Stochastic approach for the seismic analysis

Summary:

This document presents a probabilistic method of calculating to determine the answer of a structure subjected to a random excitation of seismic type starting from the interspectres of the excitation to the something to lean on of the structure. The answer itself is expressed in the form of interspectres.

Contents

1 Introduction

Classically the answer of a structure subjected to a seismic excitation can be calculated by two approaches:

- transitory calculation of dynamics if the excitation is defined by a accélérogramme (cf [R4.05.01]).
- calculation by the classical spectral method if the excitation is defined by a spectrum of answer of oscillator (SRO) (cf [R4.05.03]).

However a seismic excitation is by random nature. These two methods are not envisaged initially to take account of it: in a case it is necessary to reiterate for various excitations of many temporal calculations then to make a statistical average of it (important cost calculation), in the other case one carries out very conservative assumptions by considering averages (of quadratic type simple or supplements for example) for the maximum of the answers.

Also it was developed a method of calculating of the probabilistic type, also called “stochastic approach of seismic calculation”, based on the calculation of the dynamic response expressed in interspectres of power starting from the spectral concentrations of power of the excitation. This method has in particular the advantage of better taking into account the correlations between the excitations to the various supports of the structure.

The discussion of the various advantages of this method can be thorough in the reference [bib1].

We thus present the principle of the method and the notations retained starting from the classical approachs, then in third part probabilistic calculation itself.

Finally in fourth part the various methods will be presented to obtain the exiting interspectre.

2 Principle of the approach

2.1 Position of the problem considered and principle general

One in the case of is placed **multi-supported structure**, i.e. the structure has m degree of freedom-supports, each one being subjected to its own excitation (not necessarily equal everywhere). It is supposed that the structure is represented by a comprising model finite elements n degrees of freedom. One seeks the answer in a number finished (and low) of l degrees of freedom.

It is supposed that **size excitation is of standard imposed movement** and results in a family of accélérogrammes $g_j(t)$ for each degree of freedom-supports j, $j=1,m$.

The absolute movement of the structure is **broken up** classically in **movement of training and relative movement.**

The calculation of the answer in interspectres of power is carried out by **modal recombination**.

Following this modal calculation, a calculation of dynamic response random breaks up into three parts:

- definition of the interspectre of power discharger,
- calculation of the interspectre of power answer.

These the first two parts are the object of the order DYNA_ALEA_MODAL [U4.53.22].

The restitution of the interspectre of power answer on physical basis is carried out with the order REST_SPEC_PHYS [U4.63.22].

- calculation of statistical parameters starting from the interspectre of power result.

This last stage is treated by the order POST_DYNA_ALEA [R7.10.01] [U4.84.04].

2.2 Decomposition of the movement

The following decompositions and projections are detailed in the reference material relating to the resolution by transitory calculation of a seismic calculation [R4.05.01]. We retain only the broad outlines here of them.

That is to say $\mathbf{X_a}$ the vector absolute displacement (of dimension n) of all the degrees of freedom of the structure.

Total the answer known as **absolute** $\mathbf{X_a}$ structure is expressed as the sum of a contribution **relative** $\mathbf{X_r}$ and of the contribution **of training** $\mathbf{X_e}$ had with displacements of anchoring (subjected to the accelerations represented by a accélérogramme $g_j(t)$ in each degree of freedom-supports j, $j=1,m$).

$$\mathbf{X}_a(t) = \mathbf{X}_r(t) + \mathbf{X}_e(t)$$

Are $\mathbf{M},\mathbf{K}$ and $\mathbf{C}$ matrices of mass, rigidity and damping of the problem, limited to the not supported degrees of freedom.

The equation of the movement is written then in the reference mark related to the relative movement:

$$\mathbf{M}\ \ddot{\boldsymbol{X}}_r(t)+\mathbf{C}\ \{\dot{\boldsymbol{X}}_r(t)+\mathbf{K}\,\mathbf{X}_r(t)=-\mathbf{M}\ \ddot{\boldsymbol{X}}_e(t)+\mathbf{F}_{ext}$$

$\mathbf{F}_{ext}$: vector of the forces extéŽrieures

In general the external forces are worthless during a calculation of seismic answers.

2.3 Decomposition on the modal basis

The calculation of answer in interspectres of power is carried out by **modal recombination** and fact call, moving imposed, at a modal base which understands at the same time dynamic modes and static modes.

That is to say $\Phi=\{\phi_{i,i=1,n}\}$ the matrix (n,n) dynamic modes calculated for the associated conservative system, by maintaining the m blocked supports.

That is to say $\Psi=\{\psi_{j,j=1,m}\}$ the matrix (n,m) static modes. Mode Ψ_j corresponds to the deformation of the structure under a unit displacement imposed on the degree of freedom-support j, other degrees of freedom - supports being blocked.

The imposed displacement of anchorings $\mathbf{X_s}(t)$ is connected to $\mathbf{X_e}(t)$ by the relation: $\mathbf{X_e}(\mathbf{t})=\mathbf{\Psi}\ \mathbf{X_s}(\mathbf{t})$.

