Generation of seismic signals

Summary:

In the seismic studies “best-estimate”, one is often brought to carry out transitory dynamic analyses. The seismic signal is then modelled by a stochastic process. This process expresses acceleration on the ground according to time.

One calls trajectories the temporal signals (accélérogrammes) which are achievements of the stochastic process. These signals can come from the databases (accelerations on the ground measured at the time of earthquakes) or being obtained by digital simulation (cf also [bib17]*)*. The simulation of artificial seismic signals, as realized by the operator `GENE_ACCE_SEISME`, is described in this document. The operator proposes two modelings:

- model of DSP of Kanai-Tajimi (§2)
- model of DSP compatible with a spectrum of answer targets (§3)

In both cases, it is possible to introduce an evolution of the frequential contents and amplitude (modulation) with the duration of the earthquake.

Contents

1 Modeling of the seismic excitation by a stochastic process

The Gaussian stochastic processes are entirely characterized by their spectral concentration of power (DSP). If the process is stationary, then one can very easily simulate trajectories using the theorem of spectral representation as described in the §4. This theorem also applies to the nonstationary Gaussian processes under the condition which the process can be expressed in the form of an evolutionary DSP with low oscillation (i.e. the properties change slowly with time).

In a seismic analysis simplified, one models only the strong phase of the seismic signal. This last is then supposed to be stationary during this period. It is in particular the case within the framework of the linear seismic analyses by modal combination (CQC, SRSS).

Nevertheless, of the studies and statistical analyses showed that the seismic signal is nonstationary in amplitude and frequential contents. As for the frequential contents, its evolution can be estimated starting from natural signals. In most of the time one observes a decrease of the centre frequency with time [bib9]. This fall of the centre frequency of the seismic excitation can go hand in hand with the fall of the Eigen frequencies of the structRUE subjected to the earthquake. This coincidence can have worsening effects which it is advisable to take into account in nonlinear modelings of structures.

1.1 Modeling of the earthquake by an evolutionary process with evolutionary DSP

One models the seismic movement by achievements of a centered Gaussian process described by his spectral concentration of power évolutivE [bib8]:

$$S_X(\omega,t)=q(t)S_Y(\omega,t)q(t) \quad \in \mathbb{R} \tag{1}$$

where $S_Y(\omega,t)$ is the nonseparable part of the spectral concentration which models the evolution of the frequential contents of the seismic signal in the course of time. The function $q(t)$ east is a deterministic function of modulation which expresses the amplitude of the signal. If one has $S_Y(\omega,t)\equiv S_Y(\omega)$, i.e. the DSP does not depend on time, Y is a stationary process and $X(t)=q(t)Y(t),\ t\in T$ a quasi stationary process whose only amplitude varies with time. One speaks then separable DSP: $S_X(\omega,t)=q^2(t)S_Y(\omega)$.

In *Code_Aster* , processes are considered $Y(t)$ standardized so that the standard deviation is unit at every moment of time. It is the function of modulation which determines the pace of the signals and their energy (and thus amplitude over one duration of fixed strong phase). This is illustrated on **Figure 1a** . The amplitude of the signals to be generated is then determined by the data of the intensity of Arias I_a (energy), of the standard deviation σ_X over the duration of the strong phase, or of the median maximum a_m over the duration of the strong phase. If one wishes to generate “compatible” signals with a spectrum of answer, then the energy of the signal and its amplitude are determined by the data of the target spectrum.

ON proposes two functions of mtemporal odulation of the amplitude $q(t)$ in *Code_Aste*R: the function gamma and the function of Jennings & Housner [bib4]. The function gamma is written:

$$q(t)=\alpha_1 t^{(\alpha_2-1)}\exp(-\alpha_3 t),\ t\in T \tag{2}$$

Parameters α_2 and α_3 the form and the duration of the strong phase of the signal describe respectively. The parameter α_1 determine the energy of the signal and can be given starting from the data of the intensity of Arias. The function of Jennings & Housner [bib4] is written:

$$q(t)=\begin{cases}(t/t_1)^2 & si\ \ 0\le t<t_1\\ 1 & si\ \ t_1\le t\le t_2\\ \exp(-\alpha(t-t_2)^{\beta}) & si\ \ t_2<t\le T\end{cases} \quad (3)$$

Parameters α and β determine the pace of the slope afterwards t_2.

The parameters of these two functions are given starting from the data of the duration of strong phase T_{SM} and the moment of the beginning of the strong phase t_{ini}.

Note:

The parameters of the functions of modulation given starting from median values, like are described below. The strong, maximum phases (PGA) and intensities of Arias of the seismic signals generated with this modeling will consequently display a certain variability around the averages.

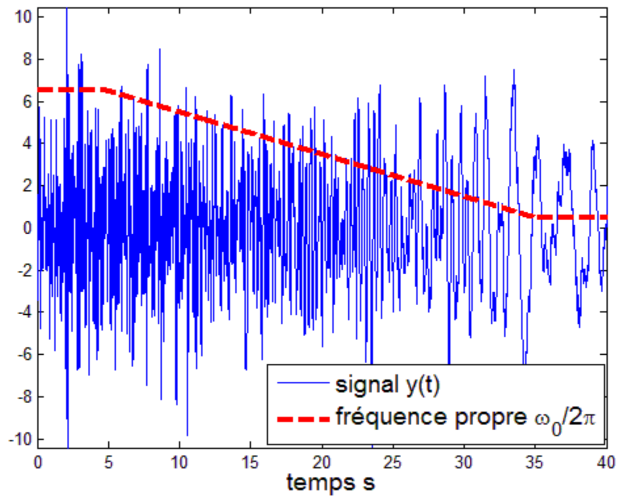

Figure 1a. Illustration of an evolution of the frequential contents of a signal without modulation (DSP of Clough & Evolutionary Penzien with $\xi_0=0.2$).

1.2 Parameter setting of the functions of modulation

1.2.1 Function of modulation Gamma

, By least squares, the parameters are identified α_2 and α_3 such as the moment $\bar{t}_1$ corresponds to the beginning of the strong phase and $\bar{t}_2=\bar{t}_{ini}+\bar{T}_{SM}$ at the end of the strong phase. The parameter α_1 is a constant of standardisation which is selected such as the equation (2) is checked if the intensity of Arias is given like parameter of entry.
In the case where a standard deviation or the median maximum is given, one standardizes the function Gamma of kind so that energy during the strong phase corresponds to the energy of a constant unit modulation: .

$\int_{\bar{t}_1}^{\bar{t}_2} q(t)^2 dt = \bar{T}_{SM}$. For the generation of seismic signals compatible with a spectrum of answer (cf §3) one chooses, in agreement with the definition of the factor of peak, the equivalence of total energy with a signal not modulated of total duration T_{SM}: $\int q(t)^2 dt = T_{SM}$.

