Modeling élasto (visco) plastic fascinating in account of the metallurgical transformations

Summary:

This document presents the modeling installation in *Code_hasster* for the mechanical analysis of operations generating of the metallurgical transformations. One presents the various mechanical effects resulting from structure transformations to take into account and their modelings.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Contents

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

1 Introduction

Certain materials undergo structure transformations when they are subjected to particular thermal evolutions [1], [2], [3]. It is for example the case of the low alloy steels during operations of type welding and heat treatment or the alloys of zircaloy of the fuel sheaths for certain cases of accidental situation (APRP).
These transformations have a more or less strong influence on the thermal and mechanical evolutions.

From a thermal point of view, structure transformations are accompanied by a modification of the thermal characteristics (voluminal heat-storage capacity, thermal conductivity) of the material which undergoes them, as well as production or of an energy absorption (latent heats of transformation) [2] . However, the latent heats of transformation in a solid state relatively weak are compared with the latent heats of change of state liquid-solid and one can thus, at first approximation, regard the thermal and structural evolutions as uncoupled. It is currently the case of the options of thermal and metallurgical calculations established in *Code_hasster*. [U4.85.01]

From a mechanical point of view, the consequences of structure transformations (at the solid state) are of four types [2] :

- The mechanical characteristics of the material which undergoes them are modified. More precisely, the elastic characteristics (Young modulus and Poisson's ratio) are not very affected whereas plastic characteristics (elastic limit in particular) and the thermal dilation coefficient are it strongly;
- The expansion or the voluminal contraction which accompanies structure transformations translates by a deformation (spherical) “of transformation” which is superimposed on the purely thermal deformation of origin. This effect is highlighted on a test of dilatometry and, in general, one gathers it with that due to the modification of the dilation coefficient and one speaks overall about the influence of the transformations on the thermal deformation;
- A transformation proceeding under constraints can give rise to an unrecoverable deformation and this, even for levels of constraints much lower than the elastic limit of material (at the temperature and in the structural state considered). One calls “ **plasticity of transformation “** this phenomenon;
- One can have at the time of the metallurgical transformation a phenomenon of **restoration of work hardening**. The work hardening of the mother phase is not transmitted to the phases lately created. Those can then be born with a virgin state of work hardening or inherit only one part, possibly totality, work hardening of the mother phase.

In addition, the mechanical state also influences the metallurgical behavior. The state of stresses can in particular accelerate or slow down the kinetics of the transformations and modify the temperatures to which they occur. However, the experimental characterization of this influence, in particular in the case of complex situations (three-dimensional, under temperature and state of variable stresses) remains very delicate and it is very frequent to regard the structural evolution as independent of the mechanical state. It is the case of the model of structure transformations established in *Code_hasster*.

If one neglects the various couplings of mechanical origin, the determination of the mechanical evolution associated with a process bringing into play structure transformations thus requires two successive and uncoupled calculations:

- A thermo-metallurgical calculation (uncoupled) allowing the determination of the thermal evolutions then structural;
- A mechanical calculation (élasto-viscoplastic) taking account of the effects due to the thermal and structural evolutions;

This document presents the mechanical modeling established in *Code_hasster*. Modeling is available for two materials:

- The steel which undergoes around $850°$ a austénito-ferritic transformation (passage of ferritic phases cold of cubic structure face centered with an austenitic phase with cubic

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

heat of centered structure). Steel presents four possible ferritic phases: ferrite, pearlite, the bainite and martensite;

- The alloys of Zircaloy which undergo around $800°C$ a transformation of cold phase α of hexagonal structure compacts with a hot phase β of centered cubic structure (cc).

The models are identical for two materials, only the number of phase changes.
The model thus comprises five phases for steel and three phases for the zircaloy. The modeling of the behavior of the zircaloy indeed requires to consider two cold phases of different mechanical behavior; a phase α regarded as pure and a phase α mixedE with β [U4.85.01], [4].

Nota bene:

The metallurgical concepts of bases necessary to the comprehension of the problem general are gathered in [1] .

The elastoplastic algorithm of resolution, without taking into account of the effects due to structure transformations is clarified in [5].

This document to some extent is extracted from [6] where one makes a more detailed presentation of the model and some elements of validation.

Presentation of models quE L'one makes in this document is illustrated with the case of L' steel.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2 General notations

All the quantities evaluated at the previous moment are subscripted by $^-$. Quantities evaluated at the moment $t+\Delta t$ are not subscripted. The increments are indicated by Δ. One has as follows:

$$\boldsymbol{Q}=\boldsymbol{Q}(t+\Delta t)=\boldsymbol{Q}(t)+\Delta\boldsymbol{Q}=\boldsymbol{Q}^{-}+\Delta\boldsymbol{Q} \quad (1)$$

For the calculation of the derivative, one will note $\dot{\boldsymbol{Q}}$ derived from $\boldsymbol{Q}$ compared to time. The tensors and the vectors will be noted in **fat**.

$Z_{k_f=1,4}$	Proportion of the ferritic phases (ferrite, pearlite, bainite and martensite)
Z_γ	Proportion of the austenitic phase
Z_f	Somme of all phases of the ferritic type
ε_γ^{th}	Thermal deformation of the austenitic phase
ε_f^{th}	Thermal deformation of the ferritic phases
$\Delta\varepsilon_{f\gamma}^{T_{ref}}$	Difference in compactness between the hot and cold phase
$\boldsymbol{\varepsilon}^{th}$	Thermal deformation
T^{ref}	Temperature of reference
α_γ	Average dilation coefficient of the austenitic phase
α_f	Average dilation coefficient of the ferritic phases
Z_γ^R	Indicator of the metallurgical phase of reference (is worth 1 when the phase of reference is the austenitic phase and 0 when the phase of reference is the ferritic phase)
$\boldsymbol{\sigma}$	Tensor of the constraints
$\sim$	Operator deviatoric
$\langle\cdot\rangle$	Positive part
$()_{eq}$	Equivalent value of a tensor within the meaning of Von Mises
$\boldsymbol{Id}$	Tensor identity
$\boldsymbol{A}$	Tensor of elasticity of Hooke
λ, μ	Coefficients of Lamé
E	Young modulus
ν	Poisson's ratio
K	Module of compressibility $3\mathrm{K}=3\lambda+2\mu$
T	Temperature
t	Time
g_Z^{pt}	Function for the plasticity of transformation
$R_{0,k}$	Linear coefficient of work hardening of the phase k
θ_{γ,k_f}	Proportion of restoration of work hardening at the time of the transformation austenite towards ferrite
$\theta_{k_f,\gamma}$	Proportion of restoration of work hardening at the time of the transformation ferrite towards austenite
C_k, m_k	Coefficients of viscous restoration of the phase k
$g^{\mathrm{re},v}$	Function for the restoration of isotropic work hardening viscous
$\boldsymbol{h}^{\mathrm{re},v}$	Function for the restoration of kinematic work hardening viscous

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$g_{k_f}^{\mathrm{re},m}$	Function for the restoration of isotropic work hardening metallurgical (cold phases)
$g_{\gamma}^{\mathrm{re},m}$	Function for the restoration of isotropic work hardening metallurgical (hot phase)
$\boldsymbol{h}_{k_f}^{\mathrm{re},m}$	Function for the restoration of kinematic work hardening metallurgical (cold phases)
$\boldsymbol{h}_{\gamma}^{\mathrm{re},m}$	Function for the restoration of kinematic work hardening metallurgical (hot phase)
$\overline{R}$	Linear coefficient of homogenized work hardening (multiphase)
$\sigma_{c,k}$	Elastic limit of the phase k
$\overline{\sigma}_c$	Elastic limit of homogenized material (multiphase)
$\boldsymbol{X}_k$	Tensor of recall for kinematic work hardening and the phase k
$\overline{\boldsymbol{X}}$	Tensor of recall for homogenized kinematic work hardening (multiphase)
η_k , n_k	Coefficients materials for the viscosity of the phase k
$\overline{\eta}$, $\overline{n}$	Coefficients materials for homogenized viscosity (multiphase)

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

3 Influence of structure transformations on the thermal deformation

3.1 Metallurgical phases

At any moment t and in each point M of material, one can detect two groups of phases:

- Phases *ferritic* , known as “cold phases”, of cubic structure centered face. Steel presents four possible ferritic phases: ferrite, pearlite, the bainite and martensite. One will note $Z_{k_f=1,4}$ the proportion of each one of these phases;
- The phase *austenitic* , known as also “hot phase”, of centered cubic structure. One will note Z_γ the proportion of austenite;

One a:

$$Z_\gamma=1-\sum_{k_f=1}^{4} Z_{k_f} \quad (2)$$

One will also note Z_f the sum of all phases of the ferritic type, is:

$$Z_f=\sum_{k_f=1}^{4} Z_{k_f} \quad (3)$$

3.2 Test of dilatometry

A test of dilatometry consists in measuring the deformation (homogeneous) of a low-size test-tube according to the temperature (or of time) at the time of an imposed thermal cycle (presumedly identical in all the points of the test-tube). One presents [Figure 2-a] a test of dilatometry of a steel. The thermal cycle comprises a heating beyond the temperature of austenitization (either $850°C$ approximately), then a maintenance at this temperature and, finally, a cooling controlled until the room temperature. One then obtains an evolution of the deformation (variable according to the kinetics of cooling imposed) as represented on the following figure.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

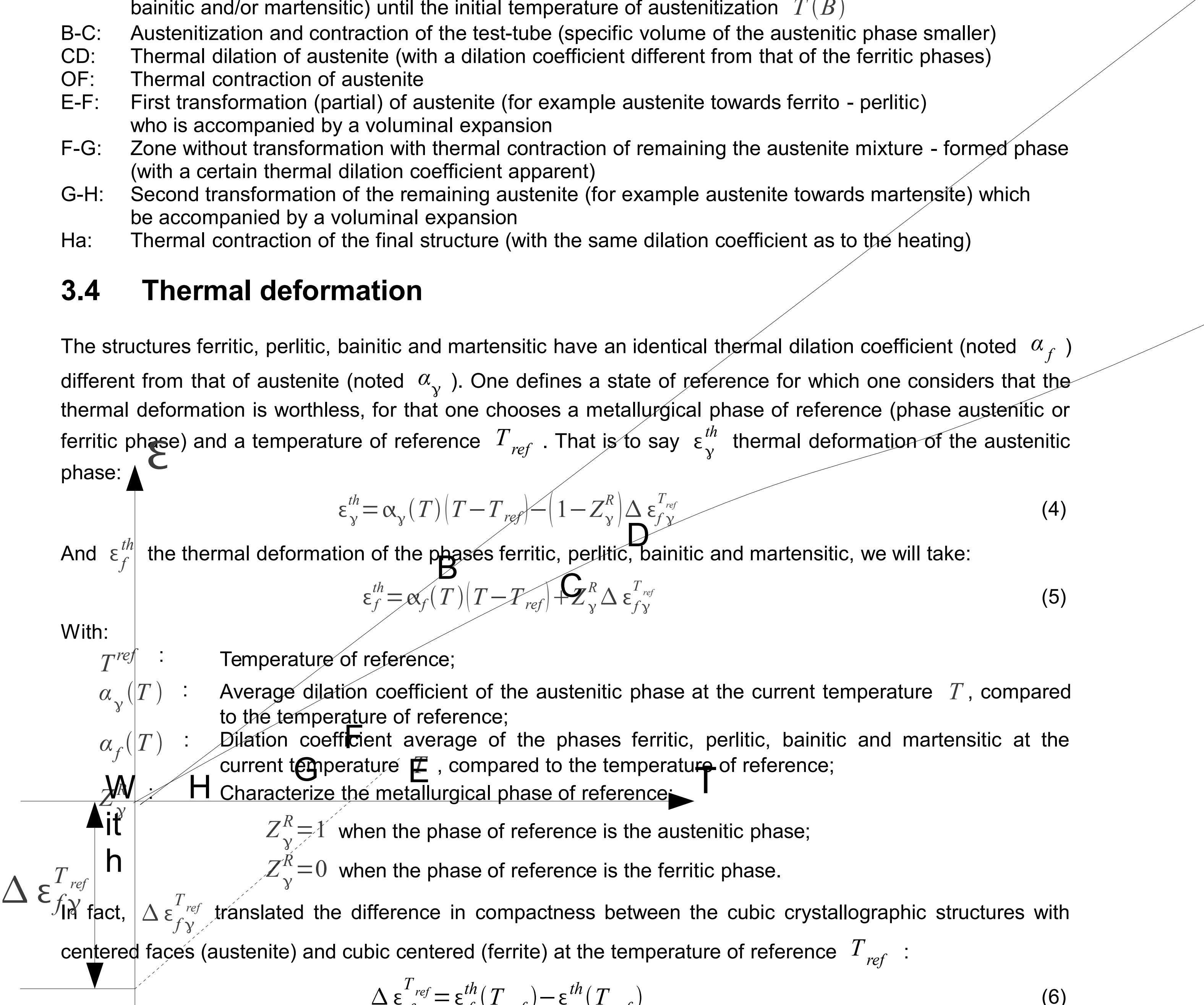
Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

3.3 Zones of thermal deformation

Various zones put in obviousness on the figure 3-1 can be interpreted as follows:

- A-B: Thermal dilation of metal in its initial metallurgical structure (of perlitic type ferrito -, bainitic and/or martensitic) until the initial temperature of austenitization $T(B)$
- B-C: Austenitization and contraction of the test-tube (specific volume of the austenitic phase smaller)
- CD: Thermal dilation of austenite (with a dilation coefficient different from that of the ferritic phases)
- OF: Thermal contraction of austenite
- E-F: First transformation (partial) of austenite (for example austenite towards ferrito - perlitic) who is accompanied by a voluminal expansion
- F-G: Zone without transformation with thermal contraction of remaining the austenite mixture - formed phase (with a certain thermal dilation coefficient apparent)
- G-H: Second transformation of the remaining austenite (for example austenite towards martensite) which be accompanied by a voluminal expansion
- Ha: Thermal contraction of the final structure (with the same dilation coefficient as to the heating)

3.4 Thermal deformation



The structures ferritic, perlitic, bainitic and martensitic have an identical thermal dilation coefficient (noted α_f) different from that of austenite (noted α_γ). One defines a state of reference for which one considers that the thermal deformation is worthless, for that one chooses a metallurgical phase of reference (phase austenitic or ferritic phase) and a temperature of reference T_{ref} . That is to say ε_γ^{th} thermal deformation of the austenitic phase:

$$\varepsilon_\gamma^{th}=\alpha_\gamma(T)\left(T-T_{ref}\right)-\left(1-Z_\gamma^R\right)\Delta\varepsilon_{f\gamma}^{T_{ref}} \tag{4}$$

And ε_f^{th} the thermal deformation of the phases ferritic, perlitic, bainitic and martensitic, we will take:

$$\varepsilon_f^{th}=\alpha_f(T)\left(T-T_{ref}\right)+Z_\gamma^R\Delta\varepsilon_{f\gamma}^{T_{ref}} \tag{5}$$

With:

- T^{ref} : Temperature of reference;
- $\alpha_\gamma(T)$: Average dilation coefficient of the austenitic phase at the current temperature T, compared to the temperature of reference;
- $\alpha_f(T)$: Dilation coefficient average of the phases ferritic, perlitic, bainitic and martensitic at the current temperature T , compared to the temperature of reference;
- Z_γ^R : Characterize the metallurgical phase of reference:
  - $Z_\gamma^R=1$ when the phase of reference is the austenitic phase;
  - $Z_\gamma^R=0$ when the phase of reference is the ferritic phase.

In fact, $\Delta\varepsilon_{f\gamma}^{T_{ref}}$ translated the difference in compactness between the cubic crystallographic structures with centered faces (austenite) and cubic centered (ferrite) at the temperature of reference T_{ref} :

$$\Delta\varepsilon_{f\gamma}^{T_{ref}}=\varepsilon_f^{th}(T_{ref})-\varepsilon_\gamma^{th}(T_{ref}) \tag{6}$$

With the help of the assumption of a law of mixture to define the thermal deformation of a multiphase mixture (characterized by Z) one a:

$$\varepsilon^{th}(\mathbf{Z},T)=Z_\gamma\left[\alpha_\gamma(T-T_{ref})-(1-Z_\gamma^R)\Delta\varepsilon_{f\gamma}^{T_{ref}}\right]+Z_f\left[\alpha_f(T-T_{ref})+Z_\gamma^R\Delta\varepsilon_{f\gamma}^{T_{ref}}\right] \tag{7}$$

α depend on the temperature and are calculated for the temperature of the point of current Gauss. The thermal deformation is purely spherical, its three-dimensional generalization is written:

$$\boldsymbol{\varepsilon}^{th}=\varepsilon^{th}\boldsymbol{Id} \text{ car } \tilde{\boldsymbol{\varepsilon}}^{th}=0 \tag{8}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Physical parameter		**Keyword ELAS_META**
Average dilation coefficient of the ferritic phases	α_f	F _ With LPHA
Average dilation coefficient of the austenitic phase	α_γ	C_ With LPHA
Metallurgical phase of reference		PHASE_REFE
Difference in compactness between the hot and cold phase	$\Delta \varepsilon_{f\gamma}^{T_{ref}}$	EPSF_EPSC_TREF

The temperature of reference T^{ref} being given by the keyword envisaged in AFFE_MATERIAU .

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4 Plasticity of transformation

In experiments, it is noted that the dilatometric statement of a test-tube in the course of structure transformation is strongly influenced by the state of stresses and that the application of pressure even lower than the elastic limit of material can nevertheless cause an unrecoverable deformation (cf [Figure 3-a]).

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

One calls **plasticity of transformation** this phenomenon and one notes ε^{pt} **unrecoverable deformation** corresponding. The model of plasticity of transformation most frequently used is, at the origin, the three-dimensional generalization of the unidimensional phenomenologic model established by Desalos [7]. If, starting from a dilatometric test, one traces the difference between lengthening ε obtained for a pressure applied different from zero and that obtained for a worthless constraint according to advance from the transformation, one notes that:

$$\varepsilon^{pt}(\sigma,b)=\varepsilon(\sigma,b)-\varepsilon(0,b)=kF(b)\sigma \qquad (9)$$

With:

- k : Homogeneous constant contrary to a constraint;
- F : Standardized function ($F(0)=0$ and $F(1)=1$);
- b : Proportion of the transformed phase;

A three-dimensional and temporal generalization of the preceding experimental model, for only one transformation, was proposed by Leblond [8], [9], [10], [11], in the form:

$$\dot{\varepsilon}_{ij}^{pt}=\frac{3}{2}K\tilde{\sigma}_{ij}F'(b)\dot{b} \qquad (10)$$

It is based on the following heuristic considerations:

- The relation must be “incremental”, i.e. to connect the rate of plastic deformation to the rate of transformation;
- The speed of plastic deformation of transformation must be, as for classical plasticity, proportional to the deviatoric part $\tilde{\boldsymbol{\sigma}}$ tensor forced $\boldsymbol{\sigma}$. The plasticity of transformation occurs without change of volume, from where a dependence compared to the diverter of the constraints rather than to the stress field itself;
- The rate of plastic deformation of transformation must be null apart from the beaches of transformations;
- The integration of this relation in the uniaxial case with constant constraint $\boldsymbol{\sigma}$ must give again the experimental relation.

On the basis of experimental tests and for transformation of a bainitic type of a steel 16MND5 for example $K=10^{-4}\,MPa^{-1}$ and $F(b)=b(2-b)$.

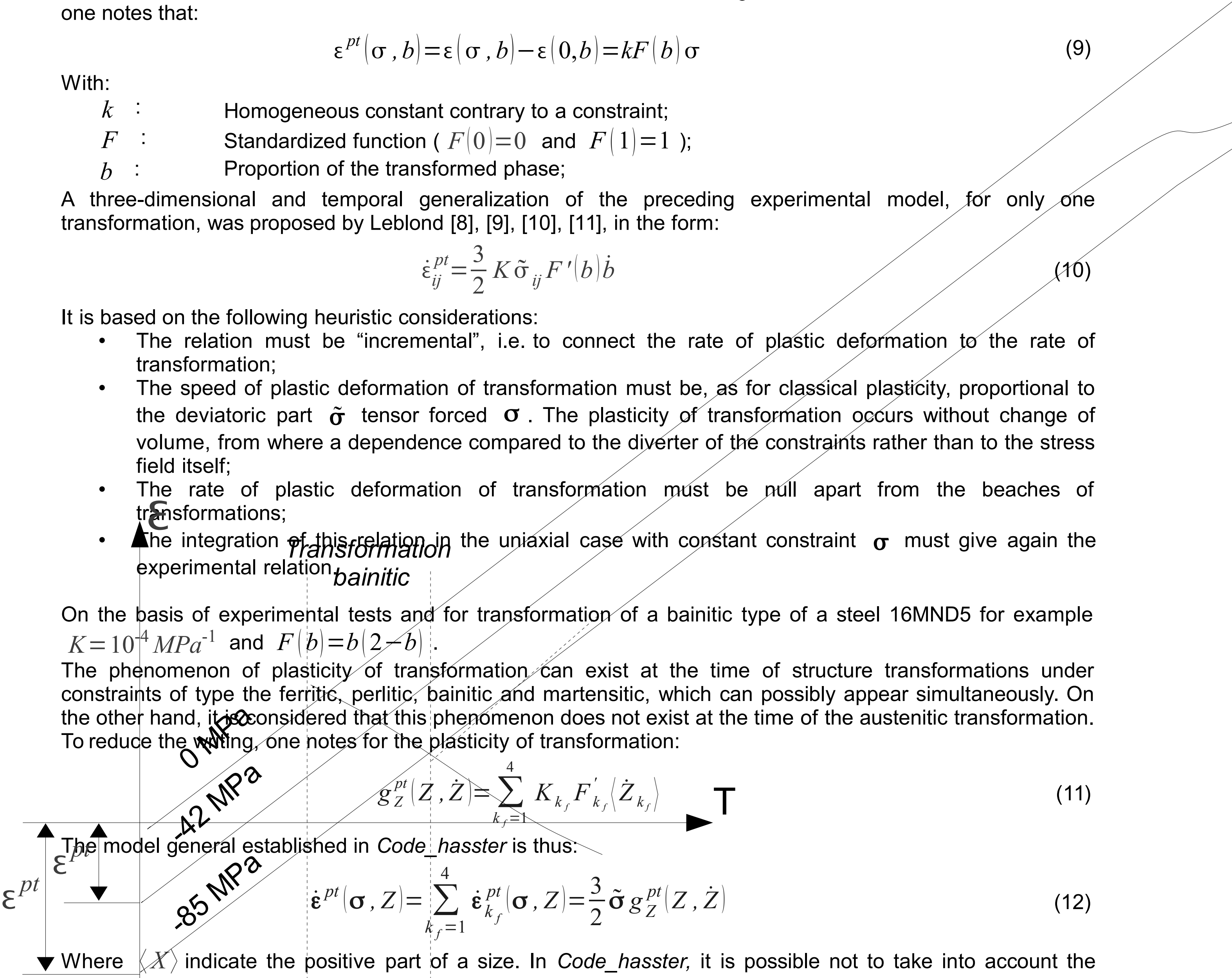
The phenomenon of plasticity of transformation can exist at the time of structure transformations under constraints of type the ferritic, perlitic, bainitic and martensitic, which can possibly appear simultaneously. On the other hand, it is considered that this phenomenon does not exist at the time of the austenitic transformation. To reduce the writing, one notes for the plasticity of transformation:

$$g_Z^{pt}(Z,\dot{Z})=\sum_{k_f=1}^{4}K_{k_f}F'_{k_f}\langle\dot{Z}_{k_f}\rangle \qquad (11)$$

The model general established in *Code_hasster* is thus:

$$\dot{\boldsymbol{\varepsilon}}^{pt}(\boldsymbol{\sigma},Z)=\sum_{k_f=1}^{4}\dot{\boldsymbol{\varepsilon}}_{k_f}^{pt}(\boldsymbol{\sigma},Z)=\frac{3}{2}\tilde{\boldsymbol{\sigma}}g_Z^{pt}(Z,\dot{Z}) \qquad (12)$$

Where $\langle X\rangle$ indicate the positive part of a size. In *Code_hasster,* it is possible not to take into account the phenomenon of plasticity of transformation. If this phenomenon is taken into account, it appears as soon as there are transformation and that even if the structure plasticizes. The model is more particularly dedicated to steel. Data K_{k_f} and F'_{k_f} are provided by the user in `DEFI_MATERIAU` under the keyword `META_PT`.



Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Physical parameter for the plasticity of transformation		**Keyword META_PT**
Constant for the ferritic phase 1	K_1	F1_K
Constant for the ferritic phase 2	K_2	F2_K
Constant for the ferritic phase 3	K_3	F3_K
Constant for the ferritic phase 4	K_4	F4_K
Derived function for the ferritic phase 1	$F_1^{'}$	F1_D_F_META
Derived function for the ferritic phase 2	$F_2^{'}$	F2_D_F_META
Derived function for the ferritic phase 3	$F_3^{'}$	F3_D_F_META
Derived function for the ferritic phase 4	$F_4^{'}$	F4_D_F_META

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

5 Restoration of work hardening

In a usual way the state of work hardening of a material is characterized by its plastic history. Thus for example in the case of plasticity with isotropic work hardening *linear*, one generally takes as variable of work hardening the noted cumulated plastic deformation p . The term of work hardening is written then:

$$R=R_o p \tag{13}$$

Where R_o is the linear coefficient of work hardening. This model is insufficient in two cases:

1. For viscoplastic materials, under the action of thermal agitation, it occurs a slow restoration of the crystalline structure of metal by annihilation of dislocations and internal stress relaxation;
2. At the time as of metallurgical transformations, there exists within material of displacements of more or less important atoms. These displacements of atoms can destroy dislocations which are at the origin of work hardening;

5.1 Viscous restoration of work hardening

5.1.1 Monophasic model

One introduces into modeling the phenomenon of viscous restoration of work hardening which leads to an evanescence partial of work hardening. The model used to describe this phenomenon is following it in the case of isotropic work hardening:

$$\dot{r}=\dot{p}-g^{\mathrm{re},v} \tag{14}$$

The term of evolution of the variable of work hardening r thus comprise a term of work hardening due to the plastic deformation $\dot{p}$ and a term of viscous restoration which one will note $g^{\mathrm{re},v}$:

$$g^{\mathrm{re},v}=(Cr)^m \tag{15}$$

The model thus makes it possible to describe the primary education phenomenon of creep (work hardening) and secondary creep (stabilization of work hardening) with the parameters C and m . In the case of kinematic work hardening, the model will be written:

$$\dot{\boldsymbol{\alpha}}=\dot{\boldsymbol{\varepsilon}}^{vp}+\boldsymbol{h}^{\mathrm{re},v} \tag{16}$$

With the term of restoration which one will note $\boldsymbol{h}^{\mathrm{re},v}$:

$$\boldsymbol{h}^{\mathrm{re},v}=\frac{3}{2}(C\alpha_{eq})^m\frac{\boldsymbol{\alpha}}{\alpha_{eq}} \tag{17}$$

5.1.2 Model multiphasic

In the multiphase case, the viscous part of the restoration is expressed by replacing the variable of work hardening by a median value on the phases:

$$r\rightarrow\bar{r}\ ;\ C\rightarrow\bar{C}\ ;\ m\rightarrow\bar{m} \tag{18}$$

With an average being done on ***five*** phases :

$$\bar{r}=\sum_{k=1}^{5}Z_k r_k\ ;\ \bar{C}=\sum_{k=1}^{5}Z_k C_k\ ;\ \bar{m}=\sum_{k=1}^{5}Z_k m_k \tag{19}$$

And thus:

$$\bar{g}^{\mathrm{re},v}=(\bar{C}\bar{r})^{\bar{m}} \tag{20}$$

In the case of kinematic work hardening, according to (17), one a:

$$\boldsymbol{h}^{\mathrm{re},v}=\frac{3}{2}(C\alpha_{eq})^m\frac{\boldsymbol{\alpha}}{\alpha_{eq}} \tag{21}$$

One proceeds in the same way as for isotropic work hardening:

$$\boldsymbol{\alpha}\rightarrow\bar{\boldsymbol{\alpha}}\ \text{ and }\ \alpha_{eq}\rightarrow\bar{\alpha}_{eq} \tag{22}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

With:

$$\bar{\boldsymbol{\alpha}}=\sum_{k=1}^{5} Z_k \bar{\boldsymbol{\alpha}}_k \text{ and } \bar{\alpha}_{eq}=\sum_{k=1}^{5} Z_k (\alpha_k)_{eq} \quad (23)$$

And thus:

$$\bar{\boldsymbol{h}}^{\mathrm{re},v}=\frac{3}{2}(\bar{C}\,\bar{\alpha}_{eq})^{\bar{m}}\frac{\bar{\boldsymbol{\alpha}}}{\bar{\alpha}_{eq}} \quad (24)$$

5.2 Restoration of isotropic work hardening metallurgical

In the case multi-phasic, the term of work hardening is written for each phase:

$$R_k=R_{0,k}\,p \quad (25)$$

Where $R_{0,k}$ is the linear coefficient of work hardening of the phase k. At the time of the metallurgical transformations, there exists within material of displacements of more or less important atoms. These displacements of atoms can destroy dislocations which are at the origin of work hardening. In these cases, the work hardening of the mother phase is not transmitted to the produced phase, it is the restoration of work hardening. The new phase can then be born with a virgin plastic state or inherit only one part, possibly totality, work hardening of the mother phase. Cumulated plastic deformation p is not characteristic any more of the state of work hardening and it is necessary to define other variables of work hardening for each phase, noted r_k who take account of the restoration of work hardening. The term of work hardening of the phase k is written then:

$$R_k=R_{0,k}\,r_k \quad (26)$$

5.2.1 Model with two phases with a direction of transformation

To define the variables r_k, one chooses the model suggested by Leblond [11]. One considers an element of two-phase volume V who undergoes a metallurgical transformation and a plastic deformation. Phase 1 is the mother phase, characterized by her voluminal fraction V_1, its proportion of phase $(1-z)$ and a variable of work hardening r_1. Phase 2 is the phase produced, with its voluminal fraction V_2, its proportion of phase z and its variable of work hardening r_2. Equations of evolution of r_i obtained by derivation compared to time are written:

$$\begin{cases}\dot{r}_1=\dot{p}\\[2ex] \dot{r}_2=\dot{p}-\dfrac{\dot{z}}{z}r_2+\dfrac{\dot{z}}{z}\theta\, r_1\end{cases} \quad (27)$$

The parameter θ characterize the proportion of work hardening transmitted of the mother phase to the produced phase and $\dot{p}$ is the rate of equivalent plastic deformation. p here is not any more one internal variable of the problem as such. Only significance of $\dot{p}$ is here to be the plastic multiplier and it is equal to the rate of equivalent plastic deformation. Sjöström obtains the same equations by using a phenomenologic reasoning that one defers here to clarify the model [12].

That is to say an increment of time Δt, such as enters t and $t+\Delta t$:

- A fraction ΔV_2 mother phase is transformed into phase 2 and thus comes to be added with volume V_2 of this phase produced;
- The element of volume V a plastic deformation undergoes Δp.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$\begin{cases} V_2, r_2(t) \\ V_1, r_1(t) \end{cases}$	$\begin{cases} \Delta t \\ \Delta p \\ \Delta V_2 \end{cases}$	$\begin{cases} V_2, r_2(t)+\Delta p \\ \Delta V_2, \theta r_1(t)+\Delta p \\ V_1, r_1(t)+\Delta p \end{cases}$

One supposes that at the time of the metallurgical transformation, the transformed fraction ΔV_2 inherit only one part θr_1 work hardening of the mother phase $0 \le \theta \le 1$. Then variables of work hardening r_i at the moment $t+\Delta t$ are such as:

$$\begin{cases} r_1(t+\Delta t) = r_1(t)+\Delta p \\ r_2(t+\Delta t) = \dfrac{V_2(r_2(t)+\Delta p)+\Delta V_2(\theta r_1(t)+\Delta p)}{V_2+\Delta V_2} \end{cases} \quad (28)$$

Maybe, by considering a standard incremental writing of the variables of work hardening such as:

$$\Delta r_i = r_i(t+\Delta_t) - r_i(t) \quad (29)$$

One obtains:

$$\begin{cases} \Delta r_1 = \Delta p \\ \Delta r_2 = \Delta p + \dfrac{\Delta V_2}{V_2+\Delta V_2}\theta r_1^- - \dfrac{\Delta V_2}{V_2+\Delta V_2} r_2^- \end{cases} \quad (30)$$

One obtains the equations (27) while passing in extreme cases. For the discretization of the laws of evolutions of r_i, one chooses an explicit diagram of integration by using the equations directly (30).

5.2.2 Generalization of the model with *N* phases with transformations with double direction

In the case of steel the existing phases are ferrite, the pearlite, the bainite, martensite and austenite γ respective proportions $Z_{k_f=1,4}$ and Z_γ. When one heats, the metallurgical transformations to consider are the transformations of the ferritic phases (ferrite, pearlite, bainite and martensite) towards the austeniqtic phase. The proportion of austenite increases and thus:

$$\begin{cases} \text{Si } Z_\gamma > 0 & \dot{r}_\gamma = \dot{p}_\gamma + g_\gamma^{\mathrm{re},m} \\ \text{Sinon} & \dot{r}_\gamma = 0 \end{cases} \quad (31)$$

With $g_\gamma^{\mathrm{re},m}$ the function of restoration of isotropic work hardening metallurgical for austenite :

$$g_\gamma^{\mathrm{re},m} = \frac{\sum_{k_f=1}^{4} \langle -\dot{Z}_{k_f} \rangle \theta_{k_f,\gamma} r_{k_f} - \sum_{k_f=1}^{4} \langle -\dot{Z}_{k_f} \rangle r_\gamma}{Z_\gamma} \quad (32)$$

Where $\theta_{k_f,\gamma}$ is the proportion of restoration of work hardening at the time of the transformation of a ferritic phase towards the austenitic phase. In the case of a cooling, the metallurgical transformations to consider are them transformations of austenite towards the ferritic phases (ferrite, pearlite, bainite and martensite). They are the proportions of phases ferritic which increase and one a:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\begin{cases} \text{Si } Z_{k_f} > 0 & \dot{r}_{k_f} = \dot{p}_{k_f} + g_{k_f}^{\text{re},m} \\ \text{Sinon} & \dot{r}_{k_f} = 0 \end{cases} \quad (33)$$

With $g_{k_f}^{\text{re},m}$ the function of restoration of metallurgical work hardening isotropic for the ferritic phases:

$$g_{k_f}^{\text{re},m} = \frac{\langle \dot{Z}_{k_f} \rangle \theta_{\gamma,k_f} r_\gamma - \langle \dot{Z}_{k_f} \rangle r_{k_f}}{Z_{k_f}} \quad (34)$$

θ_{γ,k_f} is the proportion of restoration of work hardening at the time of the transformation of the austenitic phase into a ferritic phase.

For transformations with diffusion (for example: austenite towards ferrite, pearlite and bainite), implying important displacements of atoms one will be able to take $\theta=0$; dislocations at the origin of plastic work hardening are completely destroyed by the transformation. For transformations without diffusion (for example a martensitic transformation), one will be able to take $\theta=1$, work hardening being completely transmitted.

5.2.3 Restoration of kinematic work hardening metallurgical

By analogy with the case of isotropic work hardening, the kinematic restoration of work hardening in the case multiphase will be written, in the case of austenite:

$$\begin{cases} \text{Si } Z_\gamma > 0 & \dot{\boldsymbol{\alpha}}_\gamma = \dot{\boldsymbol{\varepsilon}}^{vp} + \boldsymbol{h}_\gamma^{\text{re},m} \\ \text{Sinon} & \dot{\boldsymbol{\alpha}}_\gamma = 0 \end{cases} \quad (35)$$

With $\boldsymbol{h}_\gamma^{\text{re},m}$ the function of restoration of isotropic work hardening metallurgical for austenite :

$$\boldsymbol{h}_\gamma^{\text{re},m} = \frac{\sum_{k_f=1}^{4} \langle -\dot{Z}_{k_f} \rangle \theta_{k_f,\gamma} \boldsymbol{\alpha}_{k_f} - \sum_{k_f=1}^{4} \langle -\dot{Z}_{k_f} \rangle \boldsymbol{\alpha}_\gamma}{Z_\gamma} \quad (36)$$

And for the ferritic phases:

$$\begin{cases} \text{Si } Z_{k_f} > 0 & \dot{\boldsymbol{\alpha}}_{k_f} = \dot{\boldsymbol{\varepsilon}}^{vp} + \boldsymbol{h}_{k_f}^{\text{re},m} \\ \text{Sinon} & \dot{\boldsymbol{\alpha}}_{k_f} = 0 \end{cases} \quad (37)$$

With $\boldsymbol{h}_{k_f}^{\text{re},m}$ the function of restoration of metallurgical work hardening kinematic for the ferritic phases:

$$\boldsymbol{h}_{k_f}^{\text{re},m} = \frac{\langle \dot{Z}_{k_f} \rangle \theta_{\gamma,k_f} \boldsymbol{\alpha}_\gamma - \langle \dot{Z}_{k_f} \rangle \boldsymbol{\alpha}_{k_f}}{Z_{k_f}} \quad (38)$$

5.3 Parameters materials

Parameters θ metallurgical restoration of work hardening are provided by the user in the operator `DEFI_MATERIAU` under the keyword `META_RE`.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Physical parameter for the metallurgical restoration of work hardening		**Keyword** **MET A_RE**
Rate of work hardening transmitted of austenite to the ferritic phase 1	$\theta_{\gamma,1}$	C_ F1_ THETA
Rate of work hardening transmitted of austenite to the ferritic phase 2	$\theta_{\gamma,2}$	C_ F 2 _ THETA
Rate of work hardening transmitted of austenite to the ferritic phase 3	$\theta_{\gamma,3}$	C_ F 3 _ THETA
Rate of work hardening transmitted of austenite to the ferritic phase 4	$\theta_{\gamma,4}$	C_ F 4 _ THETA
Rate of work hardening transmitted of the ferritic phase 1 to austenite	$\theta_{1,\gamma}$	F1_ C_THETA
Rate of work hardening transmitted of the ferritic phase 2 to austenite	$\theta_{2,\gamma}$	F 2 _ C_THETA
Rate of work hardening transmitted of the ferritic phase 3 to austenite	$\theta_{3,\gamma}$	F 3 _ C_THETA
Rate of work hardening transmitted of the ferritic phase 4 to austenite	$\theta_{4,\gamma}$	F 4 _ C_THETA

Parameters C and m viscous restoration of work hardening are provided by the user in the operator DEFI_MATERIAU under the keyword META_ VISC .