Components of the acceleration of the points of anchoring $\ddot{\mathbf{X}}_s(t)$ are the accélérogrammes $g_j(t)$, $j=1,m$.

One can thus write $\ddot{\mathbf{X}}_\mathbf{e}(t)=\mathbf{\Psi}\,\ddot{\mathbf{X}}_\mathbf{s}(t)=\sum_{j=1}^{m}\boldsymbol{\psi}_j g_j(t)$.

The change of variable is carried out $\mathbf{X_r}(t)=\mathbf{\Phi}.\mathbf{q}(t)$, $\mathbf{q}(t)$ is the vector of the generalized coordinates. By prémultipliant the equation of the movement by ${}^T\mathbf{\Phi}$ ' one obtains - in the absence of external forces others that the seismic excitation - the equation projected on the basis as of dynamic modes:

$$ {}^T\mathbf{\Phi}\,\mathbf{M}\,\mathbf{\Phi}\,\ddot{\mathbf{q}}(\mathrm{t})+{}^T\mathbf{\Phi}\,\mathbf{C}\mathbf{\Phi}\,\dot{\mathbf{q}}(\mathrm{t})+{}^T\mathbf{\Phi}\,\mathbf{K}\,\mathbf{\Phi}\,\mathbf{q}(\mathrm{t})=-{}^T\mathbf{\Phi}\,\mathbf{M}\mathbf{\Phi}\,\ddot{\mathbf{X}}_s(\mathrm{t}) $$

It is supposed that the matrix of damping is a linear combination of the matrices of mass and rigidity (assumption of damping of constant Rayleigh on the structure or assumption of Basile allowing a diagonal damping). The base Φ who orthogonalise matrices M and K, orthogonalise thus also the matrix $\mathbf{C}$.

Taking into account this assumption, the preceding equation breaks up into n scalar equations uncoupled in the form:

$$ \ddot{\mathbf{q}}_\mathrm{i}+2\xi\omega_\mathrm{i}\dot{\mathbf{q}}_\mathrm{i}+\omega_\mathrm{i}^2\mathbf{q}_\mathrm{i}=-\sum_{j=1}^{m}\mathbf{p}_\mathrm{ij}g_\mathrm{j}(t) \quad \text{for } i=1,n $$

where one noted:

$\mu_i={}^T\mathbf{\Phi}_\mathrm{i}\mathbf{M}\mathbf{\Phi}_i$ modal mass

$k_i={}^T\mathbf{\Phi}_\mathrm{i}\mathbf{K}\mathbf{\Phi}_\mathrm{i}$ modal rigidity

$\omega_i=\sqrt{\frac{k_i}{\mu_i}}$ the modal pulsation

$\xi_i=\frac{{}^T\mathbf{\Phi}_\mathrm{i}.\mathbf{C}.\mathbf{\Phi}_\mathrm{i}}{2\mu_i\omega_i}$ reduced modal damping”

$\mathrm{p_{ij}}=\frac{{}^T\mathbf{\Phi}_i.\mathbf{M}.\mathbf{\Psi}_j}{\mu_i}$ the factor of modal participation of the support j on the dynamic mode i.

The solution $\mathbf{q}_\mathrm{i}(t)$ of this equation corresponds to the answer of the dynamic mode i with the whole of the seismic excitation.

One can still break up the problem by introducing the unknown factor $\mathbf{d}_{ij}(t)$ solution of the differential equation: $\ddot{\mathbf{d}}_{ij}+2\boldsymbol{\xi}_i\boldsymbol{\omega}_i\dot{\mathbf{d}}_{ij}+\omega_i^2\mathbf{d}_{ij}=g_j(t)$, this last equation corresponds to the answer of the dynamic mode i with acceleration $g_j(t)$. Relative displacement on the physical basis is expressed then:

$$\mathbf{X_r}(t)=-\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{p}_{ij}\mathbf{d}_{ij}(t)\boldsymbol{\varphi}_i$$

Information on the position of the something to lean on is contained in the factor of modal participation.

2.4 Harmonic answer

One thus broke up the total answer of the structure into a relative contribution and a differential contribution due to displacements of anchorings such as:

$$\mathbf{X}_\mathrm{a}(t)=\mathbf{X}_\mathrm{r}(t)+\mathbf{X}_\mathrm{e}(t)$$

avec

$$\begin{cases}\ddot{\mathbf{X}}_e(t)=\boldsymbol{\Psi}\,\ddot{\mathbf{X}}_\mathrm{s}(t)=\sum_{j=1}^{m}\boldsymbol{\psi}_j g_j(t)\\ \mathbf{X}_\mathrm{r}(t)=-\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{p}_{ij}\mathbf{d}_{ij}(t)\boldsymbol{\varphi}_\mathrm{i}\quad \text{où } \mathbf{d}_{ij(t)} \text{ est solution de } \ddot{\mathbf{d}}_{ij}+2\xi_i\omega_i\dot{\mathbf{d}}_{ij}+\omega_i^2\mathbf{d}_{ij}=g_j(t)\end{cases}$$

The solution of this last differential equation by the method of the transformation of Fourier utilizes the modal transfer transfer functions $h_i(\omega)$ such as: . $h_i(\omega)=\frac{1}{\omega_i^2-\omega^2+2i\xi_i\omega_i\omega}$

