Non-linear interaction ground-structure with the method Laplace-Time

Summary:

This document is a theoretical note describing the method Laplace-Time (*cf*. [U2.06.05]) developed in the operator CALC_MISS (*cf*. [U7.03.12]) for the type of result 'FICHIER_TEMPS'. It lies within the scope of calculations of interaction ground-structure (ISS) non-linear being based on the chaining *Code_Aster*- MISS3D.

The principle of resolution of such a non-linear calculation rests on a technique of under-structuring where two under-fields are generally considered: on the one hand, a non-linear limited field corresponding primarily to the structure part (and possibly, ground surrounding the foundation) and other, a field not-limited of ground with elastic linear behavior. This decomposition of fields makes it possible to apply the method finite elements (*Code_Aster*) for the transitory modeling of the non-linear problem and a method of elements of border (MISS3D) for the calculation of the matrix of impedance representing the dynamic behavior in time of the field of semi-infinite ground.

Within this framework, the method Laplace-Time makes it possible to calculate the operator of impedance in time starting from a particular sampling of the space of complex frequencies (field of Laplace). In particular, this sampling is based on the polynomial characteristic of a linear multi-step method of second order. In the same way, the method Laplace-Time makes possible the evaluation at every moment seismic calculation, integral of convolution translating the efforts of interaction ground-structure of the coupled system.

Contents

1 Introduction

The dynamic problems of interaction ground-structure (ISS), even interaction ground-fluid-structure, are often solved by using a technique of under-structuring where the complete system is broken up into two or several under-fields. In particular, in the case of non-linear ISS, it is broken up into a linear field (ground in far field) and a non-linear field (the building and possibly part of the surrounding ground).

The advantage of using this technique of decomposition rests primarily on the possibility of using the digital methods most adapted to each field. Thus, the linear ground in far field which corresponds to an infinite field can be modelled with the method of the elements of border (BEM for the English acronym) and the non-linear part which is rather confined with a limited field, often of complex geometry for the structure part, with the finite element method (FEM). This digital approach BEM-FEM is indeed that used in the chaining *Code_hasster* – MISS3D.

Within this framework, two characteristics deserve to be recalled. Initially, the fact that any non-linear problem must be solved in general with a transitory calculation. And then, the fact that MISS3D, based on a frequential formulation of the method of the elements of border, makes it possible to model the linear ground not-limited using a matrix of impedance (rigidity of the ground which depends on the frequency) project on a basis of modes representative of kinematics of the interface between the linear and non-linear field (known also by abuse language like interface ISS). For these two considerations the coupled problem is entirely formulated in the temporal field but a product of convolution coming from the frequential dependence of the impedance appears on the level of the interface.

To evaluate this product of convolution, the literature proposes the method time-frequency [1] or method of hidden variables [2] who, by using the function impedance or its reverse [3, 4], rest on a formulation in the frequential field. One also finds formulations of the function impedance in the field of Laplace [5, 6, 7] who can combine with methods of squarings of convolution [8, 9]. Nevertheless, method developed in `CALC_MISS` (the method Laplace-Time [10]) who is that introduced in this document allows to evaluate the integrals of convolution while revealing the terms of inertia, damping and stiffness which characterize the dynamic problems.

1.1 Problems: evaluation of the integrals of convolution

As previously mentioned the method Laplace-Time passes by the evaluation of a product of convolution. The main difficulty of its calculation comes from the singular character of the core of the convolution (the matrix of impedance) which indeed makes difficult its evaluation with a simpler approach and dependent classicE with algorithms of the type FFT.

To quickly illustrate the origin of this product, one will suppose that the system is linear to be able thus to leave the equations formulated in the field of Fourier:

$$-\omega^2 M\,\hat{u}(\omega)+i\,\omega\,C\,\hat{u}(\omega)+K\,\hat{u}(\omega)+\begin{bmatrix}0\\ \hat{Z}_s(\omega)\hat{u}_\Gamma(\omega)\end{bmatrix}=\begin{bmatrix}0\\ \hat{F}_s(\omega)\end{bmatrix} \quad (1)$$

where M , C and K are the matrices of mass, damping and stiffness of the system, $\hat{Z}_s(\omega)$ the matrix of impedance of ground, $\hat{F}_s(\omega)$ the equivalent seismic force and $\hat{u}_\Gamma(\omega)$, the vector of unknown factors in displacement on the level of the interface. Then, the product of convolution appears directly by applying the transform of Fourier to the preceding equation:

$$M\,\ddot{u}(t)+C\,\dot{u}(t)+K\,u(t)+\begin{bmatrix}0\\ R_\Gamma(t)\end{bmatrix}=\begin{bmatrix}0\\ F_s(t)\end{bmatrix} \quad (2)$$

with $R_\Gamma(t)=\int_0^t Z(t-\tau)u_\Gamma(\tau)\,d\tau\,,\ 0\le t\le T$ and $Z(t)$ the similar operator of the impedance of ground in time.

