Seismic answer by transitory analysis

Summary

The methods most frequently used for the seismic analysis of the structures are the spectral methods and the transitory methods.

The transitory methods (direct linear or not, by modal synthesis) make it possible to calculate the answer of structures under the effect of imposed earthquakes: single excitation (identical of each point of anchoring of the structure) or multiple and to take into account their possible nonlinear behavior.

With regard to the spectral methods, one calculates the maximum answer, for each mode of vibration, each point of anchoring. The maximum answer of the whole of the structure, on average, with the statistical direction, is then determined by combination of the maximum answers of the modes. This kind of analysis is clarified in the reference material [R4.05.03].

Contents

1 Seismic behavior of a structure

1.1 Definitions

The analysis of the seismic behavior of a structure consists in studying its answer to an imposed movement: an acceleration, in its various supports. Imposed acceleration is a temporal signal $\gamma(t)$ called accélérogramme (cf [Figure 1.1-a]).

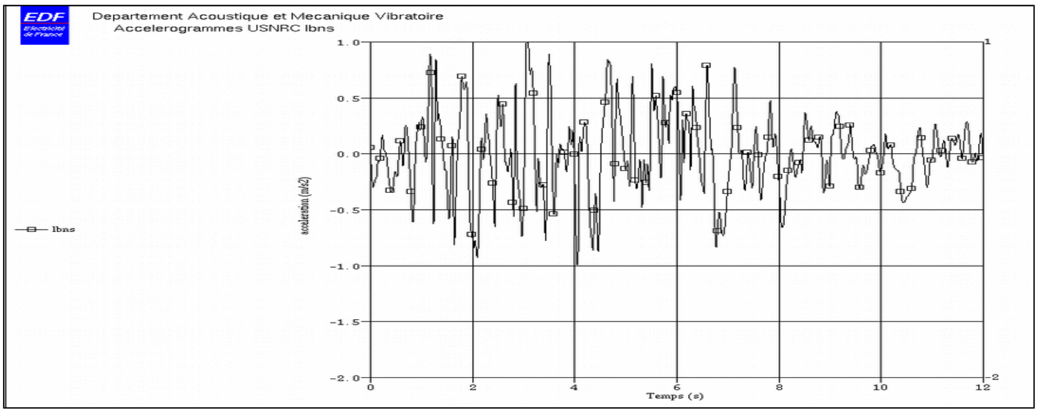

Figure 1.1-a: Accélérogramme LBNS

The seismic movement considered in calculation is a real accélérogramme known and read by the operator `LIRE_FONCTION` [U4.32.02] that is to say a synthetic accélérogramme calculated directly in the code, for example with the procedure `FORMULA` [U4.31.05].

2 Seismic response of a system to a degree of freedom

That is to say a simple oscillator made up of a mass m connected to a point fixes by a spring k and a shock absorber c being able to move in only one direction x (cf [Figure 2-a]). This oscillator with a degree of freedom is subjected to a accélérogramme $\gamma(t)$ horizontal in its support (not A).

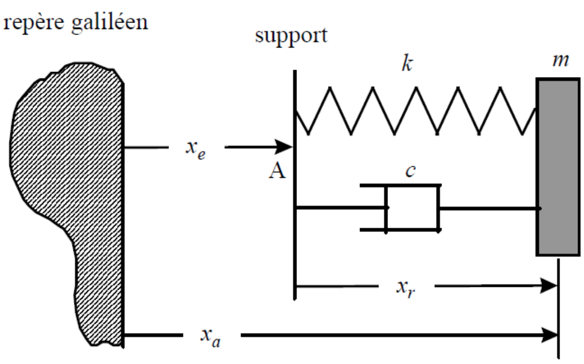

Figure 2-a : simple oscillator subjected to a seismic request.

Displacements of the oscillator are measured or calculated, that is to say in a relative reference mark related to the point A : relative displacement x_r , that is to say in an absolute reference mark (R_a) : absolute displacement x_a . Absolute displacement x_a breaks up into a uniform displacement of training in translation x_e and in a relative displacement x_r :

$$x_a(t)=x_r(t)+x_e(t)$$ **éq 2-1**

One from of deduced by derivation the relation between accelerations:

$$\ddot{x}_a(t)=\ddot{x}_r(t)+\gamma(t) \text{ with } \gamma(t)=\ddot{x}_e(t)$$ **éq 2-2**

The mass is subjected to a horizontal force of recall which is proportional to relative displacement: $F_r=-k.x_r$ and with a horizontal force of damping presumedly proportional to the relative speed: $F_v=-c.\dot{x}_r$.
The equation of the movement of the mass is written then: $-k.x_r-c.\dot{x}_r=m.\ddot{x}_a$.
Maybe, taking into account the equations [éq 2-1] and [éq 2-2]:

$$m.\ddot{x}_r+c.\dot{x}_r+k.x_r=-m.\gamma(t)=p(t)$$ **éq 2-3**

Note:

The study of the seismic response of an oscillator to a degree of freedom in the relative reference mark thus consists of the study of the response of an oscillator to a force $p(t)$ of an unspecified form. The solution of the equation of motion [éq 2-3] is then provided by the integral of Duhamel:

$$x_r(t)=\frac{1}{m.\omega_D}\int_0^t p(\tau).e^{-\xi.\omega.(t-\tau)}.\sin\left[\omega_D(t-\tau)\right].d\tau$$

for initial conditions at rest, with:

$$p(t)=-m.\gamma(t)$$
$$\omega=\sqrt{\frac{k}{m}},\ \xi=\frac{c}{2.m.\omega} \text{ et } \omega_D=\omega.\sqrt{1-\xi^2}$$

3 Seismic response of a system to several degrees of freedom

3.1 Equations of the movement in the absolute reference mark

The balance of a mechanical system consists in writing, whatever the moment of calculation t considered, that the sum of the internal forces, inertias and damping is equal to the external forces imposed on this known as system:

$$\mathbf{F}_{\text{iner}}(t)+\mathbf{F}_{\text{amo}}(t)+\mathbf{F}_{\text{int}}(t)=\mathbf{F}_{\text{ext}}(t)$$

In the case of a linear behavior, if the system is represented by a model of finite elements or discrete elements, one has (after space discretization):

$$\begin{cases} \mathbf{F}_{\text{iner}} = \mathbf{M}\ddot{\mathbf{X}}_{ab} \\ \mathbf{F}_{\text{int}} = \mathbf{K}\,\mathbf{X}_{ab} \end{cases}$$

- $\mathbf{X}_{ab}$ is the vector of nodal displacements of the discretized structure, in the absolute reference mark;
- $\mathbf{M}$ is the matrix masses structure;
- $\mathbf{K}$ is the matrix stiffness of the structure;
- $\mathbf{F}_{\text{ext}} = \mathbf{F}_e - \mathbf{F}_c$ is the vector of the forces imposed on the studied structure, $\mathbf{F}_c$ that of the possible forces of shock (cf [R5.06.03]).

One calls "support" the edges or borders of the structure on which displacements (generalized) imposed are described by functions given of time (and space); two distinct supports are thus characterized by functions describing imposed displacements distinct. To simplify the continuation of the presentation, it is considered that the structure is only requested by the displacements imposed on the level of its various supports. Thus, $\mathbf{F}_e = \mathbf{0}$.

With an aim of simplifying the presentation, one generally separates the degrees of freedom in two families, according to their type, cf [R5.05.05, § 2.6.2]:

- the degrees of freedom of structure not subjected to a movement imposed - also called "active" degrees of freedom - they are the unknown factors of the problem;
- the degrees of freedom of structure subjected to a movement imposed - also called "constrained" degrees of freedom - they are the boundary conditions in displacement of the problem (limiting conditions of Dirichlet: `DDL_IMPO`).