One thus obtains: $\mathbf{d}_{ij}(\omega)=h_i(\omega).g_j(\omega)\text{et}\,\ddot{\mathbf{d}}_{ij}(\omega)=-\omega^2h_i(\omega).g_j(\omega)$

The total harmonic answer of the structure results from the preceding formulas by modal recombination.

$$\ddot{\mathbf{X}}_a(\omega)=\ddot{\mathbf{X}}_r(\omega)+\ddot{\mathbf{X}}_e(\omega)$$

$$\ddot{\mathbf{X}}_a(\omega)=\omega^2\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{p}_{ij}\mathbf{h}_\mathrm{j}(\omega)\boldsymbol{\varphi}_j(\omega)j_i+\sum_{j=1}^{m}\boldsymbol{\Psi}_j g_j(\omega)$$

One then reveals the complex matrix (n,m), known as matrix of transfer $\mathbf{H}(\omega)$ following:

$$\mathbf{H}(\omega)=\omega^2\mathbf{P}.\mathbf{h}(\omega)\boldsymbol{\Phi}+\boldsymbol{\Psi}$$

where $\mathbf{P}$ is the matrix of the factors of participation, $\mathbf{h}(\omega)$ the vector of the modal transfer transfer functions $h_i(\omega)$.

The total answer of the structure is worth $\ddot{\mathbf{X}}_\mathrm{a}(\omega)=\mathbf{H}(\omega)\ddot{\mathbf{E}}(\omega)$ where $\ddot{\mathbf{E}}(\omega)$ is the vector of m lines made up of the transforms of Fourier of accelerations $g_j(t)$ with m degrees of freedom-apthen.

It is seen that this expression determines the answer in acceleration. This then forces to twice integrate the answer to obtain displacement, this problem is presented in [bib4]. One of the additional interests of the method which we propose here is to abstract itself from this difficulty.

3 The random dynamic response

3.1 Recall on the spectral concentrations of power [bib2]

3.1.1 Definitions

That is to say a probabilistic signal defined by its density of probability $p_x(x_1, t_1, \ldots, x_n, t_n)$. This density of probability makes it possible to calculate the functions moments of the signal.

Moment of order 1 or hope of the signal:

$$\mu_{\mathbf{X}}(t) = \mathrm{E}[\mathbf{X}(t)] = \int_{-\infty}^{+\infty} \mathbf{X}\, p_x(x, t)\, dx$$

Moments of order 2 or intercorrelation of two signals:

$$\rho_{\mathbf{XY}}(t_1, t_2) = \mathrm{E}\left[\mathbf{X}(t_1)\overline{\mathbf{Y}(t_2)}\right] = \int_{-\infty}^{+\infty} \mathbf{x}\,\bar{\mathbf{y}}\, p(x, t_1; y, t_2)\, dxdy$$

When the signal is stationary, the intercorrelation depends only on $\tau = t_2 - t_1$.

It is written $R_{\mathbf{XY}}(t) = \mathrm{E}\left[\mathbf{X}(t)\overline{\mathbf{Y}(t-\tau)}\right]$

Spectral concentration of power and interspectre

One defines $\mathbf{S_{XY}}(\omega)$ the interspectre of power or density interspectrale of power between two stationary probabilistic signals by the transform of Fourier of the function of intercorrelation, which one writes:

$$\mathbf{S_{XY}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathbf{R_{XY}}(\tau)\, e^{-i\omega\tau}\, d\tau$$

The opposite formula is written: $\mathbf{R_{XY}}(t) = \int_{-\infty}^{+\infty} \mathbf{S_{XY}}(\omega)\, e^{i\omega\tau}\, d\omega$

$\mathbf{S_{XY}}(\omega)$ is generally complex and checks the relation of symmetry: $\mathbf{S_{YX}}(\omega) = \overline{\mathbf{S_{XY}}(\omega)}$.

When $\mathbf{X} = \mathbf{Y}$, $\mathbf{S_{XX}}(\omega)$ be called **autospectre of power or density spectral of power (DSP)**. This function has the property to be real and always positive.

3.1.2 Relations between the DSP and the other characteristics of the signal

Note:

Most of the time, the signal is defined over a limited time, its transform of Fourier does not exist, one defines a transform of Fourier then estimated over one period length T by:

$$\hat{X}_T(\omega) = \frac{1}{2\pi}\int_{-T/2}^{T/2} X(t)\, e^{-i\omega\tau}\, d\tau \ .$$

One then has the following relationships to this estimated transform of Fourier:

$$\mathbf{S_{XY}}(\omega) = \lim_{T\to+\infty} \frac{2\pi}{T}\mathrm{E}\left[\hat{\mathbf{X}}_{\mathrm{T}}(w)\overline{\hat{\mathbf{Y}}_{\mathrm{T}}(\omega)}\right]$$

$$\mathbf{S_{XX}}(\omega) = \lim_{T\to+\infty} \frac{2\pi}{T}\mathrm{E}\left[\hat{\mathbf{X}}_{\mathrm{T}}(w)\overline{\hat{\mathbf{X}}_{\mathrm{T}}(\omega)}\right]$$

Link between the autospectre of power and the power of the signal:

The power of a signal is equal to its variance. For a centered signal, the variance is worth: $\sigma_X^2 = R_{XX}(0)$.