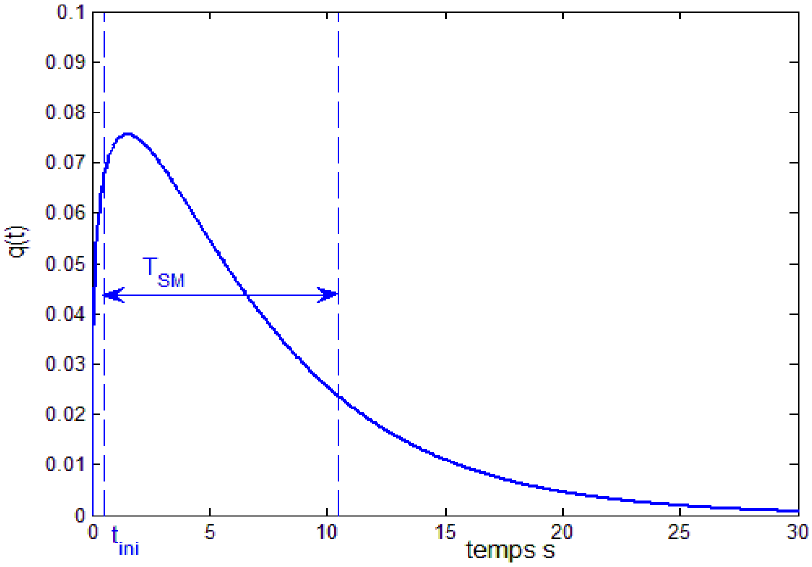

Figure 1b. Function of modulation Gamma for T_{ini}=0.5s and T_{SM}=10s, I_{has}=0.5.

1.2.2 Function of modulation of Jennings & Housner

One determines the moment $\bar{t}_{ini}$ as well as the two parameters, α and β, function of modulation of Jennings & Housner of kind so that the strong phase $\bar{T}_{SM}$ corresponds to that given. Using the duration of the strong phase one determines then the moment $\bar{t}_2$ who corresponds to the end you plate.

The length of the plate thus corresponds always to the duration of the strong phase $\bar{T}_{SM}$. Parameters α and β model are parameters which determine the pace of the function of modulation beyond the strong phase. The function is then multiplied by a constant in order to respect the average intensity of Arias $\bar{I}_a$, a standard deviation or the median maximum.

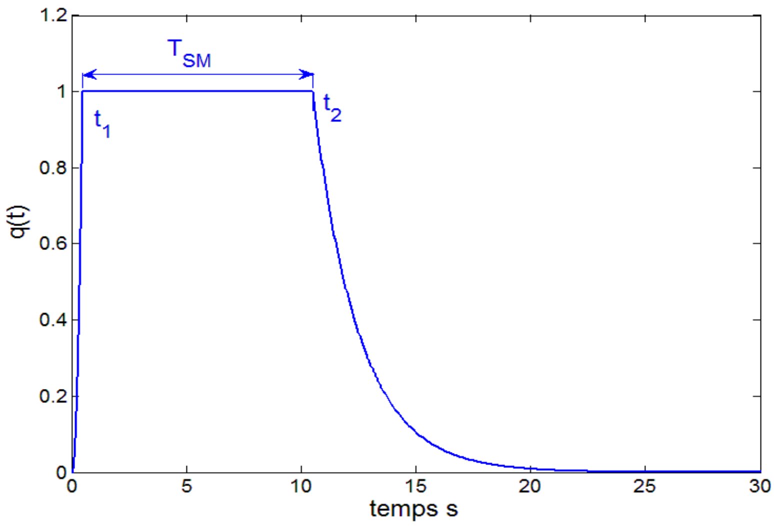

Figure 1c. Example of function of temporal modulation of Jennings & Housner for $T_{SM}=10\mathrm{s}$ **.**

2 Model of evolutionary DSP of Kanai-Tajimi

The model of DSP of Kanai-Tajimi [bib13] who expresses a filtered white vibration and models seismic acceleration in free field is one of most classical. The two parameters of the original model of Kanai-Tajimi are thus the own pulsation of the filter as well as the damping of this last. This model knew evolutions since, in particular with regard to the filtering of the contents low frequencies [bib2] and the introduction of the evolution of the frequential contents of the seismic signals (*Ahmadi & Fan* [bib1]*)*. The filtering of the low frequencies makes it possible to obtain signals in acceleration which can be integrated without drifts in displacement and of speed (it is necessary to resort to " *baseline correction* " so of the constants of integration must be eliminated). Lastly, a function of modulation is applied so D'to obtain the variation of the amplitude. This model is established in *Code_Aster and* available via the operator `GENE_ACCE_SEISME`, option `DSP`.

2.1 General formulation of the model

Modeling selected is summarized in what follows:

- The process Y is characterized by the DSP of Kanai-Tajimi (KT): this DSP models the answer in absolute acceleration of a mass subjected to an excitation by white vibration (the seismic movement with the rock). In other words, the DSP of Kanai-Tajimi describes a white vibration filtered by a clean oscillator of pulsation ω_0 and reduced damping ξ_0 .

- The fundamental pulsation ω_0 DSP of evolutionary KT is a function of time:

$$S_{KT}(\omega,t)=\frac{(\omega_0(t)^4+4\xi_0^2\omega_0(t)^2\omega^2)}{((\omega_0(t)^2-\omega^2)^2+4\xi_0^2\omega_0(t)^2\omega^2)}S_0 \qquad (10)$$

One supposes a linear evolution of the own pulsation compared to time during the strong phase of the earthquake:

$$\omega_0(t)=\begin{cases}\omega_1 & si \quad t<t_1\\ \omega_1-\omega'(t-t_m) & si \quad t_1<t<t_2\end{cases} \qquad (11)$$

where t_1 and t_m the beginning and the moment of medium of the strong phase indicate respectively: $t_m=0.5(t_1+t_2)$ (cf also §1.2 for a definition of these quantities). For $t<t_1$ the own pulsation is supposed to be constant is equal to $\omega_1=\omega_0(t_1)$.

For $t>t_2$, the own pulsation is supposed to be constant is equal to $\omega_2=\omega_1-\omega'(t_2-t_1)$. The user must take care that the own pulsation given by this relation remains positive in the course of time.

- Filtering of the DSP of KT by a clean filter of pulsation ω_f and a reduced damping $\xi_f=1.0$ according to Clough & Penzien (CP). This filtering makes it possible to very remove the contents in low frequencies which lead to nonworthless drifts (displacements and nonworthless speeds at the end of the earthquake). In the implementation *Code_Aster*, one takes $\omega_f=0.05\,\omega_0$ by default, but the user can provide another value if it wishes it. The seismologists call the frequency ω_f "Frequency corner", namely the minimal frequency below which the spectrum must tend towards 0. This leads us to the corrected DSP $S_{CP}(\omega)$:

$S_{CP}(\omega,t)=|h(\omega)|^2 S_{KT}(\omega,t)$, where the filter is written $h(\omega)=\dfrac{\omega^2}{(\omega_f^2-\omega^2+2i\,\xi_f\,\omega_f\,\omega)}$

- Two functions of temporal modulation of the amplitude are proposed $q(t)$: the function gamma and the function of Jennings & Housner [bib4]. These functions as their parameter setting are described in chapter 1.

