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Homogenisation of a network of beams bathing in a fluid

Summary:

This note describes a model obtained by a method of homogenisation to characterize the vibratory behavior of a periodic network of tubes bathed by an incompressible fluid. Then the development of a finite element associated with this homogenized model is presented.

The tubes are modelled by beams of Euler and the fluid by a model with potential.

This modeling is accessible in the order `AFFE_MODELE` by choosing modeling `3D_FAISCEAU`.

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1 Introduction

In nuclear industry, certain structures consist of networks quasi-periodicals of tubes bathed by fluids: the "combustible" assemblies, the steam generators,… to determine the vibratory behavior such structures, the classical approach (each tube is modelled, the volume occupied by the fluid is with a grid) is expensive and tiresome even impracticable (in particular, development of a complicated grid containing a large number of nodes). The studied structures presenting a character quasi-periodical, it seems interesting to use methods of homogenisation.

Techniques of homogenisation applied to a network of tubes bathed by a fluid were with various already elaborate recoveries [bib1], [bib5], [bib4]. The models obtained differ by the assumptions carried out on the fluid (compressibility, initial speed of the flow, viscosity). According to the allowed assumptions, the action of the fluid on the network of tubes corresponds to an added mass (lowers frequencies of vibration compared to those given in absence of fluid), with a damping even added to an added rigidity [bib5].

At the beginning, finite elements associated with two-dimensional models (network of runners bathed by a fluid) were elaborate [bib2]. To study the three-dimensional problems (network of tubes), a solution to consist in projecting the movement on the first mode of inflection of the beams [bib4]. Later on, three-dimensional finite elements were developed [bib3], [bib8].

2 Initial physical problem

2.1 Description of the problem

One considers a set of identical beams, of axis z , laid out periodically (either ε the period of space). These beams are located inside an enclosure filled with fluid (see [fig 2.1-a]). One wishes to characterize the vibratory behavior of such a medium, by considering for the moment only the effect of added mass of the fluid which is dominating [bib6].

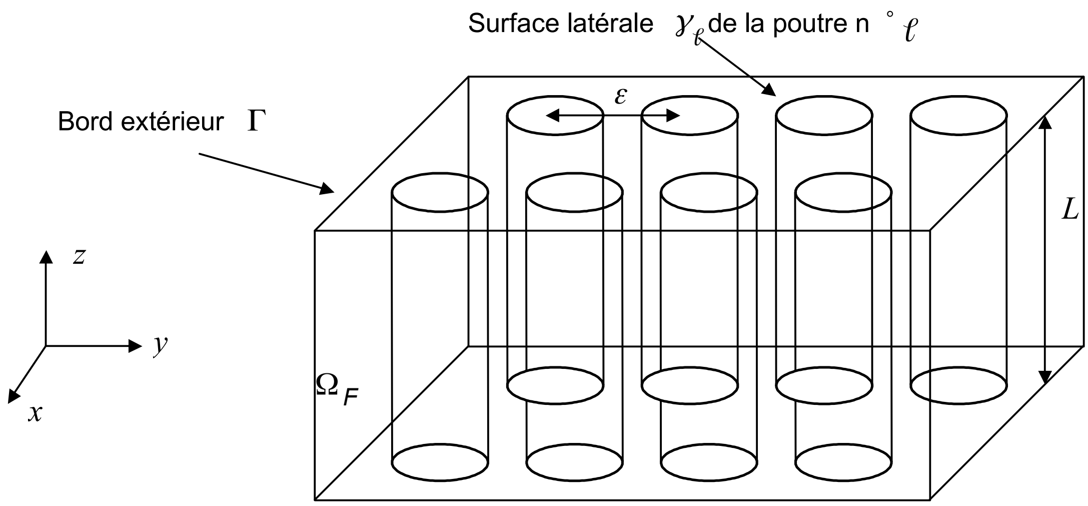

Figure 2.1-a

2.2 Assumptions of modeling

It is considered that the fluid is a true fluid initially at rest, incompressible. As the assumption of small displacements around the position of balance was carried out (fluid initially at rest), the field of displacement of the fluid particles is irrotational so that there exists a potential of displacement of the noted fluid Φ. There is no flow of fluid through external surface Γ.

It is considered that the beams are homogeneous and with constant section according to $z\in]0,L[$. To model the beams, the model of Euler is used and the movements of inflection are only taken into account. The section of beam is rigid and the displacement of any point of the section is noted:

$\mathbf{s}^l$ the inflection of the beam n° l $\left(\mathbf{s}^l(z)=\left(\mathbf{s}_x^l(z),\mathbf{s}_y^l(z)\right)\right)$.

The beams are embedded at their two ends.

The variational form of the problem fluid-structure vibroacoustic (conservation of the mass, dynamic equation of each tube) is written:

$$\int_{\Omega_F}\nabla\Phi.\nabla\Phi^*=\sum_l\int_{\gamma_l}\left(\mathbf{s}^l\cdot\mathbf{n}\right)\Phi^*\qquad\forall\Phi^*\in V_\Phi \qquad \text{éq 2.2-1}$$

$$\int_0^L\rho_S S.\frac{\partial^2\mathbf{s}^l}{\partial t^2}\cdot\mathbf{s}^{l*}+\int_0^L E\,\mathbf{I}\cdot\frac{\partial^2\mathbf{s}^l}{\partial z^2}\cdot\frac{\partial^2\mathbf{s}^{l*}}{\partial z^2}=-\int_0^L\left(\int_{\gamma_l}\rho_F.\frac{\partial^2\Phi}{\partial t^2}\cdot\mathbf{n}\right)\mathbf{s}^{l*}\qquad\forall\mathbf{s}^{l*}\in V_S \qquad \text{éq 2.2-2}$$

with:

$$V_S=\left(H_0^2(]0,L[)\right)^2 \text{ and } V_\Phi=H^1\left(\Omega_F\right)$$

where:

- $\mathbf{n}$ is the normal entering to the beam n° l,
- ρ_F is the constant density of the fluid in all the field,
- ρ_S is the density of material constituting the beam,
- S is the section of the beam,
- E is the Young modulus,
- $\mathbf{I}$ is the tensor of inertia of the section of the beam.