Physical parameter for the viscous restoration of work hardening		**Keyword** **META_VISC**
Parameter m ferritic phase 1	m_1	F1_ M
Parameter m ferritic phase 2	m_2	F 2 _ M
Parameter m ferritic phase 3	m_3	F 3 _ M
Parameter m ferritic phase 4	m_4	F 4 _ M
Parameter m austenitic phase	m_γ	C _ M
Parameter C ferritic phase 1	C_1	F1_ C
Parameter C ferritic phase 2	C_2	F 2 _ C
Parameter C ferritic phase 3	C_3	F 3 _ C
Parameter C ferritic phase 4	C_4	F 4 _ C
Parameter C austenitic phase	C_γ	C _ C

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

6 ModelS of deformation (visco) plastic

The main feature of the evolutions thermal concerned in this kind of analysis is that they sweep a broad temperature range, which has an important effect on the mechanical behavior of the material which undergoes the thermal evolution. One is in particular in temperature ranges where the phenomena of viscosity can not be negligible more. It can thus be necessary to use a élasto-viscoplastic model of behavior especially when one remains in these fields for one important length of time; for example at the time as of treatments of detensioning associated with welding.
One thus chooses a viscoplastic model whose characteristics are such as it makes it possible to describe with the same formalism, therefore without changing model:

- A classical plastic behavior; to model the cases at low temperature when the viscous effects are still negligible or to model the processes at high speed (welding);
- A hammer-hardenable viscoplastic behavior at high temperature, to model the effects of creep and relieving associated for example with the treatment with detensioning or multirun weldings;
- A behavior of the fluid type viscous for the higher temperatures at the melting point, in order to have a reasonable description of the molten zone.

The selected viscoplastic model degenerates indeed for certain borderline cases in model of plasticity independent of time, or in model of viscous fluid.

6.1 Partition of the deformation

Total deflection $\boldsymbol{\varepsilon}$ is written as the sum of four components:

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{e}+\boldsymbol{\varepsilon}^{th}+\boldsymbol{\varepsilon}^{vp}+\boldsymbol{\varepsilon}^{pt} \quad (39)$$

Where $\boldsymbol{\varepsilon}^{e}$, $\boldsymbol{\varepsilon}^{th}$, $\boldsymbol{\varepsilon}^{vp}$ and $\boldsymbol{\varepsilon}^{pt}$ are, respectively, the elastic strain, plastic thermics, visco - and of plasticity of transformation. The deformations of the plastic type are purely deviatoric and the thermal deformation is purely spherical. For the deviatoric part, one thus has:

$$\tilde{\boldsymbol{\varepsilon}}=\tilde{\boldsymbol{\varepsilon}}^{e}+\tilde{\boldsymbol{\varepsilon}}^{vp}+\tilde{\boldsymbol{\varepsilon}}^{pt} \quad (40)$$

And for the spherical part:

$$tr(\boldsymbol{\varepsilon})=tr(\boldsymbol{\varepsilon}^{e})+tr(\boldsymbol{\varepsilon}^{th}) \quad (41)$$

6.2 Laws of behavior

6.2.1 Case of isotropic work hardening

One places oneself here within the framework of (visco) the plasticity of von Mises with additive isotropic work hardening. The function threshold is written:

$$f(\sigma,r;T,Z)=\sigma_{eq}-R(r;T,Z)-\sigma_{c}(T,Z) \quad (42)$$

With σ_{eq} the equivalent constraint of Von Mises such as:

$$\sigma_{eq}=\left(\frac{3}{2}\tilde{\boldsymbol{\sigma}}:\tilde{\boldsymbol{\sigma}}\right)^{1/2} \quad (43)$$

R is the isotropic term of work hardening and σ_{c} is the initial critical stress. It corresponds to the initial minimal constraint to apply to have a flow visco-plastic. These two quantities can depend on the temperature and the metallurgical phases. One can rewrite the equation (42) in the form:

$$f^{*}=f-\sigma_{v}=\sigma_{eq}-R(r;T,Z)-\sigma_{c}(T,Z)-\sigma_{v}=0 \quad (44)$$

It is - with - to say that in this model, the constraint can be interpreted as the sum of an ultimate stress of flow (which breaks up it even into an initial ultimate stress and a term of work hardening) and a constraint “viscous” depending on the speed of deformation and worthless at worthless speed:

$$\sigma_{v}=\eta\,\dot{p}^{\frac{1}{n}} \quad (45)$$

The rate D ‘ flow (visco) plastic is written:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\dot{\boldsymbol{\varepsilon}}^{vp}=\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}=\dot{\lambda}\frac{\partial f}{\partial\boldsymbol{\sigma}}=\frac{3}{2}\dot{p}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} \quad (46)$$

This tensor is purely deviatoric. Cumulated plastic deformation $\dot{p}$ is viscous and is written:

$$\dot{p}=\left(\frac{\langle\sigma_{eq}-R(r;T,Z)-\sigma_c(T,Z)\rangle}{\eta}\right)^n \quad (47)$$

With η and n the coefficients materials of viscosity.

6.2.2 Case of kinematic work hardening

In a way equivalent to the case with isotropic work hardening, the function threshold is written:

$$f(\sigma,\boldsymbol{X};T,Z)=(\tilde{\boldsymbol{\sigma}}-\boldsymbol{X})_{eq}-\sigma_c(T,Z) \quad (48)$$

With $\boldsymbol{X}$ the tensor of work hardening associated with the variable tensor with work hardening $\boldsymbol{\alpha}$ such as:

$$\boldsymbol{X}=\frac{2}{3}H_0\boldsymbol{\alpha} \quad (49)$$

H_0 is the kinematic coefficient of work hardening. In a way similar to the isotropic case, one can rewrite the equation (48) by introducing the viscous constraint of recall:

$$f^*=f-\sigma_v=(\tilde{\boldsymbol{\sigma}}-\boldsymbol{X})_{eq}-\sigma_c(T,Z)-\sigma_v=0 \quad (50)$$

The rate D ' flow (visco) plastic is written:

$$\dot{\boldsymbol{\varepsilon}}^{vp}=\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}=\dot{\lambda}\frac{\partial f}{\partial\boldsymbol{\sigma}}=\frac{3}{2}\dot{p}\frac{\tilde{\boldsymbol{\sigma}}-\boldsymbol{X}}{(\tilde{\boldsymbol{\sigma}}-\boldsymbol{X})_{eq}} \quad (51)$$

Cumulated plastic deformation $\dot{p}$ is viscous and is written:

$$\dot{p}=\left(\frac{\langle(\tilde{\boldsymbol{\sigma}}-\boldsymbol{X})_{eq}-\sigma_c(T,Z)\rangle}{\eta}\right)^n \quad (52)$$

6.3 Borderline cases

6.3.1 Plastic model independent of time

One wants to describe an instantaneous elastoplastic behavior and to cancel the viscous effects. For that viscous parameters η and C will be taken equal to zero. To free itself from the digital problems which the taking into account can pose of η and C worthless, and in a way similar to the treatment carried out for the viscoplastic model of Taheri [R5.03.05], one rewrites the equation (44) in the form:

$$f-\sigma_v\leq 0 \quad (53)$$

The strict inequality being obtained in the case $f<0$ and $\dot{p}=0$ who corresponds to the elastic mode. In the purely plastic field of behavior ($\eta\rightarrow 0$) the inequality (53) is then reduced to:

$$f=\sigma_{eq}-R(r;T,Z)-\sigma_c(T,Z)\leq 0 \quad (54)$$

So that $\dot{p}$ can be given more only by the equation of consistency $\dot{f}=0$. One thus finds oneself well within the framework of instantaneous plasticity independent of time, with a digital processing identical to that classically used for the treatment of this one. It will be noted that σ_c corresponds then to the classical definition of the yield stress σ_y . The elastic limit will be noted σ_c in viscoplasticity and σ_y in plasticity independent of time.

6.3.2 Model of behavior of viscous fluid

At very high temperature one has $R\rightarrow 0$ and $\sigma_c\rightarrow 0$, the function threshold is thus reduced:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$f(\sigma, r; T, Z) = \sigma_{eq} \tag{55}$$

If one takes $n \to 1$, then:

$$\dot{p} = \frac{\langle \sigma_{eq} \rangle}{\eta} = \frac{\sigma_{eq}}{\eta} \tag{56}$$

The rate of flow (visco) plastic is written:

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \frac{3}{2} \frac{\sigma_{eq}}{\eta} \tag{57}$$

Shears into unidimensional:

$$\dot{\varepsilon}^{vp} = \frac{\sigma}{\eta} \tag{58}$$

One thus obtains a model of behavior of the fluid type viscous Newtonian, of viscosity η .

6.4 Multiphase plasticity

The metallurgical transformations involve modifications of the mechanical characteristics of material. The elastic characteristics (Young modulus and Poisson's ratio) are affected little by the metallurgical changes of structures. Only their dependence compared to the temperature is thus taken into account. On the other hand, the plastic characteristics (elastic limit in particular) strongly depend on the metallurgical structure. It is thus necessary to take into account the differences in characteristics plastic for each possible phase. In modeling, the strain and the stress are defined at the level of the material point (macroscopic) which can be multiphase. One seeks to define the plastic behavior are equivalent of material when it has a multiphase structure, with in particular a single criterion of plasticity.

6.4.1 Linear law of the mixtures

The definition of the behavior of material are equivalent is done using one ***law of the mixtures*** on the characteristics of the phases. More precisely the definition of this material equivalent would correspond in 1D to a rheological model of k bars in parallel such as:

$$\begin{cases} \dot{\varepsilon}^{vp} = \dot{\varepsilon}_k^{vp} \\ \sigma = \sum_k z_k \sigma_k \text{ avec } \sigma_k = \sigma_{c,k} + R_k + \eta \dot{\varepsilon}_k^{vp} \end{cases} \tag{59}$$

6.4.1.1 Viscoplasticity with isotropic work hardening

In the case of the viscoplasticity of von Mises with isotropic work hardening:

$$f(\sigma, r; T, Z) = \sigma_{eq} - R(r; T, Z) - \sigma_c(T, Z) \tag{60}$$

One writes the work hardening of multiphase material $\overline{R}$ by application of the law of the mixtures:

$$\overline{R}(r; T, Z) = \sum_{k=1}^{5} Z_k R_k (r_k; T) \tag{61}$$

Where R_k is the work hardening of the phase k . Elastic limit of multiphase material $\overline{\sigma}_c$ is written in the same way:

$$\overline{\sigma}_c(T, Z) = \sum_{k=1}^{5} Z_k \sigma_{c,k}(T) \tag{62}$$

Where $\sigma_{c,k}$ is the elastic limit of the phase k . What gives us a new function threshold on multiphase material:

$$\bar{f}(\sigma, r; T, Z) = \sigma_{eq} - \overline{R}(r; T, Z) - \overline{\sigma}_c(T, Z) \tag{63}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

The rate of plastic deformation checks the condition of consistency (44) (function f^*). With an average being done on ***five*** phases :

$$\bar{\eta}=\sum_{k=1}^{5} Z_k \eta_k \; ; \bar{n}=\sum_{k=1}^{5} Z_k n_k \tag{64}$$

One applies the law of the mixtures to the viscous constraint of recall:

$$\bar{\sigma}_v(T,Z)=\bar{\eta}\,\dot{p}^{\frac{1}{\bar{n}}} \tag{65}$$

And thus:

$$\bar{f}^*=\sigma_{eq}-\bar{R}(r;T,Z)-\bar{\sigma}_c(T,Z)-\bar{\sigma}_v(T,Z)=0 \tag{66}$$

I.e. when one is in load, $\dot{p}$ is such as:

$$\bar{f}^*=\sigma_{eq}-\bar{R}-\bar{\sigma}_c-\bar{\sigma}_v=0 \tag{67}$$

6.4.1.2 Viscoplasticity with kinematic work hardening

In the case of the viscoplasticity of von Mises with kinematic work hardening:

$$f(\sigma,\boldsymbol{X};T,Z)=(\tilde{\boldsymbol{\sigma}}-\boldsymbol{X})_{eq}-\sigma_c(T,Z) \tag{68}$$

One writes the work hardening of multiphase material $\bar{\boldsymbol{X}}$ by application of the law of the mixtures:

$$\bar{\boldsymbol{X}}(\boldsymbol{\alpha};T,Z)=\sum_{k=1}^{5} Z_k \boldsymbol{X}_k\left(\boldsymbol{\alpha}_k;T\right) \tag{69}$$

Elastic limit of multiphase material $\bar{\sigma}_c$ is written just as in the isotropic case (62):

$$\bar{\sigma}_c(T,Z)=\sum_{k=1}^{5} Z_k \sigma_{c,k}(T) \tag{70}$$

What gives us a new function threshold on multiphase material:

$$\bar{f}(\sigma,\boldsymbol{X};T,Z)=(\tilde{\boldsymbol{\sigma}}-\bar{\boldsymbol{X}})_{eq}-\bar{\sigma}_c(T,Z) \tag{71}$$

The rate of plastic deformation checks the condition of consistency (50) (function f^*). In a similar way, one applies the law of the mixtures to the viscous constraint. I.e. when one is in load, $\dot{p}$ is such as:

$$\bar{f}^*=(\tilde{\boldsymbol{\sigma}}-\bar{\boldsymbol{X}})_{eq}-\bar{\sigma}_c(T,Z)-\bar{\sigma}_v=0 \tag{72}$$

6.4.2 Non-linear law of the mixtures

It is pointed out that one notes Z_f the sum of all the phases of the ferritic type, is:

$$Z_f=\sum_{k_f=1}^{4} Z_{k_f} \tag{73}$$

One also gives the possibility of using a nonlinear law of the mixtures [9] between the hot phase (austenitic) and the cold phases (ferritic), such as one has in the unidimensional case:

$$\bar{\sigma}=(1-f_h(Z_f))\sigma_\gamma+f_h(Z_f)\sigma_\alpha \tag{74}$$

Quantity f_h is the function of mixture. There is then the work hardening which is written:

$$\bar{R}=(1-f_h(Z_f))R_\gamma+f_h(Z_f)\bar{R}_\alpha \tag{75}$$

$\bar{R}_\alpha$ is the average work hardening of the cold phases (ferritic phases):

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\overline{R}_{\alpha}=\frac{\sum_{k_f=1}^{4} Z_{k_f} R_{k_f}}{Z_f} \quad (76)$$

The initial constraint of multiphase material is written in the same way:

$$\overline{\sigma}_c=(1-f_h(Z_f))\sigma_{c,\gamma}+f_h(Z_f)\overline{\sigma}_{c,\alpha} \quad (77)$$

$\overline{\sigma}_{c,\alpha}$ is the equivalent initial constraint of the cold phases (ferritic phases):

$$\overline{\sigma}_{c,\alpha}=\frac{\sum_{k_f=1}^{4} Z_{k_f} \sigma_{c,k_f}}{Z_f} \quad (78)$$

$f_h(Z_f)$ is a function defined by the user. Elastic limit $\overline{\sigma}_y$ in the elastoplastic case (nonviscous) uses the same rule of mixture as $\overline{\sigma}_c$. The unit is indicated in the order DEFI_MATERIAU.