The evolution of the fundamental pulsation of the model of evolutionary Kanai-Tajimi is illustrated on **Figure 1a**. **Figure 2a** watch the DSP of filtered Kanai-Tajimi (DSP of Clough & Penzien) for a given Eigen frequency. For functions of regular DSP, the Eigen frequency of the filter is close to the fundamental frequency of the DSP. In figure 1, one also visualizes the bandwidth of the DSP, noted δ . bandwidth of the DSP is related to the damping of the filter as clarified in the §3.1.

Note:

A model very similar to that described above is proposed in the reference (Rezaeian & Der Kiureghian, [bib9, biberon10])*. The difference between the two approaches lies mainly in the writing of the problem in the time field by* Rezaeian & Der Kiureghian *and not in the field of the frequencies through the spectral concentration as proposed here. However, the writing in the time field has a certain number of disadvantages, like the need for evaluating the integral of convolution and for working with the process of Wiener (white vibration) which is not a process of the second order. The formulation of the problem by a filtered white vibration described by its evolutionary DSP as proposed here is, on the other hand, very easy and numerically much more effective.*

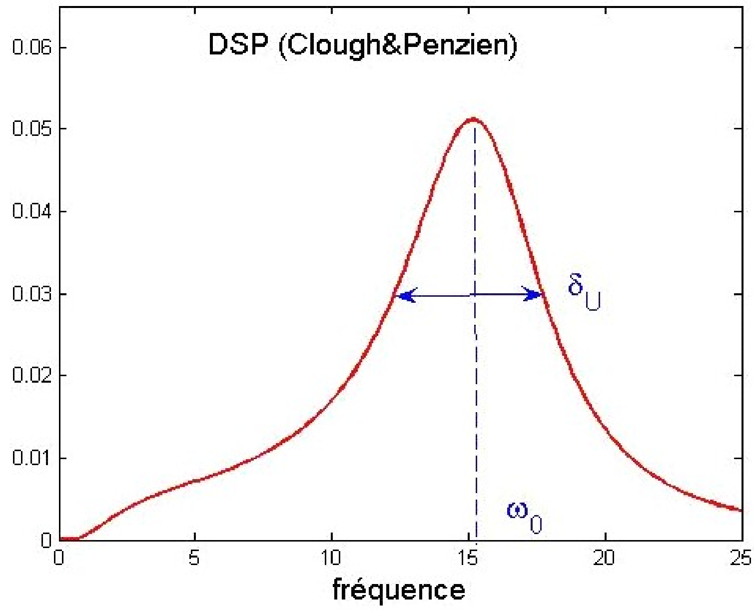

Figure 2a DSP of Kanai-Tajimi after filtering of the low frequencies according to Clough & Penzien [bib2].

2.2 Parameter setting of the functions of modulation

The parameters which characterize the pace of the function of modulation are the duration of strong phase T_{SM}, the moment of the beginning of the strong phase t_{ini}. The amplitude of the signals to be generated is determined by the data of the intensity of Arias I_a (energy), the standard deviation σ_X on the duration of the strong phase or the median maximum a_m over the duration of the strong phase.

One supposes given the averages of these parameters, in particular lasted of strong phase average $\bar{T}_{SM}$, moment beginning of the average strong phase $\bar{t}_{ini}$ and average intensity of Arias $\bar{I}_a$.

In practice, the median maximum a_m is associated with the data of the PGA (*Peak Ground Acceleration*). Thus, if the PGA is given, then it is considered that this one corresponds to the median of the maximum one $a_m = Médiane(max_{t\in T}(|X(t)|))$ trajectories of the process (during the strong phase T_{SM}). This enables us to determine the standard deviation σ_X correspondent with this maximum using the factor of peak $\eta_{T_{SM}}$:

$$a_m \approx \eta_{T_{SM}} \sigma_X \qquad (8)$$

The factor of peak is given by the formula

$$\eta^2_{T_{SM},p} = 2\ln(2N_\eta[1-\exp(-\delta^{1.2}\sqrt{\pi\ln(2N_\eta)})]) \qquad (9)$$

where $N_\eta = 1{,}4427\, T_{SM}\nu_0^+$. Variables ν_0^+ (centre frequency) and δ (bandwidth) are calculated as from the moments of the DSP saltone it expression (17) of the §3.1.

Average intensity of Arias $\bar{I}_a$ of a process $Y(t)\in\mathbb{R}$ modulated by a function $q(t)$, described by the equation (1), expresses itself like:

$$\bar{I}_a=\boldsymbol{E}\left(\frac{\pi}{2g}\int_0^{\infty}X^2(t)\,dt\right)=\frac{\pi}{2g}\int_0^{\infty}q^2(t)\boldsymbol{E}(Y^2(t))\,dt \tag{4}$$

knowing that $X(t)=q(t)Y(t)$. The operator $\boldsymbol{E}$ in the Expression above indicates the expectation and $g=9.81\mathrm{m}/s^2$. If the process is standardized $Y(t)$ of kind so that $\sigma_Y(t)=1$, one can write:

$$\bar{I}_a=\frac{\pi}{2g}\int_0^{\infty}q^2(t)\,dt \tag{5}$$

In practice integration is done until the moment of end T seismic signals. Duration of the average strong phase $\bar{T}_{SM}$ is defined starting from the intensity of Arias which expresses energy contained in the seismic signal. Thus, $\bar{T}_{SM}$ is defined like the duration of time between the moment of time $t_{0.05}$ and $t_{0.95}$ where respectively 5% and 95% of the intensity of Arias are carried out. The moment $t_{0.05}$ indicate the beginning of the average strong phase consequently, $\bar{t}_{ini}$, such as:

$$\bar{t}_{ini}:\ \frac{\pi}{2g\bar{I}_a}\int_0^{t_{ini}}q^2(t)\,dt=0.05 \tag{6}$$

and $\bar{t}_{ini}+\bar{T}_{SM}$ end of the strong phase:

$$\bar{t}_{ini}+\bar{T}_{SM}:\ \frac{\pi}{2g\bar{I}_a}\int_0^{\bar{t}_{ini}+\bar{T}_{SM}}q^2(t)\,dt=0.95 \tag{7}$$

Being given the duration of the strong phase, the average intensity of Arias $\bar{I}_a$ allows to determine the standard deviation of the process X.

Note: *The parameters of the functions of modulation given starting from the phase the strong average and average intensity of Arias, like are described above. Strong, maximum phases (PGA) and intensities of Arias of the seismic signals generated with this modeling will consequently display a certain variability around the averages.*

2.3 Uncertainties and natural variability of the signals

The parameters of the model of Kanai-Tajimi are:

- Average duration of the strong phase $\bar{T}_{SM}$ and the moment of beginning of the average strong phase $\bar{t}_{ini}$ (function of modulation),
- The own pulsation ω_1 and the slope ω' DSP of Kanai-Tajimi (or ω_0 if one considers a constant value),
- Reduced damping ξ_0 DSP of Kanai-Tajimi,
- Average intensity of Arias $\bar{I}_a$ (average energy contents in the seismic signal), the PGA (median maximum a_m) or the standard deviation σ_X.