The second member of the equation [éq 2.2-2] represents the efforts exerted on the beam by the fluid. Pressure p fluid is related to the potential of displacement by: $p=-\rho_F\frac{\partial^2\Phi}{\partial t^2}$. In the same way, the second member of the equation [éq 2.2-1] represents the flow of fluid induced by the movements of the beams. At the border of each beam l one a: $\mathbf{s}^l\cdot\mathbf{n}=\nabla\Phi\cdot\mathbf{n}$.

This formulation leads to a nonsymmetrical system matric, which is not very convenient, in individual at the time of the search for modes of vibration.

3 Homogenized problem

3.1 Homogenized problem obtained

To take account of the periodic character of the studied medium, one uses a method of homogenisation based in this precise case on an asymptotic development of the variables intervening in the physical starting problem. With regard to the operational approach, one returns the reader to the following references [bib2], [bib4], [bib5], [bib6]. One will be satisfied here to state the got results.

In the homogenized medium Ω_0 (see [fig 3.1-a]), the two following homogenized variables are considered: $\mathbf{s}_0$ (displacement of the beams) and Φ_0 (potential of displacements fluid). In variational form, these variables are connected by the equations natural vibrations :

$$\begin{cases} \int_{\Omega_0} \mathbf{A}\,.\,\nabla\Phi_0\,.\,\nabla\phi^* = -\int_{\Omega_0} \mathbf{D}\,.\,\mathbf{s}_0\,.\,\phi^* & \forall\,\phi^* \in V_\Phi^{\mathrm{hom}} \\ \int_{\Omega_0} \mathbf{M}\,.\,\dfrac{\partial^2 \mathbf{s}_0}{\partial t^2}\,\mathbf{s}^* + \int_{\Omega_0} \mathbf{K}\,.\,\dfrac{\partial^2 \mathbf{s}_0}{\partial z^2}\,\dfrac{\partial^2 \mathbf{s}^*}{\partial z^2} = \rho_F \int_{\Omega_0} \mathbf{D}\,.\,\nabla\dfrac{\partial^2 \Phi_0}{\partial t^2}\,.\,\mathbf{s}^* & \forall\,\mathbf{s}^* \in V_s^{\mathrm{hom}} \end{cases} \qquad \text{éq 3.1-1}$$

where:

$$V_s^{\mathrm{hom}} = L^2(\Omega_0)^2 \times H_0^2(\Omega_0)^2$$
$$\text{où} \quad \Omega_0 = S \times]0, L[$$
$$H_0^2(\Omega_0) = \{v;\ \forall (x, y) \in S \quad z \to v(x, y, z) \in H_0^2(]0, L[)\}$$

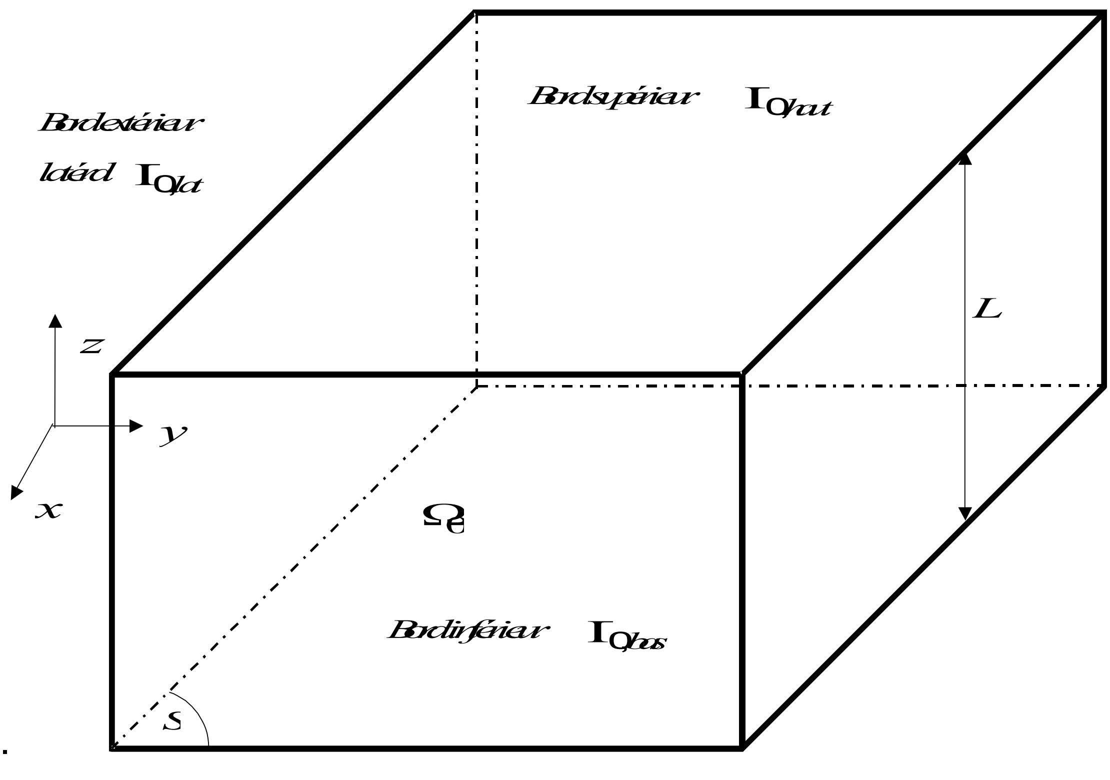

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Figure 3.1-a

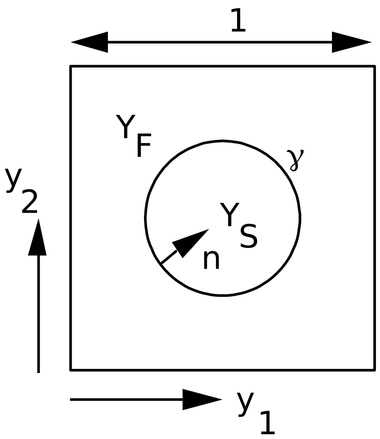

Figure 3.1-b

The various tensors which intervene in [éq 3.1-1] are defined using two functions $\chi_\alpha (\alpha=1,2)$ following way:

