Law of behavior élasto (visco) plastic in great deformations with transformations metallurgical

Summary

This document presents a model of thermo-élasto behavior (visco) plastic to isotropic work hardening with effects of the metallurgical transformations writes in great deformations. This model can be used for three-dimensional, axisymmetric modelings and in plane deformations.

One presents the writing of this model and his digital processing.

To understand this document, it is practically essential to read the two notes [R5.03.21] and [R4.04.02] devoted to the written models of behavior, respectively, in great deformations without metallurgical effects and small deformations with metallurgical effects.

Contents

1 Introduction

This document presents a law of thermo-élasto behavior (visco) plastic to isotropic work hardening in great deformations which takes into account the effects of the metallurgical transformations. This model can be used for three-dimensional, axisymmetric problems and in plane deformations.

This law represents a “assembly” of two models established in *Code_Aster*, namely a thermoelastoplastic model with isotropic work hardening written in great deformations (keyword factor `DEFORMATION: 'SIMO_MIEHE'`, cf [R5.03.21]) and a model small deformations thermo - élasto-(visco) plastic with effects of the metallurgical transformations (keyword factor `'META_P_ ** _**'` or `'META_V_ ** _**'` of `BEHAVIOR` of the operator `STAT_NON_LINE`). The first model of great deformations was thus wide to take account of the consequences of the metallurgical transformations on mechanics.

To understand this document, it is practically essential to read reference documents [R5.03.21] and [R4.04.02] which concerns, respectively, the model great deformations without metallurgical effects and the model small deformations with metallurgical effects. Nevertheless, to facilitate the reading of this note, we make some recalls on these two models.

To justify the extension of the model written in great deformations to the model great deformations with metallurgical effects, we take again some theoretical aspects extracted from [bib1] related to the writing model great deformations.

One presents then the relations of behavior of the complete model, his digital integration and the forms of the tangent matrix (options `FULL_MECA` and `RIGI_MECA_TANG`).

2 Notations

One will note by:

$\mathbf{Id}$	matrix identity
$\operatorname{tr} A$	trace of the tensor $\mathbf{A}$
A^T	transposed of the tensor $\mathbf{A}$
$\det A$	determinant of $\mathbf{A}$
$\langle X \rangle$	positive part of X
$\tilde{A}$	deviatoric part of the tensor $\mathbf{A}$ defined by $\tilde{A}=A-(\frac{1}{3}\operatorname{tr} A)\mathbf{Id}$
$:$	doubly contracted product: $A:B=\sum_{i,j} A_{ij} B_{ij}=\operatorname{tr}(\mathbf{AB}^{\boldsymbol{T}})$
$\ddot{A}$	tensorial product: $(A\ddot{A}B)_{ijkl}=A_{ij} B_{kl}$
A_{eq}	equivalent value of von Mises defined by $A_{eq}=\sqrt{\frac{3}{2}\tilde{A}:\tilde{A}}$
$\tilde{N}_X A$	gradient: $\tilde{N}_X A=\frac{\partial A}{\partial X}$
$\operatorname{div}_x A$	divergence: $(\operatorname{div}_x A)_i=\sum_j \frac{\partial A_{ij}}{\partial x_j}$
λ , μ	coefficients of Lamé: $\lambda=\frac{E\nu}{(1+\nu)(1-2\nu)}$, $m=\frac{E}{2(1+\nu)}$
E	Young modulus
ν	Poisson's ratio
K	module of rigidity to compression: $3\mathrm{K}=3\lambda+2\mathrm{m}=\frac{E}{(1-2\nu)}$
T	temperature
T_{ref}	temperature of reference
Z_g	proportion of austenite
Z_i	proportion of the four phases α : ferrite, pearlite, bainite and martensite

In addition, within the framework of a discretization in time, all the quantities evaluated at the previous moment are subscripted by $^-$, quantities evaluated at the moment $t+\Delta t$ are not subscripted and the increments are indicated by Δ . One has as follows:

$$\Delta Q=Q-Q^-$$

3 Recalls of the metallurgical model and the model great deformations

3.1 Model with metallurgical transformations

We present only here the consequences of the metallurgical transformations on the mechanical behavior.

The determination of the mechanical evolution associated with a process bringing into play metallurgical transformations requires a thermo-metallurgical calculation as a preliminary. This thermo-metallurgical calculation is uncoupled and allows the determination of the thermal evolutions then metallurgical. For the metallurgical models of behavior of steels, one will be able to consult the note [R4.04.01].

For the study of the metallurgical transformations of steel, there exist five metallurgical phases: ferrite, pearlite, the bainite, martensite (phases α) and austenite (phase γ).

The effects of the metallurgical transformations (at the solid state) are of four types:

- the mechanical characteristics of the material which undergoes the transformations are modified. More precisely, elastic characteristics (YOUNG modulus E and Poisson's ratio ν) are not very affected whereas the plastic characteristics, such as the elastic limit, are it strongly,
- the expansion or the voluminal contraction which accompanies the metallurgical transformations translates by a deformation (spherical) of "transformation" which is superimposed on the purely thermal deformation of origin. In general, one gathers this effect with that due to the modification of the thermal dilation coefficient α ,
- a transformation proceeding under constraints can give rise to an unrecoverable deformation and this, even for levels of constraints much lower than the elastic limit of material. One calls "plasticity of transformation" this phenomenon. Total deflection ε is written then:

$$\varepsilon=\varepsilon^e+\varepsilon^{th}+\varepsilon^p+\varepsilon^{pt}$$

  where ε^e , ε^{th} , ε^p and ε^{pt} are, respectively the elastic strain, thermal, plastic and of plasticity of transformation,
- one can have at the time of the metallurgical transformation a phenomenon of restoration of work hardening. The work hardening of the mother phase is not completely transmitted to the phases lately created. Those can then be born with a virgin state of work hardening or inherit only one part, even totality, work hardening of the mother phase. Cumulated plastic deformation p is not then any more characteristic of the state of work hardening and it is necessary to define other variables of work hardening for each phase, noted r_k who take account of the restoration. The laws of evolution of these work hardenings differ from the usual laws so as to allow a "return towards zero" total, or partial, of these parameters at the time of the transformations.

One will be able to find in the document [R4.04.02] the expressions of the various relations of behavior.