In order to better represent the natural variability of the seismic signals and to take account of uncertainty on the parameters of the model, one can model the latter by random variables. Each simulated seismic signal corresponds then to a particular pulling of the parameters of the model. The method of pulling of Latin Hypercube constitutes an effective method which makes it possible to sweep well the field of definition of the parameters for a reduced number of pullings.

The median values as well as the distributions of these parameters can be estimated starting from natural seismic signals corresponding to the required scenario. They are, for certain, also available in the literature. One finds of it a treatment very exhaustive in the reference [bib9]. Averages, minimal and maximum values and distributions of the parameters, identified starting from accélérogrammes recorded on average ground with hard and $D>10\text{km}$ (resulting from the base of earthquakes NGA [bib14]), are presented in the appendix of this document. These results are drawn from the report from Rezaeian & Der Kiureghian [bib9].

3 Model of DSP to generate "compatible" signals with a target SRO

If the earthquake is modelled by a Gaussian stationary process during the strong phase T_{SM}, it is possible to establish a relation (approximate) between the spectral concentration $S_Y(\omega)$ process and its spectrum of answer of oscillator (SRO), $S_a(\omega,\xi)$. This last indeed often is given by the seismologists or prescribed by codes and regulations.

3.1 Identification of a DSP compatible with a target spectrum

The problem of the first passage makes it possible to bind the SRO (for a pulsation ω_n and a reduced damping ξ_0 given) to the standard deviation of the process via the factor of peak η :

$$S_a(\omega_n,\xi_0)\approx\omega_n^2\eta_{T_{SM},p}\sigma_n \tag{12}$$

where σ_n is the standard deviation of the process answer

$$\sigma_n^2=\int\left|h_{\omega_n,\xi_0}(\omega)\right|^2 S_Y(\omega)\,d\omega \tag{13}$$

and:

$$h_{\omega_n,\xi_0}(\omega)=\frac{1}{\omega_n^2-\omega^2+2i\xi_0\omega_n\omega} \tag{14}$$

is the transfer transfer function (filter) of own pulsation ω_n and of reduced damping ξ_0.
The factor of peak makes it possible to estimate them p - fractiles of the distribution of maximum of a Gaussian process starting from the standard deviation. The factor of peak, due to Vanmarcke [bib16], is written:

$$\eta_{T_{SM},p}^2=2\ln(2\mathrm{N}_\eta[1-\exp(-\delta^{1.2}\sqrt{\pi\ln(2\mathrm{N}_\eta)})])\,. \tag{15}$$

In this expression, δ is the bandwidth of the process (confer Figure 2a) and N_η is determined from

the centre frequency ν_0^+ like

$$N_\eta=T_{SM}\nu_0^+(-\ln p)^{-1} \tag{16}$$

In this expression, p indicate the fractile of the distribution of maximum considered. In general, one takes $p=0.5$ what corresponds to the "median of maximum". The centre frequency ν_0^+ indicate the frequency where the energy of the process is concentrated. The centre frequency and the bandwidth can be expressed by the classical formulas of Rice:

$$\nu_0^+=\frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}},\quad \delta=\sqrt{\left(1-\frac{\lambda_1^2}{\lambda_0\lambda_2}\right)} \tag{17}$$

where them λ_i are the spectral moments of the DSP of the process answer:

$$\lambda_i=\int_{-\infty}^{+\infty}|\omega|^i\left|h_{\omega_n,\xi_0}(\omega)\right|^2 S_Y(\omega)\,d\omega\ , \tag{18}$$

frontEC in particular $\sqrt{\lambda_0}$ the standard deviation of the process. In order to determine DSP compatible with the SRO, it is necessary to reverse the expression (12). One can show that one has, at first approximation, for an excitation by white vibration ($S_Y(\omega)=const$) and for a filter with Eigen frequency ω_n :

$$\lambda_0=\int_{-\infty}^{+\infty}\left|h_{\omega_n,\xi_0}(\omega)\right|^2 S_Y(\omega)\,d\omega\approx S_Y(\omega_n)\int_{-\infty}^{+\infty}\left|h_{\omega_n,\xi_0}(\omega)\right|^2 d\omega=\frac{\pi S_Y(\omega_n)}{2\xi\omega_n^3} \tag{19}$$

This formula was improved in the continuation by Vanmarcke [bib15, bib16] to take account of the contribution of $S_Y(\omega)$ in the range of the pulsations $[-\omega_n,\omega_n]$ where $h_{\omega_n}(\omega)\approx 1/\omega_n^2$:

$$\lambda_0\approx\frac{\pi S_Y(\omega_n)}{2\xi\omega_n^3}+\frac{2}{\omega_n^4}\int_0^{\omega_n}S_Y(\omega_n)\,d\omega-\frac{2}{\omega_n^3}S_Y(\omega_n) \tag{20}$$

This leads us to the formula of Vanmarcke to evaluate a DSP $G_S(\omega)$ starting from a SRO $S_a(\omega,\xi)$ (for a damping ξ_0 given):

$$G_S(\omega_n)=\frac{1}{\omega_n\left(\frac{\pi}{2\xi_0}-2\right)}\left[\frac{S_a^2(\omega_n,\xi_0)}{\eta_{T_{SM},p}^2}-2\int_0^{\omega_n}G_S(\omega)\,d\omega\right],\quad \omega_n>0 \tag{21}$$

under conditioN that these two functions are sufficiently smooth. The original formula of Vanmarcke was then used in more or less close forms by many authors [bib7, biberon3]. It is established in its original version, namely formula (21), in `GENE_ACCE_SEISME`.
To determine a DSP compatible with a SRO by this formula, it is necessary to know the factor of peak (15) which, him, depends on the properties of the DSP to identify (and which is thus still unknown). With this intention, one approaches centre frequency by the Eigen frequency of the oscillator of kind so that $N_\eta=T_{SM}\frac{\omega_n}{2\pi}(-\ln p)^{-1}$. Moreover, L is expressedbandwidth according to reduced damping has ξ , always for the response of an oscillator to a white vibration, by the formula [bib15]:

$$\delta=\sqrt{1-\frac{1}{1-\xi^2}\left(1-\frac{1}{\pi}\arctan\left(\frac{2\xi\sqrt{1-\xi^2}}{1-2\xi^2}\right)\right)^2} \tag{22}$$

For low values of damping $\xi<0.1$, one can consider damping reduced by the asymptotic relation for $\xi\rightarrow 0$, following [bib15]:

$$\delta\approx\sqrt{\frac{4\xi}{\pi}}$$

A good approximation of reduced damping can be obtained while taking $\xi \approx \dfrac{\pi \delta}{4}^{2/1.2}$.

The identified DSP can then be used to generate trajectories of the stationary or quasi stationary process (variation of the amplitude by multiplication by a function of modulation Q (T)) . In order to stick as well as possible to the target spectrum, it is in general necessary to proceed to iterations to adjust the spectral contents as well as possible [bib16].