$$\mathbf{B}=(b_{ij})=\frac{1}{|Y|}\begin{pmatrix} \int_{Y_F}\frac{\partial \chi_1}{\partial y_1} & \int_{Y_F}\frac{\partial \chi_1}{\partial y_2} & 0 \\ \int_{Y_F}\frac{\partial \chi_2}{\partial y_1} & \int_{Y_F}\frac{\partial \chi_2}{\partial y_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{A}=(a_{ij})=\left(\frac{|Y_F|}{|Y|}\delta_{ij}-b_{ij}\right) \qquad \text{éq 3.1-2}$$

$$\mathbf{D}=(d_{ij})=\mathbf{B}+\frac{1}{|Y|}\begin{pmatrix} Y_S & 0 & 0 \\ 0 & Y_S & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{M}=(m_{ij})=\rho_F\mathbf{B}+\frac{\mu^2}{|Y|}\begin{pmatrix} \rho_S S & 0 & 0 \\ 0 & \rho_S S & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{K}=(k_{ij})=\frac{E\mu^2}{|Y|}\begin{pmatrix} I_{xx} & I_{xy} & 0 \\ I_{xy} & I_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{éq 3.1-3}$$

where $|Y_F|$ and $|Y_S|$ represent respectively the surfaces of the fluid fields and structure of the basic cell of reference (cf [fig 3.1-b]). $|Y|$ represent the sum of the two preceding surfaces. The basic cell of reference is homothetic of report μ with the real cell of periodicity of the heterogeneous medium.

Two functions $\chi_\alpha(\alpha=1,2)$ are solutions of a two-dimensional problem, called cellular problem. On the basic cell of reference, the functions $\chi_\alpha(\alpha=1,2)$ are defined by:

$$\begin{aligned} &\int_{Y_F}\nabla\chi_\alpha.\nabla v=\int_\gamma n_\alpha.v \qquad \forall v\in V \\ &\int_{Y_F}\chi_\alpha=0 \quad (\text{pour avoir une solution unique}) \end{aligned} \qquad \text{éq 3.1-4}$$

where:

$$V = \{ v \in H_1(Y_F), \quad v(y) \text{ périodique en } y \text{ de période } 1 \}$$

Note:

It is shown that the two-dimensional part of $\mathbf{B}$ *is symmetrical and definite positive [bib5].*

Note:

In the matrix $\mathbf{M}$*, the term* $\rho_F \mathbf{B}$ *play the part of a matrix of mass added specific to each beam in its cell.*

Note:

For the various tensors, one can put in factor the multiplicative end $\frac{1}{|Y|}$*. It was added in order to obtain the "good one masses" tubes in absence of fluid. One has then* $\int_{\Omega_0} \mathbf{M}\, dV$ = *mass of the tubes component* Ω_0 *.*

3.2 Matric problem

By discretizing the problem [éq 3.1-1] by finite elements, one leads (with obvious notations) to the following problem:

$$\begin{cases} \hat{\mathbf{A}}\,\Phi_0 = -\hat{\mathbf{D}}\,\mathbf{s}_0 \\ \hat{\mathbf{M}}\dfrac{\partial^2 \mathbf{s}_0}{\partial t^2} + \hat{\mathbf{K}}\,\mathbf{s}_0 = \rho_F\,\hat{\mathbf{D}}^{\mathbf{T}}\dfrac{\partial^2 \Phi_0}{\partial t^2} \end{cases} \quad \text{éq 3.2-1}$$

what can be put in the form (one pre - multiplies the first equation by ρ_F):

$$\tilde{\mathbf{M}}\begin{pmatrix} \dfrac{\partial^2 \mathbf{s}_0}{\partial t^2} \\ \dfrac{\partial^2 \Phi_0}{\partial t^2} \end{pmatrix} + \tilde{\mathbf{K}}\begin{pmatrix} \mathbf{s}_0 \\ \Phi_0 \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{M}} & -\rho_F\,\hat{\mathbf{D}}^{\mathbf{T}} \\ -\rho_F\,\hat{\mathbf{D}} & -\rho_F\,\hat{\mathbf{A}} \end{pmatrix}\begin{pmatrix} \dfrac{\partial^2 \mathbf{s}_0}{\partial t^2} \\ \dfrac{\partial^2 \Phi_0}{\partial t^2} \end{pmatrix} + \begin{pmatrix} \hat{\mathbf{K}} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \mathbf{s}_0 \\ \Phi_0 \end{pmatrix} = 0 \quad \text{éq 3.2-2}$$

Note:

The problem obtained is symmetrical. So instead of choosing the potential of displacement to represent the fluid, one had chosen the potential speed, one would have obtained a nonsymmetrical matric problem also revealing a matrix of damping.

Note:

It is necessary to have $\rho_F > 0$ *so the matrix of mass* $\tilde{\mathbf{M}}$ *that is to say invertible. If one wishes to do a calculation in "air", cf § 6.3, it is necessary to block them degrees of freedom in* Φ_0*.*

4 Resolution of the cellular problem

4.1 Problem to be solved

On the two-dimensional basic cell (see [fig 4.1-a]), one seeks to calculate the functions $\chi_\alpha(\alpha=1,2)$ checking:

$$\int_{Y^*}\nabla\chi_\alpha.\nabla v=\int_\gamma n_\alpha v \qquad \forall v\in V$$
$$\int_{Y^*}\chi_\alpha=0 \quad (\text{pour avoir une solution unique})$$
éq 4.1-1

where:

$$V=\{v\in H_1(Y^*),\quad v(y)\ \text{ périodique en } y \text{ de période } 1\}$$