3.2 Model written in great deformations

3.2.1 General presentation

This model is a thermoelastoplastic law of behavior eulérienne written in great deformations which was proposed by Simo and Miehe ([bib2]) which tends under the assumption of the small deformations towards the model with isotropic work hardening and criterion of von Mises describes in [R5.03.02]. It makes it possible to treat not only the great deformations, but also, in an exact way, great rotations.

The essential characteristics of this law are the following ones:

- just like in small deformations, one supposes the existence of a slackened configuration, i.e. locally free of constraint, which makes it possible to break up the total deflection into a thermoelastic part and a plastic part,
- the decomposition of this thermoelastic deformation into cubes parts and plastic is not additive any more as in small deformations (or for the models great deformations written in rate of deformation with for example a derivative of Jaumann) but multiplicative,
- as in small deformations, the constraints depend only on the thermoelastic deformations,
- to write the law of behavior, one uses the tensor of the constraints of Kirchhoff τ who is connected to the tensor of Cauchy σ by the relation $J\boldsymbol{\sigma}=\boldsymbol{\tau}$ where J represent the variation of volume between the configurations initial and current,
- the plastic deformations are done with constant volume. The variation of volume is then only due to the thermoelastic deformations,
- this model led during its digital integration to a model incrémentalement objective what makes it possible to obtain the exact solution in the presence of great rotations.

3.2.2 Kinematics

We make here some basic recalls of mechanics in great deformations and on the model of behavior.

Let us consider a solid subjected to great deformations. That is to say Ω_0 the field occupied by the solid before deformation and $\Omega(t)$ the field occupied at the moment t by the deformed solid. In the initial configuration Ω_0 , the position of any particle of the solid is indicated by $\mathbf{X}$ (Lagrangian description). After deformation, the position at the moment t particle which occupied the position $\mathbf{X}$ before deformation is given by the variable $\mathbf{x}$ (description eulérienne).

The total movement of the solid is defined, with $\mathbf{u}$ displacement, by:

$$\mathbf{x}=\hat{\mathbf{x}}(\mathbf{X},t)=\mathbf{X}+\mathbf{u}$$

To define the change of metric in the vicinity of a point, the tensor gradient of the transformation is introduced $\mathbf{F}$:

$$\mathbf{F}=\frac{\partial\hat{\mathbf{x}}}{\partial\mathbf{X}}=\mathbf{Id}+\nabla_X\mathbf{u}$$

The transformations of the element of volume and the density are worth:

$$d\,\Omega = J d\,\Omega_o \quad \text{with} \quad J = \det \mathbf{F} = \frac{\rho_o}{\rho}$$

where ρ_o and ρ are respectively the density in the configurations initial and current.

To now write the model great deformations, one supposes the existence of a slackened configuration Ω^r , i.e. locally free of constraint, which then makes it possible to break up the total deflection into cubes parts thermoelastic and plastic, this decomposition being multiplicative.

One will note by $\mathbf{F}$ the tensor gradient which makes pass from the initial configuration Ω_0 with the current configuration $\Omega(t)$, by $\mathbf{F^p}$ the tensor gradient which makes pass from the configuration Ω_0 with the slackened configuration Ω^r , and $\mathbf{F^e}$ configuration Ω^r with $\Omega(t)$. The index p refers to the plastic part, the index e with the thermoelastic part.

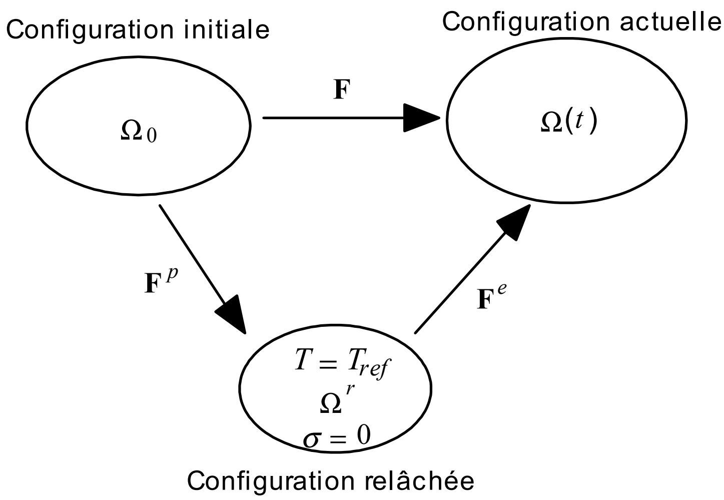


Figure 3.2.2-a: Decomposition of the tensor gradient $\mathbf{F}$ in an elastic part $\mathbf{F^e}$ and plastic $\mathbf{F^p}$

By composition of the movements, one obtains the following multiplicative decomposition:

$$\mathbf{F} = \mathbf{F^e}\,\mathbf{F^p}$$

The thermoelastic deformations are measured in the current configuration with the left tensor eulérien of Cauchy-Green $\mathbf{b^e}$ and plastic deformations in the initial configuration by the tensor $\mathbf{G^p}$ (Lagrangian description). These two tensors are defined by:

$$\mathbf{b^e} = \mathbf{F^e}\,\mathbf{F}^{eT}\ ,\ \mathbf{G^p} = (\mathbf{F}^{pT}\,\mathbf{F^p})^{-1} \quad \text{from where} \quad \mathbf{b^e} = \mathbf{F}\mathbf{G}^p\,\mathbf{F^T}$$

The model presented is written in such manner to distinguish the isochoric terms from the terms of change from volume. One introduces for that the two following tensors:

$$\bar{\mathbf{F}} = J^{-1/3}\,\mathbf{F} \quad \text{and} \quad \bar{\mathbf{b}}^e = J^{-2/3}\,\mathbf{b^e} \quad \text{with} \quad J = \det \mathbf{F}$$

By definition, one a: $\det \bar{\mathbf{F}}=1$ and $\det \bar{\mathbf{b}}^e=1$.

In this model, the plastic deformations are done with constant volume so that:

$$J^p=\det \mathbf{F^p}=1 \text{ from where } J=J^e=\det \mathbf{F^e}$$

One will find in the reference document ([R5.03.21]) the expressions of the relations of behavior.