On the supports of the structure where displacements $\mathbf{X_s}$ are imposed, one a: $\mathbf{B}\,\mathbf{X}_{ab} = \mathbf{X}_s$. $\mathbf{B}$ is the matrix of passage of all the degrees of freedom of the structure to the degrees of freedom of structure subjected to an imposed movement.

The balance of the system is written then, whatever $\mathbf{v}$ belonging to the space of displacements kinematically acceptable, but free with the supports, while noting by $\boldsymbol{\lambda}_s$ " multipliers of LAGRANGE "of the conditions $\mathbf{B}\,\mathbf{X}_{ab} = \mathbf{X}_s$:

$$\begin{cases} \langle \mathbf{M}.\ddot{\mathbf{X}}_{ab} + \mathbf{F}_{\text{amo}} + \mathbf{K}.\mathbf{X}_{ab} + \mathbf{B}^{\mathbf{T}}.\boldsymbol{\lambda} - \mathbf{F}_{\text{ext}}, \mathbf{v} \rangle = 0 \\ \langle \mathbf{B}.\mathbf{X}_{ab} - \mathbf{X}_s, \mathbf{v} \rangle = 0 \end{cases}$$

That is to say:

$$\begin{cases} \mathbf{M}.\ddot{\mathbf{X}}_{ab} + \mathbf{F}_{\text{amo}} + \mathbf{K}.\mathbf{X}_{ab} = \mathbf{F}_{\text{ext}} - \mathbf{B}^{\mathbf{T}}.\boldsymbol{\lambda}_s \\ \mathbf{B}.\mathbf{X}_{ab} = \mathbf{X}_s \end{cases} \quad \textbf{éq 3.1-1}$$

$\mathbf{F}_s = -\mathbf{B}^{\mathbf{T}}.\boldsymbol{\lambda}_s$ is the vector of the forces of reactions exerted by the supports on the structure. It is admitted that the vector of the forces of damping checks $\mathbf{B}^{\mathbf{T}}.\mathbf{F}_{\text{amo}} = \mathbf{0}$.

By taking account of the partition of the degrees of freedom in "active" degrees of freedom and in "constrained" degrees of freedom, the vector of displacements in the absolute reference mark is written: $\mathbf{X}_{ab} = \begin{Bmatrix} \mathbf{x}_a \\ \mathbf{x}_s \end{Bmatrix}$.

The operators describing the structure become: $\mathbf{M}=\begin{bmatrix}\mathbf{m} & \mathbf{m}_{xs}\\ \mathbf{m}_{sx} & \mathbf{m}_{ss}\end{bmatrix}$, $\mathbf{K}=\begin{bmatrix}\mathbf{k} & \mathbf{k}_{xs}\\ \mathbf{k}_{sx} & \mathbf{k}_{ss}\end{bmatrix}$ with $\mathbf{m}_{sx}=\mathbf{m}_{xs}^{\mathbf{T}}$ and $\mathbf{k}_{sx}=\mathbf{k}_{xs}^{\mathbf{T}}$ and the vector of the external forces (here of shock only) applied to the structure is written: $\mathbf{F}_{\text{ext}}=\begin{Bmatrix}-\mathbf{f}_c\\ 0\end{Bmatrix}$.

The fundamental equation of dynamics in the absolute reference frame is written then, by taking account of the partition of the degrees of freedom:

$$\begin{bmatrix}\mathbf{m} & \mathbf{m}_{xs}\\ \mathbf{m}_{sx} & \mathbf{m}_{ss}\end{bmatrix}.\begin{Bmatrix}\ddot{\mathbf{x}}_a\\ \ddot{\mathbf{x}}_s\end{Bmatrix}+\mathbf{F}_{\text{amo}}+\begin{bmatrix}\mathbf{k} & \mathbf{k}_{xs}\\ \mathbf{k}_{sx} & \mathbf{k}_{ss}\end{bmatrix}.\begin{Bmatrix}\mathbf{x}_a\\ \mathbf{x}_s\end{Bmatrix}=\begin{Bmatrix}-\mathbf{f}_c\\ \mathbf{f}_s\end{Bmatrix}$$

Maybe, by considering only the active degrees of freedom:

$$\mathbf{m}\,\ddot{\mathbf{x}}_a+\mathbf{F}_{\text{amo}}+\mathbf{k}\,\mathbf{x}_a=-\mathbf{f}_c-\mathbf{m}_{xs}\ddot{\mathbf{x}}_s-\mathbf{k}_{xs}\mathbf{x}_s$$

This approach requires the knowledge displacements and absolute velocities associated with the accélérogramme $\gamma(t)$ of each support; however the recorders measure either of accelerations or speeds. One can go back to displacements by simple or double integration with the order `CALC_FONCTION` [U4.32.04]. However, the integration and uncertainties of measurement give drifts which it is advisable to correct: displacements are thus less well-known than speeds and accelerations. One will keep in memory the orders of magnitude of the maximum amplitudes following:

- some tenth of " g " for accelerations;
- a few tens of cm/s for speeds;
- a few tens of cm for displacements.

One will also make sure that at the end of the earthquake speed and displacement are realistic i.e. with more few tens of cm for displacement, worthless for speed. One can also correct a accélérogramme (seismic signal in acceleration) so that the signal in displacement does not have drift: mot key `CORR_ACCE` order `CALC_FONCTION` [U4.32.04].

3.2 Equations of the movement in the relative reference mark

3.2.1 Decomposition of the absolute movement

The requests undergone by a structure at the time of an earthquake are classified in two types in the rules of construction (ASME, RCC-M):

- constraints induced by the relative movement of the structure compared to its static deformation or **primary constraints**. These requests are due to the effects inertial of the earthquake;
- constraints induced by differential displacements of anchorings or **secondary constraints**.

Generally, one thus breaks up the study of the structures with linear behavior into the study of the static deformation due to the movements of the supports (it is the movement of training) and into the study of the vibrations induced by accelerations of the supports around this deformation (it is the "relative" movement).

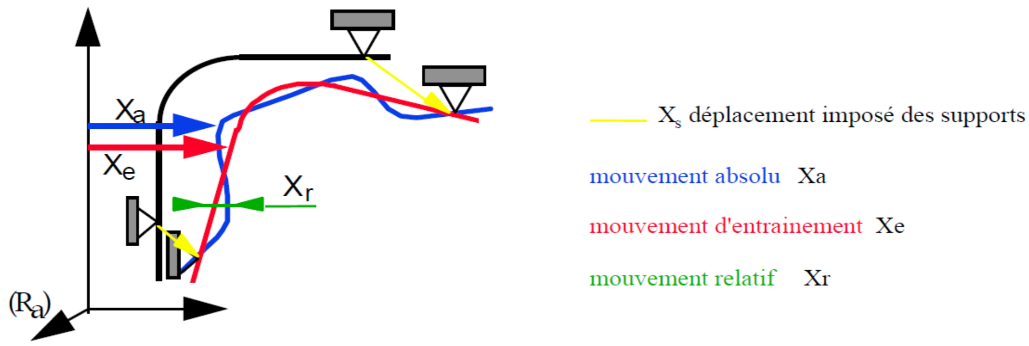

Figure 3.2.1- has : decomposition of the movement.