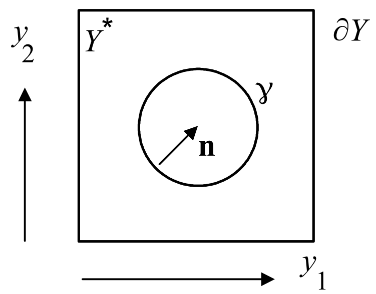

Figure 4.1-a

After having determined the functions $\chi_\alpha(\alpha=1,2)$, the homogenized coefficients are calculated defined by $b_{\alpha\beta}(\alpha=1,2\,;\beta=1,2)$ the formula:

$$b_{\alpha\beta}=\int_{Y^*}\frac{\partial\chi_\alpha}{\partial y_B}$$
éq 4.1-2

By using the formula of Green and the periodic character of χ_α, it is shown that the coefficients $b_{\alpha\beta}$ can be written differently:

$$b_{\alpha\beta}=\int_\gamma\chi_\alpha n_\beta$$
éq 4.1-3

To estimate this quantity, it is necessary at the time of a discretization by finite elements, to determine for each element the outgoing normal, which can be tiresome. Another way then is operated; while taking in the equation [éq 4.1-1] $v=\chi_\beta$, one obtains:

$$b_{\alpha\beta}=\int_{Y^*}\nabla\chi_\alpha\nabla\chi_\beta$$
éq 4.1-4

From the function potential energy defined by the classical formula:

$$W(v) = -\frac{1}{2}\int_{Y^*} \nabla v \,.\, \nabla v \qquad \text{éq 4.1-5}$$

one can rewrite the coefficients homogenized in the form:

$$b_{\alpha\beta} = -(W(\chi_\alpha+\chi_\beta) - W(\chi_\alpha) - W(\chi_\beta)) \qquad \text{éq 4.1-6}$$

In the case two-dimensional general, one must calculate three coefficients of the homogenized problem (one $b_{11,} b_{12}=b_{21,} b_{22}$ knows that the matrix $\mathbf{B}=(b_{\alpha\beta})$ is symmetrical). One must solve the two following problems:

$$\begin{cases} \text{Calculer}\, \chi_1 \in V \;\; / \;\; \int_{Y^*} \nabla \chi_1 \nabla v = \int_{Y^*} n_1 v \\ \text{Calculer}\, \chi_2 \in V \;\; / \;\; \int_{Y^*} \nabla \chi_2 \nabla v = \int_{Y^*} n_2 v \\ \text{Calculer}\, \chi^* \in V \;\; / \;\; \chi^* = \chi_1 + \chi_2 \end{cases} \qquad \text{éq 4.1-7}$$

One has then:

$$\begin{cases} b_{11} = -2\, W(\chi_1) \\ b_{22} = -2\, W(\chi_2) \\ b_{12} = b_{21} = -(W(\chi^*) - W(\chi_1) - W(\chi_2)) \end{cases} \qquad \text{éq 44.1-8}$$

Note:

If the basic cell has symmetries, that makes it possible to solve the problem on part of the cell with boundary conditions quite suitable and to calculate only certain coefficients of the homogenized problem. For example for the cell of the figure n°4.1 - there is a: $b_{11}=b_{22,}\;\; b_{12}=b_{21}=0$.

4.2 Problem are equivalent to define χ_α

In the equation [éq 4.1-1], the calculation of the second member requires the determination of the normal at the edge. To avoid a determination of the normal, one can write an equivalent problem, checked by the functions χ_α.

Are the vectors $G_1=\begin{pmatrix}1\\0\end{pmatrix}, G_2=\begin{pmatrix}0\\1\end{pmatrix}$ et $G^*=\begin{pmatrix}1\\1\end{pmatrix}$, the functions are sought $\chi^*, \chi_{1,}\chi_2 \in V$ such as:

$$\begin{cases} \int_{Y^*} \nabla \chi_1 \,.\, \nabla v = \int_{Y^*} G_1 \,.\, v \quad \forall v \in V \\ \int_{Y^*} \nabla \chi_2 \,.\, \nabla v = \int_{Y^*} G_2 \,.\, v \quad \forall v \in V \\ \int_{Y^*} \nabla \chi^* \,.\, \nabla v = \int_{Y^*} G^* \,.\, v \quad \forall v \in V \end{cases} \qquad \text{éq 4.2-1}$$

By using the formula of Green and the anti-periodicity of the normal $\mathbf{n}$, it is shown that the problems [éq 4.1-1] and [éq 4.2-1] are equivalent.

4.3 Practical application in *Code_Aster*

In *Code_Aster*, to solve the problem [éq 4.2-1], the thermal analogy by defining a material having a coefficient c_p equal to zero and one coefficient λ equal to is used. To impose the calculation of the second member utilizing the term in G_α, the keyword `PRE_GRAD_TEMP` in the order `AFFE_CHAR_THER` is selected. The thermal problem is solved by using the order `THER_LINEAIRE`. The calculation of the potential energy W is provided by the order `POST_ELEM` with the option `ENER_POT`. In the case general, three calculations are carried out to determine the values $W(\chi_1)$, $W(\chi_2)$, $W(\chi^*)$ and then, the values of the coefficients of the homogenized problem are deduced from it manually. To impose the periodic character of the space in which the solution is sought, the keyword `LIAISON_GROUP` in the order `AFFE_CHAR_THER` is used.

5 Choice of the finite element for the homogenized problem

5.1 Choice of the finite elements

In the model presented previously, the axis z a paramount role as a main axis of the beams has. The developed finite elements check this characteristic. The meshs are of the cylindrical type: the quadrangular bases are contained in plans $z=Cte$ and cylinder centers it is parallel to the axis z (see [fig 5.1-a]).