4 Extension of the model great deformations

The objective of this paragraph is to justify the extension of the model written in great deformations to take account of the metallurgical transformations. In particular, to take account of the plasticity of transformation, we cannot add as in small deformations an additional term with deformation related to plasticity with transformation. In fact, on the kinematic aspect decomposition, the taking into account of the plasticity of transformation does not change anything. There is always the decomposition $\mathbf{F}=\mathbf{F^e}\mathbf{F^p}$ where $\mathbf{F^p}$ all information contains on the “unelastic” deformation (thus including that related to the plasticity of transformation). It is only on the level behavior that is done, in particular, the treatment of the plasticity of transformation.

Initially, we point out some theoretical elements which make it possible to write the model without metallurgical effects then we show the modifications to be made to take account of the metallurgical effects and the plasticity of transformation in particular.

4.1 Thermodynamic aspect

The writing of the law of behavior great deformations is from the thermodynamic framework with internal variables. The thermodynamic formalism rests on two assumptions. First is that the free energy depends only on the elastic strain $\mathbf{b^e}$ and of the internal variables related to the work hardening of the material (here cumulated plastic deformation associated with the isotropic variable of work hardening R). This allows, thanks to the inequality of Clausius-Duhem, to obtain the laws of state. The second assumption is the principle of maximum dissipation, which corresponds to the data of a potential of dissipation, which then makes it possible to determine the laws of evolution of the internal variables.

The free energy is given by:

$$\psi=\psi(\mathbf{b^e}, p)=\psi^e(\mathbf{b^e})+\psi^p(p)$$

One obtains by the first assumption, the laws of state, that is to say:

$$\boldsymbol{\tau}=2\rho_0 \frac{\partial \psi^e}{\partial \mathbf{b^e}}\mathbf{b^e} \text{ and } R=\rho_0 \frac{\partial \psi^p}{\partial p}$$

It remains for dissipation:

$$\boldsymbol{\tau}:(-\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^p\mathbf{F}^\mathbf{T}\mathbf{b^{e-1}})-R\,\dot{p}\geq 0$$

With the help of the introduction of a function threshold such as $f(\boldsymbol{\tau}, R) \leq 0$, the principle of maximum dissipation (or an equivalent way the data of a pseudopotential of dissipation [bib3]) makes it possible to deduce some, by the property of normality, the laws of evolution, that is to say:

$$-\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^{p}\mathbf{F}^{\mathbf{T}}\mathbf{b}^{\mathbf{e-1}}=\dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\tau}} \text{ and } \dot{p}=-\dot{\lambda}\frac{\partial f}{\partial R}$$

It is here about a model of associated plasticity.

4.2 Extension

For the restoration of work hardening, there are no particular difficulties dependent on the great deformations. It is enough that the free energy depends, either to the cumulated plastic deformation, but to the internal variables of work hardening r_k associated with the variables of work hardenings $Z_k . R_k$ of each metallurgical phase.

To keep maintaining account of the deformations due to the plasticity of transformation, one proposes to add an additional term in the law with flow of the plastic deformation $\mathbf{G^p}$ who derives from a potential of dissipation Ω .

One obtains thus for the laws of state:

$$\boldsymbol{\tau}=2\rho_0\frac{\partial \psi^e}{\partial \mathbf{b^e}}\mathbf{b^e} \text{ and } Z_k . R_k=\rho_0\frac{\partial \psi^p}{\partial r_k}$$

and for the laws of evolution:

$$-\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^{p}\mathbf{F}^{\mathbf{T}}\mathbf{b}^{\mathbf{e-1}}=\dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\tau}}+\underbrace{\frac{\partial \Omega^{pt}}{\partial \boldsymbol{\tau}}}_{\text{plasticité de transformation}}$$

$$\dot{r}_k=-\dot{\lambda}\frac{\partial f}{\partial (Z_k . R_k)}-\underbrace{\frac{\partial \Omega^{r}}{\partial (Z_k . R_k)}}_{\substack{\text{restauration d'écrouissage}\\ \text{métallurgique et visqueux}}}$$

$$\Omega=\Omega^{pt}(t)+\Omega^r$$

The potentials are chosen Ω^{pt} and Ω^r , respectively related on the plasticity of transformation and the restoration of work hardening, in such manner to find, under the assumption of the small deformations, the same laws of evolution as those of the model with metallurgical effects writes in small deformations.

4.3 Relations of behavior

A linear isotropic work hardening in the case of is placed.

The partition of the deformations implies:

$$\bar{\mathbf{b}}^e=\bar{\mathbf{F}}\mathbf{G^p}\bar{\mathbf{F}}^T \text{ with } \bar{\mathbf{F}}=J^{-1/3}\mathbf{F} \text{ , } J=\det\mathbf{F} \text{ and } \bar{\mathbf{b}}^e=J^{-2/3}\mathbf{b^e}$$

The relations of behavior are given by:

- Thermoelastic relation stress-strain:

$$\tilde{\boldsymbol{\tau}}=\mu\,\tilde{\bar{\mathbf{b}}}^e$$

$$\mathrm{tr}\,\tau=\frac{3K}{2}(J^2-1)-\frac{9K}{2}\varepsilon^{th}(J+\frac{1}{J})$$

$$\epsilon^{th}=Z_\gamma\left[\alpha_\gamma(T-T_{ref})-(1-Z_\gamma^r)\Delta\varepsilon_{f\gamma}^{T\,ref}\right]+(\sum_{i=1}^{4}Z_i)\left[\alpha_f(T-T_{ref})+Z_\gamma^r\Delta\varepsilon_{f\gamma}^{T\,ref}\right]$$

where: Z_γ^r characterize the metallurgical phase of reference

$Z_\gamma^r=1$ when the phase of reference is the austenitic phase,

$Z_\gamma^r=0$ when the phase of reference is the ferritic phase.

$\Delta\varepsilon_{f\gamma}^{T_{ref}}=\varepsilon_f^{th}(T_{ref})-\varepsilon_\gamma^{th}(T_{ref})$ translated the difference in compactness between the phases ferritic and austenitic at the temperature of reference T_{ref},

α_f is the dilation coefficient of the four ferritic phases and α_γ that of the austenitic phase.