The absolute displacement of any point M structure, not subjected to an imposed displacement, is equal to the sum of “relative” displacement and the displacement of this point caused by the movement of training of the supports:

$$\mathbf{X}_{ab}(M,t)=\mathbf{X}_r(M,t)+\mathbf{X}_e(M,t)$$ **éq 3.2.1-1**

That is to say:

- $\mathbf{X}_{ab}$, the vector of displacements in the absolute reference frame;
- $\mathbf{X}_r$, the vector of definite relative displacements as the vector of displacements of the structure compared to the deformation which it would have under the static action of the displacements imposed on the level of the supports. $\mathbf{X}_r$ is thus null at the points supports: $\mathbf{B}.\mathbf{X}_r=0$;
- $\mathbf{X}_e$, the vector of the displacements of training defined as displacements of the structure requested statically by imposed displacements of the supports:

$$\begin{cases}\mathbf{B}.\mathbf{X}_e=\mathbf{X}_s\\ \mathbf{K}.\mathbf{X}_e=-\mathbf{B}^T.\boldsymbol{\lambda}_e\end{cases} \quad \text{with } \boldsymbol{\lambda}=\boldsymbol{\lambda}_r+\boldsymbol{\lambda}_e \quad \Leftrightarrow \mathbf{X}_e=\boldsymbol{\Psi}.\mathbf{X}_s$$

- $\boldsymbol{\Psi}$ is the matrix of the static modes. The static modes represent, in the absence of external forces, the linear static response of the structure to a unit displacement imposed on each degree of freedom of connection (others being blocked); they check:

$$\langle \mathbf{K}.\boldsymbol{\Psi}_k,\mathbf{v}\rangle=0 \text{ whatever } \mathbf{v} \text{ such as } \mathbf{B}.\mathbf{v}=0$$

In particular, $\langle \mathbf{K}.\boldsymbol{\Psi}_k,\boldsymbol{\Phi}_j\rangle=0$, whatever the oscillatory mode $\boldsymbol{\Phi}_j$ on blocked supports considered.

3.2.2 Simple or multiple excitation

To clarify more in detail the approach moving relative, and more particularly the calculation of the components of training, for example under earthquake, it is necessary to introduce concept of the simple or multiple excitation.

3.2.2.1 Simple excitation: case “mono-support”

It is considered that the movement of imposed training is a solid movement of body. It is generally said that **the structure mono-is supported**.

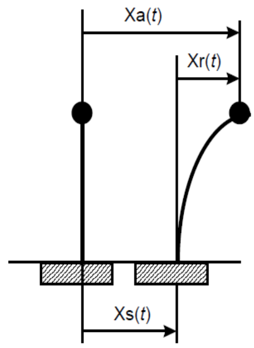

Figure 3.2.2- has : case “mono-support”.

absolute displacement of any point M structure, not subjected to an imposed displacement thus breaks up into one **relative displacement compared to a pointer related to the support** where the seismic movement and in one is imposed **rigid displacement of training**.

In this case, the static modes correspond to the six modes of rigid body. As the structure is linear rubber band, one separately studies the effects of the six components of the seismic movement. For each seismic direction, one writes simply the inertial forces of training induced by the seismic accélérogramme in the following form:

$$\mathbf{P}(t)=-\mathbf{M}.\mathbf{\Psi}.\ddot{\mathbf{X}}_s(t)=-\gamma(t).\mathbf{M}.\mathbf{\Delta}$$

- $\gamma(t)$ is the accélérogramme of the seismic movement in a direction;
- $\mathbf{\Delta}$ is the mode of body rigid in this direction and unit on the support considered;
- The seismographs measure only signals of translation. To consider that the studied structure mono-is supported amounts supposing that all its supports undergo the same translation. In this case, components of $[\Delta]$ are worth 1 for the degrees of freedom which correspond to displacements in the seismic direction considered and 0 for the degrees of freedom which correspond to displacements in seismic directions perpendicular to that considered or with rotations.
- However, considering the size of the models, the complete seismic analysis of a superstructure is generally carried out in several stages. The detailed seismic analysis of the superstructure considered then uses like excitations the accelerations calculated on its structure support. They are composed of the six accélérogrammes of translation and rotation. One thus calculates the three modes corresponding to imposed displacements of translation and the three modes corresponding to imposed displacements of rotation. If the movement of training is a rotation $\vec{\Omega}$ imposed, of axis passing in a point O, displacements of translation are in a point M : $\vec{\Delta}_M=\vec{MO}\wedge\vec{\Omega}$ for the degrees of freedom which correspond to displacements of translation and the degrees of freedom which correspond to rotations are equal to Ω on the axis considered.

3.2.2.2 Multiple excitation: case “multi-support”

One cannot always only consider:

- the accelerations of training undergone by the whole of the supports of the studied structure are identical and in phase;
- the supports indeformable and are actuated by the same movement of rigid body.

In this case, it is said that **the structure is multimedia**. Static modes $\mathbf{\Psi}=\begin{Bmatrix}\Psi\\ \mathbf{Id}\end{Bmatrix}$ correspond then to $6.\mathrm{nb}_{supports}$ static modes (case of a model beam or plate) or $3.\mathrm{nb}_{supports}$ modes (case threedimensional continuous medium) where $nb_{supports}$ is the number of accélérogrammes different undergone simultaneously by the structure. They are calculated by the operator MODE_STATIQUE [U4.52.14] with the option DDL_IMPO. They are solution of the following equation:

$$\begin{cases} \mathbf{\Psi}\,\mathbf{X}_s=\mathbf{X}_e \\ \mathbf{K}\,\mathbf{X}_e=-\mathbf{B}^T.\boldsymbol{\lambda}_e \end{cases} \quad \text{that is to say} \quad \begin{bmatrix} \mathbf{k} & \mathbf{k}_{xs} \\ \mathbf{k}_{sx} & \mathbf{k}_{ss} \end{bmatrix} . \begin{Bmatrix} \mathbf{\Psi} \\ \mathbf{Id} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_{se} \end{Bmatrix} \qquad \textbf{éq 3.2.2.2 - 1}$$

Maybe, by considering only the active degrees of freedom: $\mathbf{k}.\mathbf{\Psi}+\mathbf{k}_{xs}.\mathbf{Id}=\mathbf{0}$.

The inertial forces induced by the excitation multi-support are written then simply:

$$\mathbf{P}(t)=-\sum_{m=1}^{\text{nb\_supports}} \mathbf{M}.\mathbf{\Psi}_m.\ddot{\mathbf{X}}_{s_m}(t)$$

3.2.3 Modeling of damping

It is considered that the damping dissipated by the structure is of viscous type and that the force of damping is proportional to the only relative speed of the structure:

$$\mathbf{F}_{\text{amo}}=\mathbf{C}.\dot{\mathbf{X}}_r$$

where $\mathbf{C}$ is the matrix of damping of the structure.

That amounts neglecting the effect imposed speed. Indeed, one can more generally write:

$$\mathbf{F}_{\text{amo}}=\mathbf{C}.\dot{\mathbf{X}}_{ab}=\mathbf{C}.\dot{\mathbf{X}}_r+\mathbf{C}.\mathbf{\Psi}.\dot{\mathbf{X}}_s$$

In the case of a uniform excitation at the base (case mono-support), damping intervenes only on relative displacements (the forces of damping are indeed worthless for a rigid movement of body). In the case of a multiple excitation (case multi-support) where the static solution is not any more one movement of rigid body, to consider that the force of damping is proportional to the relative speed of the structure is a simplifying assumption.