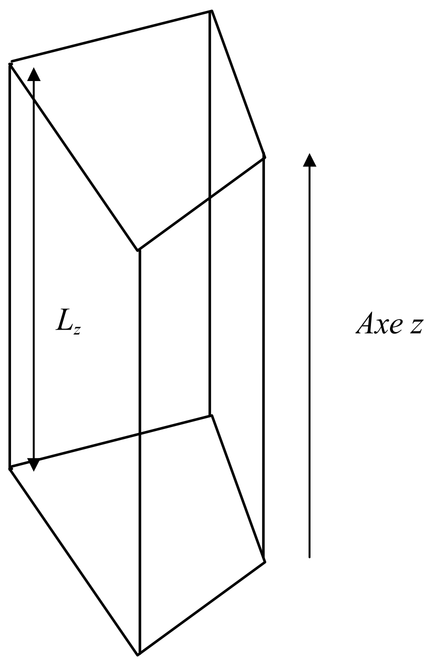

Figure 5.1-a

According to the equations [éq 3.1-1], derivative second following the coordinate z intervene in the model, which requires finite elements C^1 in the direction z. Functions of form of the type Hermit to represent the variations of s along the axis z are thus used. At the points of discretization, displacements s_x, s_y but also the derivative $\frac{\partial s_y}{\partial z}, \frac{\partial s_x}{\partial z}$ who are related to the degrees of freedom of rotation θ_x, θ_y by the formulas $\theta_x = \frac{\partial s_y}{\partial z}$, $\theta_y = -\frac{\partial s_x}{\partial z}$ must be known. With regard to the variations according to x, y, one limits oneself for the moment to functions of form Q_1.

For the degree of freedom of potential, functions of form Q_1 or Q_2 according to the three directions x, y, z space are used.

The finite element thus has as unknown factors the following degrees of freedom: $s_x, s_y, \theta_x, \theta_y, \Phi$.

Note:

The order of the nodes of the meshs support is very important. Indeed, edges parallel with the axis z only the edges contained in the plans are represented in the same way $z = Cte$. The nodes of the meshs are thus arranged in a quite precise order: list of the nodes of the lower base, then list of with respect to the higher base (or vice versa).

With regard to the geometry, the functions of form making it possible to pass from the element of reference to the element running are Q_1. The finite element is thus under-parametric.

Two finite elements were developed:

- a associate with a mesh HEXA8. In each node of the mesh, the unknown factors are $s_x, s_y, \theta_x, \theta_y, \Phi$. Functions of form associated with the potential Φ are Q_1.
- another associate with a mesh HEXA20. In each node top of the mesh, the unknown factors are $s_x, s_y, \theta_x, \theta_y, \Phi$. In each node medium of the edges, the unknown factor is Φ. Functions of form associated with the potential Φ are Q_2.

5.2 Finite elements of reference

5.2.1 Mesh HEXA8

On the finite element of reference HEXA8 (see [fig 5.2-a]), the following functions of form are defined:

$$N^L_{\pm1,\pm1,\pm1}(\xi) = P_{\pm1}(\xi_1) P_{\pm1}(\xi_2) P^L_{\pm1}(\xi_3) \quad \text{avec } L = \Phi \text{ ou } D \text{ ou } R \qquad \textbf{éq 5.2.1-1}$$

Indices ±1 represent the coordinates of the nodes of the mesh support of reference.

The functions which make it possible to define the functions of form write:

$$\begin{aligned}
&P_{(-1)}(\varsigma) = \frac{1-\varsigma}{2} && P_{(+1)}(\varsigma) = \frac{1+\varsigma}{2} \\
&P^{\Phi}_{(-1)}(\varsigma) = \frac{1-\varsigma}{2} && P^{\Phi}_{(+1)}(\varsigma) = \frac{1+\varsigma}{2} \\
&P^{D}_{(-1)}(\varsigma) = \frac{1}{2}\left(1 - \frac{3}{2}\varsigma + \frac{1}{2}\varsigma^3\right) && P^{D}_{(+1)}(\varsigma) = \frac{1}{2}\left(1 + \frac{3}{2}\varsigma - \frac{1}{2}\varsigma^3\right) \qquad \varsigma \in [-1,1] \qquad \textbf{éq 5.2.1-2} \\
&P^{R}_{(-1)}(\varsigma) = \frac{1}{4}\left(1 - \varsigma - \varsigma^2 + \varsigma^3\right) && P^{R}_{(+1)}(\varsigma) = \frac{1}{4}\left(-1 - \varsigma + \varsigma^2 + \varsigma^3\right)
\end{aligned}$$

Functions P^D, P^R are the functions of Hermit.

The unknown factors of the homogenized problem, on a mesh, break up in the following way:

$$\begin{cases} s_x(\xi)=\sum_{i=1}^{8} DX_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRX_i N_i^R(\xi) \\ s_y(\xi)=\sum_{i=1}^{8} DY_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRY_i N_i^R(\xi) \quad \xi=(\xi_{1,}\xi_{2,}\xi_3) \\ \Phi(\xi)=\sum_{i=1}^{8} \Phi_j N_j^{\Phi}(\xi) \end{cases} \qquad \textbf{éq 5.2.1-3}$$

where $DX_i, DY_i, DRX_i, DRY_i, \Phi_i$ are the values of displacement according to x, displacement according to y, rotation around the axis x, rotation around the axis y and of the potential of displacement at the top i mesh. In *Code_Aster*, for each node, the degrees of freedom are arranged in the order quoted previously.

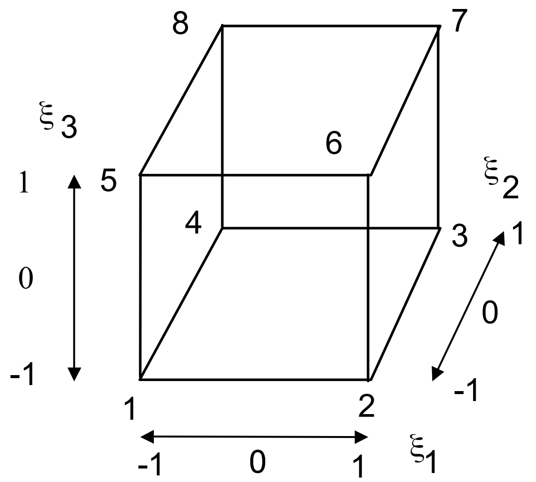

Figure 5.2.1-a

5.2.2 Mesh HEXA20

On the finite element of reference HEXA20 (see [fig 5.2-b]), the following functions of form are defined:

$$N^L_{\pm1,\pm1,\pm1}(\xi)=P_{\pm1}(\xi_1)P_{\pm1}(\xi_2)P^L_{\pm1}(\xi_3) \quad \text{avec } L=\Phi \text{ ou } D \text{ ou } R \qquad \textbf{éq 5.2.2-1}$$

$$N_j^{\Phi}(\xi)=Q_j(\xi_3) \quad j=1,\dots,20 \qquad \textbf{éq 5.2.2-2}$$

Indices ± 1 represent the coordinates of the nodes tops of the mesh support of reference.

