Calculation of matrix of added mass on modal basis

Summary:

This document presents an aspect of the fluid coupling/structure: when a vibrating structure is immersed in a fluid that one supposes at rest, incompressible and nonviscous, it feels compressive forces whose resultant is proportional to the acceleration of the structure in the fluid: the proportionality factor is homogeneous with a mass: it is called **added mass**. One specifies here the means of estimating a matrix of mass added for one (or of) structure (S) at several degrees of freedom on the modal basis of (of) the structure (S) in the vacuum.

Contents

1 Notations

p	:	fluctuating pressure in the fluid,
γ_l	:	contour of the structure indexed by l
$\ddot{x}_{s_l}$	:	the field of displacements in the structure l,
ρf, ρs	:	density of the fluid, the structure,
$\boldsymbol{X}_{il}$	:	clean mode of order I of the structure l in air
a_{il}, $\dot{a}_{il}$ $\ddot{a}_{il}$	:	coordinates, speeds, generalized accelerations relating to mode I of the structure l in air
$\bar{\bar{\sigma}}$	:	the tensor of the constraints in the structure
$\mathbf{\Phi}$	:	the fluid vector of flow
$\mathbf{H}$	:	the matrix of rigidity of the fluid
$\mathbf{v}$	:	the field fluid speeds
$\mathbf{n}$	:	the interior normal of the fluid.

2 Introduction

Many industrial components are in contact with fluid environments, which more is often in flow. These surrounding fluid environments disturb the vibratory characteristics of the structures, in particular their modal characteristics. This action of the fluid on the structure results in effects of fluid coupling/structure.

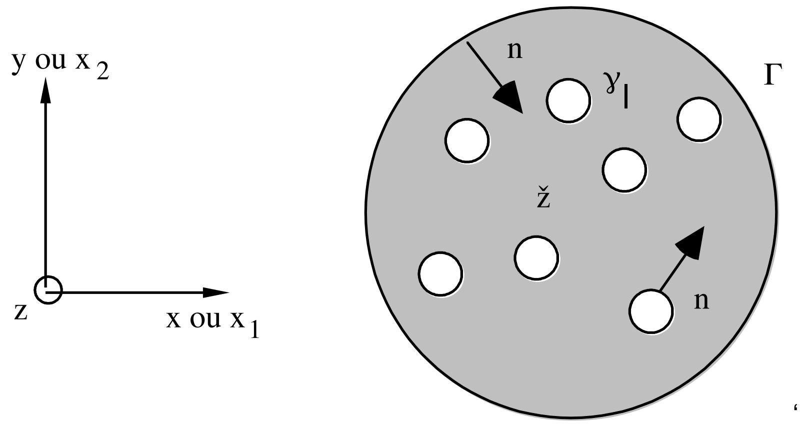
One supposes the incompressible, perfect fluid environment here surrounding and at rest. One will show that then, a structure which vibrates with a small amplitude in this fluid modifies the field of pressure in the fluid at rest, and thus feels a compressive force, proportional to his acceleration. The proportionality factor is a mass. It describes the inertial effect of the fluid on the structure: this is why this mass is named **added mass** fluid on the structure.

When several structures are in contact of the same fluid, when one of the structures starts to vibrate, not only it feels the inertia of the fluid, but it modifies the field of pressure around the interfaces with the fluid of all the other structures. The efforts that each one feels are proportional to the acceleration of the vibrating structure: there still the proportionality factors are masses called **added masses of coupling**.