Physical parameter		**Keyword ELAS_META**
Elastic limit of the ferritic phase 1	$\sigma_{y,1}$	F1_ SY
Elastic limit of the ferritic phase 2	$\sigma_{y,2}$	F 2 _ SY
Elastic limit of the ferritic phase 3	$\sigma_{y,3}$	F 3 _ SY
Elastic limit of the ferritic phase 4	$\sigma_{y,4}$	F 4 _ SY
Elastic limit of the austenitic phase	$\sigma_{y,\gamma}$	C _ SY
Initial critical stress of the ferritic phase 1	$\sigma_{c,1}$	F1_ S_VP
Initial critical stress of the ferritic phase 2	$\sigma_{c,2}$	F 2 _ S_VP
Initial critical stress of the ferritic phase 3	$\sigma_{c,3}$	F 3 _ S_VP
Initial critical stress of the ferritic phase 4	$\sigma_{c,4}$	F 4 _ S_VP
Initial critical stress of the austenitic phase	$\sigma_{c,\gamma}$	C _ S_VP
Function of mixture	$f_h(Z_f)$	SY_MELANGE
Function of mixture for the viscous case		S_VP_MELANGE

Parameters η_k and n_k are defined in DEFI_MATERIAU under the keyword factor META_VISC .

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Physical parameter		**Keyword** **META_VISC**
Parameter η ferritic phase 1	η_1	F1_ ETA
Parameter η ferritic phase 2	η_2	F 2 _ ETA
Parameter η ferritic phase 3	η_3	F 3 _ ETA
Parameter η ferritic phase 4	η_4	F 4 _ ETA
Parameter η austenitic phase	η_γ	C _ ETA
Parameter n ferritic phase 1	n_1	F1_ NR
Parameter n ferritic phase 2	n_2	F 2 _ NR
Parameter n ferritic phase 3	n_3	F 3 _ NR
Parameter n ferritic phase 4	n_4	F 4 _ NR
Parameter n austenitic phase	n_γ	C _ NR

6.5 Summary of the models available

6.5.1 Viscoplastic model with isotropic work hardening, viscous restoration and plasticity of transformation

While joining together:

- The partition of the deformations (39);
- Thermal deformation (7) ;
- Deformation for the plasticity of transformation (12) ;
- Viscoplastic deformation with isotropic work hardening (46);
- The law of Hooke;

One obtains the following system to solve:

$$\begin{cases} \tilde{\boldsymbol{\varepsilon}} = \tilde{\boldsymbol{\varepsilon}}^{e} + \tilde{\boldsymbol{\varepsilon}}^{vp} + \tilde{\boldsymbol{\varepsilon}}^{pt} \\ \boldsymbol{\sigma} = \boldsymbol{A} : \boldsymbol{\varepsilon}^{e} \\ \boldsymbol{\varepsilon}^{th} = \left(Z_\gamma \left[\alpha_\gamma (T - T_{ref}) - (1 - Z_\gamma^R) \Delta \varepsilon_{f\gamma}^{T_{ref}} \right] + Z_f \left[\alpha_f (T - T_{ref}) + Z_\gamma^R \Delta \varepsilon_{f\gamma}^{T_{ref}} \right] \right) \boldsymbol{Id} \\ \dot{\tilde{\boldsymbol{\varepsilon}}}^{pt} = \frac{3}{2} \tilde{\boldsymbol{\sigma}} \sum_{k_f=1}^{4} K_{k_f} F'_{k_f} \langle \dot{Z}_{k_f} \rangle \\ \dot{\tilde{\boldsymbol{\varepsilon}}}^{vp} = \frac{3}{2} \dot{p} \frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} \end{cases} \quad (79)$$

While adding, the expression of the viscoplastic law with isotropic work hardening, is:

- The function threshold (42) by applying the law of the mixtures to work hardening (61) and the constraint criticizes (62) ;
- The function threshold modified (67);

There is the expression of the criterion of (visco-) plasticity:

$$\begin{cases} \bar{f} = \sigma_{eq} - \bar{R} - \bar{\sigma}_c \\ \bar{f}^* = \sigma_{eq} - \bar{R} - \bar{\sigma}_c - \bar{\sigma}_v \\ \dot{p} = 0 \text{ si } \bar{f} < 0 \\ \dot{p} \geq 0 \text{ si } \bar{f} = 0 \text{ avec } \bar{f}^* = 0 \end{cases} \quad (80)$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Lastly, the update of work hardening by phases with taking into account of the metallurgical restoration of work hardening and the viscous restoration enable us to write the put-with-day of the variables of work hardening:

$$\begin{cases} \dot{r}_\gamma = \dot{p}_\gamma + g_\gamma^{\mathrm{re},m} - \overline{g}^{\mathrm{re},v} \text{ si } Z_\gamma > 0 \text{ et } \dot{r}_\gamma = 0 \text{ sinon} \\ \dot{r}_{k_f} = \dot{p}_{k_f} + g_{k_f}^{\mathrm{re},m} - \overline{g}^{\mathrm{re},v} \text{ si } Z_{k_f} > 0 \text{ et } \dot{r}_{k_f} = 0 \text{ sinon} \end{cases} \quad (81)$$

6.5.2 Viscoplastic model with kinematic work hardening, viscous restoration and plasticity of transformation

While joining together:

- The partition of the deformations (39);
- Thermal deformation (7) ;
- Deformation for the plasticity of transformation (12) ;
- Viscoplastic deformation with kinematic work hardening (51);
- The law of Hooke;

One obtains the following system to solve:

$$\begin{cases} \tilde{\boldsymbol{\varepsilon}} = \tilde{\boldsymbol{\varepsilon}}^e + \tilde{\boldsymbol{\varepsilon}}^{vp} + \tilde{\boldsymbol{\varepsilon}}^{pt} \\ \boldsymbol{\sigma} = \boldsymbol{A} : \boldsymbol{\varepsilon}^e \\ \boldsymbol{\varepsilon}^{th} = \left(Z_\gamma \left[\alpha_\gamma (T - T_{ref}) - (1 - Z_\gamma^R) \Delta \varepsilon_{f\gamma}^{T_{ref}} \right] + Z_f \left[\alpha_f (T - T_{ref}) + Z_\gamma^R \Delta \varepsilon_{f\gamma}^{T_{ref}} \right] \right) \boldsymbol{Id} \\ \dot{\tilde{\boldsymbol{\varepsilon}}}^{pt} = \frac{3}{2} \tilde{\boldsymbol{\sigma}} \sum_{k_f=1}^{4} K_{k_f} F'_{k_f} \langle \dot{Z}_{k_f} \rangle \\ \dot{\tilde{\boldsymbol{\varepsilon}}}^{vp} = \frac{3}{2} \dot{p} \frac{\tilde{\boldsymbol{\sigma}} - \boldsymbol{X}}{(\tilde{\boldsymbol{\sigma}} - \boldsymbol{X})_{eq}} \end{cases} \quad (82)$$

While adding, the expression of the viscoplastic law with isotropic work hardening, is:

- The function threshold (48) by applying the law of the mixtures to work hardening (69) and the constraint criticizes (70) ;
- The function threshold modified E (72);

There is the expression of the criterion of (visco-) plasticity:

$$\begin{cases} \overline{f} = (\tilde{\boldsymbol{\sigma}} - \overline{\boldsymbol{X}})_{eq} - \overline{\sigma}_c \\ \overline{f}^* = (\tilde{\boldsymbol{\sigma}} - \overline{\boldsymbol{X}})_{eq} - \overline{\sigma}_c - \overline{\sigma}_v \\ \dot{p} = 0 \text{ si } \overline{f} < 0 \\ \dot{p} \geq 0 \text{ si } \overline{f} = 0 \text{ avec } \overline{f}^* = 0 \end{cases} \quad (83)$$

Lastly, the update of work hardening by phases with taking into account of the metallurgical restoration of work hardening and the viscous restoration enable us to write the put-with-day of the variables of work hardening:

$$\begin{cases} \dot{\boldsymbol{\alpha}}_\gamma = \dot{\boldsymbol{\varepsilon}}^{vp} + \boldsymbol{h}_\gamma^{\mathrm{re},m} + \overline{\boldsymbol{h}}^{\mathrm{re},v} \text{ si } Z_\gamma > 0 \text{ et } \dot{\boldsymbol{\alpha}}_\gamma = \boldsymbol{0} \text{ sinon} \\ \dot{\boldsymbol{\alpha}}_{k_f} = \dot{\boldsymbol{\varepsilon}}^{vp} + \boldsymbol{h}_{k_f}^{\mathrm{re},m} + \overline{\boldsymbol{h}}^{\mathrm{re},v} \text{ si } Z_{k_f} > 0 \text{ et } \dot{\boldsymbol{\alpha}}_{k_f} = \boldsymbol{0} \text{ sinon} \end{cases} \quad (84)$$

6.5.3 Selection of the behavior

In term of relations of behavior available, the modeling installation gives several opportunities:

- Choice of the type of behavior for the plastic deformation: plastic independent of time or with taking into account of the viscous effects;
- Choix of a work hardening isotropic linear, isotropic nonlinear or kinematic;
- Taking into account or not of the plasticity of transformation;
- Taking into account or not of the metallurgical restoration of work hardening.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

The choice of the material (steel or zircaloy) and thus amongst phase is done by informing the keyword KIT of BEHAVIOR. STEEL for steel with five phases and ZIRC for the zircaloy with three phases.
There are on the whole 24 combinations.

Behavior	Plastic	Visco.	Work hardening			Plasticity Transform.	Restoration
			Isotropic		Kinematics		
			Linear	Not Linear	Linear		
META_P_CL_PT	YES	NOT	NOT	NOT	YES	YES	NOT
META_P_CL_PT_RE	YES	NOT	NOT	NOT	YES	YES	YES
META_P_CL	YES	NOT	NOT	NOT	YES	NOT	NOT
META_P_CL_RE	YES	NOT	NOT	NOT	YES	NOT	YES
META_P_IL_PT	YES	NOT	YES	NOT	NOT	YES	NOT
META_P_IL_PT_RE	YES	NOT	YES	NOT	NOT	YES	YES
META_P_IL	YES	NOT	YES	NOT	NOT	NOT	NOT
META_P_IL_RE	YES	NOT	YES	NOT	NOT	NOT	YES
META_P_INL_PT	YES	NOT	NOT	YES	NOT	YES	NOT
META_P_INL_PT_RE	YES	NOT	NOT	YES	NOT	YES	YES
META_P_INL	YES	NOT	NOT	YES	NOT	NOT	NOT
META_P_INL_RE	YES	NOT	NOT	YES	NOT	NOT	YES
META_V_CL_PT	NOT	YES	NOT	NOT	YES	YES	NOT
META_V_CL_PT_RE	NOT	YES	NOT	NOT	YES	YES	YES
META_V_CL	NOT	YES	NOT	NOT	YES	NOT	NOT
META_V_CL_RE	NOT	YES	NOT	NOT	YES	NOT	YES
META_V_IL_PT	NOT	YES	YES	NOT	NOT	YES	NOT
META_V_IL_PT_RE	NOT	YES	YES	NOT	NOT	YES	YES
META_V_IL	NOT	YES	YES	NOT	NOT	NOT	NOT
META_V_IL_RE	NOT	YES	YES	NOT	NOT	NOT	YES
META_V_INL_PT	NOT	YES	NOT	YES	NOT	YES	NOT
META_V_INL_PT_RE	NOT	YES	NOT	YES	NOT	YES	YES
META_V_INL	NOT	YES	NOT	YES	NOT	NOT	NOT
META_V_INL_RE	NOT	YES	NOT	YES	NOT	NOT	YES

For the whole of the relations, the internal variables produced in *Code_hasster* are:

- r_k variables of effective work hardening for k phases. They are named by phase. For example, for an isotropic work hardening of a steel alloy, one a: FERRITE#EPSPEQ, PERLITE#EPSPEQ, BAINITE#EPSPEQ, MARTENSITE#EPSPEQ and AUSTENITE#EPSPEQ. ;
- d indicator of plasticity (0 if the last calculated increment is elastic; 1 if not); C'is a total quantity;
- R the term of work hardening of the function threshold. It is a total quantity calculated by a law of the mixtures (see § 6.4);

In addition, these modelings can be realized with the geometrical functionality of reactualization PETIT_REAC. For the Relations with isotropic work hardening, the model of great deformations SIMO_MIEHE is also available (see [R4.04.03]).