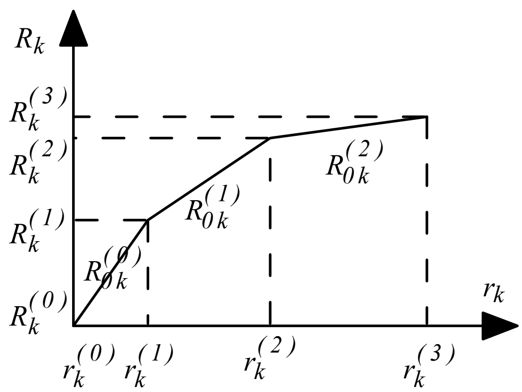
- Threshold of plasticity:

$$f=\tau_{eq}-R-\sigma_y$$

R is the variable of work hardening of the multiphase material, which is written:

$$R=(1-\bar{f}(Z))R_\gamma+\frac{\bar{f}(Z)}{Z}\sum_{i=1}^{4}Z_i.R_i\ ,\ Z=\sum_{i=1}^{4}Z_i$$

where R_k is the variable of work hardening of the phase k who can be linear or not linear compared to r_k and $\bar{f}(Z)$ a function depending on Z such as $\bar{f}(Z)\in[0,1]$.
In the linear case, one has $R_k=R_{0k}\,r_k$ where R_{0k} is the slope of work hardening of the phase k .
In the nonlinear case, one writes: $R_k=R_k^{(i)}+R_{0k}^{(i)}(r-r_k^{(i)})$ where significances of $R_k^{(i)}$, $R_{0k}^{(i)}$ and $r_k^{(i)}$ are represented on the figure below.



Nonlinear curve of work hardening

Elastic limit σ_y is worth:

If $Z \neq 0$, $\sigma_y=(1-\bar{f}(Z))\sigma_{y\gamma}+\bar{f}(Z)\sigma_{y\alpha}$, $\sigma_{y\alpha}=\frac{\sum_{i=1}^{4} Z_i s_{y\alpha i}}{Z}$

If $Z=0$, $\sigma_y=\sigma_{y\gamma}$

where $\sigma_{y\alpha i}$ are the four elastic limits of the ferritic phases, $\sigma_{y\gamma}$ that of the austenitic phase.

- Laws of evolution:

$$\bar{\mathbf{F}}\dot{\mathbf{G}}^p\bar{\mathbf{F}}^T=-\dot{p}\frac{3}{\tau_{eq}}\tilde{\boldsymbol{\tau}}\,\bar{\mathbf{b}}^e-3\tilde{\boldsymbol{\tau}}\,\bar{\mathbf{b}}^e\sum_{i=1}^{4}K_i F_i'(1-Z_\gamma)\langle\dot{Z}_i\rangle$$

$$\dot{r}_\gamma=\dot{p}+\frac{\sum_{i=1}^{4}\langle-\dot{Z}_i\rangle(\theta_{i\gamma}r_i-r_\gamma)}{Z_\gamma}\underbrace{-(Cr_{moy})^m}_{\text{uniquement en viscosité}}\qquad \text{si}\quad Z_\gamma>0$$

$$\dot{r}_i=\dot{p}+\frac{\langle\dot{Z}_i\rangle(\theta_{\gamma i}r_\gamma-r_i)}{Z_i}\underbrace{-(Cr_{moy})^m}_{\text{uniquement en viscosité}}\qquad \text{si}\quad Z_i>0$$

$$r_{\text{moy}}=\sum_{k=1}^{5}Z_k r_k\ ,\ C=\sum_{k=1}^{5}Z_k C_k\ ,\ m=\sum_{k=1}^{5}Z_k m_k$$

where K_i , F_i' , C_i and m_i are data of material associated with the phase i , $\theta_{\gamma i}$ the coefficient of restoration of work hardening at the time of the transformation γ in i ($\theta_{\gamma i}\in[0,1]$) and $\theta_{i\gamma}$ the coefficient of restoration of work hardening at the time of the transformation i in γ ($\theta_{i\gamma}\in[0,1]$).

All the data material are indicated in the operator `DEFI_MATERIAU` ([U4.43.01]) under different the keyword factors `ELAS_META (_F0)` and `META_ **`.

For a model of plasticity, the plastic multiplier is obtained by writing the condition of coherence $\dot{f}=0$ and one a:

$$\dot{p}\geq 0, f\leq 0 \ \text{ et } \ \dot{p}\,f=0$$

In the viscous case, $\dot{p}$ is written:

$$\dot{p}=\left(\frac{\langle f\rangle}{h}\right)^n$$

or in an equivalent way:

$$\langle f\rangle=(1-\bar{f}(Z))\eta_\gamma\,\dot{p}^{1/n_\gamma}+\frac{\bar{f}}{Z}\sum_{i=1}^{4}Z_i\eta_i\,\dot{p}^{1/n_i}$$

where n_i and η_i are the viscosity coefficients of material associated with the phase i who possibly depend on the temperature.

The calculation of $\bar{\mathbf{F}}\dot{\mathbf{G}}^p\bar{\mathbf{F}}^T$ give:

$$\bar{\mathbf{F}}\dot{\mathbf{G}}^p\bar{\mathbf{F}}^T=-3(A\tau_{eq}+\dot{p})(\frac{1}{3}\mathrm{tr}\,\bar{\mathbf{b}}^e\frac{\tilde{\boldsymbol{\tau}}}{\tau_{eq}}+\frac{\tau_{eq}}{\mu}\frac{\tilde{\boldsymbol{\tau}}\tilde{\boldsymbol{\tau}}}{t_{eq}^2})$$

where one posed $A=\sum_{i=1}^{4} K_i F_i^{'}\langle \dot{Z}_i \rangle$.

Since $\|\tilde{\boldsymbol{\tau}}/\tau_{eq}\|\le 1$ and $\|\tilde{\boldsymbol{\tau}}\,\tilde{\boldsymbol{\tau}}/\tau_{eq}^2\|\le 1$, the second term of the expression above can be neglected (in front of 1) for metallic materials insofar as:

$$\frac{\tau_{eq}}{\mu}=\frac{R+\sigma_y}{\mu}\approx 10^{-3}\ll 1\le \mathrm{tr}\,\bar{\boldsymbol{b}}^e$$

$\mathrm{tr}\,\bar{\mathbf{b}}^e\ge 1$ because the tensor $\bar{\mathbf{b}}^e$ is symmetrical, definite positive and $\det \bar{\mathbf{b}}^e=1$.