One thus has: $\sigma_X^2 = R_{XX}(0) = \int_{-\infty}^{+\infty} S_{XX}(\omega) d\omega$.

3.2 Equations of motion

The total answer of the structure is determined by the relation: $\ddot{\mathbf{X}}_a(\omega) = \mathbf{H}(\omega)\ddot{\mathbf{E}}(\omega)$,

where $\ddot{\mathbf{E}}(\omega)$ is the vector of $\mathbf{m}$ lines made up of the excitations represented by the transforms of Fourier of the accélérogrammes $g_j(t)$ with $\mathbf{m}$ degrees of freedom-supports,

$\mathbf{H}(\omega)$ is the matrix of transfer defined by $\mathbf{H}(\omega) = \omega^2 \mathbf{ph}(\omega)\mathbf{\Phi} + \mathbf{\Psi}$

where $\mathbf{p}$ is the matrix of the factors of participation,

$\mathbf{H}(\omega)$ the vector of the modal transfer transfer functions $h_i(\omega)$

$\mathbf{\Phi}$ base dynamic modes

$\mathbf{\Psi}$ base static modes

it comprises n lines (= many free degrees of freedom of the structures) and m columns.

3.2.1 Matrix "interspectrale-excitation"

NB:

This name "stamps interspectrale-excitation" is abusive: it means "matrix of density interspectrale of power of the excitation".

It is supposed that the seismic excitation can be regarded as a stationary signal - taking into account the relationship between representative times - and centered. This makes it possible to use a certain number of result of the probabilistic analysis. One is interested then in the stationary answer of the system to a stationary excitation.

One notes $\mathbf{S}_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\omega)$ the matrix of the interspectres of power corresponding to the excitation. Its data is clarified in chapter 4.

For memory we recall here that it is calculated starting from transforms of Fourier of accelerations. It is a matrix ($m \times m$). The ij term corresponds to the interspectre between the signals $\ddot{\mathbf{E}}^i$ and $\ddot{\mathbf{E}}^j$ that is to say still between the transforms of Fourier of the accélérogrammes g_i and g_j .

3.2.2 Random dynamic response

It was seen that the interspectre of power between two probabilistic signals is the transform of Fourier of the function of intercorrelation of the two signals. One applies it to the total answer of the structure:

$$S_{\ddot{X}_a\ddot{X}_a}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{\ddot{X}_a\ddot{X}_a}(\tau) e^{-i\omega\tau} d\tau = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{E}\left[\ddot{\mathbf{X}}_a(t)^T \overline{\ddot{\mathbf{X}}_a(t-\tau)}\right] e^{-i\omega\tau} d\tau$$

One works then in the temporal field to express the function of intercorrelation of the answer total $R_{\ddot{X}_a\ddot{X}_a}(t,t')$.

One notes $h(t)$ the impulse response of the system: $\mathbf{h}(t) = \mathrm{TF}^{-1}[\mathbf{H}(\omega)]$

and $\ddot{\mathbf{e}}(t)$ the transform of Fourier reverses exiting DSP: $\ddot{\mathbf{e}}(t) = \mathrm{TF}^{-1}[\ddot{\mathbf{E}}(\omega)]$

By transform of Fourier relation reverses: $\ddot{\mathbf{X}}_a(\omega) = \mathbf{H}(\omega)\ddot{\mathbf{E}}(\omega)$

one has $\ddot{\mathbf{X}}_a(t)=\mathbf{h}\times\ddot{\mathbf{e}}(t)=\int_R \mathbf{h}(u)\ddot{\mathbf{e}}(t-u)du$

$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\mathrm{E}\left[\ddot{\mathbf{X}}_a(t)^T\,\overline{\ddot{\mathbf{X}}_a(t')}\right]$$
$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\mathrm{E}\left[\int_R \mathbf{h}(u)\ddot{\mathbf{e}}(t-u)du^T\,\overline{\int_R \mathbf{h}(v)\ddot{\mathbf{e}}(t'-v)dv}\right]$$
$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\mathrm{E}\left[\int_R\int_R \mathbf{h}(u)\ddot{\mathbf{e}}(t-u)^T\,\overline{\ddot{\mathbf{e}}(t'-v)}^T\,\overline{\mathbf{h}(v)}dv\,du\right]$$

One supposes in this analysis the deterministic system, one can thus leave the impulse response the calculation of the expectation. It comes:

$$R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\int_R\int_R \mathbf{h}(u)\mathrm{E}[\ddot{\mathbf{e}}(t-u)^T\,\overline{\ddot{\mathbf{e}}(t'-v)}]^T\,\overline{\mathbf{h}(v)}dv\,du$$

The excitation is supposed a stationary process, the intercorrelation thus depends only on the variation of time $\tau=t\text{-}t'$:

$$R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(t-t'-u+v)=\mathrm{E}[\ddot{\mathbf{e}}(t-u)^T\,\overline{\ddot{\mathbf{e}}(t'-v)}]=R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\theta)\,\mathrm{pour}\,\theta=t-t'-u+v=\tau-u+v$$

from where $R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(t,t')=\int_R\int_R \mathbf{h}(u)\,R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\theta)^T\,\overline{\mathbf{h}(v)}dv\,du=R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(\tau)$ what justifies a posteriori the approach.