If the part structure (more possibly ground in close field) were non-linear, the product of convolution also appears in the equations of the unknown factors of interface ISS because the efforts of interaction ground-structure depend only on nature of the field on not-limited ground which him, is supposed always linear.

2 The method Laplace-Time

2.1 Principle general

Let us consider the typical case of two under-fields separated by an interface Γ : one noted Ω_1 , which not-is limited and which presents a linear behavior and the other, Ω_2 , limited and possibly non-linear. The effects of interaction between the two under-fields can be represented on the interface by an impedance which we suppose defined in the field of Fourier or Laplace. Within this framework, one solves the total problem in time in Ω_2 by taking of account effects of the under-field Ω_1 by means of an external force applied to the border. This force of interaction, which brings into play the function impedance in the temporal field $Z(t)$ corresponds to an integral of convolution with the unknown field $u_\Gamma(t)$, that one will note $(Z*u_\Gamma)(t)$.

Thus, in a general way, the evaluation of the efforts of interaction between the under-fields passes by the calculation of an integral of convolution between two causal functions definite like:

$$(Z*u_\Gamma)(t)=\int_0^t Z(t-\tau)u_\Gamma(\tau)\,d\tau,\ 0\le t\le T \tag{3}$$

That is to say $\hat{Z}(s)$ the transform of Laplace of $Z(t)$, it will be supposed that it is analytical in the complex half-plane $\Re e(s)>\sigma_0$ and with slow growth for $|s|$ large:

$$\|\hat{Z}(s)\|\le C(\sigma_0)|s|^{\mu} \text{ avec } C(\sigma_0) \text{ et } \mu\in\mathbb{R} \tag{4}$$

Within this framework, that is to say $\hat{Z}(s)=\hat{Z}_m(s)\hat{P}(s)$ with $\hat{P}(s)$ a function polynomial in s with matric values[1] of degree $m\ge\mu$:

$$\hat{P}(s)=\sum_{p=0}^{m}\Lambda_p s^p \tag{5}$$

If this decomposition polynomial is considered, the core of convolution $Z(t)$ defined within the meaning of the distributions can express itself in the shape of an operator of differentiation of order p like:

$$Z(t)=\left(Z_m*\sum_{p=0}^{m}\Lambda_p\frac{d^p\delta}{dt^p}\right)(t) \tag{6}$$

Consequently, the product of convolution can be finally written like:

$$(Z*u_\Gamma)(t)=\sum_{p=0}^{m}\left(\int_0^t Z_m(\tau)\Lambda_p u_\Gamma^{(p)}(t-\tau)\,d\tau\right) \tag{7}$$

with $u_\Gamma(t)$ a sufficiently differentiable causal function.

While making use of the method of squarings of Lubich [8, 9] , one can express the preceding equation in the form of a discrete convolution. Indeed, if one notes Δt the step of time of discretization, this one can be written at the moments time $n\Delta t$ ($0\le n\Delta t\le t$) as follows:

$$(Z*u_\Gamma)(n\Delta t)=\sum_{k=1}^{n}\left(\Psi_1^{n-k+1}u_{\Gamma,k}+\ldots+\Psi_p^{n-k+1}u_{\Gamma,k}^{(p)}+\ldots+\Psi_m^{n-k+1}u_{\Gamma,k}^{(m)}\right) \tag{8}$$

where coefficients Ψ_k^j contain the contribution of the matrices Λ_k .

2.2 Application to the operators of impedance of ground

1To simplify the theoretical approach, we consider in the continuation all the sizes already discretized in space (for example, with a method finite elements).

