whose trajectories are almost surely continuous. Consequently, for all ξ fixed in an interval J_ξ given, $\omega\to S_j(\xi,\omega)$ is a stochastic process indexed on the tape of analysis J_ω with value in $\mathbb{R}^+$. It is admitted that this process is of the second order, i.e.:

$$E\{S_j(\xi,\omega)^2\}<+\infty, \quad \forall\,\omega\in J_\omega . \qquad \text{éq 3.4.1.2 - 1}$$

3.4.2 Harmonic case

In the paragraph [§3.1], random matrices and parameters were introduced into the reduced matric model. The harmonic answer $t\mapsto Q^n(\omega)$ is thus a stochastic process indexed on $\left[\omega_1,\omega_2\right.$ with value in R^n.

Consequently, with the vector of m free degrees of freedom $\underline{y}^n(\omega)$ corresponds the stochastic process $Y^n(\omega)$ indexed on $\left[\omega_1,\omega_2\right]$ and with value in R^m such as

$$Y^n(\omega)=\left[\underline{\Phi}_n\right]\,Q^n(\omega) \qquad \text{éq 3.4.2-1}$$

$Y_j^n(\omega)$ j ème component of the vector $Y^n(\omega)$ is a random variable which one will admit of the second order.

3.4.3 Construction of the stochastic answer by the method of Monte Carlo

3.4.3.1 Choice and implementation of the method of Monte Carlo

The answers and the spectra of answer correspond to strongly nonlinear transformations of the random matrices and random parameters which result from the probabilistic modeling of uncertainties. Moreover, one can of course build only digital approximations as of these answers and these spectra of answer. The statistics (first statistical moments, probability of going beyond a threshold,…) are written formally as multiple integrals of very great dimension because the number of random variables of the probabilistic model is by construction high. Lastly, the number of sizes observed is very large (several ddl for several frequencies). For all these reasons, the method most adapted to calculate the probabilistic solution (stochastic answer and associated statistics) is the method of digital simulation of Monte Carlo.

The method of simulation of Monte Carlo has the advantage of giving results which one can control the precision (checking of convergence, cf [§4.1]), contrary to the majority of the methods based on approximations. It can be expensive in computing times, but the use of the techniques of reduction of the variance can make it possible to reduce the number of simulations necessary (cf [bib8] or [bib9]).

The implementation of the method of Monte Carlo consists for the problem which concerns us to generate n_s achievements of the random matrices $\left[\mathbf{M}_n\right]$, $\left[\mathbf{K}_n\right]$ and $\left[\mathbf{D}_n\right]$ dynamic system and/or n_s achievements of the vectorial random variable W. Resolutions of the deterministic dynamic system for each one of n_s achievements of ($\left[\mathbf{M}_n\right]$, $\left[\mathbf{K}_n\right]$, $\left[\mathbf{D}_n\right]$, W) produce n_s achievements of the stochastic process solution $t\mapsto Q^n(t)$ (resp. $t\mapsto Q^n(\omega)$) and in consequence of $t\to\mathbf{Y_n}(t)$, of $t\to\mathbf{Z_n}(t)$ and of $\omega\to S_j(\xi,\omega)$ (resp. $\omega\mapsto Y_j^n(\omega)$). The generation of the random matrices is treated in the following paragraph; the generation of the random variable W is more classical and is not recalled.

3.4.3.2 Generation of the pseudo-random matrices

In order to generate the achievements of the random matrix $[G_A]$, one uses the following algebraic representation of the random matrix $[G_A]$ the law of probability is defined by the equations [éq 3.2.1 - 2], [éq 3.2.2-1]:

$$[G_A]=[L]^T[L] \quad \text{éq 3.4.3.2 - 1}$$

the triangular random matrix $[L]$ being such as:

- Random variables $[L]_{ij}, i \leq j$ are independent.
- For $i<j$, real random variables $[L]_{ij}$ are written $[L]_{ij}=\sigma_n U_{ij}$ where $\sigma_n=\delta_A(n+1)^{-1/2}$ and where U_{ij} is a Gaussian real random variable of average 0 and variance 1.
- For $i=j$, real random variables $[L]_{ij}$ are written $[L]_{ij}=\sigma_n(2V_j)^{1/2}$ where σ_n is defined previously and where V_j is variable real positive random of law gamma of which the function of density of probability $p_{v_j}(v)$ compared to measurement dv is written:
-

$$p_{v_j}(v)=\frac{\mathbf{1}_{[0,+\infty[}(v)}{\Gamma((n+1)/(2\delta_A^2)+(1-j)/2)}v^{(n+1)/(2\delta_A^2)+(1-j)/2}e^{-v} \quad \text{éq 3.4.3.2 - 2}$$

where $\mathbf{1}_{[0,+\infty[}$ is the indicating function of $[0,+\infty[$.