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

7 Digital formulation

One will treat only the viscoplastic law of behaviour with isotropic work hardening. One obtains the same thing in the case of kinematic work hardening, R_0 is then replaced by the kinematic coefficient of work hardening H_0.

7.1 Discretization

Knowing the fields $\boldsymbol{\sigma}$, $\boldsymbol{u}$ and p at the moment t, one chooses an implicit scheme to discretize in time the equations of the continuous problem, except for the parameters of work hardening. It is noticed that with an implicit discretization, only two points differentiate the two types of viscoplastic and plastic behavior independent of time:

- The form of the function of load, for which one has a complementary term in the case of viscosity (see 6.4.1.1 and 6.4.1.2);
- The presence of the term of restoration of work hardening in the evolution of the variable of work hardening for the viscoplastic case.

Moreover, incremental classical plasticity seems the borderline case (without associated digital difficulty) of incremental viscoplasticity when $\eta, C \to 0$ and $\sigma_c \to \sigma_y$. This kind of treatment was already carried out by Lorentz [R5.03.05]. To calculate the tangent operators, one will adopt the convention of writing of the symmetrical tensors of order two in the form of vectors with six components. Thus, for a tensor $\boldsymbol{a}$:

$$\boldsymbol{a}={}^t\left[a_{xx} \quad a_{yy} \quad a_{zz}\ \sqrt{2}\,a_{xy} \quad \sqrt{2}\,a_{xz} \quad \sqrt{2}\,a_{yz}\right] \tag{85}$$

7.2 Integration of the relations metallurgical

7.2.1 Expression of the constraints

One gives the expression of $\boldsymbol{\sigma}$ according to:

- $\Delta\,\boldsymbol{\varepsilon}$ unknown factor of the problem;
- Known terms such variables calculated with the preceding step (internal constraints, variables…), characteristic materials, variables of orders (temperature, phases metallurgical);

One points out the partition of the deviatoric deformations:

$$\tilde{\boldsymbol{\varepsilon}}=\tilde{\boldsymbol{\varepsilon}}^{e}+\tilde{\boldsymbol{\varepsilon}}^{vp}+\tilde{\boldsymbol{\varepsilon}}^{pt} \tag{86}$$

One separates the tensor from the constraints between his deviatoric components and his spherical component:

$$\boldsymbol{\sigma}=\tilde{\boldsymbol{\sigma}}+tr(\boldsymbol{\sigma})\,\boldsymbol{Id} \tag{87}$$

By applying the law of Hooke:

$$\boldsymbol{\sigma}=\boldsymbol{A}\,\boldsymbol{\varepsilon}^{e} \tag{88}$$

With its deviatoric part:

$$\tilde{\boldsymbol{\sigma}}=2\,\mu\,\tilde{\boldsymbol{\varepsilon}}^{e} \tag{89}$$

And its spherical part:

$$tr(\boldsymbol{\sigma})=3\mathrm{K}\,tr(\boldsymbol{\varepsilon}^{e}) \tag{90}$$

By implicit discretization, one obtains:

$$\tilde{\boldsymbol{\sigma}}=\frac{\mu}{\mu^{-}}\tilde{\boldsymbol{\sigma}}^{-}+2\mu\left(\Delta\,\tilde{\boldsymbol{\varepsilon}}-\Delta\,\tilde{\boldsymbol{\varepsilon}}^{vp}-\Delta\,\tilde{\boldsymbol{\varepsilon}}^{pt}\right) \tag{91}$$

While injecting (12) and (46):

$$\tilde{\boldsymbol{\sigma}}=\frac{\mu}{\mu^{-}}\tilde{\boldsymbol{\sigma}}^{-}+2\,\mu\left(\Delta\,\tilde{\boldsymbol{\varepsilon}}-\frac{3}{2}\Delta\,p\,\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}-\frac{3}{2}\,\tilde{\boldsymbol{\sigma}}\,g_Z^{pt}\right) \tag{92}$$

For the spherical part of the tensor of the constraints, one a:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$tr(\boldsymbol{\sigma})=\frac{3K}{3K^-}tr(\boldsymbol{\sigma}^-)+3K\,tr(\Delta\boldsymbol{\varepsilon}) \quad (93)$$

Because the deformations of the plastic type are purely deviatoric and thus $tr(\Delta\boldsymbol{\varepsilon}^{vp})=tr(\Delta\boldsymbol{\varepsilon}^{pt})=0$. While reorganizing (92):

$$\tilde{\boldsymbol{\sigma}}=\frac{1}{1+3\mu g_Z^{pt}}\left(\frac{\mu}{\mu^-}\tilde{\boldsymbol{\sigma}}^-+2\mu\Delta\tilde{\boldsymbol{\varepsilon}}-3\mu\Delta p\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right) \quad (94)$$

One needs to develop the expression of $\frac{\tilde{\sigma}}{\sigma_{eq}}$. One leaves the expression of the partition of the deviatoric deformations, that is to say:

$$\Delta\tilde{\boldsymbol{\varepsilon}}=\Delta\tilde{\boldsymbol{\varepsilon}}^e+\Delta\tilde{\boldsymbol{\varepsilon}}^{vp}+\Delta\tilde{\boldsymbol{\varepsilon}}^{pt} \quad (95)$$

By injecting the values of the increments of deformation:

$$\Delta\tilde{\boldsymbol{\varepsilon}}=\left(\frac{\tilde{\boldsymbol{\sigma}}}{2\mu}-\frac{\tilde{\boldsymbol{\sigma}}^-}{2\mu^-}\right)+\frac{3}{2}\Delta p\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}+\frac{3}{2}\tilde{\boldsymbol{\sigma}}g_Z^{pt} \quad (96)$$

While multiplying by 2μ :

$$2\mu\Delta\tilde{\boldsymbol{\varepsilon}}+\frac{2\mu}{2\mu^-}\tilde{\boldsymbol{\sigma}}^-=\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\left[\left(1+3\mu g_Z^{pt}\right)\sigma_{eq}+3\mu\Delta p\right] \quad (97)$$

The deviatoric elastic constraint is posed:

$$\tilde{\boldsymbol{\sigma}}^e=2\mu\Delta\tilde{\boldsymbol{\varepsilon}}+\frac{2\mu}{2\mu^-}\tilde{\boldsymbol{\sigma}}^- \quad (98)$$

And the equivalent elastic constraint within the meaning of Von Mises:

$$\sigma_{eq}^e=\left[\left(1+3\mu g_Z^{pt}\right)\sigma_{eq}+3\mu\Delta p\right] \quad (99)$$

The expression of $\frac{\tilde{\sigma}}{\sigma_{eq}}$ is worth then:

$$\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}=\frac{\tilde{\boldsymbol{\sigma}}^e}{\sigma_{eq}^e} \quad (100)$$

7.2.2 Expression of the plastic increment

Quantities (98) and (99) are all known at the moment running, except the increment Δp . To find this value, the condition of coherence is used:

$$\begin{cases}\bar{f}=\sigma_{eq}-\bar{R}-\bar{\sigma}_c\\ \bar{f}^*=\sigma_{eq}-\bar{R}-\bar{\sigma}_c-\bar{\sigma}_v\\ \Delta p=0 \text{ si } \bar{f}<0\\ \Delta p\geq 0 \text{ si } \bar{f}=0 \text{ avec } \bar{f}^*=0\end{cases} \quad (101)$$

It is thus enough to express $\bar{f}^*=0$ to find the expression of Δp . Only σ_{eq} , $\bar{R}$ and the viscous constraint $\bar{\sigma}_v$ depend on Δp . Maybe in incremental form:

$$\bar{f}^*=\sigma_{eq}-\bar{R}(p^-+\Delta p)-\bar{\sigma}_c-\eta\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{\bar{n}}}=0 \quad (102)$$

By using the expression (99), one finds:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\sigma_{eq}=\frac{\sigma_{eq}^{e}-3\mu\Delta p}{1+3\mu g_{Z}^{pt}} \quad (103)$$

It is thus necessary to find the zero of the following function to determine Δp :

$$\bar{f}^{*}=\frac{\sigma_{eq}^{e}-3\mu\Delta p}{1+3\mu g_{Z}^{pt}}-\bar{R}(p^{-})-\bar{R}(\Delta p)-\bar{\sigma}_{c}-\bar{\eta}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{\bar{n}}}=0 \quad (104)$$

This equation is non-linear in Δp . The resolution is made in *Code_hasster* by a method of the secants with interval of research [R5.03.05].

7.3 Quantities in prediction

The goal of this paragraph is to calculate quantities necessary in prediction. Inter alia, the tangent operator K_{i-1} (option of calculation `RIGI_MECA_TANG` called with the first iteration of a new increment of load) is evaluated starting from the results known at the previous moment t_{i-1} :

$$\boldsymbol{K}_{i-1}=\left.\frac{d\boldsymbol{\sigma}}{d\boldsymbol{\varepsilon}}\right|_{t_{i-1}} \text{ that is to say } \Delta\boldsymbol{\sigma}=\boldsymbol{K}_{i-1}\Delta\boldsymbol{\varepsilon} \text{ or } \dot{\boldsymbol{\sigma}}=\boldsymbol{K}_{i-1}\dot{\boldsymbol{\varepsilon}} \quad (105)$$

Like one makes the quasi-static assumption, the various quantities depend on the time that in manner *implicit* , via the dependence of the parameters materials to the variables of order [1] $\beta(t)$, themselves functions of time. In the situation present, there are two variables of orders: the temperature T and metallurgical phases Z . For n_{varc} variables of orders, one writes the total differential of a quantity a :

$$\frac{d\boldsymbol{a}}{dt}=\frac{\partial\boldsymbol{a}}{\partial t}+\sum_{j=1,n_{varc}}\frac{\delta\boldsymbol{a}}{\delta\beta^{j}}\cdot\frac{\delta\beta^{j}}{\delta t} \quad (106)$$

One leaves the expression of the incremental law of Hooke, for his deviatoric part:

$$\dot{\tilde{\boldsymbol{\sigma}}}=2\mu\left(\dot{\tilde{\boldsymbol{\varepsilon}}}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{pt}\right) \quad (107)$$

The main difficulty of this expression is that the viscoplastic deformation $\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}$ has as an unknown factor the plastic multiplier which is itself an unknown factor of the problem. For the spherical part on the other hand, the expression is commonplace because it depends only on the thermal deformation and the elastic strain:

$$tr(\dot{\boldsymbol{\sigma}})=3\mathrm{K}\,tr(\dot{\boldsymbol{\varepsilon}})-3\mathrm{K}\,tr(\dot{\boldsymbol{\varepsilon}}^{th}) \quad (108)$$

With the expression of the thermal deformation given by (7).

7.3.1 Plastic multiplier

To establish the expression of the plastic multiplier, the condition of consistency is written $\dot{\bar{f}}=0$:

$$\dot{\bar{f}}=\frac{d}{dt}\left(\sigma_{eq}-\bar{R}-\overline{\sigma_{c}}\right)=0 \quad (109)$$

The function $\bar{f}$ depends on the equivalent constraint of Von Mises who is purely deviatoric and who is worth:

$$\sigma_{eq}=\left(\frac{3}{2}\tilde{\boldsymbol{\sigma}}:\tilde{\boldsymbol{\sigma}}\right)^{1/2} \quad (110)$$

The contribution is written then:

$$\frac{d\sigma_{eq}}{dt}=\frac{\partial\sigma_{eq}}{\partial t}+\frac{\delta\sigma_{eq}}{\delta T}\cdot\frac{\delta T}{\delta t}+\frac{\delta\sigma_{eq}}{\delta Z}\cdot\frac{\delta Z}{\delta t} \quad (111)$$

1 A variable of order is a scalar quantity, function of time and space, data a priori by the user via the keyword `AFFE_VARC` in the operator `AFFE_MATERIAU` . It is one *parameter* problem and not one *unknown factor* .