3.2.4 Fundamental equation of dynamics

The fundamental equation of dynamics [éq 3.1-1], in **relative reference mark**, is written then, taking into account the equations [éq 3.2.1-1] and [éq 3.2.2.2 - 1], and by admitting that the force of damping is proportional to the only relative speed of the structure:

$$\mathbf{M}.\ddot{\mathbf{X}}_r+\mathbf{C}\dot{\mathbf{X}}_r+\mathbf{K}.\mathbf{X}_r=-\mathbf{M}.\mathbf{\Psi}.\ddot{\mathbf{X}}_s+\mathbf{F}_{\text{ext}}-\mathbf{B}^T.\boldsymbol{\lambda}_r \qquad \textbf{éq 3.2.4-1}$$

Maybe, by partitionnant the degrees of freedom:

$$\begin{bmatrix} \mathbf{m} & \mathbf{m}_{xs} \\ \mathbf{m}_{sx} & \mathbf{m}_{ss} \end{bmatrix} . \begin{Bmatrix} \ddot{\mathbf{x}}_r \\ \mathbf{0} \end{Bmatrix} + \begin{bmatrix} \mathbf{c} & \mathbf{c}_{xs} \\ \mathbf{c}_{sx} & \mathbf{c}_{ss} \end{bmatrix} . \begin{Bmatrix} \dot{\mathbf{x}}_r \\ \mathbf{0} \end{Bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{k}_{xs} \\ \mathbf{k}_{sx} & \mathbf{k}_{ss} \end{bmatrix} . \begin{Bmatrix} \mathbf{x}_r \\ \mathbf{0} \end{Bmatrix} = \begin{Bmatrix} -\mathbf{f}_c \\ \mathbf{f}_{s_r} \end{Bmatrix} - \begin{Bmatrix} (\mathbf{m}.\mathbf{\Psi}+\mathbf{m}_{xs}.\mathbf{Id})\ddot{\mathbf{x}}_s \\ (\mathbf{m}_{sx}.\mathbf{\Psi}+\mathbf{m}_{ss}.\mathbf{Id})\ddot{\mathbf{x}}_s \end{Bmatrix}$$

with $\mathbf{c}_{sx}=\mathbf{c}_{xs}^{\mathbf{T}}$.

Maybe, by considering only the active degrees of freedom:

$$\mathbf{m}.\ddot{\mathbf{x}}_r+\mathbf{c}.\dot{\mathbf{x}}_r+\mathbf{k}.\mathbf{x}_r=-\mathbf{f}_c-(\mathbf{m}.\mathbf{\Psi}+\mathbf{m}_{xs}.\mathbf{Id})\ddot{\mathbf{x}}_s$$

The main advantages of the approach moving “relative” compared to that in absolute displacement are the following:

- it is not necessary to integrate the accélérogrammes $\boldsymbol{\gamma}(t)$ imposed on the supports;
- relative displacements obtained make it possible to directly determine the induced primary constraints by the earthquake.

3.3 Calculation of the seismic loading

The seismic loading (cf [§3.2]) $-\mathbf{M}.\boldsymbol{\Psi}$ that is to say $-\left(\mathbf{m}.\boldsymbol{\Psi}+\mathbf{m}_{xs}.\mathbf{Id}\right)\ddot{\mathbf{x}}_s$ on the active degrees of freedom is built by the operator CALC_CHAR_SEISME [U4.63.01]. It is usable directly during a direct transitory analysis or of a transitory analysis by modal synthesis with DYNA_VIBRA [U4.53.03]. On the other hand, during a nonlinear direct transitory analysis with DYNA_NON_LINE [U4.53.01], it should be transformed into a concept of the type load. This is carried out starting from the operator AFFE_CHAR_MECA [U4.44.01] in the following way:

```
char_sei      = CALC_CHAR_SEISME (...)
load          = AFFE_CHAR_MECA (MODEL =…, VECT_ASSE = char_sei)
dyna_nlin     = DYNA_NON_LINE (
                    EXCIT= _F (   CHARGE= con_lim,)
                           _F (   CHARGE= cham_no,
                              FONC_MULT= acceler)
              …)
```

In the case of a mono-supported structure, it is enough to indicate the direction of the movement of training:

```
mono_x = CALC_CHAR_SEISME (MATR_ASSE = mass,
              DIRECTION (...), MONO_APPUI = ' OUI')
```

In the case of a multimedia structure, it is necessary as a preliminary to have calculated the unit static modes with the operator MODE_STATIQUE [U4.52.14]. One calculates as many loadings of training than of supports which undergo a different acceleration.

```
multi_xi = CALC_CHAR_SEISME (MATR_ASSE = mass, DIRECTION (...),
                  NODE = NOI, MODE_STAT = mode_stat,)
```

3.4 Loading of type incidental wave

It is also possible to impose a seismic loading by plane wave via the order AFFE_CHAR_MECA [U4.44.01] and the keyword factor ONDE_PLANE. That corresponds to the loadings usually met during calculations of dynamic interaction ground-structure by integral equations of border with the operator CALC_MISS [U7.03.12].

In harmonic, a plane wave in an isotropic springy medium is characterized by its direction, its pulsation and its type (wave P for the compression waves, waves SV or SH for the waves of shearing). In transient, the data of the pulsation corresponding to a stationary harmonic wave in time, must be replaced by the data of a profile of displacement which one will take into account the propagation in the course of time on the direction of the vector of wave $\mathbf{k}$ of propagation.

More precisely, one characterizes:

- a wave P by the function $\mathbf{u}_S(\mathbf{x},t)=f\left(\mathbf{k}.\mathbf{x}-C_p t\right)\mathbf{k}/\|\mathbf{k}\|$
- a wave S by the function $\mathbf{u}_S(\mathbf{x},t)=\mathbf{f}_S\left(\mathbf{k}.\mathbf{x}-C_s t\right)\wedge\mathbf{k}/\|\mathbf{k}\|$

with:

- $\mathbf{k}/\|\mathbf{k}\|$, unit vector of direction according to the vector of wave (one recalls that $\|\mathbf{k}\|=\omega/C$ for a harmonic wave);
- f or { $\mathbf{f}_S$ reference markthe profile of the wave feels then given.

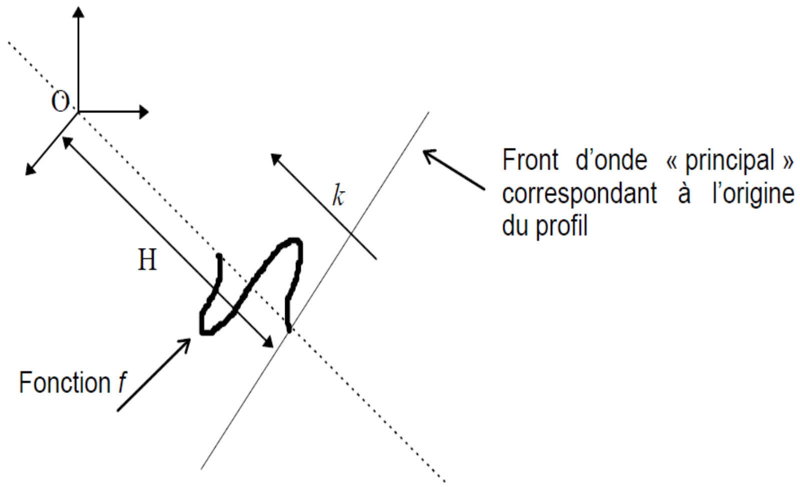

Figure 3.4- has : incidental plane wave.

H_0 is the distance from the principal wave front in the beginning O , carried by the directing vector of the wave at the initial moment of calculation, H the distance from the principal wave front in the beginning O , at one unspecified moment.

Note:

This kind of load is available in a linear direct transitory calculation `DYNA_VIBRA` or not `DYNA_NON_LINE`.
The use of this kind of loading is detailed in [R4.02.05], to also see [V2.04.120] and [V2.04.121].