3.4.4 Statistics on the spectra

In this chapter, one presents the definition of the statistics of the spectra of elastic answer $S_j(\xi,\omega)$, in the case of a transitory resolution. In the harmonic case, the statistics on the random variables $Y_j^n(\omega)$ are defined in the same way and are thus not presented.

3.4.4.1 Estimate of the quantiles

For all $(\xi,\omega)\in J_\xi \times J_\omega$, $S_j(\xi,\omega)$ is a random variable with value in $\mathbb{R}^+$. One seeks to consider the quantile associated with the probability α noted $S_{j,\alpha}(\xi,\omega)$ and defined by:

$$S_{j,\alpha}(\xi,\omega)=F^{-1}_{\xi,\omega}(1-\alpha) \quad \text{éq 3.4.4.1 - 1}$$

where $F_{\xi,\omega}$ is the function of unknown distribution of $S_j(\xi,\omega)$.

That is to say $(S_j(\xi,\omega;\theta_1), \ldots, S_j(\xi,\omega;\theta_r), \ldots, S_j(\xi,\omega;\theta_{n_s}))$ the sample made up of n_s achievements of $S_j(\xi,\omega)$ and $(S_j(\xi,\omega;\theta_{(1)}), \ldots, S_j(\xi,\omega;\theta_{(r)}), \ldots, S_j(\xi,\omega;\theta_{(n_s)}))$ the associated ordered sample.

A natural estimator of the quantile $S_{j,\alpha}(\xi,\omega)$ for $\alpha=\frac{r}{n_S}$, $1\leq r\leq n_S$ is:

$$\hat{S}_{j,\frac{r}{n_S}}(\xi,\omega)=S_j(\xi,\omega;\theta_{(r)}) \quad \text{éq 3.4.4.1 - 2}$$

To obtain a more robust estimator of the quantile, one can “realise” the estimator on several series of n_s achievements. If the desired probability is such as $\alpha<1/n_S$, or if one wishes to reduce the number of simulations, it is possible to use more sophisticated estimators, for example by supposing

that the function of distribution $F_{\xi,\omega}$ belongs to a field of attraction given (extreme theory of value) or for example by using a method of regularization bayésienne (cf [bib12]).

3.4.4.2 Extreme values of sample

For a sample of n_s achievements of $S_j(\xi,\omega)$ noted $S_j(\xi,\omega;\theta_1)$, ... , $S_j(\xi,\omega;\theta_{n_s})$ one defines the extreme values of sample by:

$$\omega \mapsto dB_{j,min}(\xi,\omega;n_S)=\log_{10}\left(\min_{r=1,\ldots,n_s} S_j(\xi,\omega;\theta_r)\right) \quad \text{éq 3.4.4.2 - 1}$$

$$\omega \mapsto dB_{j,max}(\xi,\omega;n_S)=\log_{10}\left(\max_{r=1,\ldots,n_s} S_j(\xi,\omega;\theta_r)\right) \quad \text{éq 3.4.4.2 - 2}$$

3.4.4.3 "Field of confidence" established starting from the inequality of Tchebychev

For a sample of n_s achievements of the process $\omega \mapsto S_j(\xi,\omega)$ noted $\omega \mapsto S_j(\xi,\omega;\theta_1)$,..., $\omega \mapsto S_j(\xi,\omega;\theta_{n_s})$, one can build the "field of confidence" of the random variable for all, by using the inequality of Tchebychev associated on a level with probability P_C :

$$\text{Proba}\left[dB_j^-(\xi,\omega)<dB_j(\xi,\omega)<dB_j^+(\xi,\omega)\right]\geq P_C \ , \quad \text{éq 3.4.4.3 - 1}$$

where the lower envelope and the higher envelope are defined by:

$$dB_j^+(\xi,\omega)=\log_{10}\left(m_{1j}(\xi,\omega)+\frac{\sigma_j(\xi,\omega)}{\sqrt{1-P_C}}\right) , \quad \text{éq 3.4.4.3 - 2}$$

$$dB_j^-(\xi,\omega)=2\log_{10}\left(m_{1j}(\xi,\omega)\right)-dB_j^+(\xi,\omega) \ . \quad \text{éq 3.4.4.3 - 3}$$

with the average and the standard deviation of $dB_j(\xi,\omega)$:

$$m_{1j}(\xi,\omega)= E\{\ S_j(\xi,\omega)\} \quad \text{éq 3.4.4.3 - 4}$$

$$\sigma_j(\xi,\omega)=E\left\{\left(S_j(\xi,\omega)-m_{1j}(\xi,\omega)\right)^2\right\}^{1/2} \ . \quad \text{éq 3.4.4.3 - 5}$$

The "field of confidence" thus built proved to be a good approximation of the extreme values of sample for the case treated in [bib21]. However, this "field of confidence" utilizes only the first two moments of which consistent estimators more quickly with respect to the number n_s simulations that extreme values of sample. It can thus be interesting to rather use this construction of the "field of confidence" than a construction based on the more expensive estimate of the quantiles of many simulations.