Functions $P_{\pm 1}, P_{\pm 1}^{L}$ were already defined in the paragraph [§5.2.1]. Functions Q_i are defined by:

$$
\begin{aligned}
Q_i(\xi) &= \frac{1}{8}\left(1+\xi_1\xi_1^i\right)\left(1+\xi_2\xi_2^i\right)\left(1+\xi_3\xi_3^i\right)\left(\xi_1\xi_1^i+\xi_2\xi_2^i+\xi_3\xi_3^i-2\right) && i=1,\ldots,8\\
Q_i(\xi) &= \frac{1}{4}\left(1-\left(\xi_1\xi_1^i\right)^2\right)\left(1+\xi_2\xi_2^i\right)\left(1+\xi_3\xi_3^i\right) && i=9,11,17,19\\
Q_i(\xi) &= \frac{1}{4}\left(1-\left(\xi_2\xi_2^i\right)^2\right)\left(1+\xi_1\xi_1^i\right)\left(1+\xi_3\xi_3^i\right) && i=10,12,18,20\\
Q_i(\xi) &= \frac{1}{4}\left(1-\left(\xi_3\xi_3^i\right)^2\right)\left(1+\xi_1\xi_1^i\right)\left(1+\xi_2\xi_2^i\right) && i=13,14,15,16
\end{aligned}
\qquad \textbf{éq 5.2.2-3}
$$

where $(\xi_{1,}^i\xi_{2,}^i\xi_3^i)$ are the coordinates of the node i mesh.
The unknown factors of the homogenized problem, on a mesh, break up in the following way:

$$
\begin{cases}
s_x(\xi)=\sum_{i=1}^{8} DX_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRX_i N_i^R(\xi)\\
s_y(\xi)=\sum_{i=1}^{8} DY_i N_i^D(\xi)+\frac{L_S}{2}\sum_{i=1}^{8} DRY_i N_i^R(\xi) \qquad \xi=(\xi_{1,}\xi_{2,}\xi_3)\\
\Phi(\xi)=\sum_{i=1}^{8} \Phi_j N_j^{\Phi}(\xi)
\end{cases}
\qquad \textbf{éq 5.2.2-4}
$$

where $DX_i, DY_i, DRX_i, DRY_i, \Phi_i$ are the values of fluid displacement according to x, displacement according to y, rotation around the axis x, rotation around the axis y and of the potential of displacement at the top i mesh $(i=1,\ldots,8)$ and Φ_j fluid potential of displacement to the node medium of the edges $(j=9,\ldots,20)$.

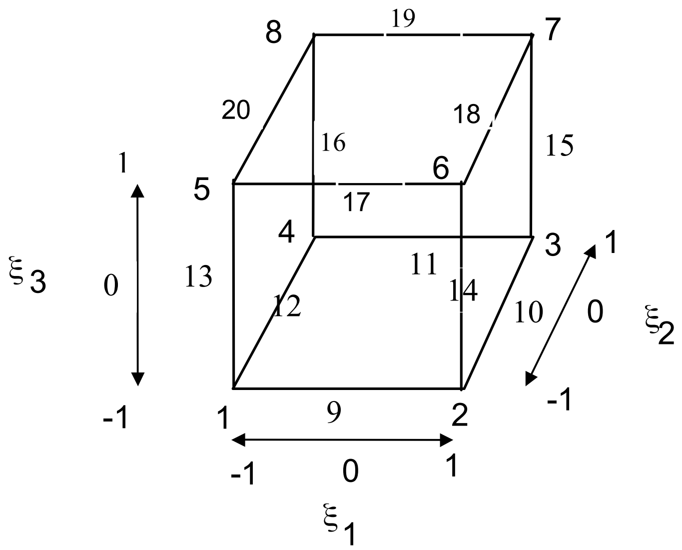

Figure 5.2.2-a

5.3 Choice of the points of Gauss

Each integral which intervenes in the forms of the elementary matrices, is transformed into an integral on the element of reference (a change of variable is carried out) who is then calculated by using a formula of squaring of the GAUSS type.

The points of Gauss are selected in order to integrate exactly the integrals on the element of reference. Families of different points of integration are used to calculate the matrices of mass and the matrices of rigidity (the degrees of the polynomials to be integrated are different). But here, to calculate the various contributions of the matrix of mass, various families of points of Gauss can still be used.

The element of reference being a HEXA8 or a HEXA20, the integral on volume can be separate in a product of three integrals which correspond each one to a direction of the space of reference. The number of points of integration necessary is determined by direction.

According to the mesh of reference, the number of points of integration by direction is the following:

	Mesh HEXA8		**Mesh HEXA20**	
	direction x **or** y	**direction** z	**direction** x **or** y	**direction** z
matrix $\hat{\mathbf{K}}$	2	2	2	2
matrix $\hat{\mathbf{A}}$	2	2	3	3
matrix $\hat{\mathbf{D}}$	2	3	2	3
matrix $\hat{\mathbf{M}}$	2	4	2	4

Four families of points of Gauss were defined. Each family corresponds to one of the matrices of the problem to solve.

On the segment [- 1.1], the coordinates of the points of integration and their weights are the following [bib7]:

Many points of integration	**Coordinates**	**Weight**
2	$\pm 1/\sqrt{3}$	1
3	0 $\pm\sqrt{3/5}$	$8/9$ $5/9$
4	$\pm\sqrt{\dfrac{3-2\sqrt{6/5}}{7}}$ $\pm\sqrt{\dfrac{3+2\sqrt{6/5}}{7}}$	$\dfrac{1}{2}+\dfrac{1}{6\sqrt{6/5}}$ $\dfrac{1}{2}-\dfrac{1}{6\sqrt{6/5}}$

The weight of a point of Gauss in the three-dimensional element of reference is obtained by multiplying the three weights corresponding to each coordinate of the point of Gauss.