It is this simplification of the law of evolution of $\mathbf{G}^\mathbf{p}$ who allows to integrate the law of behavior easily i.e. to bring back it to the solution of a nonlinear scalar equation. One will thus take thereafter:

$$\bar{\mathbf{F}}\dot{\mathbf{G}}^p\bar{\mathbf{F}}^T\approx-\left(\dot{p}+\tau_{eq}A\right)\frac{\mathrm{tr}\,\bar{\boldsymbol{b}}^e}{\tau_{eq}}\tilde{\tau} \qquad \text{éq 4.3-1}$$

4.4 Various relations

In the operator STAT_NON_LINE, one reaches these various models by using the keyword following factors:

```
| BEHAVIOR:   (
                  RELATION:         / 'META_P_IL'
                                    / 'META_P_INL'
                                    / 'META_P_IL_PT'
                                    / 'META_P_INL_PT'
                                    / 'META_P_IL_RE'
                                    / 'META_P_INL_RE'
                                    / 'META_P_IL_PT_RE'
                                    / 'META_P_INL_PT_RE'
                                    / 'META_V_IL'
                                    / 'META_V_INL'
                                    / 'META_V_IL_PT'
                                    / 'META_V_INL_PT'
                                    / 'META_V_IL_RE'
                                    / 'META_V_INL_RE'
                                    / 'META_V_IL_PT_RE'
                                    / 'META_V_INL_PT_RE'

                  DEFORMATION: /'SIMO_MIEHE'

              )
```

We point out only here the significance of the letters for the behaviors META :

- P_IL : plasticity with linear isotropic work hardening,
- P_INL : plasticity with nonlinear isotropic work hardening,
- V_IL : viscoplasticity with linear isotropic work hardening,
- V_INL : viscoplasticity with nonlinear isotropic work hardening,
- Pt : plasticity of transformation,
- RE : restoration of metallurgical work hardening of origin.

Example: 'META_V_INL_RE' = elastoviscoplastic law with nonlinear isotropic work hardening with restoration of work hardening but without taking into account of the plasticity of transformation

The various characteristics of material are given in the operator DEFI_MATERIAU. One returns the reader to the note [R4.04.02] for the significance of the keyword factors of this operator.

Caution:

If isotropic work hardening is linear, one informs under the keyword META_ECRO_LINE of DEFI_MATERIAU, the module of work hardening i.e. the slope in the stress-strain plan.
On the other hand, if isotropic work hardening is nonlinear, one gives directly under the keyword META_TRACTION of DEFI_MATERIAU, curved isotropic work hardening R *(* $R=\tau-\sigma_y$ *) according to the cumulated plastic deformation* p *(* $p=\varepsilon-\frac{\tau}{E}$ *).*

Notice :

The user must make sure well that the "experimental" traction diagram used to deduce the slope from it from work hardening is well given in the plan forced rational $\sigma=F/S$ *- deformation logarithmic curve* $\ln(1+\Delta l/l_0)$ *where* l_0 *is the initial length of the useful part of the test-tube,* Δl *variation length after deformation,* F *the force applied and* S *current surface. It will be noticed that* $\sigma=F/S=\frac{F}{S_0}\frac{l}{l_0}\frac{1}{J}$ *from where* $\tau=J\sigma=\frac{F}{S_0}\frac{l}{l_0}$ *. In general, it is well the quantity* $\frac{F}{S_0}\frac{l}{l_0}$ *who is measured by the experimenters and this gives the constraint of Kirchhoff directly used in the model of Simo and Miehe.*

4.5 Internal constraints and variables

The constraints of exit are the constraints of Cauchy σ, therefore measured on the current configuration.
For the whole of the relations META_ **, internal variables produced in *Code_Aster* are:

- V1 : r_1 variable of work hardening for ferrite,
- V2 : r_2 variable of work hardening for the pearlite,
- V3 : r_3 variable of work hardening for bainite,
- V4 : r_4 variable of work hardening for martensite,
- V5 : r_5 variable of work hardening for austenite,
- V6 : indicator of plasticity (0 if the last calculated increment is elastic; 1 if not),
- V7 : R the isotropic term of work hardening of the function threshold,
- V8 : the trace divided by three of the tensor of elastic strain $\bar{b}^e$ that is to say $\frac{1}{3}tr\,\bar{\boldsymbol{b}}^e$.

5 Digital formulation

For the variational formulation, it is same as that given in the note [R5.03.21] and which refers to the law of behavior great deformations. We recall only that it is about a eulérienne formulation, with reactualization of the geometry to each increment and each iteration, and that one takes account of the rigidity of behavior and geometrical rigidity.
We now present the digital integration of the law of behavior and give the form of the tangent matrix (options `FULL_MECA` and `RIGI_MECA_TANG`).

5.1 Integration of the various relations of behavior

In the case of an incremental behavior, keyword factor `BEHAVIOR`, knowing the tensor of the constraints $\boldsymbol{\sigma}^-$, internal variables r_k^-, the trace divided by three of the tensor of elastic strain $\frac{1}{3} tr\ \bar{\boldsymbol{b}}^e$, displacements $\mathbf{u}^-$ and $\Delta\mathbf{u}$, temperatures T^- and T, and proportions of the various metallurgical phases $Z_k,\ \Delta Z_k$, one seeks to determine $(\sigma, r_k, \frac{1}{3} tr\ \bar{\boldsymbol{b}}^e)$.