Initially, one improves the adjustment the DSP by calling on the formulas of Rice. These last make it possible to bind the DSP to the SRO, through the factor of peak. One determines the moments of the DSP for then calculating the factor of peak (15) thanks to the expressions (17). Thus, starting from the DSP with iteration I, $G_S^{i+1}(\omega_n)$, the corresponding SRO is determined $S_a^i(\omega_n)$ by the formula (12), and one evaluates the new estimate of the DSP, $G_S^{i+1}(\omega_n)$, while calculating:

$$G_S^{i+1}(\omega_n) = G_S^i(\omega_n)\left(S_a(\omega_n)/S_a^i(\omega_n)\right)^2 \qquad (23)$$

In the expression (23), $S_a(\omega_n)$ is the target spectrum of answer and $S_a^i(\omega_n)$ is the spectrum of answer, within the meaning of the median of maximum, determined by the formulas of Rice for the DSP $G_S^i(\omega_n)$.

In the second time, one can directly adjust the spectral contents of the temporal signal or a set of signals. This is the approach generally adopted in the literature, for example [bib16]. In this approach, less rigorous from a point of view of the treatment of the signal and stochastic processes, one generates a signal, one calculates his spectrum of answer, and one reiterates directly on his spectral contents. The same formula is used that before (expression (23)), which leads to:

$$A_S^{i+1}(\omega_n) = A_S^i(\omega_n) S_a(\omega_n)/S_a^i(\omega_n) , \qquad (24)$$

but where the spectrum $S_a^i(\omega_n)$ with the iteration i is the spectrum of answer determined starting from the accélérogramme to the iteration i and $A_S^i(\omega_n)$ indicate the spectral contents of the signal.

Notice*: From a mathematical point of view, the spectral contents of a realization of the process (or accélérogramme) correspond to the realization of the incremental process dZ defined in the §4. It is the latter which is updated in the iterations in order to improve the adjustment with the target SRO.*

If one wishes to generate several compatible signals in median with a target SRO, it is necessary to reiterate on the spectral contents of this whole of signals (the sample). Considering the case where one wishes to generate L signals. The spectrum of answer to be calculated with each iteration is the median of the SRO of L signals. These is then the median SRO which is used in the formula (24) instead of $S_a^i(\omega_n)$. This in particular makes it possible to obtain a set of independent signals (and not correlated). This property is not guaranteed while generating and by carrying out the iterations on the distinct signals.

3.2 Nonstationary parametric model for the generation of signals compatible with a spectrum

The DSP compatible with the spectrum identified according to the formula (21) does not make it possible to generate nonstationary signals with evolutionary frequential contents. For that, it is necessary to return to a parametric model in which the fundamental pulsation ω_0 fact part of the parameters. However, any process (Gaussian) physical can be described by a rational spectral concentration. One thus works with a rational DSP which constitutes a generalization of the model of Kanai-Tajimi.

More precisely, the model selected is the following:

$$S_{FR}(\omega, t)=\frac{(R_0^2+R_2^2\omega^2)}{((\omega_0(t)^2-\omega^2)^2+4\xi_0^2\omega_0(t)^2\omega^2)} \quad (25)$$

Parameters of this model, ω_0, ξ_0 and R_0, R_1, are identified by an algorithm of optimization of kind to minimize the error between the DSP compatible with the spectrum, G^S, (equation (21)) and the rational DSP, S_{FR} (equation (25)). The method of Simplex is used by `GENE_ACCE_SEISME`. To initialize the algorithm, one chooses the parameters corresponding to the model classic of Kanai-Tajimi. In addition, the filter of Cough & Penzien describe in the §2 is applied.

Notice: *The formulation (25) was retained because itintroduction of additional parameters, by increasing the order of the polynomials, does not lead to better results.*

The rational DSP makes it possible to generate signals to which the average spectrum is close to the target. In the case general, there remains a difference because of to have chosen a parametric model. In order to stick as well as possible to the target spectrum, one applies the equivalence of the energy of the stationary process on the duration of the strong phase and the evolutionary process:

$$\int_0^{T_{SM}} S_{FRC}(\omega, t)\,dt=G^S(\omega)\int_0^{T_{SM}} dt \quad (26)$$

where $S_{FRC}(\omega, t)=S_0(\omega)S_{RC}(\omega, t)$ is the rational DSP compatible with the spectrum. If one considers a nonevolutionary rational DSP this amounts applying a “corrective” term, $S_0(\omega)=G^S(\omega)/S_{RC}(\omega)$, which expresses the difference between the DSP compatible with the SRO and the model of evolutionary rational DSP. It is this last expression which is implemented by choosing the value of reference to the moment t_m (medium of the strong phase): $S_{RC}(\omega, t_m)$.

The compatible rational model summarizes itself as follows:

- The process Y is characterized by the DSP of the rational type (FR) corrected to adjust itself as well as possible with the DSP compatible with the SRO.
- The centre frequency ω_0 DSP of KT evolutionary is a function of time
- A function of temporal modulation of the amplitude is applied $q(t)$ (cf section 1) to obtain the evolutionary DSP (frequential contents and amplitude) S_X.

3.3 Introduction of the natural variability of the signals (record-to-record variability)

According to the laws of attenuation, spectral accelerations are supposed to follow a law lognormale. Consequently, the spectra of answer of the movement of the ground can be modelled by vectors lognormales. To facilitate the mathematical notations and handling, one brings back oneself to Gaussian vectors by considering the log-spectra rather $ln S_a$. The components of this vector are spectral accelerations at the frequencies $\omega=\omega_{1,}\dots,\omega_N$. Thus, always according to the laws of attenuation (for example [bib12]), one has $ln S_a(\omega_n)=f(\omega_n, x)+\epsilon_n\sigma(\omega_n)$ where X indicates the set of parameters characterizing the seismic scenario (magnitude, distance etc) and ϵ_n are standardized Gaussian random variables. This can be also written in the form: $S_a(\omega_n)=\bar{S}_a(\omega_n, x)\alpha_n$ where $\bar{S}_a(\omega_n, x)=e^{f(\omega_n, x)}$ are median spectral accelerations and $\alpha_n=e^{\sigma(\omega_n)\epsilon_n}$ are random variables lognormales of unit median. This simple model neglects the correlations between acceleration spectral. Models of coefficients of correlation developed by Baker [bib3] starting from American databases NGA [bib12, bib14], allows to introduce this correlation.