One now defers this expression in the expression of the spectral concentration of power of the answer:

$$S_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}R_{\ddot{\mathbf{X}}_a\ddot{\mathbf{X}}_a}(\tau)e^{-i\omega\tau}d\tau=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_R\int_R \mathbf{h}(u)R_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\tau-u+v)^T\,\overline{\mathbf{h}(v)}e^{-i\omega\tau}dv\,du\,dt$$

By distributing the dummy variables of integration one reveals the respective transforms of Fourier of $h(u), R_{\ddot{E}\ddot{E}}(t-u+v), {}^T\overline{h(v)}$, it comes finally:

$$S_{\ddot{X}_a\ddot{X}_a}=\mathbf{H}(\omega).S_{\ddot{E}\ddot{E}}(\omega).{}^T\overline{\mathbf{H}(\omega)}$$

with $\mathbf{H}(\omega)=\omega^2\mathbf{P.h}(\omega)\mathbf{\Phi}+\mathbf{\Psi}$

Taking into account the relations between the transforms of Fourier of displacement, speed and acceleration, one has moreover:

$$S_{\dot{X}_a\dot{X}_a}=\frac{-1}{\omega^2}\mathbf{H}(\omega)S_{\ddot{E}\ddot{E}}(\omega)^T\overline{\mathbf{H}(\omega)}$$
$$S_{X_aX_a}=\frac{1}{\omega^4}\mathbf{H}(w)S_{\ddot{E}\ddot{E}}(\omega)^T\overline{\mathbf{H}(\omega)}$$

These relations make it possible to express the answer of the structure by the DSP of displacement or speed.

Note:

- *According to the expression given to* $\mathbf{H}(\omega)$*, one expresses the DSP of displacement (respectively the speed or of acceleration) total, relative or differential:*

  absolute movement: $\mathbf{H}(\omega)=\omega^2\mathbf{P.\,h}(\omega)\mathbf{\Phi}+\mathbf{\Psi}$

  relative movement: $\mathbf{H}(\omega)=\omega^2\mathbf{P.\,h}(\omega)\mathbf{\Phi}$

  differential movement (IE of training): $\mathbf{H}(\omega)=\mathbf{\Psi}$

- *It is of use, during a calculation with Code_Aster, to restrict the matrix of the transfer transfer function to the lines of l degrees of freedom of observation. This makes it possible to reduce of as much calculations as soon as l is small in front n .*

3.3 Application in *Code_Aster*

The whole of the spectral approach for seismic calculation is treated in the order DYNA_ALEA_MODAL [U4.53.22]. The data are gathered under three keywords factors and a simple keyword.

The modal base is made up by the dynamic modes calculated by the order CALC_MODES [U4.52.02] stored in a concept of the type mode_meca recovered by the keyword factor BASE_MODALE, on the one hand; static modes calculated by the order MODE_STATIQUE [U4.52.14] stored in a concept of the type mode_stat recovered by the simple keyword MODE_STAT, in addition. The keyword factor BASE_MODALE also have the arguments which make it possible to determine the waveband or the modes retained for corresponding calculation and depreciation.

The data corresponding to the excitation are gathered under the keyword factor EXCIT (cf paragraph [§4]): one specifies there the type of excitation within the meaning of SIZE : excitation in displacement or effort, nodes NODE and components NOM_CMP excited, the name of the interspectres or autospectres INTE_SPEC, complex functions read beforehand or calculated, respectively by the operators LIRE_INTE_SPEC [U4.36.01] or CALC_INTE_SPEC [U4.36.03] and stored in a table of interspectre of concept tabl_intsp who apply in each excited degree of freedom.

Under the keyword factor ANSWER are the data related to the choice of the discretization.

The order DYNA_ALEA_MODAL provides the answer in the form of spectral concentration of power on modal basis. To obtain the restitution of the DSP on physical basis, one will use REST_SPEC_PHYS [U4.63.22] which makes it possible to specify the type of size of the answer (displacement or effort), at the "points of observation" (node-component) of the result. In the presence of an answer of type displacement, one will specify here also if the answer corresponds to absolute displacement, relative or differential.

REST_SPEC_PHYS provides a table of interspectres which contains according to the request of the user, the matrix interspectrale in displacement $S_{\mathbf{XX}}$, of speed $S_{\dot{\mathbf{X}}\dot{\mathbf{X}}}$, **or** in acceleration $S_{\ddot{\mathbf{X}}\ddot{\mathbf{X}}}$ for an expression in the absolute reference mark (index a), the relative reference mark (index r) or of training (index e).

Each preceding "combination" requires a call specific to the order REST_SPEC_PHYS.

4 Definition of the matrix interspectrale of exiting power

The seismic excitation is by nature, we said it, random. Also it can be known not by its temporal expression but in frequential form by a spectral concentration of power also said interspectre.