Displacements being known, gradients of the transformation of Ω_0 with Ω^-, noted $\mathbf{F}^-$, and of Ω^- with $\Omega(t)$, noted $\Delta\mathbf{F}$, are known.
One will pose thereafter:

$$DA=\sum_{i=1}^{4} K_i F'_i \langle DZ_i\rangle\ ,\quad \Delta G_\gamma=\frac{\sum_{i=1}^{4}\langle -\Delta Z_i\rangle(\theta_{i\gamma} r_i^- - r_\gamma^-)}{Z_\gamma}\quad \text{and}$$

$$\Delta G_i=\frac{\langle\Delta Z_i\rangle(\theta_{\gamma i} r_\gamma^- - r_i^-)}{Z_i}\quad (i=1,4)$$

Discretization **implicit** law gives:

$$\mathbf{F}=\Delta\mathbf{F}\ \mathbf{F}^-$$
$$J=\det\mathbf{F}$$
$$\bar{\mathbf{F}}=J^{-1/3}\mathbf{F}$$
$$\bar{\mathbf{b}}^e=J^{-2/3}\mathbf{b}^e$$
$$J\boldsymbol{\sigma}=\boldsymbol{\tau}$$
$$\tilde{\boldsymbol{\tau}}=\mu\,\tilde{\bar{\mathbf{b}}}^e$$
$$\mathrm{tr}\,\boldsymbol{\tau}=\frac{3K}{2}(J^2-1)-\frac{9K}{2}\varepsilon^{th}(J+\frac{1}{J})$$
$$\varepsilon^{th}=Z_\gamma\left[\alpha_\gamma(T-T_{ref})-(1-Z_\gamma^r)\Delta\varepsilon_{f\gamma}^{T\,ref}\right]+(\sum_{i=1}^{4} Z_i)\left[\alpha_f(T-T_{ref})+Z_\gamma^r\Delta\varepsilon_{f\gamma}^{T\,ref}\right]$$
$$f=\tau_{eq}-(1-\bar{f})R_\gamma-\frac{\bar{f}}{Z}\sum_{i=1}^{4} Z_i R_i-\sigma_y$$
$$\bar{\mathbf{b}}^e=\bar{\mathbf{F}}\mathbf{G}^p\bar{\mathbf{F}}^T=\bar{\mathbf{F}}\mathbf{G}^{p-}\bar{\mathbf{F}}^T-\frac{\Delta p\,\mathrm{tr}\ \bar{\mathbf{b}}^e}{\tau_{eq}}\tilde{\boldsymbol{\tau}}-\Delta A\,\mathrm{tr}\ \bar{\boldsymbol{b}}^e\tilde{\boldsymbol{\tau}}$$

If $Z_\gamma>0$ then $\Delta r_\gamma=\Delta p+\Delta G_\gamma-\underbrace{\Delta t(Cr_{moy}^-)^m}_{\text{uniquement en viscosité}}$, if not $r_\gamma^-=0$ and $\Delta r_\gamma=0$

If $Z_i>0$, $\Delta r_i=\Delta p+\Delta G_i-\underbrace{\Delta t(Cr_{moy}^-)^m}_{\text{uniquement en viscosité}}$, if not $r_i^-=0$ and $\Delta r_i=0$

In the resolution of this system, only the deviatoric constraint $\tilde{\boldsymbol{\tau}}$ because the trace is unknown of $\boldsymbol{\tau}$ is function only of J (known).

One introduces $\boldsymbol{\tau}^{\boldsymbol{Tr}}$, the tensor of Kirchhoff which results from an elastic prediction (Tr: trial, in English test):

$$\tilde{\boldsymbol{\tau}}^{Tr}=\mu\,\tilde{\bar{\mathbf{b}}}^{eTr}$$

where

$$\bar{\mathbf{b}}^{eTr}=\bar{\mathbf{F}}\,\mathbf{G}^{\mathbf{p}-}\,\bar{\mathbf{F}}^{T}=\Delta\,\bar{\mathbf{F}}\,\bar{\mathbf{b}}^{e-}\,\Delta\,\bar{\mathbf{F}}^{T}\ ,\ \Delta\,\bar{\mathbf{F}}=(\Delta\,J)^{-1/3}\,\Delta\,\mathbf{F}\ \text{ and }\ \Delta\,J=\det(\Delta\,\mathbf{F})$$

One obtains $\bar{\mathbf{b}}^{e-}$ starting from the constraints $\boldsymbol{\tau}^{-}$ by the thermoelastic relation stress-strain and starting from the trace of the tensor of the elastic strain.

$$\bar{\mathbf{b}}^{e-}=\frac{\tilde{\boldsymbol{\tau}}^{-}}{\mu^{-}}+\frac{1}{3}\,tr\ \boldsymbol{b}^{\bar{e}-}$$

One obtains for the tensor of Kirchhoff $\tilde{\boldsymbol{\tau}}$:

$$\tilde{\boldsymbol{\tau}}=\mu\,\tilde{\bar{\mathbf{b}}}^{eTr}-\mu\,\Delta\,p\,\frac{\mathrm{tr}\ \mathbf{b}^{\bar{e}Tr}}{\tau_{eq}}\,\tilde{\boldsymbol{\tau}}-\mu\,\Delta\ \mathrm{tr}\ \boldsymbol{b}^{\bar{e}\boldsymbol{Tr}}\,\tilde{\boldsymbol{\tau}}$$

If $f<0$, one has then $\Delta\,p=0$ and:

$$\tilde{\boldsymbol{\tau}}=\frac{\tilde{\boldsymbol{\tau}}^{Tr}}{1+\mu\,\Delta\,\mathrm{A}\ \mathrm{tr}\ \boldsymbol{b}^{\bar{e}\boldsymbol{Tr}}}$$

if not one obtains:

$$\mathrm{tr}\ \bar{\boldsymbol{b}}^{\boldsymbol{e}}=\mathrm{tr}\ \boldsymbol{b}^{\bar{e}\boldsymbol{Tr}}$$

$$1+\mu\,\Delta\,p\,\frac{\mathrm{tr}\ \bar{\boldsymbol{b}}^{eTr}}{\tau_{eq}}$$

¿

$$\tilde{\boldsymbol{\tau}}\left[+\mu\,\Delta\ \mathrm{tr}\ \bar{\boldsymbol{b}}¿¿eTr\right]=\tilde{\tau}^{Tr}$$

By calculating the equivalent constraint, one obtains the scalar equation in $\Delta\,p$ following:

$$\tau_{eq}+\mu\,\Delta\,p\,\mathrm{tr}\ \boldsymbol{b}^{\bar{e}\boldsymbol{Tr}}+\mu\,\Delta\,A\,\tau_{eq}\,\mathrm{tr}\ \boldsymbol{b}^{\bar{e}\boldsymbol{Tr}}=\tau_{eq}^{Tr}$$

Expression of τ_{eq} :