Note:

The term "field of confidence", can be regarded by certain as an abuse language. One should rather use the less intuitive terminology "inter-quantiles field". Indeed, in the statistical literature, a confidence interval is theoretically the interval in which is the true value of a parameter of a random variable (for example its average) with a given probability. This terminology is employed within the very precise framework of the theory of the estimate ensemblist. The confidence interval is not a characterization of the variability of a random variable, contrary to a standard deviation or quantiles. One nevertheless uses " field of confidence" with parsimony in the continuation, because it is certainly a little more speaking for the non-specialists of the statistics.

4 Implementation in *Code_Aster*

4.1 Study of the stochastic convergence of the digital model

4.1.1 Transitory case

The convergence of the stochastic solution must be studied compared to the number n modes and with the number n_s simulations of Monte Carlo. As the stochastic solution is a process of second order (by assumption, cf [§ 3.4.1.2]), its convergence can be analyzed by studying the applications such as:

$$|||\ddot{Z}_j^n|||^2 = \int_0^T E\left\{\ddot{Z}_j^n(t)\right\}^2 dt \ , \qquad \text{éq 4.1.1-1}$$

where is a stochastic process of the second order indexed by and with value in R representing the acceleration of j ème degree of freedom of the structure.
In the framework of simulations of Monte Carlo, this standard is estimated for n fixed starting from a whole of n_s random achievements by the approximation with

$$\mathrm{conv}_j(n, n_S)^2 = \int_0^T \left(\frac{1}{n_S} \sum_{i=1}^{n_S} \ddot{Z}_j^n(t;\theta_i) \right)^2 dt \qquad \text{éq 4.1.1-2}$$

The stochastic convergence of the model is thus analyzed according to the dimension of the scale model (i.e. the number of mode n under clean space of the average model finite elements on which the stochastic nonlinear dynamic system was project in the paragraph [§2.2]) and the number n_s simulations of Monte Carlo by studying the function.

4.1.2 Harmonic case

Convergence in the case of a transitory resolution can transpose a harmonic resolution directly in the case of, with the standard:

$$|||Z_j^n|||^2 = \int_{\omega_1}^{\omega_2} E\left\{Z_j^n(\omega)\right\}^2 d\omega \ , \qquad \text{éq 4.1.2-1}$$

4.2 Choice of the parameters of dispersion

To use the method, the parameters of dispersion δ must be fixed. Two approaches can be *a priori* used to fix the value of these parameters.

The first approach consists in identifying the value of the parameters δ for a given structure or a class of structure using suitable methods. For that, one can use experimental results of the dynamic responses of the structure. One can also use digital simulations built by using a parametric approach of uncertainties. In this last case, it should be noted that only the errors on the data of the model are taken into account, since the errors of modeling cannot be taken into account by the parametric approach.

The second approach consists in not fixing *a priori* a value fixes parameters δ but to vary them in a given beach (only 3 scalars to vary for the matrices of mass, stiffness and damping on the not-parametric part in comparison with the very a large number of parameters to vary simultaneously in a classical parametric study). This approach makes it possible to carry out a total analysis of sensitivity to uncertainties. In the case of absence of objective information on the parameters dispersion to be chosen, it is preferable to use such an approach. The non-parametric method suggested then seems a robust and simple approach of analysis of sensitivity to uncertainties.

4.3 Principal stages

The implementation in *Code_Aster* is made up of three principal stages: the construction of the average reduced matric model, the generation of the achievements of the answer seen like a stochastic process, and finally the statistical postprocessing of these achievements. The two last stages constitute in fact the method of digital simulation of direct Monte Carlo.