Thus, for a set of frequencies given, the log-spectrum is a correlated Gaussian vector, of average μ and of matrix of covariance Σ. The latter expresses the correlation between various spectral accelerations. The vector average μ contains the spectral log-accelerations given by the laws of attenuation, $f(\omega_n, x)$, whereas the matrix of covariance can be built starting from the standard deviations $\sigma(\omega_n)$ and coefficients of correlation ρ_{ij} according to the relation:

$$\Sigma_{ij} = \rho_{ij}\,\sigma(\omega_i)\,\sigma(\omega_j) \quad (27),$$

This same distribution lognormale can be introduced into the model of DSP to simulate accélérogrammes whose SRO respect the distribution of the target. Following the example of spectral accelerations, one models the DSP by a random vector $\boldsymbol{S}_{FRC} = (S_{FRC}(\omega_1), \ldots, S_{FRC}(\omega_N))$ of median $\bar{\boldsymbol{S}}_{FRC}$. The terms of this “random” DSP are written:

$$S_{FRC}(\omega_n) = \alpha_n^2 \bar{S}_{FRC}(\omega_n) \quad (28)$$

where $\boldsymbol{\alpha} = (\alpha_{1,} \ldots, \alpha_N)$ lognormal is a random vector correlated, of unit median and matrix of log-covariance Σ.

The choice to use, for the DSP, the same distribution (lognormale) that observed for the SRO is justified by the fact that, for depreciation ξ_0 weak, one has the following approximation:

$$S_a^2(\omega_n, \xi_0) \approx \omega_n^4 \eta_{T_{SM}, p=0,5}^2 \frac{\pi S_Y(\omega_n)}{2\xi_0 \omega_n^3}$$

where $S_a(\omega_n, \xi_0)$ is a target spectrum and $S_Y(\omega_n)$ the DSP compatible with this last. This indicates a relation proportional between the DSP and the square of the SRO. This relation suggests that the DSP compatible with the spectrum must follow, except for a factor, the same distribution as the target SRO. From where the relation (28).

Notice: *The approach for the generation of accélérogrammes whose median spectrum as well as the fractiles are compatible with the distribution of the target SRO (given by the laws of attenuation) is described in the reference [bib18].*

3.4 Simulation of seismic signals compatible with a SRO: features implemented in `GENE_ACCE_SEISME`

For the generation of seismic signals “compatible with a spectrum”, the three following configurations can be considered:

1. Generation of a distinct accélérogramme compatible with the target SRO: one carries out iterations on the spectral contents of the accélérogramme by using the formula (24).
2. Generation of accélérogrammes whose median spectrum is compatible with the target SRO: the median spectrum is considered for the iterations according to the formula (24).
3. Generation of accélérogrammes compatible with the distribution (lognormale) of the SRO: the median spectrum is informed and the spectrum with a sigma, namely the fractile to 84 %, this makes it possible to introduce the variability of the natural accélérogrammes (record-to-record variability) (§3.3).

`GENE_ACCE_SEISME` allows to generate signals according to these three criteria. As for item 2, it is possible to determine the compatible DSP and then to generate signals seismic without iterations on the spectral contents. If the sample of the generated signals is sufficiently large and if the target SRO is a physical spectrum, then the median spectrum is close to the target. For nonphysical spectra of answers, which were widened and undergo other treatments of adjustment, one can always proceed to iterations, the criterion relates then to the median spectrum. The same factor of correction will be applied to all the generated accélérogrammes. Larger sample in general makes it possible to obtain a good adjustment with the SRO with less iterations.

When one integrates the signals generated for obtaining displacement, one can sometimes observe a divergent behavior: displacement are not cancelled at the end of the earthquake. In order to avoid this kind of problem, it is enough to filter the low frequencies of the signals in acceleration (filter passes high). This makes it possible to remove a possible drift in displacement. The choice of the frequency of the filter is done by FREQ_FILTRE.

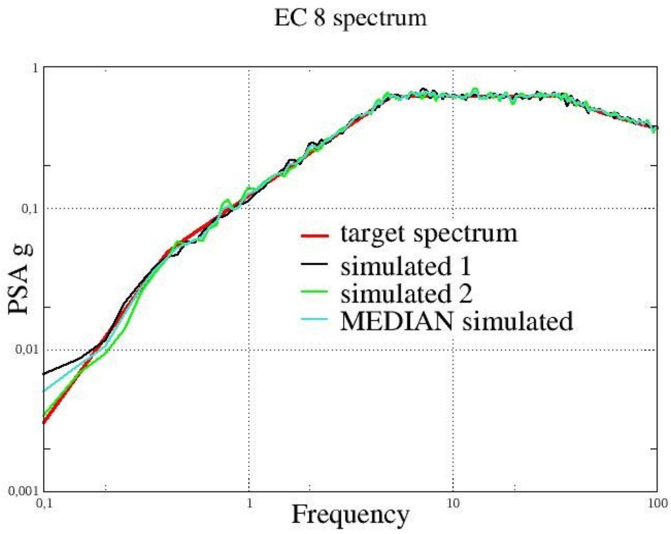

Figure 3a. Illustration of the configuration 1) for a target spectrum EC8: simulation of two accélérogrammes whose spectrum of answer is adjusted as well as possible with the target SRO

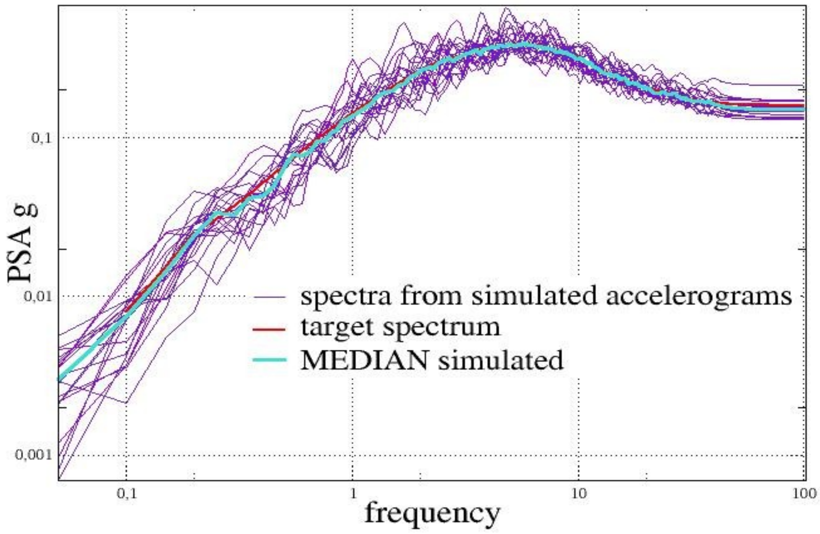

Figure 3b. Illustration of the configuration 2) for a spectrum resulting from base NGA (law of attenuation of Campbell & Bozorgnia): simulation of a game accélérogrammes whose median spectrum of answer (curve blue) is adjusted as well as possible with the target SRO.