3 Recalls of the equations of the problem

3.1 Equations in the fluid

It is supposed that K vibrating structures are immersed in a true fluid (nonviscous), incompressible and at rest. One neglects the effect of gravity. One can thus write the equations of Euler associated with the fluid at rest:



- conservation of the mass:

$$\frac{\partial \rho_f}{\partial t}+div(\rho_f \mathbf{v})=0 \qquad \text{éq 3.1-1}$$

- conservation of the momentum:

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}.\boldsymbol{grad})\mathbf{v}+\frac{1}{\rho_f}\boldsymbol{grad}\ p=0 \qquad \text{éq 3.1-2}$$

Because of incompressibility of the fluid, the equation [éq 3.1-1] becomes:

$$\text{div}\ \mathbf{v}=0 \qquad \text{éq 3.1-3}$$

In volume Ω fluid, one neglects the convection induced by the movement of low amplitude of the structure. The equation [éq 3.1-2] thus becomes:

$$\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{\rho_f}\boldsymbol{grad}\ p=0 \qquad \text{éq 3.1-4}$$

While deriving [éq 3.1-3] compared to time and by deferring the expression of $\frac{\partial \mathbf{v}}{\partial t}$ according to the pressure in this equation, one obtains:

$$\text{div}\,\boldsymbol{grad}\ p=0$$

that is to say:

$$\Delta p=0 \text{ in } \Omega$$

who is the equation of Laplace in a fluid at rest.

With the fluid interface/structure, one can write that the normal acceleration of the wall of the structure is equal to the normal acceleration of the fluid (continuity of the normal accelerations - condition of impermeability of the structure). One uses here following convention for the normal: it is the normal **external** with the structure, **directed structure towards the fluid.**

$$\frac{\partial \mathbf{v}}{\partial t}.\boldsymbol{n}=\ddot{\mathbf{x}}_{S_l}.\boldsymbol{n}$$

With the equation [éq 3.1-4], one obtains:

$$\mathbf{grad}\, p.\boldsymbol{n}=-\rho_f\frac{\partial \mathbf{v}}{\partial t}.\boldsymbol{n}=-\rho_f\ddot{\mathbf{x}}_{S_l}.\boldsymbol{n}$$

That is to say:

$(\frac{\partial p}{\partial n})_{\gamma_l}=-\rho_f\ddot{\mathbf{x}}_{S_l}.\boldsymbol{n}$ on γ_l, fluid interface/structure of the structure indexed by l.

In short, the fluid problem consists in solving an equation of Laplace with boundary conditions of the type von Neumann:

$$\begin{cases}\Delta p=0 \quad \text{dans} \quad \Omega \\ (\frac{\partial p}{\partial n})_{\Gamma_1}=-\rho_f\ddot{\mathbf{x}}_s.\mathbf{n} \quad \text{sur} \quad \Gamma_1, \quad \Gamma_1=\bigcup_{l=1,K}\gamma_l \\ (\frac{\partial p}{\partial n})_{\Gamma_2}=0 \quad \text{sur} \quad \Gamma_2, \quad \Gamma_2=\partial\Omega-\Gamma_1\end{cases} \qquad \text{éq 3.1-5}$$

3.2 Equations in the structures

Let us consider K elastic structures divings in a fluid environment. The equation of their movement in the presence of fluid is written:

$$\begin{cases}\forall l \quad \text{indice de structure}, l\in\{0,\ldots,K\}, \quad \mathbf{M_l}\ddot{\mathbf{X}}_l+\mathbf{K_l}\mathbf{X_l}=0 \quad \text{dans} \quad \Omega_{S_l}, \text{volume de la structure} \quad l \\ \forall l, \bar{\bar{\boldsymbol{\sigma}}}\boldsymbol{n}=-p\boldsymbol{n} \quad \text{sur}\,\gamma_l, \text{contour de la structure} \quad l\end{cases}$$

$\mathbf{M_l}$ is the matrix of mass of the structure, $\mathbf{K_l}$ its matrix of rigidity. The boundary condition on the contour of the structures translates the continuity of the normal constraint to the fluid interface/structure (the tensor of the fluid constraints being tiny room to its nondeviatoric part, fluid being perfect). By integrating on the contour of each structure this normal constraint, one obtains a force $\mathbf{F_l}$ resultant of the structure/compressive forces of the fluid to the fluid interface. This force is the integral of the field of pressure on contour γ_l of each structure:

$$\forall l \quad \text{indice de structure}, l\in\{0,\ldots,K\}, \quad \mathbf{F_l}=-\int_{\gamma_l} p\,\boldsymbol{n}\,d\Gamma$$

The field of pressure checks the problem [éq 3.1-5].