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

We make the assumption (usual in *Code_hasster*) that variation of the matrix of Hooke A according to the variables of order is negligible. I.e.:

$$\frac{\delta\sigma_{eq}}{\delta T}=\frac{\delta\sigma_{eq}}{\delta Z}=0 \tag{112}$$

With final, the contribution $\dot{\sigma}_{eq}$ is worth then:

$$\frac{d\sigma_{eq}}{dt}=\dot{\sigma}_{eq}=\frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}:\boldsymbol{A}\dot{\boldsymbol{\varepsilon}}^{e}}{\sigma_{eq}} \tag{113}$$

We treat from now on the case of the term of work hardening $\bar{R}$. We will neglect the variation of the module of work hardening compared to the variables of orders. I.e.:

$$\frac{\delta\bar{R}}{\delta T}=\frac{\delta\bar{R}}{\delta Z}=0 \tag{114}$$

The total differential in time is thus written:

$$\frac{d\bar{R}}{dt}=\sum_{k=1}^{5}\dot{Z}_k R_{0,k} r_k+\sum_{k=1}^{5}\dot{r}_k R_{0,k} Z_k \tag{115}$$

One neglects the variation of the metallurgical phases. The second term implies to use the multiphase law of work hardening written in (81) without the term of viscous restoration, which one one will write in condensed form:

$$\dot{r}_k=\dot{p}_k+g_k^{\mathrm{re},m} \tag{116}$$

One thus has:

$$\frac{d\bar{R}}{dt}=\sum_{k=1}^{5}\dot{Z}_k R_{0,k} r_k+\sum_{k=1}^{5}\left(\dot{p}+g_k^{\mathrm{re},m}\right)R_{0,k} Z_k \tag{117}$$

The equivalent module is recognized $\bar{R}_0$:

$$\bar{R}_0=\sum_{k=1}^{5} R_{0,k} Z_k \tag{118}$$

The other terms will be gathered in the function B :

$$\frac{d\bar{R}}{dt}=\bar{R}_0\dot{p}+B \tag{119}$$

As one neglects L are effects of the variation of the metallurgical phases, this term B is supposed to be null. Finally:

$$\frac{d\bar{R}}{dt}=\dot{\bar{R}}=\bar{R}_0\dot{p} \tag{120}$$

Lastly, we consider the case of the elastic limit σ_y . We will make a last simplification by supposing that this term does not vary according to the metallurgical structure and from the temperature. I.e.:

$$\frac{d\bar{\sigma}_y}{dt}=\dot{\sigma}_y=\frac{\partial\bar{\sigma}_y}{\partial t}\dot{t}+\frac{\partial\bar{\sigma}_y}{\partial T}\dot{T}+\frac{\partial\bar{\sigma}_y}{\partial Z}\dot{Z}=C \tag{121}$$

Quantity $\bar{\sigma}_y$ is a time-constant. Its partial derivative compared to time is thus worthless. The other quantities (dependence at the temperature and the metallurgical phases are gathered in the function C . This term C will be supposed to be null. With final, one will have crudely:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\frac{d\overline{\sigma}_y}{dt}=\dot{\sigma}_y=0 \quad (122)$$

While beginning again (109) and by injecting there the results of the differentiation of the terms (113), (120) and (122), one has finally:

$$\dot{\overline{f}}=\frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}:\boldsymbol{A}\dot{\tilde{\boldsymbol{\varepsilon}}}^e}{\sigma_{eq}}-\overline{R}_0\dot{p}=0 \quad (123)$$

The deviatoric component of the law of Hooke is written in incremental form:

$$\dot{\tilde{\boldsymbol{\sigma}}}=2\mu\dot{\tilde{\boldsymbol{\varepsilon}}}^e \quad (124)$$

What gives a new expression (deviatoric) of the condition of consistency:

$$\dot{\overline{f}}=3\mu\frac{\tilde{\boldsymbol{\sigma}}:\left(\dot{\tilde{\boldsymbol{\varepsilon}}}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{pt}\right)}{\sigma_{eq}}-\overline{R}_0\dot{p}=0 \quad (125)$$

It is necessary to express the tensor of the elastic strain starting from the expression (86) :

$$\dot{\tilde{\boldsymbol{\varepsilon}}}^e=\dot{\tilde{\boldsymbol{\varepsilon}}}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{pt} \quad (126)$$

One develops the first term for the plastic deformation:

$$3\mu\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}}{\sigma_{eq}}=3\mu\frac{\tilde{\boldsymbol{\sigma}}:\frac{3}{2}\dot{p}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}}{\sigma_{eq}} \quad (127)$$

While using (110):

$$3\mu\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}}{\sigma_{eq}}=3\mu\dot{p} \quad (128)$$

One develops the second term for the plasticity of transformation (12) :

$$3\mu\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}^{pt}}{\sigma_{eq}}=3\mu\frac{\tilde{\boldsymbol{\sigma}}:\frac{3}{2}\tilde{\boldsymbol{\sigma}}\,g_Z^{pt}(Z,\dot{Z})}{\sigma_{eq}} \quad (129)$$

It is important to note that the term g_Z^{pt} depends on the increment of the metallurgical phases $\dot{Z}$. Thereafter, to reduce the writing, we will not write it explicitly. Always while using (110):

$$3\mu\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}^{pt}}{\sigma_{eq}}=3\mu\sigma_{eq}g_Z^{pt} \quad (130)$$

Finally:

$$\dot{\overline{f}}=3\mu\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}}-3\mu\sigma_{eq}g_Z^{pt}-\left(\overline{R}_0+3\mu\right)\dot{p}=0 \quad (131)$$

What enables us to express $\dot{p}$:

$$\dot{p}=\frac{3\mu}{\overline{R}_0+3\mu}\left(\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}}-\sigma_{eq}g_Z^{pt}\right) \quad (132)$$

This expression is different from that given by (104) because it is about an approximation. Indeed, one considered the condition of consistency without the viscous term as it is usual to proceed. But this expression does not need to be exact since it is the tangent matrix in prediction and has the advantage of being an analytical formula, contrary to (104) who requires the use of an algorithm of search for zero on a nonlinear function.

7.3.2 Increment of constraint

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

From the definition of the viscoplastic increment of deformation $\dot{\boldsymbol{\varepsilon}}^{vp}$:

$$\dot{\boldsymbol{\varepsilon}}^{vp}=\begin{cases}\frac{3}{2}\dot{p}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} & \text{si} \quad \sigma_{eq}-\overline{R}-\overline{\sigma}_c=0\\ 0 & \text{si} \quad \sigma_{eq}-\overline{R}-\overline{\sigma}_c<0\end{cases} \tag{133}$$

And of the expression of $\dot{p}$ obtained previously, O N obtains:

$$\dot{\boldsymbol{\varepsilon}}^{vp}=\begin{cases}\frac{9\mu}{2\left(\overline{R}_0+3\mu\right)}\left\langle\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}}-\sigma_{eq}g_Z^{pt}\right\rangle\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} & \text{si} \quad \sigma_{eq}-\overline{R}-\overline{\sigma}_c=0\\ 0 & \text{si} \quad \sigma_{eq}-\overline{R}-\overline{\sigma}_c<0\end{cases} \tag{134}$$

Thanks to the expression of the incremental law of Hooke:

$$\dot{\tilde{\boldsymbol{\sigma}}}=2\mu\left(\dot{\tilde{\boldsymbol{\varepsilon}}}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{vp}-\dot{\tilde{\boldsymbol{\varepsilon}}}^{pt}\right) \tag{135}$$

By injecting the expression of the plastic deformation (134) and from the plasticity of transformation, one obtains the expression of the increment of constraint:

$$\dot{\tilde{\boldsymbol{\sigma}}}=2\mu\left[\dot{\tilde{\boldsymbol{\varepsilon}}}-\frac{9\mu}{2\sigma_{eq}\left(\overline{R}_0+3\mu\right)}\left\langle\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}}-\sigma_{eq}g_Z^{pt}\right\rangle\tilde{\boldsymbol{\sigma}}-\frac{3}{2}g_Z^{pt}\tilde{\boldsymbol{\sigma}}\right] \tag{136}$$

The expression of $\dot{\tilde{\boldsymbol{\sigma}}}$ depends on the sign of the term (criterion of load-discharge) according to:

$$\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}}-\sigma_{eq}g_Z^{pt} \tag{137}$$

One can simplify by saying that the discharge or the load does not depend on the plasticity of transformation, one will thus approximate:

$$\left\langle\frac{\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}}{\sigma_{eq}}-\sigma_{eq}g_Z^{pt}\right\rangle\approx\frac{\left\langle\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}\right\rangle}{\sigma_{eq}}-\sigma_{eq}g_Z^{pt} \tag{138}$$

In discharge, the second term will be neglected and charges some, it will be added to the other terms of (136):

$$\dot{\tilde{\boldsymbol{\sigma}}}=\begin{cases}2\mu\left[\dot{\tilde{\boldsymbol{\varepsilon}}}-\frac{9\mu}{2}\frac{\left\langle\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}\right\rangle}{\left(\overline{R}_0+3\mu\right)\sigma_{eq}^2}\tilde{\boldsymbol{\sigma}}-\frac{3}{2}g_Z^{pt}\tilde{\boldsymbol{\sigma}}\right] & \text{en décharge}\\ 2\mu\left[\dot{\tilde{\boldsymbol{\varepsilon}}}-\frac{9\mu}{2}\frac{\left\langle\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}\right\rangle}{\left(\overline{R}_0+3\mu\right)\sigma_{eq}^2}\tilde{\boldsymbol{\sigma}}-\frac{3\overline{R}_0}{2\left(\overline{R}_0+3\mu\right)}g_Z^{pt}\tilde{\boldsymbol{\sigma}}\right] & \text{en charge}\end{cases} \tag{139}$$

The parameter is introduced d , which is worth 1 if one plasticizes and if one is in load at the moment t and 0 in the contrary case:

$$\dot{\tilde{\boldsymbol{\sigma}}}=2\mu\left[\dot{\tilde{\boldsymbol{\varepsilon}}}-\frac{9\mu}{2}\frac{\left\langle\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}\right\rangle}{\left(\overline{R}_0+3\mu\right)\sigma_{eq}^2}\tilde{\boldsymbol{\sigma}}-\frac{3}{2}g_Z^{pt}\left(Z,\dot{Z}\right)\left(1-d\frac{3\mu}{3\mu+\overline{R}_0}\right)\tilde{\boldsymbol{\sigma}}\right] \tag{140}$$

It is noticed that $\dot{\tilde{\boldsymbol{\sigma}}}$ is a function closely connected of $\dot{\tilde{\boldsymbol{\varepsilon}}}$ but also that there exists a part which does not depend on $\dot{\tilde{\boldsymbol{\varepsilon}}}$ but of the increment of the metallurgical phases $\dot{Z}$.

7.3.3 Second member in prediction – Contribution of the metallurgy

Because of dependence of (140) with the increment of the metallurgical phases, there is a contribution $\Delta \boldsymbol{L}^{pt}$ with the second member [2] temporal variation of the tensor of the constraints. On simple cases tests for which

2 What reverses the sign in front of this contribution!

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

there exists an analytical solution, one noted that the fact of neglecting the second member $\Delta \boldsymbol{L}^{pt}$ could lead, to converge, with a significant number of iterations. This is why this term is taken into account for the phase of prediction. The contribution comes from the plasticity of transformation, which is a purely deviatoric quantity. With final, one obtains:

$$\Delta \boldsymbol{L}^{pt}=\int_{\Omega} 3\mu\left[g_Z^{pt}(Z,\dot{Z})\left(1-d\frac{3\mu}{3\mu+\bar{R}_0}\right)\tilde{\boldsymbol{\sigma}}\right]\delta\boldsymbol{\varepsilon}\, d\Omega \tag{141}$$

With $\delta\boldsymbol{\varepsilon}$ virtual deformation.

7.3.4 Tangent matrix in prediction (option RIGI_MECA_TANG)

It is pointed out that the tangent operator K_{i-1} (option of calculation RIGI_MECA_TANG called with the first iteration of a new increment of load) is evaluated starting from the results known at the previous moment t_{i-1} :

$$\dot{\boldsymbol{\sigma}}=\boldsymbol{K}_{i-1}\dot{\boldsymbol{\varepsilon}} \tag{142}$$

In load, it is supposed that one plasticizes and thus one a:

$$\langle\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}}\rangle=\tilde{\boldsymbol{\sigma}}:\dot{\tilde{\boldsymbol{\varepsilon}}} \tag{143}$$

The deviatoric part of the matrix of prediction is thus worth:

$$\tilde{\boldsymbol{K}}_{i-1}=2\mu\left[\boldsymbol{I}-\frac{9\mu}{2(\bar{R}_0+3\mu)}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\otimes\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right] \tag{144}$$

The expression (100) give the value of $\frac{\tilde{\sigma}}{\sigma_{eq}}$. The spherical part is worth:

$$\frac{1}{3}tr(\boldsymbol{K}_{i-1})=K\boldsymbol{I} \tag{145}$$

Finally:

$$\boldsymbol{K}_{i-1}=2\mu\left[\boldsymbol{I}-\frac{9\mu}{2(\bar{R}_0+3\mu)}\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\otimes\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right]+K\boldsymbol{I} \tag{146}$$

7.4 Quantities in correction

7.4.1 Tangent matrix in correction (option FULL_MECA)

The tangent operator K^n (option of calculation FULL_MECA called with each iteration of Newton) is such as:

$$\boldsymbol{K}^n=\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}} \tag{147}$$

This matrix has two contributions, deviatoric and spherical:

$$\boldsymbol{K}^n=\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}=\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial\boldsymbol{\varepsilon}}+\frac{1}{3}\frac{\partial tr(\boldsymbol{\sigma})}{\partial\boldsymbol{\varepsilon}}\boldsymbol{I} \tag{148}$$

The spherical part is commonplace:

$$\frac{1}{3}\frac{\partial tr(\boldsymbol{\sigma})}{\partial\boldsymbol{\varepsilon}}\boldsymbol{I}=K\boldsymbol{I} \tag{149}$$

With K the module of compressibility. One must now express the deviatoric part of the tangent matrix. One a:

$$\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial\boldsymbol{\varepsilon}}=\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial\tilde{\boldsymbol{\varepsilon}}}\frac{\partial\tilde{\boldsymbol{\varepsilon}}}{\partial\boldsymbol{\varepsilon}} \tag{150}$$