The matrix of impedance of ground, i.e. the version discretized in space of the operator of impedance, connects a vector displacement defined on the degrees of freedom of the interface ground-structure (or a way generalized within the framework of the chaining Aster-MISS3D, on border FEM-BEM) with the vector forces definite on the same interface, noted Γ in what follows:

$$\hat{Z}(s)\hat{u}_\Gamma(s)=\hat{F}_\Gamma(s) \tag{9}$$

In a general way, the matrix of impedance of ground, which is supposed to be analytical in the half space $\Re e(s)>\sigma_0$ for reasons of causality, can express itself in the following form:

$$\hat{Z}(s)=\hat{Z}_{sing}(s)+\hat{Z}_{nsing}(s) \tag{10}$$

where $\hat{Z}_{nsing}(s)$ corresponds to the regular part of the impedance of which the opposite transform of Laplace $Z_{nsing}(t)$, which exists with the classical direction, is defined like a function exponentially limited and continuous in time which is cancelled for $t<0$:

$$Z_{nsing}(t)=\frac{1}{2\pi i}\int_{\sigma_0+iR} e^{st}\hat{Z}_{nsing}(s)\,ds \tag{11}$$

On the other hand, $\hat{Z}_{sing}(s)$ indicate the singular part of the impedance, heard here like a function with growth slow and not-limited in high frequency. In particular, by analogy with a formulation FEM [2], this singular part of the impedance can be written as follows:

$$\hat{Z}_{sing}(s)=M_\Gamma s^2+C_\Gamma s+K_\Gamma \tag{12}$$

where M_Γ , C_Γ and K_Γ model respectively the contributions of the ground in terms of inertia, damping and rigidity on the level of the interface Γ . One highlight two points interesting to mention. The first aims at stressing that this formulation makes it possible to show easily that the impedance explicitly checks the condition clarified in the equation (5) for $m=2$. The second relates rather to the character non-singulieR opposite transform of Laplace which this time exists only within the meaning of the distributions and which consequently, is written by means of the delta of Dirac and its derivative of first and second order:

$$Z_{sing}(t)=M_\Gamma\ddot{\delta}(t)+C_\Gamma\dot{\delta}(t)+K_\Gamma\,\delta(t) \tag{13}$$

Lhas digital handling of $\hat{Z}_{sing}(t)$ is very delicate and at the same time essential, because this evolution must be calculated for obtaining the transitory efforts of interaction ground-structure. Consequently, one can affirm that the special character of the temporal impedance makes necessary the use of approaches of discretization in more effective times and among those one finds the method Laplace-Time resting on a method of squarings of convolution.

Within this framework, method of squaring of convolution presented by Lubich [8] can be applied for the digital evaluation of an integral of convolution such as data in the equation (3), where one considera for $Z(t)$ the transform of Laplace reverses impedance of ground $\hat{Z}(s)$ and $u_\Gamma(t)$ the field displacement on the interface. This method makes it possible to evaluate in an approximate way the equation (3) by a discrete convolution (with a step of time $\Delta t>0$):

$$(Z*u_\Gamma)(n\Delta t)=\sum_{0\leq n\Delta t\leq t}\Phi_k u_\Gamma(t-n\Delta t) \quad \text{for} \quad t=\Delta t, 2\Delta t, 3\Delta t, \ldots \tag{14}$$

where coefficients Φ_k correspond to the weights of the following series of powers:

$$\sum_{k=0}^{+\infty}\Phi_k\zeta^k=\hat{Z}(s_{\Delta t}) \tag{15}$$

Points of sampling $s_{\Delta t}$ dynamic impedance of ground are given in the following section.

Arrived at this point and while taking account that the expression (14) is homogeneous with a force, it appears judicious to express this convolution not only in terms of displacement, but also according to accelerations and speeds. The approach suggested is registered accordingly and rested on the factorization of the polynomial part $\hat{P}(s)$ impedance:

$$\hat{Z}(s)=\hat{Z}_m(s)\hat{P}(s)=\hat{Z}_m(s)(\widetilde{M}_\Gamma s^2+\widetilde{C}_\Gamma s+\widetilde{K}_\Gamma) \quad (16)$$

Where $\widetilde{K}_\Gamma$, $\widetilde{C}_\Gamma$ and $\widetilde{M}_\Gamma$ are respectively estimators of the matrices of rigidity, damping and inertia of the ground. Thus, the convolution can be written by means of the transform of Laplace reverses as follows:

$$(Z*u_\Gamma)(t)=\frac{1}{2\pi i}\int_{\sigma_0+iR}\hat{Z}_m(s)\hat{P}(s)\hat{u}_\Gamma(s)e^{st}ds \quad (17)$$

Consequently, the polynomial function $\hat{P}(s)$ being seen as an operator of differentiation which acts on the field displacement, the equation (14) becomes:

$$(Z*u_\Gamma)(t)=(Z_m*\widetilde{M}_\Gamma\ddot{u}_\Gamma)(t)+(Z_m*\widetilde{C}_\Gamma\dot{u}_\Gamma)(t)+(Z_m*\widetilde{K}_\Gamma u_\Gamma)(t) \quad (18)$$

where efforts of interaction (noted in the continuation by $R_\Gamma(t)$) are calculated by convolutions with accelerations, speeds and displacements of the interface.