3.3 Equations of the coupled problem - description of the matrix of added mass

Ultimately, the fluid coupled problem/structure is written:

$$\begin{cases} \Delta p = 0 \text{ dans } \Omega \\ \forall l \in \{0,\ldots,K\}, \left(\dfrac{\partial p}{\partial n}\right)_{\gamma_l} = -\rho_f \ddot{\mathbf{x}}_{S_l}.\boldsymbol{n} \text{ sur } \gamma_l \\ \forall l \in \{0,\ldots,K\}, \ \mathbf{M_l}\ddot{\mathbf{X}}_l + \mathbf{K_l}\mathbf{X_l} = 0 \text{ dans } \Omega_{S_l} \\ \forall l \in \{0,\ldots,K\}, \ F_l = -\int_{g_l} p\,\boldsymbol{n}\,d\Gamma \text{ sur } \gamma_l \end{cases}$$

éq 3.3-1

One will show from now on that the effort that feel the immersed structures is proportional to their acceleration. A good means of showing that is to place itself in the modal base of the structures in the vacuum. One can thus break up acceleration on this basis (which is in fact the meeting of the modal bases of each structure). As follows:

$$\mathbf{x}_{\mathbf{S_l}}(r,t) = \sum_{i=1}^{\infty} a_{il}(t)\mathbf{X}_{il}(\mathbf{r})$$

By deferring this expression in the second equation of the system [éq 3.3-1], one is brought to search the field of pressure in the form:

$$p = \sum_{l=1,\ldots,K} \sum_{i=1,\ldots,\infty} \ddot{a}_{il}(t)\, p_{il}(\mathbf{r})$$

By deferring in the problem [éq 3.3-1] these expressions, one has to as many solve in the fluid problems of Laplace than one chose modes for each structure. This results in:

$$\forall l \in \{1,\ldots,K\}, \quad \forall i\,\{1,\ldots,\infty\}, \quad \begin{cases} \Delta p_{il} = 0 \quad \text{dans} \quad \Omega \\ \left(\dfrac{\partial p_{il}}{\partial n}\right)_{\gamma_l} = -\rho_f \mathbf{X_{il}}.\boldsymbol{n} \text{ sur } \gamma_l \\ \left[\mathbf{m_{il}}\right](\ddot{\mathbf{a}}_l) + \left[\mathbf{k_{il}}\right](\mathbf{a_l}) = (\mathbf{f_{il}}) \text{ dans } \Omega_l \end{cases}$$

The “matrices” of mass and rigidity written in these bases are diagonal.

Each component of the effort of pressure resulting project on modal basis is written:

$$\forall i\,\{1,\ldots.,\infty\}, \forall l\,\{1,\ldots.,K\}, \ (f_{il}) = -\sum_{k=1}^{K}\sum_{j=1}^{\infty} \ddot{a}_{jk}\int_{\gamma_l} p_{jk}\boldsymbol{X_{il}}.\boldsymbol{n}\,N_j\,d\Gamma$$

One can then write the vector of the effort generalized of pressure on a structure immersed in matric form:

$$(\mathbf{f}_{il}) = -\left[\boldsymbol{m_{il\,jk}}\right]\ddot{\boldsymbol{a}}_{jk} \text{ avec } m_{il\,jk} = \int_{\gamma_l} p_{jk}\boldsymbol{X_{il}}.\boldsymbol{n}\,d\Gamma$$

Here, l is fixed: the matrix $[\boldsymbol{m}_{il\,jk}]$ be called **matrix of added mass** fluid on the structure of contour γ_l. When one considers the modal base of the whole of K structures, one generalizes the notation of **matrix of added mass** $[\boldsymbol{m}_{il\,jk}]$ on modal basis in the vacuum, l varying 1 with K. This matrix is in general not diagonal.