In plasticity : $\tau_{eq}=\sigma_y+R' Dp+D(r^- ;T,Z)$
with

$$R'=(1-\bar{f})R_{0\gamma}+\frac{\bar{f}}{Z}\sum_{i=1}^{4} Z_i R_{0i}$$

and $D(r^-;\mathrm{T,Z})=[1-\bar{f}]R_\gamma(r_\gamma^-+\Delta G_\gamma)+\frac{\bar{f}}{Z}\sum_{i=1}^{4} Z_i R_i(r_i^-+\Delta G_i)$

In viscosity :

$$\tau_{eq}=\sigma_y+R' \Delta p+D(r^- ;T,Z)+(1-\bar{f}(Z))\eta_\gamma(\Delta p/\Delta t)^{1/n_\gamma}+\frac{\bar{f}}{Z}\sum_{i=1}^{4} Z_i\eta_i(\Delta p/\Delta t)^{1/n_i}$$

with

$$D(r^-;\mathrm{T,Z})=[1-\bar{f}]R_\gamma(r_\gamma^-+DG_\gamma-\Delta t(\mathrm{Cr}_{\mathrm{moy}}^-)^m) +\frac{\bar{f}}{Z}\sum_{i=1}^{4} Z_i R_i(r_i^-+\Delta G_i-\Delta t(\mathrm{Cr}_{\mathrm{moy}}^-)^m)$$

Δp check:

$$(1-\bar{f}(Z))\eta_\gamma(\Delta p/\Delta t)^{1/n_\gamma}+\frac{\bar{f}}{Z}\sum_{i=1}^{4} Z_i\eta_i(\Delta p/\Delta t)^{1/n_i}=\frac{\tau_{eq}^{Tr}-\mu\Delta p\,\mathrm{tr}\;\bar{\boldsymbol{b}}^{eTr}}{1+\mu\Delta A'\mathrm{tr}\;\bar{\boldsymbol{b}}^{eTr}}-D(r^- ;T,Z)-\sigma_y-R'\Delta p$$

The resolution is made in *Code_Aster* by a method of the secants with interval of research [bib4].

Note:

In the case of a nonlinear isotropic work hardening, slopes of work hardening R_{0k} *and work hardenings* R_k *in the expressions of* R' *and* $D(r^- ;T,Z)$ *correspond to the variables* r_k *catches at the moment* t *, i.e.* $r_k=r_k^-+\Delta G_k+\Delta p-\Delta t(Cr_{moy}^-)^m$ *. However, as one does not know a priori the value of these variables* r_k *, one solves the equation in* Δp *by taking the slopes* R_{0k} *and work hardenings* R_k *for the quantities* $r_k^-+\Delta G_k-\Delta t(Cr_{moy}^-)^m$ *. Once solved the equation in* Δp *, one checks, for each phase, which one is well in the good interval during the calculation of work hardening and the slope. In the contrary case, for the phases concerned, one takes the following interval and one solves the equation again in* Dp *. One continues this process until finding the good interval for all the phases.*

One finds then for the diverter of the constraints:

$$\tilde{\boldsymbol{\tau}}=\frac{1}{1+\mu\Delta A\,\mathrm{tr}\;\bar{\mathbf{b}}^{eTr}}\left[1-\mu\frac{\Delta p\,\mathrm{tr}\;\bar{\mathbf{b}}^{eTr}}{\tau_{eq}^{Tr}}\right]\tilde{\boldsymbol{\tau}}^{Tr}$$

Once calculated the cumulated plastic deformation, the tensor of the constraints and the tangent matrix, one carries out a correction on the trail of tensor of the elastic strain $\bar{\mathbf{b}}^e$ to take account of the plastic incompressibility, which is not preserved with the simplification made on the law of flow [éq 4.3.1]. This correction is carried out by using a relation between the invariants of $\bar{\mathbf{b}}^e$ and $\tilde{\bar{\mathbf{b}}}^e$ and by exploiting the plastic condition of incompressibility $J^p=1$ (or in an equivalent way $\det\ \bar{\mathbf{b}}^e=1$). This relation is written:

$$x^3-\bar{J}_2^e x-(1-\bar{J}_3^e)=0$$

with $\bar{J}_2^e=\frac{1}{2}(\tilde{\bar{b}}^e)_{\text{eq}}^2=\frac{(\tau_{\text{eq}})^2}{2(\mu)^2}$, $\bar{J}_3^e=\det\ \tilde{\bar{\boldsymbol{b}}}^e=\det\frac{\tilde{\boldsymbol{\tau}}}{\mu}$ and $x=\frac{1}{3}tr\ \bar{\boldsymbol{b}}^e$

The solution of this cubic equation makes it possible to obtain $tr\ \bar{\boldsymbol{b}}^e$ and consequently thermoelastic deformation $\bar{\mathbf{b}}^{e-}$ with the step of next time. If this equation admits several solutions, one takes the solution nearest to the solution of the step of previous time. It is besides why one stores in an internal variable $\frac{1}{3}tr\ \bar{\boldsymbol{b}}^e$.

Notice :

If the plasticity of transformation is not taken into account, the expressions obtained are the same ones while taking $\Delta A=0$.
If it is the restoration of work hardening which is neglected then one also has the same expressions but by taking all them θ equal to 1.

5.2 Form of the tangent matrix

We give only here the forms of the tangent matrix (option FULL_MECA during iterations of Newton, option RIGI_MECA_TANG for the first iteration). For the assumptions concerning the metallurgical part, they are the same ones as those of the document [R4.04.02]. For the part great deformations, one will find in appendix of [bib1], the detail of the linearization of the law of behavior.