One sets out again of the expression of the deviatoric constraints:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\tilde{\boldsymbol{\sigma}}=2\mu\left(\tilde{\boldsymbol{\varepsilon}}-\tilde{\boldsymbol{\varepsilon}}^{vp}-\tilde{\boldsymbol{\varepsilon}}^{pt}\right) \tag{151}$$

And one derives:

$$\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial\tilde{\boldsymbol{\varepsilon}}}=\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left[2\mu\left(\tilde{\boldsymbol{\varepsilon}}-\tilde{\boldsymbol{\varepsilon}}^{vp}-\tilde{\boldsymbol{\varepsilon}}^{pt}\right)\right] \tag{152}$$

One finds:

$$\frac{\partial\tilde{\boldsymbol{\varepsilon}}}{\partial\tilde{\boldsymbol{\varepsilon}}}=\boldsymbol{I} \tag{153}$$

For the plasticity of transformation:

$$\frac{\partial\tilde{\boldsymbol{\varepsilon}}^{pt}}{\partial\tilde{\boldsymbol{\varepsilon}}}=\frac{3}{2}g_Z^{pt}\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial\tilde{\boldsymbol{\varepsilon}}} \tag{154}$$

For viscoplasticity:

$$\frac{\partial\tilde{\boldsymbol{\varepsilon}}^{vp}}{\partial\tilde{\boldsymbol{\varepsilon}}}=\frac{3}{2}\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left(\Delta p\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)=\frac{3}{2}\left[\frac{\partial(\Delta p)}{\partial\tilde{\boldsymbol{\varepsilon}}}\otimes\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}+\Delta p\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left(\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)\right] \tag{155}$$

One needs to evaluate $\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left(\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)$ however the expression (100) the value established of $\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}$:

$$\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}=\frac{\tilde{\boldsymbol{\sigma}}^e}{\sigma_{eq}^e} \tag{156}$$

By observing the usual rules of derivation:

$$\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left(\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)=\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left(\frac{\tilde{\boldsymbol{\sigma}}^e}{\sigma_{eq}^e}\right)=\frac{1}{\left(\sigma_{eq}^e\right)^2}\left(\sigma_{eq}^e\frac{\partial\tilde{\boldsymbol{\sigma}}^e}{\partial\tilde{\boldsymbol{\varepsilon}}}-\tilde{\boldsymbol{\sigma}}^e\frac{\partial\sigma_{eq}^e}{\partial\tilde{\boldsymbol{\varepsilon}}}\right) \tag{157}$$

One a:

$$\tilde{\boldsymbol{\sigma}}^e=2\mu\Delta\tilde{\boldsymbol{\varepsilon}}+\frac{2\mu}{2\mu^-}\tilde{\boldsymbol{\sigma}}^-\rightarrow\frac{\partial\tilde{\boldsymbol{\sigma}}^e}{\partial\tilde{\boldsymbol{\varepsilon}}}=2\mu\boldsymbol{I} \tag{158}$$

By the law of Hooke:

$$\frac{\partial\sigma_{eq}^e}{\partial\tilde{\boldsymbol{\varepsilon}}}=3\mu\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} \tag{159}$$

Finally:

$$\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left(\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right)=\frac{1}{\sigma_{eq}^e}\left(2\mu\boldsymbol{I}-3\mu\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\otimes\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}}\right) \tag{160}$$

It remains to find the expression of $\frac{\partial(\Delta p)}{\partial\tilde{\boldsymbol{\varepsilon}}}$. For that, the condition of consistency is taken again $\bar{f}^*=0$:

$$\bar{f}^*=\frac{\sigma_{eq}^e-3\mu\Delta p}{1+3\mu g_Z^{pt}}-\bar{R}(p^-)-\bar{R}(\Delta p)-\bar{\sigma}_c-\bar{\eta}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{\bar{n}}}=0 \tag{161}$$

And one derives compared to $\tilde{\boldsymbol{\varepsilon}}$:

$$\frac{\partial\bar{f}^*}{\partial\tilde{\boldsymbol{\varepsilon}}}=\frac{\partial}{\partial\tilde{\boldsymbol{\varepsilon}}}\left[\frac{\sigma_{eq}^e-3\mu\Delta p}{1+3\mu g_Z^{pt}}-\bar{R}(\Delta p)-\bar{\eta}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{\bar{n}}}\right] \tag{162}$$

Thanks to (159), one has already $\frac{\partial\sigma_{eq}^e}{\partial\tilde{\boldsymbol{\varepsilon}}}$. It remains to calculate the other terms. The derivative of the metallurgical term of work hardening is worth:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

$$\frac{\partial \overline{R}}{\partial \tilde{\varepsilon}}=\overline{R_0}\frac{\partial(\Delta p)}{\partial \tilde{\varepsilon}} \qquad (163)$$

And that of the viscous constraint:

$$\frac{\partial}{\partial \tilde{\varepsilon}}\left[\overline{\eta}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1}{\overline{n}}}\right]=\frac{\overline{\eta}}{\overline{n}\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-\overline{n}}{\overline{n}}}\left(\frac{\partial(\Delta p)}{\partial \tilde{\varepsilon}}\right) \qquad (164)$$

Finally, (162) has as an expression:

$$\frac{\partial \overline{f}^*}{\partial \tilde{\varepsilon}}=\frac{3\mu}{1+3\mu g_Z^{pt}}\frac{\tilde{\sigma}}{\sigma_{eq}}-\frac{3\mu}{1+3\mu g_Z^{pt}}\frac{\partial(\Delta p)}{\partial \tilde{\varepsilon}}-\overline{R_0}\frac{\partial(\Delta p)}{\partial \tilde{\varepsilon}}-\frac{\overline{\eta}}{\overline{n}\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-\overline{n}}{\overline{n}}}\left(\frac{\partial(\Delta p)}{\partial \tilde{\varepsilon}}\right)=0 \qquad (165)$$

While rearranging, one finds the expression final of $\frac{\partial(\Delta p)}{\partial \tilde{\varepsilon}}$:

$$\frac{\partial(\Delta p)}{\partial \tilde{\varepsilon}}=\frac{3\mu}{3\mu+\left[\overline{R_0}+\frac{\overline{\eta}}{\overline{n}\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-\overline{n}}{\overline{n}}}\right]\left[1+3\mu g_Z^{pt}\right]}\frac{\tilde{\sigma}}{\sigma_{eq}} \qquad (166)$$

Therefore, for the viscoplastic term:

$$\frac{\partial \tilde{\varepsilon}^{vp}}{\partial \tilde{\varepsilon}}=\frac{3}{2}\left[\frac{3\mu}{3\mu+\left[\overline{R_0}+\frac{\overline{\eta}}{\overline{n}\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-\overline{n}}{\overline{n}}}\right]\left[1+3\mu g_Z^{pt}\right]}\frac{\tilde{\sigma}}{\sigma_{eq}}\otimes\frac{\tilde{\sigma}}{\sigma_{eq}}+\Delta p\frac{1}{\sigma_{eq}^e}\left(2\mu \boldsymbol{I}-3\mu\frac{\tilde{\sigma}}{\sigma_{eq}}\otimes\frac{\tilde{\sigma}}{\sigma_{eq}}\right)\right] \qquad (167)$$

If one sets out again of the formal expression of the deviatoric tangent matrix:

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{\varepsilon}}=\frac{\partial}{\partial \tilde{\varepsilon}}\left[2\mu\left(\tilde{\varepsilon}-\tilde{\varepsilon}^{vp}-\tilde{\varepsilon}^{pt}\right)\right] \qquad (168)$$

One injects the expression of the term there corresponding to the plasticity of transformation and elasticity:

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{\varepsilon}}=2\mu \boldsymbol{I}-2\mu\frac{\partial \tilde{\varepsilon}^{vp}}{\partial \tilde{\varepsilon}}-2\mu\frac{3}{2}g_Z^{pt}\frac{\partial \tilde{\sigma}}{\partial \tilde{\varepsilon}} \qquad (169)$$

And thus:

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{\varepsilon}}=\frac{1}{1+3\mu g_Z^{pt}}\left[2\mu \boldsymbol{I}-2\mu\frac{\partial \tilde{\varepsilon}^{vp}}{\partial \tilde{\varepsilon}}\right] \qquad (170)$$

The deviatoric tangent matrix breaks up into two terms:

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{\varepsilon}}=\frac{1}{1+3\mu g_Z^{pt}}\left[\alpha \boldsymbol{I}+\beta\frac{\tilde{\sigma}}{\sigma_{eq}}\otimes\frac{\tilde{\sigma}}{\sigma_{eq}}\right] \qquad (171)$$

With the term α :

$$\alpha=2\mu\left(1-\frac{3\mu\Delta p}{\sigma_{eq}^e}\right) \qquad (172)$$

And the term β :

$$\beta=-9\mu^2\left[\frac{1}{3\mu+\left[\overline{R_0}+\frac{\overline{\eta}}{\overline{n}\,\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-\overline{n}}{\overline{n}}}\right]\left[1+3\mu g_Z^{pt}\right]}+\frac{\Delta p}{\sigma_{eq}^e}\right] \qquad (173)$$

7.5 Synthesis of the matrices

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

The matrices of prediction and correction can be written in the same form, while varying following coefficients the cases. The spherical part is the same one in the two matrices. The deviatoric part is written in general form:

$$\tilde{\boldsymbol{K}}=\left[\frac{2\mu}{a}\left(1-c_3\frac{3\mu\Delta p}{\sigma_{eq}^{e}}\right)\boldsymbol{I}-\frac{c_p}{a}\tilde{\boldsymbol{\sigma}}\otimes\tilde{\boldsymbol{\sigma}}\right] \quad (174)$$

With the table following for the coefficients:

	RIGI_MECA_TANG
a	1
c_3	0
c_p	$c_1\frac{(3\mu)^2}{(\sigma_{eq}^{e})^2}\left[\frac{1}{3\mu+\overline{R_0}}\right]$
c_1	1 is worth if plasticization, 0 if not
c_2	1 is worth if one charges, 0 if not

	FULL_MECA
a	$1+3\mu g_Z^{pt}$
c_3	1
c_p	$c_2\frac{(3\mu)^2}{(\sigma_{eq}^{e})^2}\left[\frac{1}{3\mu+\left[\overline{R_0}+\frac{\overline{\eta}}{\overline{n}\Delta t}\left(\frac{\Delta p}{\Delta t}\right)^{\frac{1-\overline{n}}{\overline{n}}}\right]\left[1+3\mu g_Z^{pt}\right]}-\frac{\Delta p}{\sigma_{eq}^{e}}\right]$
c_1	1 is worth if plasticization, 0 if not
c_2	1 is worth if one charges, 0 if not

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
3.0	F.Waeckel	Initial version
6.3	*Razakanaivo, A., A. - M. Donore, F.Waeckel* EDF-R&D/AMA	Fifth version of the document
12.2	*M.Abbas* EDF-R&D/AMA	Rewriting of documentation Correction of some shells in the equations

9 Bibliography

[1] **Waeckel, F.** (**1994**). *A thermo-metallurgical law of behavior of steels for the mechanical calculation of the structures*, doctorate, ENSAM.

[2] **Andrieux, S. and Waeckel, F.** (**1994**). *Thermal, metallurgical modeling and mechanics of an operation of welding - Study Bibliographical*, EDF R & D, Internal report.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

[3] **Andrieux, S. and Waeckel, F.** (**1994**). *A thermo-metallurgical law of behavior to cooling for steels slightly allied*, EDF R & D, Internal report.

[4] **Razakanaivo, A.** (**2000**). *Introduction of model EDGAR into Code_Aster*, EDF R & D, Internal report.

[5] **Mialon, P.** (**1986**). *Elements of analysis and digital resolution of the relations of elastoplasticity*, EDF R & D, Internal report.

[6] **Razakanaivo, A.** (**1997**). *Introduction into Code_Aster of a model of fascinating élasto-viscoplastic behavior of account metallurgy*, EDF R & D, Internal report.

[7] **Desalos, Y.** (**1981**). *N° report 9534901 MET44*, IRSID, Internal report.

[8] **Leblond, J. - B.; Mottet, G. and Devaux, J. - C.** (**1986**). *With theorical (and numerical) approach to the plastic behaviour of steels during phase transformations. Derivation of general equations*, Newspaper of the Mechanics and Physics Solids 34, pp. 395-406.

[9] **Leblond, J. - B.; Mottet, G. and Devaux, J. - C.** (**1986**). *With theorical (and numerical) approach to the plastic behaviour of steels during phase transformations – Study of classical plasticity for ideal plastic phases*, Newspaper of the Mechanics and Physics Solids 34, pp. 411-432.

[10] **Leblond, J. - B. and Devaux, J. - C.** (**1989**). *Mathematical modelling of transformation plasticity in steels. Box of ideal plastic phases*, International Newspaper of Plasticity 5, pp. 551-572.

[11] **Leblond, J. - B.** (**1989**). *Mathematical modelling of transformation plasticity in steels. Coupling with strain hardening phenomena*, International Newspaper of Plasticity 5, pp. 573-591.

[12] **Sjöström, S.** (**1987**). *The problem of calculating quench stress*, In: *Swedish symposium one residual stresses*.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)