3.4 Some definitions

3.4.1 Definition 1

When $l=k$ (even structure) and $i=j$ (even order of mode), the coefficient m_{ilil} is it**car - added mass** mode i structure l. It is additional inertia due to the fluid moved by the mode of order i structure, taking into account the geometrical containments induced in the fluid by the presence of the other presumedly fixed structures.

3.4.2 Definition 2

When $l=k$ (even structure) and $i\neq j$ (different orders of mode), the coefficient m_{iljl} is **added mass of coupling** between the modes of order i and j structure l. In air, these extra-diagonal terms of mass are worthless, because the modes are orthogonal between them. Taking into account the general expression of the coefficient $m_{il\,jk}$, modes i and j can be coupled in mass, because the field of pressure p_{jl} created by the mode j structure l is not necessarily orthogonal with the mode of order i of this same structure. It is enough that this structure is immersed in an environment not comprising geometrical symmetry so that this coefficient is nonnull. In a symmetrical environment, on the other hand, the orthogonality of the field of pressure with the mode is observed.

3.4.3 Definition 3

When $l\neq k$ (different structures) and $i\neq j$ (different orders of mode), the coefficient $m_{il\,jk}$ is **added mass of coupling** between the modes of order i and j respectively structures l and k. This coefficient translates the inertial effort which the structure makes undergo k vibrating on its mode of order J to the structure l vibrating on its mode i.

3.5 Properties of the matrix of added mass

3.5.1 Theorem 1: the matrix of added mass is symmetrical

To simplify the demonstration, we will consider a single structure immersed in a true, incompressible and nonviscous fluid. We break up the movement of the structure on its modal basis (truncated with n modes), but the result can be just as easily shown in “physical” base (*i.e.* the base of the nodal functions of interpolation). Lastly, the result spreads with the case of K structures immersed in the same fluid.

One must show that: $m_{ij}=\int_\Gamma p_i \boldsymbol{X}_j.\boldsymbol{n}\,d\Gamma=m_{ji}=\int_\Gamma p_j \boldsymbol{X}_i.\boldsymbol{n}\,d\Gamma$

- p_i (respectively p_j) represent the field of pressure created in the fluid and to the interface with the structure by the mode of order i (respectively of order j) structure,
- $\boldsymbol{X_j}$ (respectively $\boldsymbol{X_i}$) represent the modal deformation of the mode of order j (respectively of order i).

However:

$$\begin{cases} \Delta p_i = 0 \quad \text{dans} \quad \Omega \quad \text{volume fluide} \\ \dfrac{\partial p_i}{\partial n} = -\rho_f X_i . \boldsymbol{n} \quad \text{sur} \quad \Gamma \end{cases} \quad \text{and} \quad \begin{cases} \Delta p_j = 0 \quad \text{dans} \quad \Omega \ \text{volume fluide} \\ \dfrac{\partial p_j}{\partial n} = -\rho_f \mathbf{X}_\mathrm{j} . \boldsymbol{n} \quad \text{sur} \quad \Gamma \end{cases}$$

From where, by using the formula of Green with a normal directed of the structure towards the fluid and the harmonicity of p_i and of p_j :

$$\begin{aligned} m_{ij} &= \int_\Gamma p_i \boldsymbol{X}_j . \boldsymbol{n} \, d\Gamma = -\frac{1}{\rho_f} \int_\Gamma p_i \frac{\partial p_j}{\partial n} d\Gamma \\ &= -\frac{1}{\rho_f} \Big(\underbrace{\int_\Omega p_i \Delta p_j d\Omega}_{0} - \int_\Omega \boldsymbol{grad}\, p_i . \boldsymbol{grad}\, p_j d\Omega \Big) \\ &= -\frac{1}{\rho_f} \Big(\underbrace{\int_\Omega p_j \Delta p_i d\Omega}_{0} - \int_\Omega \boldsymbol{grad}\, p_j . \boldsymbol{grad}\, p_i d\Omega \Big) \\ &= -\frac{1}{\rho_f} \int_\Gamma p_j \frac{\partial p_i}{\partial n} d\Gamma = \int_\Gamma p_j \boldsymbol{X_i} . \boldsymbol{n} \, d\Gamma \\ &= m_{ji} \end{aligned}$$

3.5.2 Theorem 2: the matrix of added mass is definite positive

One returns to the reference [bib1] for the complete demonstration.