By taking a step of time of discretization $\Delta t>0$, the integral of convolution becomes a typical case of the equation (8) for $m=2$:

$$R_n=(Z*u_\Gamma)(n\Delta t)=\sum_{k=1}^{n}(\Psi_2^{n-k+1}\ddot{u}_{\Gamma,k}+\Psi_1^{n-k+1}\dot{u}_{\Gamma,k}+\Psi_0^{n-k+1}u_{\Gamma,k}) \quad (19)$$

where the matrices multiplying the vectors of displacement, speed and acceleration are given by:

$$\Psi_0^k=Z_m^k\widetilde{K}_\Gamma \qquad \Psi_1^k=Z_m^k\widetilde{C}_\Gamma \qquad \Psi_2^k=Z_m^k\widetilde{M}_\Gamma \quad (20)$$

Indeed, DE manner similar hasU reasoning followed for the equations (14) and (15), coefficients matrices Ψ_0^k, Ψ_1^k and Ψ_2^k correspond to the weights of the series of following powers:

$$\sum_{k=0}^{+\infty}\Psi_j^k z^k=\hat{Z}_m(s_{\Delta t})\Lambda_j \quad \text{with} \quad j=0,1,2 \quad (21)$$

With $\Lambda_0=\widetilde{K}_\Gamma$, $\Lambda_1=\widetilde{C}_\Gamma$ and $\Lambda_2=\widetilde{M}_\Gamma$.

3 Calculation of the impedance in time

As mentioned previously, the method Laplace-Time is a digital approach making it possible to discretize in time an integral of convolution.

3.1 Establishment in *Code_hasster*

The equation (21) watch that the digital evaluation of the weights Ψ_k^j master key initially by that of $\hat{Z}_m^k$. In its turn, this one can be obtained starting from an integral of Cauchy steady to a contour $|z|=\rho$:

$$Z_m(k\,\Delta t)=\frac{1}{2\pi i}\int_{|z|=\rho}\hat{Z}_m\left(\frac{\delta(z)}{\Delta t}\right)z^{-k-1}dz \quad (22)$$

If one expresses this integral in polar coordinates $z=\rho e^{i\theta}$, then one applies the rule of trapezoids to the phase to discretize it in L not identical of value $\Delta\theta=\frac{2\pi}{L}$, one obtains:

$$Z_m(k\,\Delta t)\approx Z_m^k=\frac{\rho^{-k}}{L}\sum_{l=0}^{L-1}\hat{Z}_m(s_l)e^{-i\frac{2\pi l}{L}k} \quad \text{with} \quad k=0,1,\ldots,N \quad (23)$$

where the operator defined in the field of Laplace is sampled on $s_l=\frac{\delta(\rho e^{2\pi i l/L})}{\Delta t}$ with $\delta(z)=\frac{3}{2}-2z+\frac{1}{2}z^2$ the polynomial characteristic of a multi-step straight-line method of order two. Let us note that N and L are not forcing equal, N being the number of steps of time of the window of time of interest (for example, according to the duration of the seismic signal) and L being the number of steps of computing time of the impedance in time.

If it is considered that $\hat{Z}_m(s_l)$ is evaluated with a variability[2] ϵ_{CQM}, coefficients $\hat{Z}_m^k$ will be calculated with a precision[3] $O(\sqrt{\epsilon_{CQM}})$ S I $L=N$ and $\rho^N=\sqrt{\epsilon_{CQM}}$. With the values of L and ρ fixed, the weights of the series can be obtained starting from a classical FFT, which brings back the complexity of the calculation algorithm to $O(L\log L)$ instead of $O(L^2)$. Nevertheless, the value of ϵ_{CQM} is not obvious to gauge and partly depends on the nature of the problem to solve. Indeed, too low values of ϵ_{CQM}, such as 10^{-20} or 10^{-30} can involve rays of integration $|z|=\rho$ too much small. Considering the intégrande of Cauchy contains a singularity into zero, values too smallES of the ray can be translated into a bad evaluation of the integral. For this reason, parametric studies were carried out in [10] to conclude that a good compromise is reached when $\epsilon_{CQM}=10^{-10}$[4].