4 Transitory seismic answer by modal synthesis

4.1 Description of the method

The method of modal recombination consists in breaking up the relative movement of the structure on the basis of the clean modes. As this one is null on the level of the supports, one projects the equation of dynamics on the basis of blocked clean mode (clean modes obtained by blocking all the degrees of freedom of connection).

$$\mathbf{X}_r=\mathbf{\Phi}.\mathbf{Q}$$

- $\mathbf{\Phi}$ is the matrix of the blocked clean modes;
- $\mathbf{Q}$ the vector of the unknown factors generalized of the relative movement on the basis of blocked clean mode.

The blocked clean modes are solution of:

$\begin{cases}\left(\mathbf{K}-\Omega_i^2.\mathbf{M}\right).\mathbf{\Phi}_i=-\mathbf{B}^T.\boldsymbol{\lambda}_i \\ \mathbf{B}.\mathbf{\Phi}_i=\mathbf{0}\end{cases}$ where $\boldsymbol{\lambda}_i$ are the modal reactions at the fulcrums.

The equation of the relative movement projected on the basis of blocked dynamic mode is written then (one recalls that for the blocked oscillatory modes: $\mathbf{\Phi}^T.\mathbf{K}.\mathbf{\Psi}=0$ and that one neglects the forces of damping associated at the speed of training):

$$\mathbf{M}_G\ddot{\mathbf{Q}}(t)+\mathbf{C}_G\dot{\mathbf{Q}}(t)+\mathbf{K}_G\mathbf{Q}(t)=-\mathbf{\Phi}^T.\mathbf{M}.\mathbf{\Psi}.\ddot{\mathbf{X}}_s+\mathbf{\Phi}^T.\mathbf{F}_{\text{ext}}-\mathbf{\Phi}^T.\mathbf{B}^{\mathbf{T}}.\boldsymbol{\lambda}_r$$

where $\mathbf{M}_G$, $\mathbf{C}_G$ and $\mathbf{K}_G$ are the matrices of mass, damping and stiffness generalized. To simplify, it is considered that they are diagonal. The matrix of damping generalized $\mathbf{C}_G$ as because it is supposed as the assumption of Basile is checked (the matrix of damping is a linear combination of the matrices masses and stiffness).

Maybe, by considering only the active degrees of freedom:

$$\mathbf{m}_G.\ddot{\mathbf{q}}(t)+\mathbf{c}_G.\dot{\mathbf{q}}(t)+\mathbf{k}_G.\mathbf{q}(t)=-\mathbf{\Phi}^T.\mathbf{f}_c-\mathbf{\Phi}^T.\left(\mathbf{m}.\mathbf{\Psi}+\mathbf{m}_{xs}.\mathbf{Id}\right)\ddot{\mathbf{x}}_s$$

In the absence of shock or another nonlinear force, to see § 4.3, and of exiting force given, one is thus led to solve a set of uncoupled equations (there is as much as clean modes), of which the second member is directly built using the only signals of acceleration of training which are applied to the supports and of the static modes applied to the matrix of mass.

Note:

It is possible to calculate a modal base with nondiagonal matrices. It is enough to specify it during construction to the classification generalized by the keyword `STORAGE = 'FULL'` order `NUME_DDL_GENE` [U4.65.03].

4.2 Choice of the modal base

For the seismic analysis of a linear structure, it would be necessary in theory to retain all the modes whose Eigen frequencies are lower than the cut-off frequency (generally about $33\,Hz$). In practice, one is often satisfied to preserve in the modal base only the modes which contribute to a significant degree to the answer. One then preserves only the modes whose unit effective mass in a direction is higher than 1 ‰ and one also makes sure that, for the whole of these modes selected, the unit effective mass cumulated in each direction is not very different from the total mass of the structure (higher than 90%). The criterion of office plurality of the effective modal masses is reached by connecting the following operators:

- Calculation of the total mass of the structure: `POST_ELEM` [U4.81.22]
```
masse_in = POST_ELEM (MASS_INER =_F (ALL = 'YES'))
```
- Calculation of the blocked dynamic clean modes: they are calculated in the operator `CALC_MODES` [U4.52.02].
```
mode = CALC_MODES(...);
```
- Standardisation of the modes compared to the generalized mass: `NORM_MODE` [U4.52.11]
```
NORM_MODE(MODE = mode, STANDARD = ' MASSE_GENE', MASSE_INER =
masse_in);
```
- Extraction of the modal base of the modes whose unit effective mass exceeds a certain threshold (1 ‰ for example) and checking which extracted modes represent at least 90% of the total mass: `EXTR_MODE` [U4.52.12]
```
EXTR_MODE(
    FILTRE_MODE(MODE= mode, CRIT_EXTRE= 'MASSE_EFFE_UN',
THRESHOLD =1.e-3)
```

```
IMPRESSION (OFFICE PLURALITY = ' OUI');
```

Note:

The sum of the effective modal masses is worth in fact the total mass which works on the selected modal basis. In other words, this working total mass is worth the total mass minus the contributions in mass which are carried by embedded degrees of freedom (which thus do not work on the modal basis). Thus, for example, on a system with 1 degree of freedom mass-arises with a mass $M1$ at the top and another mass $M2$ at the level to erase it, then the working mass will be worth $M1$ and total mass $M1+M2$. Consequently, modal mass effective unit for the only mode of the system will be worth $M1/(M1+M2)$. The total office plurality will thus have the same value and, according to the ratio in $M1$ and $M2$, one will not be able inevitably to thus reach 90% of the total mass $(M1+M2)$, even by considering all the modes (there is only one only mode on this example). In practice, more the model with the finite elements will be so and realistic, more the difference between the working mass and the total mass will be weak.

The macro order `CALC_MODES` [U4.52.02], option '`BAND`' with cutting in several sub-bands, allows to directly connect the whole of the three last preceding orders.

Attention, certain local answers (in the typical case of nonlocalised linearities) can be strongly influenced by modes of a higher nature whose frequency is beyond the cut-off frequency and whose effective modal mass is low (lower than 1 ‰). The keyword `VERI_CHOC` order `DYNA_TRAN_MODAL` [U4.53.21] allows to check a posteriori that the selected modal base is sufficient. If it is not the case, one highly advises to supplement it.

4.3 Taking into account of nonlinear forces in the dynamic response of the structure studied by modal synthesis

It is possible to take into account nonlinear forces, resulting from shocks, buckling of support, or antiseismic supports, cf [R5.03.17], applied into cubes particular nodes, which are evaluated using displacements and speeds in local physical reference mark reconstituted starting from the unknown factors generalized of the relative movement on the basis of blocked clean mode to which one adds displacements and speeds of training to the supports, which the user must then provide.

4.4 Calculation of the dynamic response of the structure studied by modal synthesis

After having calculated the base of the dynamic clean modes and having built a classification generalized by `NUME_DDL_GENE` [U4.65.03], one projects then the matrices of mass, damping and stiffness, on this same basis with the operator `PROJ_MATR_BASE` [U4.63.12], vectors second member with `PROJ_VECT_BASE` [U4.63.13].

Note:

The macro order `PROJ_BASE` [U4.63.11] allows to directly connect all three operation.

The matrices and vectors thus projected, one calculates the generalized answer of the system mono or multi - excited using the operator `DYNA_TRAN_MODAL` [U4.53.21].

4.5 Taking into account of the modes neglected by static correction

The truncation error resulting from the taking into account of an incomplete modal base can be corrected by means of a static correction. The static correction can be realized in two manners:

– *a priori* , by constitution of a base of Ritz starting from the base of the dynamic clean modes and of static modes, which they are with imposed acceleration (pseudo-mode) or imposed force;
– *a posteriori,* via the pseudo-mode (possible only in mono-support).