3.5.3 Theorem 3

Let us suppose that one has K structures having identical properties of elasticity linear and which are immersed in the same fluid. Moreover, these structures admit two degrees of freedom of displacement in the plan Oxy (cf diagram). Each one of these structures admits the same spectrum $f_1, \ldots, f_n, \ldots$ Eigen frequencies in the vacuum.

For any Eigen frequency f_n , there exists $2\mathrm{K}$ Eigen frequencies $\{\omega_1, \ldots, \omega_{2\mathrm{K}}\}$ fluid coupled system/structure checking $\forall i \in [1, \ldots, 2K], \ \omega_i \leq f_n$

One returns to the reference [bib1] for the complete demonstration.

3.5.4 Other properties

- the coefficients of added auto--mass are always positive

One always supposes that one has only one structure immersed in a true fluid, incompressible and at rest. The demonstration spreads without difficulty with K immersed structures.

One must show that:

$$\forall i \text{ indice de mode} \in [1, \ldots, n], m_{ii} = \int_\Gamma p_i \boldsymbol{X_i} . \boldsymbol{n} \, d\Gamma \geq 0$$

However:

$$m_{ii} = \int_\Gamma p_i \boldsymbol{X_i} . \boldsymbol{n} \, d\Gamma = -\frac{1}{\rho_f} \int_\Gamma p_i \frac{\partial p_i}{\partial n} d\Gamma$$

$$= -\frac{1}{\rho_f} \Big(\underbrace{\int_\Omega p_i \Delta p_i \, d\Omega}_{0} - \int_\Omega \boldsymbol{grad}\, p_i . \boldsymbol{grad}\, p_i \, d\Omega \Big)$$

$$= \frac{1}{\rho_f} \int_\Omega (\boldsymbol{grad}\, p_i)^2 d\Omega$$

$$\geq 0$$

- let us suppose that one has K structures immersed in the same fluid. It is supposed that they have only one degree of freedom of translation according to Ox. Then the sum of all the coefficients of mass added of this matrix gives the auto--mass added on the whole of K structures moving same sinusoidal rectilinear motion very.

One returns to the reference [bib2] for the complete demonstration.

4 Numeric work implementation

4.1 Resolution of the equation of Laplace by finite elements of volume

Let us take again the fluid problem of Laplace with boundary conditions of the type von Neumann:

$$\begin{cases} \Delta p = 0 \quad \text{dans}\, \Omega \\ \left(\frac{\partial p}{\partial n}\right)_{\Gamma_1} = -\rho_f \ddot{\mathbf{x}}_s . \boldsymbol{n} \quad \text{sur}\, \Gamma_1, \quad \Gamma_1 = \bigcup_{l=1,K} \gamma_l \\ \left(\frac{\partial p}{\partial n}\right)_{\Gamma_2} = 0 \quad \text{sur}\, \Gamma_2, \quad \Gamma_2 = \partial\Omega - \Gamma_1 \end{cases}$$

Let us write a variational formulation of this problem:

$$\int_\Omega \mathbf{v} \Delta p \, d\Omega = 0$$

By using the formula of Green with a normal which one supposes directed of the structure towards the fluid (thus interior with fluid volume) and while posing $\Gamma=\Gamma_1 \cup \Gamma_2$:

$$\int_\Omega \boldsymbol{grad}\, v . \boldsymbol{grad}\, p\, d\Omega + \int_\Gamma v \frac{\partial p}{\partial n} d\Gamma = 0$$

That is to say:

$$\int_\Omega \boldsymbol{grad}\, v . \boldsymbol{grad}\, p\, d\Omega = \rho_f \int_{\Gamma_1} v\, \ddot{x}_n\, d\Gamma \qquad \text{éq 4.1-1}$$

A partition of volume is considered Ω in a finished number of elements. On this discretization of the field, one can write the approximate shape of the hydrodynamic field of pressure:

$$p=\sum_{i=1}^{N} N_i(\mathbf{r})\, p_i$$

N_i represent the nodal functions of interpolation definite on the elements: they are worth 1 with the node number i , and 0 on all the others.