In the same way, it will be noticed that the dependence in ρ^{-k} carry out, for values of k large, with inaccuracies on the coefficients $\hat{Z}_m^k$ calculated. Knowing that the use of the more precise methods of integration, of Simpson type for example, do not correct this problem, the idea is to widen the beach of time (i.e. $L=mN, m\in\mathbb{R}$) on which one calculates the impedance in order to reduce the disturbances on the solution of interest. For this reason, it was observed that, in general, the inaccuracies of $\hat{Z}_m^k$ are propagated on the answer only beyond $t\approx 0{,}7\,T_{FIN}$, where T_{FIN} is the final moment of calculation of the impedance[5].

3.2 Principal digital considerations

We saw previously that in general, the singular part of the impedance of ground can be approximate by a polynomial of order two. If the coefficients of this polynomial are real, it is easy to show that the impedance of

2 Referring to the concept of reproducibility (*precision* in English).
3 Referring to the concept of exactitude (*accuracy* in English).
4 This value initializes by default the keyword `PRECISION` of the operator `CALC_MISS` (*cf.* [U7.03.12]) when the type of expected result is `'FICHIER_TEMPS'`.
5 With this intention, `CALC_MISS` (`TYPE_RESU = 'FICHIER_TEMPS'`) the keyword envisages `COEF_SURECH` (*cf.* [U7.03.12]) initialized at 1.35, that is to say the equivalent to make $L=1.35N$ in the equation 23.

ground checks $\hat{Z}(\sigma-i\omega)=conj[\hat{Z}(\sigma+i\omega)]$ in the analytical plan of the field of Laplace[6]. It will be noticed that when a damping hysteretic is used in the ground (case of MISS3D), the imaginary part of its static stiffness is not-worthless and the assumption of real coefficients is not checked any more.

A second interesting property coming directly from the definition from transformed unilateral of Laplace is the following one:

$$\hat{Z}(s=0)=\int_0^{\infty} Z(t)dt \approx \sum_k^N Z(k\,\Delta t) \quad (24)$$

for N chosen sufficiently large (| a hundred steps of time). Let us note that the impedance of ground evaluated with $s=0$ corresponds to the static stiffness of ground (left real) which it, is very different from the instantaneous stiffness of the ground. This must be taken well into account when a dynamic calculation ($Z(t=0)]$ is assembled with the first member) is realized following a static calculation ($\Re[\hat{Z}(s=0)]$ is assembled with the first member)[7].

6 This property is exploited in Code_hasster: the impedance is échantillonnée on the higher half-plane, then supplemented numerically with combined. Consequently, it is necessary to pay attention with the grounds presenting of very strong depreciation hysteretic (superior with 25%).

7 There exists in Code_hasster an macro-order (`PRE_SEISME_NONL`, *cf*. [U4.63.02])) allowing to carry out the transition statics-dynamics from a calculation of non-linear interaction ground-structure.

4 Seismic analysis

The seismic calculation of non-linear interaction ground-structure must be solved in the temporal field. Some elements on the calculation of the seismic force and the discretization in time of the equations of the coupled system are provided below.

4.1 Calculation of the seismic force in time

The method Laplace-Time could be also applied for the calculation of the seismic force, nevertheless the nature of the incidental field developed in MISS3D invalidates the assumptions on the fields solution searched in the field of Laplace. Within this framework, one cannot guarantee that the calculation of the seismic force at complex frequency with MISS3D is correct for any type of foundation (surface and buried) and stratigraphy. The approach recommended in the studies passes thus by the opposite transform of Fourier of the seismic force evaluated by MISS3D in the frequential field.

4.2 Diagram of integration in time

This part seeks to illustrate the digital processing of the integral of convolution within the framework of a resolution with a diagram of integration in time of the family of Newmark. Nevertheless, the reasoning also applies to the diagram of integration in time α - HHT, also available in Code_hasster .