5.4 Addition of the problems of traction and torsion

To supplement the problem of inflection homogenized described previously, the problem of traction and the problem of torsion are added in an uncoupled way (these problems do not make into tervenir the fluid).

5.4.1 Problem of traction

The problem of traction homogenized is written in the following form:

$$\int_\Omega \frac{E\,S\,\mu^2}{|Y|}\frac{\partial s_z}{\partial z}\frac{\partial v}{\partial z}+\int_\Omega \frac{\mu^2 \rho_S S}{|Y|}\frac{\partial^2 s_z}{\partial t^2}v=0 \quad \forall v\in V \text{ with } V=H^1([0,L])$$

μ being the report on side of the basic cell of reference, surface $|Y|$, compared to the real cell of periodicity of the heterogeneous medium.
The finite element of reference is a HEXA8 having for unknown factor displacement DZ in each node. The associated functions of form are Q_1 .

5.4.2 Problem of torsion

The problem of torsion homogenized is written in the following form:

$$\int_\Omega \frac{E\,J_z\,\mu^2}{2(1+\nu)|Y|}\frac{\partial \theta_z}{\partial z}\frac{\partial v}{\partial z}+\int_\Omega \frac{\mu^2 \rho_S J_z}{|Y|}\frac{\partial^2 \theta_z}{\partial t^2}v=0 \quad \forall v\in V \text{ with } V=H^1([0,L])$$

where J_z is the constant of torsion.

The finite element of reference is a HEXA8 having for unknown factor displacement DRZ in each node. The associated functions of form are Q_1

5.5 Integration in *Code_Aster* of this finite element

The finite element is developed in *Code_Aster* in 3D. A modeling was added in the catalogue of modelings:

- 'FAISCEAU_3D'for the 3D.

In the catalogue of the elements, the element can apply to the two following meshs:

Mesh	Many nodes in displacement and rotation	Many nodes in fluid potential	Name of the element in the catalogue
HEXA8	8	8	meca_poho_hexa8
HEXA20	8	20	meca_poho_hexa20

In the routines of initialization of this element, one defines:

- two families of functions of form respectively associated with displacements and rotation with the beams (linear function of form in x, y and cubic in z) and under the terms of potential of the fluid (function linear in x, y, z),
- four families of points of Gauss to calculate the matrix of rigidity and the various parts of the matrix of mass.

During the calculation of the elementary terms, the derivative first or seconds of the functions of form on the element running is calculated. In spite of the simplified geometry of the finite element (the axis of the cylindrical mesh is parallel to the axis z and the sections lower and higher are in plaNS $z=Cte$), a general subroutine to calculate the derivative second was written [bib7]. In addition, two news subroutines was developed starting from the subroutines existing for the isoparametric elements to take account of the under-parametric character of the element.

6 Use in *Code_Aster*

6.1 Data necessary

Characteristics of the beams (section S, tensor of inertia $\mathbf{I}$, constant of torsion J_z) are directly well informed under the keyword factor BEAM order AFFE_CARA_ELEM.

The characteristics of the homogenized coefficients and the cell of reference are indicated under the keyword factor POUTRE_FLUI order AFFE_CARA_ELEM. For the simple keywords, the correspondence is the following one:

B_T : b_{11}
B_N : b_{22}
B_TN : b_{12}
A_FLUI : Y_F
A_CELL : $Y=Y_F+Y_S$
COEF_ECHELLE : μ

The characteristics of materials are indicated in the order DEFI_MATERIAU. For the tubes, the keyword factor ELAS is used to indicate the Young modulus (E: E), the Poisson's ratio (NAKED : ν) and density (RHO : ρ_S). For the fluid, the keyword factor FLUID is used to indicate the density of the fluid (RHO : ρ_F).

6.2 Orientation of the axes of the beams

The generators of the cylindrical meshs are obligatorily parallel to the axis of the beams and the bases of the meshs perpendicular to this same axis. During the development of the grid, it should be made sure that the order of the nodes (local classification) of each cylindrical mesh is correct: nodes of the lower base then nodes of the higher base (or vice versa). The direction of the axis of the beams is indicated under the keyword factor ORIENTATION order AFFE_CARA_ELEM.
The following assumption was carried out: the reference mark of reference is the same one as the principal reference mark of inertia of the characteristic tube representing the homogenized medium. That means that in the equations [éq 3.1-3], the term I_{xy} is null.

6.3 Modal calculation

The developed finite element makes it possible to characterize the vibratory behavior of a network of beams bathed by a fluid. It is interesting to determine the frequencies of vibration of such a network in air and water.

To carry out a modal calculation in air ($\rho_F=0$), it is necessary to block all the degrees of freedom corresponding to the fluid potential of displacement Φ, if not rigidity stamps it (and even the shiftée matrix of the modal problem) is noninvertible [R5.01.01].

To carry out a modal water calculation ($\rho_F\neq 0$), it is necessary to use in the order CALC_MODES with OPTION='CENTER'. The matrix shiftéE $(\tilde{\mathbf{K}}-\sigma\,\tilde{\mathbf{M}})$ is then invertible if σ is not eigenvalue or if σ is different from zero.

7 Characterization of the spectrum of the homogenized model

7.1 Heterogeneous model

That is to say a network with square step of n beams fixed in their low ends and whose higher ends move in the same way (uniform movement) (cf appears [fig 7.1-a]). Only the movements of inflection are considered.