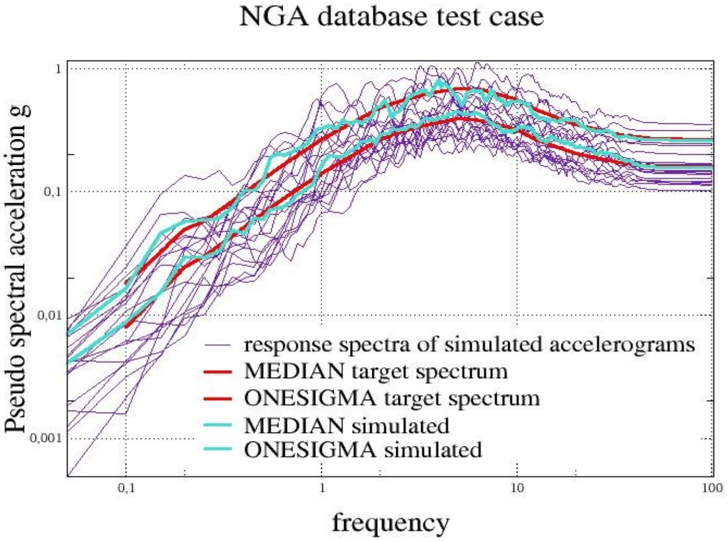

Figure 3c. Illustration of the configuration 3) for a spectrum resulting from base NGA (law of attenuation of Campbell & Bozorgnia): simulation of a game accélérogrammes whose spectra of answer have the distribution of the target SRO; one checks the adjustment of the median and the fractile to a sigma (curves blue).

3.5 Model of DSP of Kanai-Tajimi in agreement with a target SRO

Alternatively, one can estimate the parameters of the DSP of Kanai-Tajimi, namely the Eigen frequency ω_0 and reduced damping ξ_0 , as from the moments of the DSP compatible with the spectrum: $\lambda_i=\int_{-\infty}^{+\infty}|\omega|^i G^S(\omega)\,d\omega$. This makes it possible to build a nonstationary model of earthquake (nonstationary in amplitude and frequential contents) by introducing the evolution of the centre frequency.

The parameter should then be identified ω_0 and ξ_0 so that moments of order 1 and 2 of the DSP of Kanai-Tajimi and of DSP $G^S(\omega)$, compatible with the SRO, are close or coincide. In practice, it is appropriate of identifier ω_0 and ξ_0 such as the centre frequency ν_0^+ and the bandwidth δ DSP, defined in section 3.1, coincide. These two parameters are illustrated on figure 1 and can be calculated as from the moments by the classical formulas of Rice of the expression (17). To determine these quantities, it is necessary to calculate them moments what can be carried out with *Code_Aster* using the operator `POST_DYNA_ALEA`. The own pulsation ω_0 and reduced damping ξ_0 identified are then used to build the model of earthquake with `GENE_ACCE_SEISME`.

Notice : *For the response in displacement of an oscillator to a white vibration, one knows analytical expressions of the parameters* ω_0 *and* ξ_0 *according to the moments and thus of the centre frequency and bandwidth. These expressions also apply to the DSP of Kanai-Tajimi, if damping is weak. The DSP of Kanai-Tajimi corresponds indeed to the answer in acceleration (absolute) of an oscillator subjected to a white vibration and not the answer in displacement.*
Thus, if damping is weak, the Eigen frequency of the DSP of Kanai-Tajimi can be taken equal to the centre frequency. But the approximation is not in general step very good. Studies showed that a better approximation can be obtained in prenaNT the frequency corresponding to the centre of gravity of the positive part of the DSP.
Let us recall that L centre frequency has ν_0^+ *characterize the frequency where the energy of the process is concentrated. In the temporal field, it corresponds to the rate of passages by zero with*

positive slope of the process. It can also be given starting from a accélérogramm E by counting of passages positive by zé ro.

4 Case of vectorial processes

4.1 Seismic signals 2D and 3D

Knowing the coefficient of correlation ρ, it is possible to build a matrix of spectral concentration $\boldsymbol{S}_X(\omega,t)\in Mat_{\mathbb{C}}(M,M)$ describing a vectorial process 2D or 3D starting from the scalar DSP $S_X(\omega,t)\in\mathbb{R}$. In the case of two horizontal components, $M=2$, the matrix of DSP is written:

$$\boldsymbol{S}_X(\omega,t)=\begin{bmatrix} S_X(\omega,t) & \rho \\ \rho & S_X(\omega,t) \end{bmatrix}$$

For the earthquake, the vertical component is generally considered not correlated with the two horizontal components. `GENE_ACCE_SEISME` allows to generate signals 2D with correlated horizontal components (defined by the coefficient of correlation `COEF_CORR`, which can be null) and a vertical component not correlated to both others, but to which applies the horizontal/vertical ratio.

4.2 Variable seismic field in space

Within the framework of a temporal study with a variable seismic excitation in space, the seismic movement is described by a space-time field. After discretization, the problem is reduced to the simulation of a vectorial process, described by its matric DSP as for the problem 2D.
In the literature, functions of coherence γ are proposed to define the space correlation of the seismic movement. Two types of functions of coherences are available for the operator `GENE_ACCE_SEISME` : the function of exponentionnelle coherence of Became moth-eaten & Luco and the empirical function of Abrahamson. One finds more details on the functions of coherence in the documentation of the operator `DYNA_ISS_VARI`, in particular R4.05.04. One builds a matrix of spectral concentration $\boldsymbol{S}_X(\omega,t)\in Mat_{\mathbb{C}}(M,M)$, where M is the number of nodes for which it is necessary to evaluate the field. The components of this matrix are written:

$$S_{ij}(\omega,t)=\gamma(\omega,d_{ij})S_X(\omega,t)$$

where d_{ij} is the horizontal distance between two nodes i and j grid.

5 Simulation of trajectories of the process

Classical method for simulation of a process stochastic Gaussian stationary, centered be based on the spectral representation of the processes, cf for example [bib11] . The spectral representation as well as the algorithm of digital simulation are pointed out in what follows.
One treats the case general of a vectorial Gaussian process $\boldsymbol{Y}(t)\in\mathbb{R}^M, t\in[0,T]$ characterized by its matric DSP $\boldsymbol{S}_Y(\omega)\in Mat_{\mathbb{C}}(M,M)$. This method can be wide with the case of the evolutionary processes with evolutionary DSP $\boldsymbol{S}_X(\omega,t)\in Mat_{\mathbb{C}}(M,M)$ if the process is with slow variation.

5.1 Discretization

The digital simulation requires the discretization of the temporal field. One uses a temporal discretization with step Δt constant. The cut-off frequency results directly from this value: $\Omega_C=\pi/(\Delta t)$. For a discretization by NR not of time, one notes $t_j = j\Delta t,\ j=0,\dots N-1$ points of temporal sampling. In addition, one defines the step of frequency by $\Delta\omega = 2\Omega_C/N$ and the duration of the temporal signal is $T=\Delta t(N-1)$.

The spectral concentrations are defined on the interval $F=[-\Omega_C, +\Omega_C]$ and the points of frequential discretization are selected like the mediums of the paving stones:: $\omega_j = -\Omega_C + (0.5+j)\Delta\omega,\quad j=0,\dots N-1$.