One will thus seek to solve in time the linear dynamic problem according to, presumedly discretized in space with the classical method of the finite elements:

$$\begin{bmatrix} L_{bb}(\cdot) & L_{b\Gamma}(\cdot) \\ (sym.) & L_{\Gamma\Gamma}(\cdot) \end{bmatrix} \begin{bmatrix} u_b(t) \\ u_\Gamma(t) \end{bmatrix} + \begin{bmatrix} 0 \\ R_\Gamma(t) \end{bmatrix} = \begin{bmatrix} F_b(t) \\ F_\Gamma(t) \end{bmatrix} \quad (25)$$

where an operator of differentiation $L_{\alpha\beta}(\cdot)$ in time is introduced for $\alpha, \beta \in \{b, \Gamma\}$, Γ for the degrees of freedom of the interface in contraposition to the rest of the degrees of freedom of the system noted b and where the vector of efforts of interaction ground-structure noted $R_\Gamma(t)$, corresponds to the integral of convolution which one will discretize with the method Laplace-Time.

The resolution of the system coupled ground-structure passes by the discretization of all the sizes in time and by to isolate the unknown factors at the moment $t = n\Delta t$, where the step of time is noted Δt . In particular, for the efforts of interaction ground-structure, it is interesting to leave the equation (18) and to gather the not-unknown terms corresponding to the previous moments to $t = n\Delta t$ in only one term in order to separate them from the unknown factors to be solved with the step of time n . The vector $R_n \approx R_\Gamma(n\Delta t)$ corresponding is written thus as follows:

$$R_n = \Psi_2^1 \ddot{u}_{\Gamma,n} + \Psi_1^1 \dot{u}_{\Gamma,n} + \Psi_0^1 u_{\Gamma,n} + R_{\Sigma(n-1)} \quad (26)$$

With Ψ_0^1, Ψ_1^1 and Ψ_2^1 referring respectively to the stiffness, instantaneous damping and inertia [8] and $R_{\Sigma(n-1)}$ depending only on the quantities calculated at the previous moments [9] :

$$R_{\Sigma(n-1)} = \sum_{k=1}^{n-1} \left(\Psi_2^{n-k+1} \ddot{u}_{\Gamma,k} + \Psi_1^{n-k+1} \dot{u}_{\Gamma,k} + \Psi_0^{n-k+1} u_{\Gamma,k} \right) \quad (27)$$

This regrouping of terms makes it possible to introduce with the second member the part of the convolution coming from the previous moments and in the first member the terms relating to unknown factors. Indeed, this

8 In Code_hasster, these terms Instantanés are assembled with the first member via the operator `MACR_ELEM_DYNA` (*cf.* [U4.65.01]) and possibly a connection `LIAISON_INTERF` in the operator `AFFE_CHAR_MECA` (*cf.* [U4.44.01]) if a reduced modal base of interface were used.

9 The term $R_{\Sigma(n-1)}$ is modelled in Code_hasster during the non-linear dynamic analysis like a load with the second member obtained with the option “ `CHARGE_SOL` ” of the operator `AFFE_CHAR_MECA` (*cf.* [U4.44.01]).

can easily highlighted on the basis of a formulation in displacement of a diagram of unconditionally stable Newmark integration ($\beta=0.25$, $\gamma=0.5$) applied to the equation (25) :

$$\begin{bmatrix} \tilde{K}_{11} & \tilde{K}_{12} \\ \tilde{K}_{21} & \tilde{K}_{22} \end{bmatrix} \begin{bmatrix} u_{b,n} \\ u_{\Gamma,n} \end{bmatrix} = \begin{bmatrix} F_{b,n} \\ F_{\Gamma,n} \end{bmatrix} - \begin{bmatrix} 0 \\ R_{\Sigma(n-1)} \end{bmatrix} \quad (28)$$

where the matrix $\tilde{K}$ corresponds to the operator of Newmark which it will be necessary to reverse in each step of time and the second member with the sum of the equivalent vector of Newmark and the known part of the discrete product of convolution. In particular, the term $\widetilde{K_{22}}$ who applies to the degrees of freedom of the interface Γ , is written:

$$\tilde{K}_{22} = \frac{1}{\beta \Delta t^2}(M_{22} + \Psi_2^1) + \frac{\gamma}{\beta \Delta t}(C_{22} + \Psi_1^1) + (K_{22} + \Psi_0^1) \quad (29)$$

It will be noticed that the terms Ψ_0^1, Ψ_1^1 and Ψ_2^1 play a part on the conditioning of the operator of Newmark $\tilde{K}$ and consequently, their value have an impact on the convergence of the solution. Also let us note that $\widetilde{K_{22}}$, constant remainder during calculation, it is calculated only once.

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