Then, while taking as function-tests v successively the nodal functions of interpolation, one obtains a system of N equations while deferring in [éq 4.1-1]:

$$j=1,\ldots,N\,;\;\; \int_\Omega \sum_{i=1}^{N} p_i\, \boldsymbol{grad}\, N_i(\boldsymbol{r}) . \boldsymbol{\Delta}\, N_j(\boldsymbol{r})\, d\Omega = \rho_f \int_{\gamma_1} N_j\, \ddot{x}_n\, d\Gamma$$

what can be written in the form:

$\mathrm{HP}=\boldsymbol{\Phi}$ with $\boldsymbol{\Phi}$ vector of components $\Phi_j=\rho_f \int_\Gamma N_j\, \ddot{x}_n\, d\Gamma$

with $\boldsymbol{H}$ matrix of coefficients $H_{ij}=\int_\Omega \boldsymbol{grad}\, N_i \cdot \boldsymbol{grad}\, N_j\, d\Omega$ éq 4.1-2

In any rigour, this system is singular. He admits an infinity of solutions differing from a constant. It is thus necessary to impose a pressure (boundary condition of the Dirichlet type) in a point of the fluid to raise the indetermination on the solution.

These precautions taken, by reversing the system [éq 4.1-2], one obtains the field of pressure in all volume Ω of fluid, including with the fluid interface/structure, where it interests us obviously.

4.2 Calculation of the coefficients of the matrix of mass added on modal basis

It is necessary to estimate the value of the integral numerically:

$$m_{il\,jk}=\int_{\gamma_l} p_{jk}\, \boldsymbol{X_{il}} . \boldsymbol{n}\, d\Gamma \qquad \text{éq 4.2-1}$$

starting from a field with the nodes of pressure represented by a vector column noted $\boldsymbol{P}_{jk}$ and of a field to the nodes of displacement corresponding to a modal deformation of structure in air and represented by the vector column $\boldsymbol{X}_{il}$.

However, on the fluid interface/structure, the approximate field of pressure p_{jk} had with the discretization of the interface in N elements of edge can be written:

$$p_{jk}=\sum_{m=1}^{N} N_m(\boldsymbol{r})\, p_{jk_m}$$

while the field of “modal” displacement is written on this same discretization:

$$\mathbf{X}_{il}=\sum_{n=1}^{N} N_n(\mathbf{r})\, \boldsymbol{X}_{il_n}$$

Thus, by deferring these two expressions in the integral [éq 4.2-1], one obtains:

$$m_{il\,jk}\simeq\int_{\gamma_l}\Big(\sum_{m=1}^{N} N_m(\boldsymbol{r})\, p_{jk_m}\Big)\left[\sum_{n=1}^{N} N_n(\boldsymbol{r})\, X_{ilx_n} A\, n_x+\sum_{n=1}^{N} N_n(\boldsymbol{r})\, X_{ily_n} A\, n_y\right] d\Gamma$$

$$m_{il\,jk}\simeq\sum_{m=1}^{N}\sum_{n=1}^{N} p_{jk_m}\Big(\int_{\gamma_l} N_m(\boldsymbol{r})\, N_n(\boldsymbol{r})\, n_x\, d\Gamma\Big) X_{ilx_n}+\sum_{m=1}^{N}\sum_{n=1}^{N} p_{jk_m}\Big(\int_{\gamma_l} N_m(\boldsymbol{r})\, N_n(\boldsymbol{r})\, n_y\, d\Gamma\Big) X_{ily_n}$$

One supposes in the demonstration that the problem is two-dimensional.