One poses:

$$J=\det \mathbf{F}\ ,\ J^{-}=\det \mathbf{F}^{-}\ \text{ and }\ \Delta J=\det \Delta \mathbf{F}$$

- For the option FULL_MECA , one a:

$$\bar{\mathbf{A}}=\frac{\partial \boldsymbol{\sigma}}{\partial \Delta \mathbf{F}}=\frac{(\Delta J)^{-1/3}}{J}\mathbf{H}-\frac{1}{3J\Delta J}(\mathbf{H}\Delta \bar{\mathbf{F}})\otimes \mathbf{B}-\frac{J^{-}}{J^{2}}\boldsymbol{\tau}\otimes\mathbf{B}$$
$$+\frac{J^{-}}{J}\left[KJ-\frac{3}{2}K\,\varepsilon^{th}(1-J^{-2})\right]\mathbf{Id}\otimes\mathbf{B}$$

where $\mathbf{B}$ is worth:

$$B_{11}=\Delta F_{22}\Delta F_{33}-\Delta F_{23}\Delta F_{32}$$
$$B_{22}=\Delta F_{11}\Delta F_{33}-\Delta F_{13}\Delta F_{31}$$
$$B_{33}=\Delta F_{11}\Delta F_{22}-\Delta F_{12}\Delta F_{21}$$
$$B_{12}=\Delta F_{31}\Delta F_{23}-\Delta F_{33}\Delta F_{21}$$
$$B_{21}=\Delta F_{13}\Delta F_{32}-\Delta F_{33}\Delta F_{12}$$
$$B_{13}=\Delta F_{21}\Delta F_{32}-\Delta F_{22}\Delta F_{31}$$
$$B_{31}=\Delta F_{12}\Delta F_{23}-\Delta F_{22}\Delta F_{13}$$
$$B_{23}=\Delta F_{31}\Delta F_{12}-\Delta F_{11}\Delta F_{32}$$
$$B_{32}=\Delta F_{13}\Delta F_{21}-\Delta F_{11}\Delta F_{23}$$

and where $\mathbf{H}$ and $\mathbf{H}\Delta\bar{\mathbf{F}}$ are given by:

In the elastic case ($f<0$):

$$H_{ijkl}=\frac{\mu}{(1+\mu\Delta A\,\mathrm{tr}\ \overline{\boldsymbol{b}}^{e\boldsymbol{Tr}})}(\delta_{ik}\bar{b}^{e-}_{lp}\Delta\bar{F}_{jp}+\Delta\bar{F}_{ip}\bar{b}^{e-}_{pl}\delta_{jk}-\frac{2}{3}\delta_{ij}\Delta\bar{F}_{kp}\bar{b}^{e-}_{lp}-2\Delta A\tilde{\tau}_{ij}\Delta\bar{F}_{kp}\bar{b}^{e-}_{pl})$$

and

$$\mathbf{H}\Delta\bar{\mathbf{F}}=\frac{2\mu}{(1+\mu\Delta A\,\mathrm{tr}\ \overline{\boldsymbol{b}}^{e\boldsymbol{Tr}})}(\tilde{\bar{\mathbf{b}}}^{eTr}-\Delta A\,\mathrm{tr}\ \overline{\boldsymbol{b}}^{e\boldsymbol{Tr}}\tilde{\boldsymbol{\tau}})$$

if not in plastic or viscoplastic load, one a:

$$H_{ijkl}=\frac{\mu}{a}(\delta_{ik}\bar{b}^{e-}_{lp}\Delta\bar{F}_{jp}+\Delta\bar{F}_{ip}\bar{b}^{e-}_{pl}\delta_{jk})$$
$$-2\mu\left[\frac{\delta_{ij}}{3a}+\frac{\bar{R}'(\tau_{eq}\Delta A+\Delta p)\tilde{\tau}_{ij}}{\tau_{eq}(\bar{R}'+\mu\,\mathrm{tr}\,\bar{\boldsymbol{b}}^{eTr}(1+\bar{R}'\Delta A))}\right]\Delta\bar{F}_{kp}\bar{b}^{e-}_{lp}$$
$$+\frac{3\mu^2\,\mathrm{tr}\;\bar{\boldsymbol{b}}^{eTr}(\bar{R}'\Delta p-\tau_{eq})}{a\tau^3_{eq}(\bar{R}'+\mu\,\mathrm{tr}\,\bar{\boldsymbol{b}}^{eTr}(1+\bar{R}'\Delta A))}\tilde{\tau}_{ij}\tilde{\tau}_{kq}\Delta\bar{F}_{qp}\bar{b}^{e-}_{lp}$$

and

$$\mathbf{H}\Delta\bar{\mathbf{F}}=\frac{2\mu}{a}\bar{\mathbf{b}}^{eTr}-2\mu\,\mathrm{tr}\;\bar{\mathbf{b}}^{eTr}\left[\frac{\mathbf{Id}}{3a}+\frac{\bar{R}'(\tau_{eq}\Delta A+\Delta p)\tilde{\boldsymbol{\tau}}}{\tau_{eq}(\bar{R}'+\mu\,\mathrm{tr}\;\bar{\mathbf{b}}^{eTr}(1+\bar{R}'\Delta A))}\right]$$
$$+\frac{3\mu^2\,\mathrm{tr}\,\bar{\mathbf{b}}^{eTr}(\bar{R}'\Delta p-\tau_{eq})}{a\tau^3_{eq}(\bar{R}'+\mu\,\mathrm{tr}\,\bar{\boldsymbol{b}}^{eTr}(1+\bar{R}'\Delta A))}(\tilde{\boldsymbol{\tau}}:\bar{\mathbf{b}}^{eTr})\tilde{\boldsymbol{\tau}}$$

with

$$\bar{R}'=(1-\bar{f})R_{0\gamma}+\frac{\bar{f}}{Z}\sum_{i=1,4}Z_iR_{0i}+\underbrace{(1-\bar{f}(Z))\eta_\gamma(\Delta p/\Delta t)^{(1-n_\gamma)/n_\gamma}/n_\gamma\Delta t+\frac{\bar{f}}{Z}\sum_{i=1}^{4}Z_i\eta_i(\Delta p/\Delta t)^{(1-n_i)/n_i}/n_i\Delta t}_{\text{uniquement en viscosite}}$$

$$a=\frac{\tau^{Tr}_{eq}}{\tau_{eq}}$$

- For the option `RIGI_MECA_TANG`

  for the plastic model: it is the same expressions as those given for `FULL_MECA` but with $\Delta p=0$ and $\Delta A=0$, all variables and coefficients of material being taken at the moment t^- . In particular, one will have $\Delta\bar{\mathbf{F}}=\mathbf{Id}$.

  for the viscous model: one takes only the expressions of `FULL_MECA` in the elastic case with $\Delta A=0$, all variables being taken at the moment t^- .

6 Bibliography

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7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5	V.CANO, E.LORENTZ-EDF-R&D/AMA	Initial text
6.3	V.CANO EDF-R&D/AMA	Corrections