Stage 1: construction of the average reduced matric model
The average reduced matric model is built using a classical sequence operators depending on the precise analysis carried out whose principal ones can be: ASSE_MATRICE, CALC_MODES, MODE_STATIQUE, CALC_CHAR_SEISME, PROJ_BASE, …

Stage 2: generation of the achievements of the transitory answer
n_s achievements of the stochastic transitory answer are calculated in a loop in language Python made up of:

- Generation of $p^{ième}$ achievements of the random generalized matrices of mass, stiffness and damping by GENE_MATR_ALEA (Doc. [U4.36.06]). These matrices are not diagonal and thus require a full storage.
- Generation of $p^{ième}$ achievements of the random variables of the parameters of non-linearities by GENE_VARI_ALEA (Doc. [U4.36.07]).
- Calculation of $p^{ième}$ realization $Q^n(t;p)$ or $Q^n(\omega;p)$ solution of the stochastic matric system s. This realization is the solution of the classical matric system whose matrices and the second members are the achievements previously generated. Calculation is thus carried out by DYNA_TRAN_MODAL or DYNA_LINE_HARM (with matr_asse_GENE_R and vect_asse_GENE as starter).
- 1 - Extraction of the temporal observations of the preset physical degrees of freedom (for example $\ddot{Z}_i^n(t;p)$ or $Y_j^n(\omega;p)$, but also possibly fields of displacement, speed, constraints, etc) *via* RECU_FONCTION (after one REST_GENE_PHYS for $Y_j^n(\omega;p)$).
  2 - Calculation of the spectra corresponding (by CALC_FONCTION (SPEC_OSCI) for $\omega \to S_j(\xi,\omega;p)$ and CALC_FONCTION (MODULE) for $Y_j^n(\omega;p)$).
- Evaluation, *via* CALC_FONCTION keywords COMB or POWER or ENVELOPE, contributions to the estimators of the averages, moments of order two, max. extreme values and min. of sample for the standardized spectra:

$$\hat{m}_{1j}(x,w;p)=S_j(x,w;p)+\hat{m}_{1j}(x,w;p-1),$$
$$\hat{m}_{2j}(x,w;p)=S_j(x,w;p)^2+\hat{m}_{2j}(x,w;p-1),$$
$$\hat{S}_{j,\max}(x,w;p)=\mathrm{Max}\left\{S_j(x,w;p),\hat{S}_{j,\max}(x,w;p-1)\right\},$$
$$\hat{S}_{j,\min}(x,w;p)=\mathrm{Min}\left\{S_j(x,w;p),\hat{S}_{j,\min}(x,w;p-1)\right\}.$$

Stage 3: statistical postprocessings
The averages, the standard deviations, the max. extreme values and min. of sample for the standardized spectra can be evaluated *via* CALC_FONCTION (COMB) :

$$m_{1j}(x,w)=\frac{1}{n_s}\hat{m}_{1j}(x,w;n_s), \qquad m_{2j}(x,w)=\frac{1}{n_s}\hat{m}_{2j}(x,w;n_s).$$

The confidence intervals can then be traced starting from the extreme values of sample or the terminals obtained by Tchebychev cf [§3.4.4].

In the transitory case, an example is given by a case test of a flexbeam with nonlinearities of shock, cf Doc. [V5.06.001] [bib1]. Other details are given in Doc. [U2.08.05] [bib2].

4.4 Digital effectiveness of the nonparametric approach

The nonparametric approach is more economic in computing times than a purely parametric approach in which the parameters of geometry, materials, etc are random variables.
In the purely parametric approach, the model finite elements depends on the dubious parameters. For each simulation of Monte Carlo, the model finite elements is different. It is thus necessary, for each simulation, to calculate the elementary matrices, to carry out the assemblies, to pass in relative coordinates, to solve the problem with the eigenvalues, to project on modal basis, to solve the reduced system and to return in physical base then in relative coordinates.

In the nonparametric approach, only the reduced system is different with each simulation. It is thus simply necessary, with each simulation, to solve the reduced system and to return in physical base then in relative coordinates. In particular, the resolution of the problem to the eigenvalues of the model average finite elements is carried out once and for all, before simulations of Assembles - Carlo.

The saving of time of calculation which results from it is variable, but it can be important. At first approximation, this saving of time calculation depends on the ratio between time CPU necessary to the resolution with the eigenvalues and time CPU necessary to the resolution of the reduced system. The larger this ratio is, the more the nonparametric approach is advantageous compared to the purely parametric approach. In particular, the saving of time of calculation can be very important for structures with one very a large number of degrees of freedom and a modal base low-size.

5 Checking

The features and the methods presented in this document are checked by the cases following tests:

SDNS01	Nonparametric probabilistic model – parametric of a flexbeam with nonlocalised linearities of shock	[V5.06.001]
SHLS200	Nonparametric probabilistic model: Harmonic answer of a under-structured plate	[V2.06.200]

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7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6.4	*S. CAMBIER, C. DESCELIERS EDF-R&D/AMA*	Initial text