5.2 Spectral representation of the stationary processes

The theorem of the spectral representation (cf. [bib11]) known as that there exists a vectorial stochastic process $d\boldsymbol{Z}(\omega)\in\mathbb{C}^M$ with orthogonal increments such as:

$$\boldsymbol{Y}(t) = \int_{\mathbb{R}^M} e^{i\omega t}\, d\boldsymbol{Z}(\omega) \tag{29}$$

This process, called spectral process associated with $\boldsymbol{Y}$, checks:

$$\boldsymbol{E}(d\boldsymbol{Z}(\omega)\, d\boldsymbol{Z}(\omega')^*) = \begin{cases} 0 & si\ \ \omega\neq\omega' \\ \boldsymbol{S}_{\boldsymbol{Y}}(\omega)\, d\omega & si\ \ \omega=\omega' \end{cases}, \tag{30}$$

where $\boldsymbol{E}$ is the operator of the expectation and $d\boldsymbol{Z}$ is a process with orthogonal increments.

The matrix of spectral concentration $\boldsymbol{S}_{\boldsymbol{Y}}(\omega)$ being, by construction, a positive square matrix for all $\omega\in F$, there exists a matrix $\boldsymbol{L}(\omega)\in\mathbb{C}^M$ such as:

$$\boldsymbol{S}_{\boldsymbol{Y}}(\omega) = \boldsymbol{L}(\omega)\boldsymbol{L}(\omega)^* \tag{31}$$

The matrix $\boldsymbol{L}(\omega)$ can be obtained by decomposition of Cholesky SI $\boldsymbol{S}_{\boldsymbol{Y}}(\omega)$ is definite positive. $\boldsymbol{L}(\omega)$ is then a lower triangular matrix. In the case plus general, the decomposition in clean vectors can be used. If the process is scalar, the problem is reduced to the calculation of the root of the DSP. One can then simulate trajectories of the process by the following formula:

$$\boldsymbol{Y}(t) = \sqrt{\Delta\omega}\,\Re e \sum_{j=0}^{N-1} \boldsymbol{L}(\omega_j)\chi_j e^{i\omega_j t} \tag{32}$$

where them χ_j are complex Gaussian random vectors of which the components are independent.

The use of the IFFT makes it possible to reduce considerably the digital cost of simulation. Indeed, one can write the expression (26) in the form

$$\boldsymbol{Y}(t_k) = \sqrt{\Delta\omega}\,\Re e(\boldsymbol{V}_{\boldsymbol{k}} e^{-i\pi k(1-1/N)}) \tag{33}$$

where V_k is calculable by IFFT:

$$\boldsymbol{V}_{\boldsymbol{k}} = \sum_{j=0}^{N-1} \boldsymbol{L}(\omega_j)\chi_j e^{2i\pi k j N^{-1}}$$

This algorithm can be used for the stationary and nonstationary Gaussian processes with separable DSP. In these cases, one obtains the seismic signals modulated by applying the function of modulation of kind so that $X(t)=q(t)Y(t)$. This is carried out by the operator `GENE_ACCE_SEISME`.

5.3 Spectral representation of the evolutionary processes with evolutionary DSP

The spectral representation always applies to evolutionary processes described by their nonseparable evolutionary DSP:

$$Y(t)=\int_{\mathbb{R}^M} e^{i\omega t} d\,Z(\omega,t) \quad (34)$$

under condition which the DSP evolves slowly with time [bib8,bib6]. There exists a stochastic process then $d\,Z(\omega,t)\in\mathbb{C}^M$ with orthogonal increments such as

$$E(d\,Z(\omega,t)\,d\,Z(\omega',t)^*)=\begin{cases}0 & si\ \ \omega\neq\omega'\\ S_Y(\omega,t)\,d\omega & si\ \ \omega=\omega'\end{cases} \quad (35)$$

where E is the operator of the expectation and $d\,Z$ is a process with orthogonal increments. One can then simulate trajectories of the process by the formula

$$Y(t)=\sqrt{\Delta\omega}\,\Re e\sum_{j=0}^{N-1} L(\omega_j,t)\chi_j e^{i\omega_j t} \quad (36)$$

where them χ_j are complex Gaussian random vectors of which the components are independent. The matrix $L(\omega,t)$ can always be obtained by decomposition of Cholesky if the row of $S_Y(\omega,t)$ is maximum. On the other hand, it is not possible to call on the fast transform of Fourier as in the stationary case (cf equation (33)) what increases the computing times.

One obtains the seismic signals modulated by applying the function of modulation of kind so that $X(t)=q(t)\,Y(t)$. This is carried out in *Code_Aster* by the operator `GENE_ACCE_SEISME`.

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Documentation of Code_Aster:

[R4.05.02]: Stochastic approach for the seismic analysis.

[R4,05,04]: Interaction ground-structure with space variability (operator `DYNA_ISS_VARI`).

[R7.10.01]: Examination of random answers.

[U4.84.04]: Operator `POST_DYNA_ALEA`

[U4.84.01]:: Operator `COMB_SISM_MODAL`

[U4.32.05]: Operator `INFORMATION_FONCTION`

[U4.36.05]: Operator `GENE_FONC_ALEA`

[U4.36.04]: Operator `GENE_ACCE_SEISME`

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
11.2	I.ZENTNER EDF/R & D /AMA	Initial version of the document.
13.1	I.ZENTNER EDF/R & D /AMA	Introduction of space variability: simulation of a vectorial process starting from the data of the DSP and the function of coherence

8 Appendix

One reproduces below tables of the PEER carryforward of Rezaeian & Der Kiureghian ([bib9], p.93-94) where median values as well as the distributions of the parameters of the model of earthquake were estimated starting from American database NGA for a selection of recorded earthquakes on average ground with hard and $D>10\,km$.

Legend:

I_a : intensity Arias; D_{5-95} : duration of the strong phase (here T_{SM}); ζ_f : reduced damping of the filter (here ξ_0); ω_{mid} : Eigen frequency of the filter at the moment $t_{0.45}$ (one works here with the pulsation ω_0 at the moment $t_{0.05}$); ω' : ratio of variation of the Eigen frequency of the filter (the slope of the function describing the evolution of the own pulsation of the filter).

Parameter	Minimum	Maximum	Sample Mean	Sample Standard Deviation	Coefficient of Variation
I_a (s.g.)	0.000275	2.07	0.0468	0.164	3.49
D_{5-95} (s)	5.37	41.29	17.25	9.31	0.54
t_{mid} (s)	0.93	35.15	12.38	7.44	0.60
$\omega_{mid}/2\pi$ (Hz)	1.31	21.6	5.87	3.11	0.53
$\omega'/2\pi$ (Hz/s)	−1.502	0.406	−0.089	0.185	2.07
ζ_f (Ratio)	0.027	0.767	0.213	0.143	0.67

Parameter	Fitted Distribution[8]	Distribution Bounds
$\bar{I}_a$ (s.g.)	Lognormal	$(0,\infty)$
D_{5-95} (s)	Beta	[5,45]
t_{mid} (s)	Beta	[0.5,40]
$\omega_{mid}/2\pi$ (Hz)	Gamma	$(0,\infty)$
$\omega'/2\pi$ (Hz)	Two-sided Truncated Exponential	[−2,0.5]
ζ_f (Ratio)	Beta	[0.02,1]

[8] Means and standard deviations of these distributions are according to columns 4 and 5 of Table 4.3.