4.5.1 Static correction a priori

Lbe statics modesS with imposed acceleration (pseudo-mode) or imposed force are calculated beforehand according to:

```
    MODE_FO=MODE_STATIQUE (    MATR_RIGI=RIGIDITE,
                            MATR_MASS=MASSE,
               FORCE_NODALE=_F (  ALL = 'YES', AVEC_CMP = 'DX'))
or
    MODE_CO=MODE_STATIQUE (    MATR_RIGI=RIGIDITE,
                               MATR_MASS=MASSE,
                        PSEUDO_MODE=_F (  NODE = ('P1', 'P4',),
                                             AVEC_CMP = ('DX',))
                            )
```

C be modes S one T then associated with the dynamic modes to constitute one base of Ritz. The assistant modes can be:

- the statics modes S with imposed acceleration (pseudo-mode) and static modes with imposed force:

```
BAM0=DEFI_BASE_MODALE (RITZ= (_F (MODE_MECA=mode,),
                         _F (MODE_INTF=MODE_FO,),
                         _F (MODE_INTF=MODE_CO,),),
                ORTHO=' OUI', MATRICE=MASSE',
                         NUME_REF=N_DDL,);
```

- or static modes with force imposed only:

```
BAM1=DEFI_BASE_MODALE (RITZ= (_F (MODE_MECA=mode,),
                         _F (MODE_INTF=MODE_FO,),
                ORTHO=' OUI', MATRICE=MASSE',
                NUME_REF=N_DDL,);
```

- or static modes with acceleration imposed only:

```
BAM2=DEFI_BASE_MODALE (RITZ= (_F (MODE_MECA=mode,),
                         _F (MODE_INTF=MODE_CO,),
                ORTHO=' OUI', MATRICE=MASSE',
                NUME_REF=N_DDL,);
```

One insists on the need for D-orthogonalise the base thus obtained, preferably by the matrix of mass (the matrix of rigidity can also be appropriate, but does not allow to standardize the eigenvalues).

4.5.2 Static correction a posteriori

In this case, once reconsidered the physical base one corrects the value of the relative displacement calculated (respectively relative speed and relative acceleration) by the contribution of a pseudo-mode. The pseudo-mode is defined by the difference between the static mode associated with the unit loading of standard imposed constant acceleration and projection on the calculated dynamic modes of displacement (respectively relative speed and relative acceleration).

One has then:

$$\begin{cases} \mathbf{X}_{\text{r\_corrigé}} = \mathbf{X}_r + \sum_i f_i(t) . \left(\mathbf{\Psi}_i - \sum_{j=1}^{p} \eta_j . \mathbf{\Phi}_j \right) \\ \dot{\mathbf{X}}_{\text{r\_corrigé}} = \dot{\mathbf{X}}_r + \sum_i \dot{f}_i(t) . \left(\mathbf{\Psi}_i - \sum_{j=1}^{p} \dot{\eta}_j . \mathbf{\Phi}_j \right) \\ \ddot{\mathbf{X}}_{\text{r\_corrigé}} = \ddot{\mathbf{X}}_r + \sum_i \ddot{f}_i(t) . \left(\mathbf{\Psi}_i - \sum_{j=1}^{p} \ddot{\eta}_j . \mathbf{\Phi}_j \right) \end{cases}$$

Multiplicative functions of time $f_i(t)$ correspond to the imposed accélérogramme $\gamma_i(t)$ in each direction i considered.

The approach to be followed is the following one:

- Calculation of the unit loading of type forces imposed (constant acceleration) in the direction of the earthquake: `AFFE_CHAR_MECA` [U4.44.01]. One will pay attention to permute the sign of the direction since the seismic inertial force is form $\mathbf{P}(t) = -\mathbf{M\Psi} . \ddot{\mathbf{X}}_s$
```
cham_no = AFFE_CHAR_MECA(MODELE=modèle, PESANTEUR= (VALE,
DIRECTION)) ;
```
- Calculation of the linear static response of the structure to the preceding loading case: `MACRO_ELAS_MULT` [U4.51.02].
```
mode_cor = MACRO_ELAS_MULT(CHAR_MECA_GLOBAL = con_lim,…
                CAS_CHARGE =_F (NOM_CAS = ' xx', CHAR_MECA = cham_no)) ;
```

It will be noted that there is as much loading case than of direction of earthquake

- Calculation of the derivative first and second of the accélérogramme: `CALC_FONCTION` [U4.32.04].
`deri_pre` and `deri_sec = CALC_FONCTION (OPTION = DRIFT);`
- Calculation of the answer generalized by taking of to account the modes neglected by static correction:
```
dyna_mod = DYNA_TRAN_MODAL (MASS_GENE =… , RIGI_GENE =…
                MODE_CORR = mode_cor
                EXCIT =_F(CORR_STAT = 'YES'
                       D_FONC_DT = deri_pre, D_FONC_DT2 = deri_sec.)
                …) ;
```

- Return towards the physical base: the static correction is not implicitly taken into account. It is necessary to specify `CORR_STAT=' OUI'` in `RECU_FONCTION` or `REST_GENE_PHYS` so that the static correction is taken into account.

Note:

In the case of an multi-excited structure, the taking into account of the modes neglected by static correction a posteriori is not available. One post-draft absolute displacement in this case.

4.6 Taking into account of the multimedia character of a structure

It was seen previously (cf [§3.3]) that to calculate the seismic loading in the case of a multimedia structure, should as a preliminary have been calculated the static modes $\mathbf{\Psi}$, which will control the inertial forces of training of the supports, amplified by the signals of acceleration of training which theirs are applied.

If one wants to be able to restore the sizes calculated in the absolute reference mark or if one wants to be able to take into account nonlocalised linearities (shock, antiseismic supports…), cf § 4.3, it is also necessary to specify in `DYNA_TRAN_MODAL` that the studied structure is multi-excited. Indeed, in this last case, one compared to each moment, the vector of displacements and absolute velocities of each point of shock considered, in order to determine if there is shock and to calculate the corresponding forces of shock. It is advisable to make sure that displacements and absolute velocities of the supports are coherent with the signals of acceleration of training considered.

The approach to be followed is the following one:

- Calculation of the static modes: `MODE_STATIQUE` [U4.52.14].
```
mode_stat = MODE_STATIQUE(DDL_IMPO = (...));
```
- Calculation of the answer generalized by taking of account the component of training:
```
dyna_mod = DYNA_TRAN_MODAL (MASS_GENE =… , RIGI_GENE =…
              MODE_STAT = mode_stat
              EXCIT =_F(MULT_APPUI = 'YES'
                     ACCE = accelero, QUICKLY = speed, DEPL = moves
                     DIRECTION = (...), NODE =NO1
                     …)
              …) ;
```

4.7 Postprocessings

Operators `REST_GENE_PHYS` [U4.63.31] or `RECU_FONCTION` [U4.32.03] can then restore in physical space the calculated evolutions:

- the operator `REST_GENE_PHYS` restore overall (the complete field) displacements, speeds and accelerations;
- the operator `RECU_FONCTION` locally restore (temporal evolution of a degree of freedom) displacements, speeds and accelerations.

One can restore the relative sizes while specifying (`MULT_APPUI = 'NOT'`) or absolute sizes by (`MULT_APPUI = 'YES'`).
One obtains then displacements of training necessary to the calculation of the secondary sizes by withdrawing from absolute displacements relative displacements. This is carried out by the order `CALC_FONCTION` [U4.32.04] option `COMB`.

From the preceding evolutions, one can also extract the values maximum and RMS and to calculate the spectrum of answer of associated oscillator. This is carried out by the order `CALC_FONCTION` options `MAX`, `RMS` and `SRO`.