This can be put in the shape of a scalar product, utilizing a product stamps vector:

$m_{il\,jk}=\mathbf{P}_{jk}^{\mathbf{T}}\mathbf{A_x}\mathbf{X_{ilx}}+\mathbf{P}_{jk}^{\mathbf{T}}\mathbf{A_y}\mathbf{X_{ily}}$ with $\boldsymbol{A_x}$ matrix of coefficients $\int_{\gamma_l} N_i N_j n_x\, d\Gamma$

and $\boldsymbol{A_y}$ matrix of coefficients $\int_{\gamma_l} N_i N_j n_y\, d\Gamma$

5 Implementation in *Code_Aster*

5.1 Thermal analogy

To solve the problem of Laplace in pressure, a thermal analogy is used: it is a question of in hover solving the equation of heat with a material of thermal conductivity equal to the unit. As follows:

$$\begin{cases}\Delta p=0 \text{ dans } \Omega\\ \left(\dfrac{\partial p}{\partial n}\right)_G=-\rho_f\,\ddot{\mathbf{x}}_s\cdot\mathbf{n} \text{ dans } \Gamma\end{cases}\Leftrightarrow\begin{cases}div(\lambda\, grad\, T)=0 \quad \text{dans } \Omega \Leftarrow \Delta T=0 \text{ si } \lambda=1\\ \left(\dfrac{\partial T}{\partial n}\right)_G=\phi_n \text{dans } \Gamma\end{cases}$$

T represent the temperature in the medium, it plays the part of the pressure in the fluid environment. ϕ_n is the normal heat flow to the wall, it plays the part of the term $-\rho_f\,\ddot{\mathbf{x}}_s.\boldsymbol{n}$ who is comparable to the variation in the course of the time of the flow of mass (fluid) to the wall of the structure. This quantity $-\rho_f\,\ddot{\mathbf{x}}_s.\boldsymbol{n}$ is indeed homogeneous with a mass divided by a surface and a time squared.

5.2 Implementation practical

The operator CALC_MATR_AJOU [U4.66.01] was developed to take into account the inertial coupling (mass added : OPTION = 'MASS_AJOU') between structures bathed in the same true fluid, incompressible and at rest. The fluid is described by equivalent thermal characteristics (operator DEFI_MATERIAU [U4.43.01]) and the part of the grid representing is affected by thermal elements (operator AFFE_MODELE [U4.41.01]). This operator CALC_MATR_AJOU also allows to calculate the stiffness or added damping. In order to facilitate its use in certain cases, there exists also the macro-order MACRO_MATR_AJOU [U4.66.11].

The operator uses five obligatory keywords:

- the keyword MODELE_FLUIDE : it is on this model that one solves the problem of Laplace with boundary conditions of Von Neumann (or his thermal problem are equivalent),
- the keyword MODE_MECA (or CHAM_NO, or MODELE_GENE): this keyword makes it possible to calculate the boundary conditions of type flow to the wall of the structure,
- the keyword MODELE_INTERFACE : it is on this model which understands all the thermal elements of edge of the fluid interface/structure that one calculates the scalar product mentioned in the paragraph [§4.2],
- the keyword CHAM_MATER : it is fluid material (described by equivalent thermal characteristics),
- the keyword LOAD : it is a thermal load (temperature imposed in an unspecified node of the fluid grid) which corresponds to the boundary condition of Dirichlet to raise the singularity of the problem of Laplace (see [§4.1]).

One thus obtains a matrix of generalized added mass. This matrix having a profile line of full sky but (operator NUME_DDL_GENE [U4.65.03]) can be summoned with the matrix of mass generalized of the structure by using the operator COMB_MATR_ASSE [U4.72.01]. This makes it possible to calculate the coupled modes fluid/structure of the immersed structures ("wet" modes) (operator CALC_MODES [U4.52.02]).

6 Bibliography

1) C. CONCA, J. PLANCHARD, B. THOMAS, MR. VANNINATHAN: "Mathematical problems in fluid coupling/structure" _EYROLLES (1994).

2) F. BEAUD, G. ROUSSEAU: "Validation inter-software of the calculation of mass added with *Code_Aster* and codes it CALIPH", HT-32/95/004/A

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
3	***G.ROUSSEAU*** *(EDF/EP/AMV)*	Initial text