When there are several supports, they can be excited by excitations identical or different, this last case is that of the multi-supports.

For $\mathbf{m}$ supports, one defines the matrix of density interspectrale of power of order $\mathbf{m}$, or by abuse language the interspectre of order $\mathbf{m}$, which is a matrix ($m \times m$) complex functions depending on the frequency.

The diagonal terms represent the "auto-" densities spectral of powers - or autospectres- at the points of excitation, the extra-diagonal terms correspond to the densities interspectrales between the excitations in two distinct something to lean on (each line or column of the matrix represents in fact a something to lean on in physical grid or a mode in modal calculation). By definition of these terms, it from of deduced that the matrices of density interspectrales of power handled are square. (See [bib2] or reference material associated with the order POST_DYNA_ALEA [R7.10.01])

We present the various orders hereafter of *Code_Aster* who allow to obtain a matrix of density interspectrale of power.

4.1 Reading on a file

The most elementary way to define a matrix of density interspectrale of power is to give, "with the hand", the values with the various steps of frequency.

The operator then is used LIRE_INTE_SPEC [U4.36.01].

LIRE_INTE_SPEC bed in a file "interspectre excitation". The format of the file in which the matrix interspectrale is consigned is simple: one describes successively the function of each term of the matrix interspectrale; for each function, one gives a line by frequency by indicating the frequency, the parts real and imaginary of the complex number; or the frequency, the module and the phase of the complex number (keyword FORMAT).

Example of file interspectre excitation (for a matrix reduced in the term):

```
INTERSPECTRE
DIM = 1
FONCTION_C
I = 1
J = 1
NB_POIN = 4
VALUE =
      2.9999   0.  0.
      3.       1.  0.
      13.      1.  0.
      13.0001  0.  0.
FINSF
END
```

4.2 Obtaining a interspectre starting from functions of time

One can deduce the matrix from density interspectrale of power starting from functions of time. The operator then is used CALC_INTE_SPEC [U4.36.03] in *Code_Aster* [bib3].

Starting from a list of N functions of time, this operator allows to calculate the interspectre of power $N \times N$ who corresponds to them.
For each term of the matrix interspectrale ($N \times N$) the following approach is used [bib3].

To calculate the interspectre of two signals one uses the relation of Wiener-Khintchine [bib7] which makes it possible to establish a formula of computation of the spectral concentration of power by the transform of Fourier of samples finished of the signals $\mathbf{x}(t)$ and $\mathbf{y}(t)$.

It comes then:

$$S_{\mathbf{xy}}(f)=\lim_{T\to\infty}\frac{1}{T}\mathrm{E}[\mathbf{X}_{\mathrm{k}}(f,T).\mathbf{Y}_{\mathrm{k}}\times(f,T)]$$

where $$\begin{cases} \mathbf{X}_k(f,T)=\mathrm{TF}[\mathbf{x}_k](f)=\int_0^T \mathbf{x}_k(t)\,e^{-i2\pi f}\,dt \\ \mathbf{Y}_k(f,T)=\mathrm{TF}[\mathbf{y}_k](f)=\int_0^T \mathbf{y}_k(t)\,e^{-i2\pi f}\,dt \end{cases}$$

are the discrete transforms of Fourier of $«x»$ and $«y»$.

When one is interested in signals resulting from measurements, one has most of the time only known signals in a discrete way, in the same way a transitory computation result is a discrete signal.

An approximation of the interspectre of the discrete signals $x[n]$ and $y[n]$ defined on L points spaced of Δt, cut out in p blocks of q points is obtained by the relation:

$$\hat{\mathbf{S}}_{xy}[k]=\frac{1}{pq\,\Delta t}\sum_{i=1}^{p}\mathbf{X}^{(i)}[k]\,\mathbf{Y}^{(i)*}[k]$$
$$\mathbf{X}^{(i)}[k]=\Delta t\sum_{n=0}^{q}\mathbf{x}^{(i)}[n]\,e^{-2i\pi kn/q}$$
$$\mathbf{Y}^{(i)}[k]=\Delta t\sum_{n=0}^{q}\mathbf{y}^{(i)}[n]\,e^{-2i\pi kn/q}$$

The various blocks can or not overlap. Values p and q are with the choice of the user.

This method is that of the periodogram of WELCH [bib8].

Calculation is done on a window which moves on the field of definition of the functions. The user specifies in the order the length of the window of analysis, the shift between two successive windows of calculation and the number of points per window.

4.3 Excitations preset or reconstituted starting from existing complex functions

One can wish to define a matrix of density interspectrale of power in various ways:

- by a white vibration: the values are constant
- according to the analytical formula of useful KANAI-TAJIMI in seismic calculation (white vibration filtered),
- or by taking again existing complex functions.

The operator then is used `DEFI_INTE_SPEC` [U4.36.02].

4.3.1 Existing complex functions

It is enough under the keyword factor `PAR_FONCTION` to give the name of the function for each pair of index `NUME_ORDRE_I, NUME_ORDRE_J`, corresponding to the higher triangular matrix (because of its hermiticity).