5 Direct transitory seismic answer

Direct integration is realizable is with assumptions of linear behavior: operator DYNA_LINE_TRAN [U4.53.02] that is to say with assumptions of nonlinear behavior: operator DYNA_NON_LINE [U4.53.01]. Setting except for the way of taking into account the seismic loading (cf [§3.3]), syntaxes of DYNA_NON_LINE and DYNA_LINE_TRAN are identical.

5.1 Taking into account of a damping are equivalent to modal damping

Generally, the most precise information that one has on damping comes from the tests of vibration which make it possible to determine, for a frequency of resonance given f_i , the width of corresponding resonance and thus reduced damping ξ_i with this resonance. **It is thus necessary to be able to take into account, in a direct transitory calculation, a damping equivalent to modal damping**.

From the spectral development of the matrix identity:

$$\mathbf{Id}=\sum_{i=1}^{n\_\text{modes}}\frac{\mathbf{X}_i\mathbf{X}_i^T\mathbf{K}}{\mathbf{X}_i^T\mathbf{K}\mathbf{X}_i}=\sum_{i=1}^{n\_\text{modes}}\frac{\mathbf{X}_i\mathbf{X}_i^T\mathbf{K}}{M_{G\_i}.\omega_i^2}$$

one shows:

- that one can develop the matrix of damping of the structure $\mathbf{C}$ in series of clean modes:

$$\mathbf{C}=\sum_{i=1}^{n\_\text{modes}}a_i.(\mathbf{K}.\mathbf{\Phi}_i)(\mathbf{K}.\mathbf{\Phi}_i)^T$$

- and that, account held of the definition of the critical percentage of damping:

$$\mathbf{\Phi}_i^T.\mathbf{C}.\mathbf{\Phi}_i=2.\mathbf{M}_{G\_i}.\omega_i.\xi_i.a_i=2.\frac{\xi_i}{K_{G\_i}.\omega_i}$$

It is thus advised with the user to specify (syntaxes of DYNA_NON_LINE and DYNA_LINE_TRAN are identical), the values of modal depreciation for each Eigen frequency via the keyword factor AMOR_MODAL.

That amounts imposing a force of damping proportional to the relative speed of the structure:

$$\mathbf{F}_\text{amo}=\mathbf{C}\dot{\mathbf{X}}_r \text{ with } \mathbf{C}=\sum_{i=1}^{n\_\text{modes}}2.\frac{\xi_i}{K_{G\_i}.\omega_i}.(\mathbf{K}.\mathbf{\Phi}_i)(\mathbf{K}.\mathbf{\Phi}_i)^T$$

5.2 Taking into account of a request multi-supports with restitutions of the relative and absolute fields

By defaults, the sizes are calculated in the relative reference mark. In DYNA_NON_LINE and DYNA_LINE_TRAN, one uses a syntax identical to that of DYNA_TRAN_MODAL (presence of the keywords MODE_STAT and MULT_APPUI = 'YES') to calculate them in the absolute reference mark.

6 Interaction ground-structure

The seismic behavior of a building depends on the characteristics of the ground on which it is posed since it depends on the seismic movement imposed on the ground and the dynamic behavior of the building and its foundations in the ground. The interaction ground-structure most frequently contributes to decrease the answer of the studied structure.

6.1 Impedance of a foundation

That is to say a surface rigid foundation without mass, subjected to a harmonic force of pulsation ω : $P(t)=P_0.e^{iwt}$ It is thus actuated by a movement $X(t)$ of the same frequency. One calls **impedance of the foundation**, or dynamic stiffness, the complex number $K(\omega)$, function of the frequency ω such as: $K(\omega)=\frac{P(t)}{X(t)}$.

Several analytical or digital methods make it possible to calculate the impedance of a foundation according to the complexity of the foundation and the ground on which it is posed or partially hidden. Among most frequently used, one quotes:

- analytical methods within the competences of WOLF or DELEUZE where it is supposed that the foundation raft is circular, rigid and posed on a homogeneous ground. The foundation must be surface;
- digital method of code CLASSI where one supposes that the foundation raft is of an unspecified form, rigid and posed on a possibly laminated ground. The foundation must be surface;
- digital method of code MISS3D where the foundation raft can be of an unspecified form, possibly deformable and posed on a possibly laminated ground.

It is possible to treat the interaction ground-foundation by **frequential method of coupling** (taking into account of the frequency response of the matrix of impedance) by carrying out a calculation coupled MISS3D / *Code_Aster*. This kind of calculations is not detailed in this reference material. One presents here only the case more the current where the interaction ground-foundation is treated by **method within the competences of ground** (it is considered that the terms of the matrix of impedance are independent of the frequency).

In the case of a surface rigid foundation, the impedance is calculated in the centre of gravity of surface in contact in a reference mark related to the main axes of inertia of this surface. For each frequency, it is expressed in the shape of a matrix of dimension $(6,6)$. One adjusts then the value of each term according to a particular clean mode of the building studied in blocked base:

- frequency of the first mode of swinging ω_0 for the horizontal stiffnesses $K_x(\omega_0), K_y(\omega_0)$ and of rotation $K_{rx}(\omega_0), K_{ry}(\omega_0)$;
- frequency of the first mode of pumping ω_1 for the vertical stiffness $K_z(\omega_1)$ and of torsion $K_{rz}(\omega_1)$.

As the Eigen frequencies of the building depend on the stiffnesses of ground, the calculation of the global values within the six competences of ground results from an illustrated iterative process appears [Figure 6.1-a]. First stiffnesses of ground $K_x(\omega_0), K_y(\omega_0), K_z(\omega_1), K_{rx}(\omega_0), K_{ry}(\omega_0)$ and $K_{rz}(\omega_1)$ are selected according to the first Eigen frequencies of swinging (ω_0) and of pumping (ω_1) structure in blocked base. The stiffnesses of grounds are then adjusted at the first significant Eigen frequencies of the structure on spring until correspondence of the frequencies to which the functions of impedance are calculated with the values of the Eigen frequencies of the system coupled ground - building.

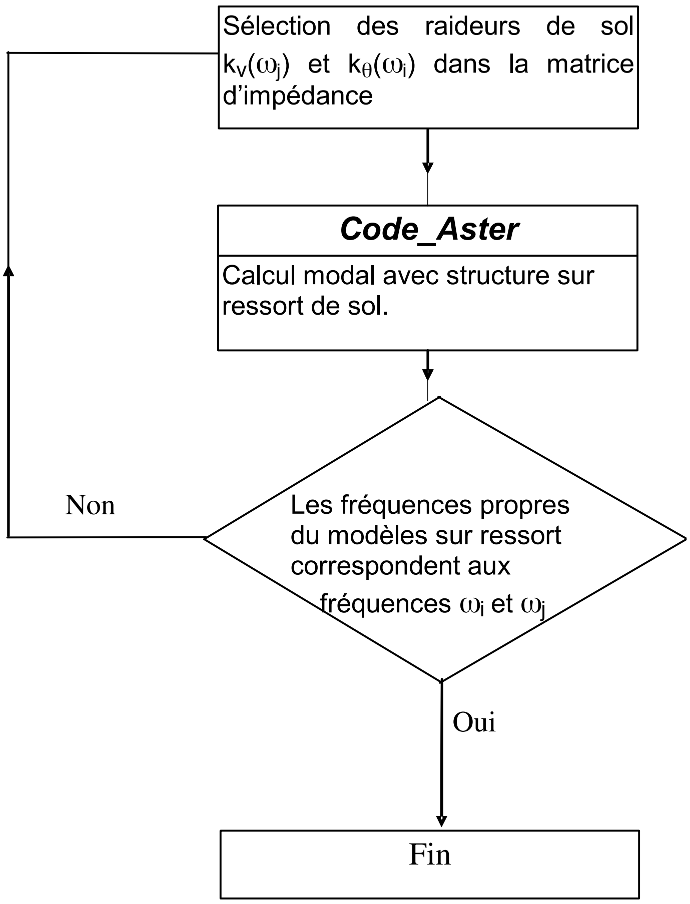

Figure 6.1-a : Process of adjustment of the stiffnesses of ground.