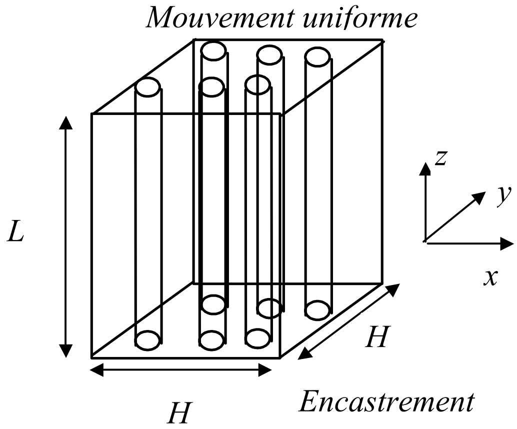

Figure 7.1-a

The spectrum of vibration in air of this network to the following form. For each order of mode of vibration of inflection, the modal structure consists of a frequency doubles correspondent with a mode in x and with a mode in y where all the upper part moves (all the beams have the same deformation) and of a frequency of multiplicity $(2n-2)$ correspondent with modes where all the upper part of the beams is motionless and where beams move in opposition of phase.

In the presence of fluid, the spectrum is modified. For each order of mode of vibration in inflection, them $2n$ frequencies of vibration are lower than the frequencies of vibration obtained in air. The effect of the incompressible fluid is comparable to an added mass. There is always a frequency doubles correspondent with a mode in x and with a mode in y where all the upper part moves (all the beams have the same deformation). On the other hand, one obtains $(n-1)$ couples different of frequency doubles (one in x and one in y) correspondent with modes where all the upper part of the beams is motionless and where beams move in opposition of phase.

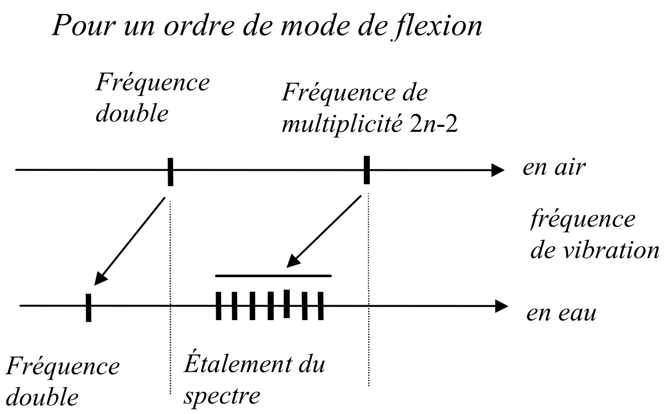

Figure 7.1-b

7.2 Homogeneous model

The heterogeneous medium was replaced by a homogeneous medium.

7.2.1 Continuous problem

Recent work, concerning a problem of plane homogenisation of a network of runners retained by springs and bathed by a fluid, shows that the spectrum of the continuous homogeneous model consists of a continuous part and two frequencies of infinite multiplicity [bib10]. The spectrum of the Eigen frequencies of the water problem is also contained in a well defined interval limited exceptionally by the fréqUin it of vibration in air of a runner [bib5].

These results are transposable for each order of inflection of the network of tubes.

7.2.2 Discretized problem

That is to say the homogeneous field with a grid by hexahedrons. That is to say p the number of generators parallel with the axis z network of beams.

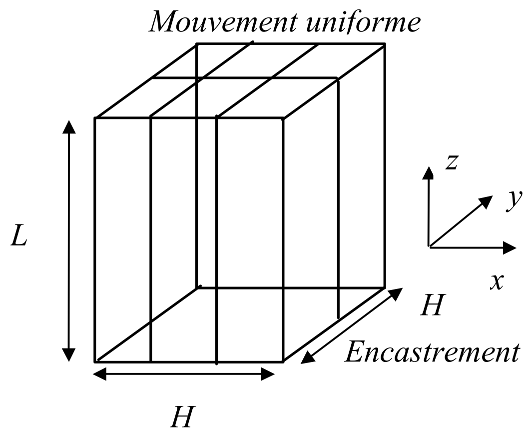

Figure 7.2.2-a

One finds results similar to those obtained for the heterogeneous model. It is enough to replace n by p . For an order of inflection of beam, the number of frequencies corresponding to modes where the beams do not vibrate all in the same direction, depends on the discretization used in the transverse directions with the axis of the beams.

According to the finite element used (mesh HEXA8 or mesh HEXA20), the distribution of $(2p-2)$ last frequencies is different. The first double frequency (that corresponding to the mode where the upper part moves) is the same one for the two finite elements.

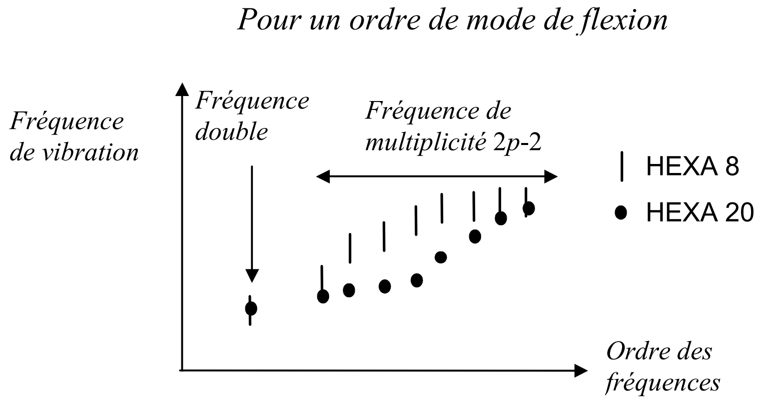

Figure 7.2.2-b

All in all, the homogeneous model makes it possible to obtain the frequencies of vibration easily corresponding to modes where all the beams vibrate in the same direction. The other modes obtained provide only one vision partial of the spectrum. In the discretized spectrum, one can turn over one or the two frequencies of infinite multiplicity present in the spectrum of the continuous model.

8 Conclusion

The use of the finite elements developed associated with the homogenized model of a periodic tube bundle bathed by a fluid makes it possible to characterize the overall vibratory movements (all the structure moves in the same direction) of such a structure.

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10 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
4.0	*B. QUINNEZ (EDF/IMA/MMN)*	Initial text
10.2	F.VOLDOIRE (EDF/AMA)	Corrections of working ; correction equation 3.2-2.