4.3.2 White vibration

A white vibration is characterized by a constant value on all the field of definition considered. Under the keyword factor `CONSTANT`, one gives this value (`VALE_R` or `VALE_C`) on the waveband `[FREQ_MIN, FREQ_MAX]` for each pair of index `INDI_I, INDI_J`, corresponding to the higher triangular matrix (because of its hermiticity). To define the function perfectly, one specifies the interpolation and the prolongations.

4.3.3 White vibration filtered by KANAI-TAJIMI [bib9]

For a structure pressed on the ground, it is current to take as excitation the spectral concentration of power of Kanaï-Tajimi. This spectral concentration represents the filtering of a white vibration by the ground, considered as a system with a degree of freedom. The parameters of the formula make it possible to exploit the centre frequency and the bandwidth of the spectrum.

The spectrum $G(\omega)$ express yourself by the following relation:

$$G(\omega)=\frac{\omega_g^4+4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\xi_g^2\omega_g^2\omega^2}G_0$$

$\omega_g=2\pi f$ pulsation propre

ξ_g *amortissement total*

G_0 *niveau du bruit blanc avant filtrage*

The user must specify the Eigen frequency f_g filter, modal damping ξ_g and the white sound level G_0 (= VALE_R) before filtering; like as for any function: the interpolation, profiles external and the field of definition (waveband).

By default a ground running is well represented by the values $f_g=2.5\,Hz$ and $\xi_g=0.6$.

Example of use for a white vibration filtered by KANAI_TAJIMI:

```
Interex =     DEFI_INTE_SPEC (
              DIMENSION: 1
              KANAI_TAJIMI: (
                 NUME_ORDRE_I: 1             indices of the term of the matrix of density
                 NUME_ORDRE_J: 1                interspectrale of power
                 FREQ_MOY  : 2.5             Eigen frequency
                 AMOR: 0.6                modal damping
                 VALE_R: 1                white sound level
                 Interpol: 'FLAX'            linear interpolation
                 PROL_GAUCHE: 'CONSTANT' prolongation
                 PROL_DROIT: 'CONSTANT'
                 FREQ_MIN  : 0.              field of definition
                 FREQ_MAX  : 200.
                 NOT: 1.
              ) );
```

4.4 Other types of excitation

Calculations of the preceding paragraphs were carried out within the framework of the assumption of an excitation in **imposed movement** on a degree of freedom. With the help of some modifications it is possible to use the same approach for an excitation **in effort** [§4.4.1] or **by fluid sources** [§ 4.4.2], this one being expressed in a finite element [§4.4.3] or on a function of form of the structure [§4.4.4].

In the continuation of this paragraph, one supposes the random excitation known and provided by the user in the form of a DSP, spectral concentration of power.

4.4.1 Case of the excitation in imposed forces

Under the keyword `EXCIT` one has `SIZE = EFFO`.

When the excitation with the supports is of type forces imposed, the general equation of the movement is:

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\sum_{j=1}^{m}\mathbf{F}_j$$

The answer of the structure is then calculated on one **base dynamic modes** $\mathbf{\Phi}=\{\phi_{i,\,i=1,n}\}$, these modes being calculated by supposing them **free exiting supports.** One does not distinguish, in this case of movement absolute, relative and differential and one does not use static modes.

One defines the factor of modal participation in the form: $P_{ij}=\frac{{}^T\phi_i F_j}{\mu_i}$

The transitory, harmonic and random answers have them **same expressions** that answers of **relative movement** excitation multi-support in the case general [§3]. (What corresponds to the absence of static modes). The exiting force is represented in each degree of freedom-support by its DSP in the form of a term are equivalent to $\mathbf{S}_{\ddot{\mathbf{E}}\ddot{\mathbf{E}}}(\omega)$.

4.4.2 Excitation by fluid sources

The fluid sources appear, for example, in the study of a network of pipings. They correspond to active bodies or connections of secondary pipings. They are generally sources of pressure or sources of flow. These various types of source are presented hereafter according to their mathematical working and what makes *Code_Aster* in each configuration.

These fluid sources are not directly seismic excitations but can be induced by an earthquake. The resolution of the mechanical problem makes call with the very methods, because of their randomness, which justifies their presentation here.

The modeling of the network of piping is supposed to be realized using vibroacoustic beam of *Code_Aster*.

The answer to fluid sources is calculated within the framework of the answer to imposed forces (cf [§4.4.1]), within this framework one is interested in answers of size of type "displacement" (`SIZE = DEPL_R` under the keyword `ANSWER`).

sources of pressure and force, for reasons of modeling of the fluid sources are represented by dipoles [bib5], it is thus necessary to give **two points of application**.

Source of flow-volume: `SIZE = SOUR_DEBI_VOLU` under the keyword `EXCIT`

A volume flow rate is expressed in m^3/s , its spectral concentration of power in $(m^3/s)^2/Hz$.

A source of flow-volume is considered, in the formulation $P-\phi$ pipes with fluid, like an effort imposed on the degree of freedom ϕ node of application of the source [R4.02.02].