6.2 Taking into account of a modal damping calculated according to the rule of the RCC-G

One breaks up damping due on the ground into part of material origin and a geometrical part: damping due to the reflection of the elastic waves in the ground.
The rule of the RCC-G consists in summoning, for each mode, depreciation of each under structure constitutive of the building considered and depreciation structural and geometrical of the ground balanced by their respective rate of potential energy compared to total potential energy:

$$\eta_i = \frac{\sum_k E_{ki} \cdot \eta_k + \sum_s E_{si} \cdot \eta_{si}}{\sum_k E_{ki} + \sum_s E_{si}}$$

with:

- η_i , damping reduces average mode i ;
- η_k , the reduced damping of k ème element of the structure;
- η_{si} , the reduced damping within the competence of ground S for the mode i ;
- E_{ki} , potential energy of k ème element of the structure for the mode i ;
- and E_{si} , potential energy within the competence of ground s for the mode i .

In the regulation, modal damping is limited to a maximum value of 0.3.

The part of material origin of the damping of the ground is calculated by balancing the damping of each under structure by the report: rate of potential energy on total potential energy. As for the geometrical part of damping, it is calculated by distributing the values of damping for each direction (three translations and three rotations) balanced by the rate of potential energy in the ground of the direction. The directional values of damping are obtained while interpolating, for each calculated Eigen frequency, the directional functions of damping exit of a code of interaction ground-structure (PARASOL, CLASSI or MISS3D). The report of the imaginary part on twice the real part of the matrix of impedance: $\frac{\mathrm{Im}(K(\omega))}{2.\mathrm{Re}(K(\omega))}$, provides the values of this radiative damping.

The approach to be followed is the following one:

•Calculation of the potential energy dissipated in the studied structure: `POST_ELEM` [U4.81.22]

E_k = `POST_ELEM (ENER_POT=_F (ALL = 'YES')) ;`

•Calculation of modal damping by the rule of the RCC-G: `CALC_AMOR_MODAL` [U4.52.13]

```
l_amor = CALC_AMOR_MODAL (
    ENER_SOL =_F (MODE_MECA = base_modale, GROUP_NO_RADIER =… ,
                  KX = Kx(ω0) , KY = Ky(ω0) , KZ = Kz(ω1) ,
                  KRX = Krx(ω0) , KRY = Kry(ω0) , KRZ: Krz(ω1) )) ;
    AMOR_INTERNE =_F (GROUP_MA =…, ENER_POT = Ek , AMOR_REDUIT = ηk )
    AMOR_SOL =F (FONC_AMOR_GEO = Im(K(ω)) / 2.Re(K(ω)) )
    );
```

The calculation of the contribution of the ground to the potential energy E_s (keyword factor `ENER_SOL`) is calculated starting from the values of impedance of ground determined previously (cf [§6.1]). It can be calculated according to two different methods according to whether one average the modal efforts (keyword `RIGI_PARASOL`) or modal displacements with the nodes of the foundation raft.

The reduced damping within the competence of ground η_s (keyword factor `AMOR_SOL`) is calculated starting from the values of radiative damping.

6.3 Distribution of the stiffnesses and damping of ground

If one wants to study the effect of an earthquake on the possible separation of the foundation raft for example, one can have to model the ground either by a single spring in the centre of gravity of the interface ground - building but by a carpet of springs. This is possible thanks to the order `AFFE_CARA_ELEM` [U4.42.01] option `RIGI_PARASOL`.

The approach consists in calculating in each node of the grid of the foundation raft the elementary stiffnesses $(k_x, k_y, k_z, kr_x, kr_y, kr_z)$ to apply starting from the global values within the three competences of translations: kx, ky, kz and within the three competences of rotations: krx, kry, krz exits of a code of interaction ground-structure (or calculated analytically).
It is supposed that the elementary stiffnesses of translation are proportional to surface $S(P)$ represented by the node P and with a function of distribution $f(r)$ depending on the distance r node P in the centre of gravity of the foundation raft O :

$$\begin{cases} K_x = \sum_P k_x(P) = k_x . \sum_P S(P) . f(OP) \\ K_y = \sum_P k_y(P) = k_y . \sum_P S(P) . f(OP) \\ K_z = \sum_P k_z(P) = k_z . \sum_P S(P) . f(OP) \end{cases}$$

One from of deduced then k_x then $k_x(P)$ starting from calculation:

$$k_x(P) = k_x . S(P) . f(OP) = K_x . \frac{S(P) . f(OP)}{\sum_P S(P) . f(OP)} .$$

One from of deduced in the same way $k_y(P)$ and $k_z(P)$.

For the elementary stiffnesses of rotation, one distributes what remains after having removed the contributions due to the translations in the same way that translations:

$$\begin{cases} K_{rx} = \sum_P k_{rx}(P) + \sum_P \left[k_y(P) . z_{OP}^2 + k_z(P) . y_{OP}^2 \right] = k_{rx} . \sum_P S(P) . f(OP) + \sum_P \left[k_y(P) . z_{OP}^2 + k_z(P) . y_{OP}^2 \right] \\ K_{ry} = \sum_P k_{ry}(P) + \sum_P \left[k_x(P) . z_{OP}^2 + k_z(P) . x_{OP}^2 \right] = k_{ry} . \sum_P S(P) . f(OP) + \sum_P \left[k_x(P) . z_{OP}^2 + k_z(P) . x_{OP}^2 \right] \\ K_{rz} = \sum_P k_{rz}(P) + \sum_P \left[k_x(P) . y_{OP}^2 + k_y(P) . x_{OP}^2 \right] = k_{rz} . \sum_P S(P) . f(OP) + \sum_P \left[k_x(P) . y_{OP}^2 + k_y(P) . x_{OP}^2 \right] \end{cases}$$

One from of deduced then k_{rx} then $k_{rx}(P)$ starting from calculation:

$$\begin{aligned} k_{rx}(P) &= k_{rx} . S(P) . f(OP) \\ &= \left(K_{rx} - \sum_P \left[k_y(P) . z_{OP}^2 + k_z(P) . y_{OP}^2 \right] \right) . \frac{S(P) . f(OP)}{\sum_P S(P) . f(OP)} \end{aligned}$$

One from of deduced in the same way $k_{ry}(P)$ and $k_{rz}(P)$.

Note:

By default, one considers that the function of distribution is constant and unit i.e. each surface is affected same weight.

One can distribute in the same way six global values of damping, analytical or calculated by a code of interaction ground-structure.

6.4 Taking into account of an absorbing border

If one wants to calculate the seismic answer of a stopping for example, it is necessary, amongst other things, power to take into account it not reflection of the waves on the arbitrary border of the model finite elements within the ground or reserve. This functionality is not detailed in this document. It is the object of documentation [R4.02.05].

7 Bibliography

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[6] "Seismic calculation of the buildings" - Rules RCC-G, appendix A.

[7] [R5.03.17] Relations of behavior of the discrete elements.

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
01/05/00	Fe. WAECKEL- EDF/R & D /AMV	Initial text
01/12/13	F. VOLDOIRE- EDF/R & D /AMA	Corrections of form and addition of explanations.
V15.0	F. VOLDOIRE- EDF/R & D /ERMES	Corrections of form and addition of explanations.