The user provides the DSP of volume flow rate $\mathbf{S}_{vv}(\omega)$, the DSP $\mathbf{S}'_{vv}(\omega)$ applied in effort to the degree of freedom ϕ is: $\mathbf{S}'_{vv}(\omega)=(\rho\,\omega)^2\mathbf{S}_{vv}(\omega)$

where ρ is the density of the fluid.

Source of flow-mass: `SIZE = SOUR_DEBI_MASS` under the keyword `EXCIT`

A flow-mass is expressed in kg/s , its spectral concentration of power in $(kg/s)^2/Hz$. flow - mass is the product of flow-volume by the density of the fluid.

The user provides the DSP of flow-mass $\mathbf{S}_{mm}(\omega)$, the DSP $\mathbf{S}'_{mm}(\omega)$ applied in effort to the degree of freedom ϕ is: $\mathbf{S}'_{mm}\omega=\omega^2\mathbf{S}_{mm}\omega$

Source of pressure: `SIZE = SOUR_PRESS` under the keyword `EXCIT`

A source of pressure is applied in *Aster* in one **dipole** $P_1 P_2$.

For a source of pressure whose DSP is $\mathbf{S}_{PP}(\omega)$, expressed in Pa^2/Hz, *Aster* built a matrix of density interspectrale of power $\mathbf{S}'_{pp}(\omega)$ who is applied in force imposed to the degree of freedom ϕ points P_1 and P_2.

$$\mathbf{S}'_{PP}(\omega)=\mathbf{S}_{PP}(\omega)\begin{pmatrix}\left(\frac{S}{dx}\right)^2 & -\left(\frac{S}{dx}\right)^2\\ -\left(\frac{S}{dx}\right)^2 & \left(\frac{S}{dx}\right)^2\end{pmatrix}$$

where S is the fluid section, dx the distance enters the two points P_1 and P_2.

Source of force : `SIZE = SOUR_FORCE` under the keyword `EXCIT`

The force corresponds simply to the product of the pressure by the fluid section of the tube: $F=PS$. It thus is also applied to one **dipole** $P_1 P_2$.

For a source of force whose DSP is $\mathbf{S}_{FF}(\omega)$, expressed in N^2/Hz, *Aster* bracket in force imposed on the degree of freedom f points P_1 and P_2, (distant of dx), the matrix of density interspectrale of power $\mathbf{S}'_{FF}(\omega)$ such as:

$$\mathbf{S}'_{FF}(\omega)=\mathbf{S}_{FF}(\omega)\begin{pmatrix}\left(\frac{1}{dx}\right)^2 & -\left(\frac{1}{dx}\right)^2\\ -\left(\frac{1}{dx}\right)^2 & \left(\frac{1}{dx}\right)^2\end{pmatrix}$$

4.4.3 Excitation distributed on a function of form

If spectral concentration of power of the excitation $E(\omega)$ corresponds to an effort imposed on a function of form f_i, $E(\omega)$ give the frequential dependence of the level of the excitation.

The space weighting of the effort is represented in *Code_Aster* by a field with the nodes which does not depend on the frequency: keyword `CHAM_NO` under the keyword factor `EXCIT`. This field with the nodes is a "assembled vector". From the theoretical point of view the formalism of calculation is the same one as previously (excitation in imposed force [§4.4.1]), for a vector of force in second member equal to f_i.

4.5 Applications

These various types of excitation are included in the tests of validation, and are presented for examples in the report [bib6]. In particular the excitations of the fluid type are in the test: pipe subjected to random fluid excitations [V2.02.105] (SDLL105). The excitations on functions of form are tested in the case test: beam subjected to a random excitation distributed [V2.02.106] (SDLL106).

5 Bibliography

1) P. LABBE and H. NOAH: "Stochastic approach for the seismic design of nuclear power seedling equipment". Nuclear Engineering and Design 129 (1991) 367 - 379.

2) A. DUMOND Report EDF DER HP62/95.021B: Post treatment of a vibratory calculation of mechanics under random excitations in *Code_Aster*. Note of reference of order POST_DYNA_ALEA.

3) G. JACQUART Report EDF DER HP61/93.073: Random generations of signals of spectral concentration given: Note of principle and specifications of ASTER integration.

4) Fe. WAECKEL Report EDF DER HP62/95.017B: Method for calculation by modal superposition of the seismic answer of a multimedia structure.

5) P. THOMAS: Taking into account of the acoustic sources in the models of pipings in mechanics. Bulletin of DER - series A. Thermal Hydraulic Nucléaire n° 2 1991 pp19-36.

6) C. DUVAL Report EDF DER HP-61/92.148: Dynamic response under random excitations in *Code_Aster* : theoretical principles and examples of use.

7) BENDAT and PIERSOL: Spectral engineering applications of correlation and analysis. John Wiley and His 1980.

8) MARPLE: DIGITAL spectral analysus with applications. Prentice Hall 1987.

9) H. TAJIMI A statistical method of determining the maximum answer of has building structure during year earthquake. Proc 2Nd world Conf. Earthquake Eng. Tokyo and Kyoto, Japan (1960) pp 751-797.

6 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
4	A.Dumond EDF-R&D/AMA	Initial text
14	F.Voldoire EDF-R&D/ERMES	Small corrections