Identification of efforts on a modal model

Summary:

The macro-order CALC_ESSAI gather the features of Code_Aster for the correlation calculation-tests. One describes within the framework of this documentation the functionality "identification of efforts", which must make it possible to identify the efforts to apply to a structure in the shape of a inter-spectrum, starting from the data of the inter-spectrum of measurements under operation, of a modal model, identified or calculated, and choice of the localization of the points of load application.

Contents

1 Introduction, assumptions design

This documentation describes the methods used in the macro one `CALC_ESSAI`.

1.1 Position of the problem

One considers a structure, which constitutes a dynamic system that one supposes **linear (H1 assumption).**

It is supposed that this structure is subjected to a presumedly random excitation (typical example: turbulent fluid efforts), but that one can describe perfectly using his DSP (spectral concentration of power, to see the document "Modeling of the turbulent excitations" R4.07.02, section 2.1). This vibration is measured in a certain number of points of X-coordinates $x_k(1 \le k \le nmes)$. The efforts applied to the structure are not, as for them, measurable. One thus wishes to find the efforts starting from measurement. It is about an opposite problem, for the resolution of which it is necessary to take many precautions in order to get a result which, if it is not exact, is nevertheless relevant.

1.2 Assumption of modal behavior

The structure is supposed to be linear (H1), one can thus describe his mechanical behavior on modal basis. Each mode I of the structure, is defined by the following parameters:

- ω_i : own pulsation, f_i associated Eigen frequency,
- ξ_i : reduced damping,
- m_i : modal mass,
- $\varphi_i(\underline{x})$: modal deformation.

The flow exerts on the structure a force $f(\underline{x}, t)$. It is supposed that these efforts are applied in a direction. One will treat the two directions of measurement separately, by supposing that those are not coupled.

Displacement on modal basis: it is supposed that the movement of the structure is rather well described by its N first modes:

$$u(\underline{x}, t) \approx \sum_{i=1}^{N} q_i(t) . \varphi_i(\underline{x})$$

The generalized coordinates then check the uncoupled equations:

$$m_i\left(\ddot{q}_i + 2\xi_i \omega_i \dot{q}_i + \omega_{i^2} q_i\right) = \int_0^L \varphi_i(\underline{x}) f(\underline{x}, t) dx = Q_i(t) \Leftrightarrow m_i\left(-\omega^2 q_i + 2\xi_i \omega_i \omega q_i + \omega_i^2 q_i\right) = Q_i(\omega),\ 1 \le i \le N$$

2 Modeling and calculation of the forces

2.1 Modeling on modal base, observability and commandability

The supposed linearity of the behavior of the structure allows the decomposition of the model on modal basis:

$$\underline{y} = \underline{\underline{C\,\Phi}} . \underline{\underline{Z}}^{-1} . \underline{\underline{\Phi^T B}} . \underline{f}$$

By using the concept of inter-spectrum, privileged in the macro one[1] :

$$\underline{\underline{S_{yy}}}=\underline{\underline{C\,\Phi}}.\underline{\underline{Z}}^{-1}.\underline{\underline{\Phi^T B}}.\underline{\underline{S_{ff}}}.\underline{\underline{\Phi^T B^H}}.\underline{\underline{Z}}^{-1}.\underline{\underline{C\,\Phi^H}} \quad (1)$$

$\underline{\underline{C}}$ and $\underline{\underline{B}}$ are the matrices of observability and order. $\underline{\underline{C}}$ allows to project a field defined on a model, a more restricted field of sensors. $\underline{\underline{B}}$ project this same field on the points of load application. Matrices $\underline{\underline{C\,\Phi}}$ and $\underline{\underline{\Phi^T B}}$ are the matrices of modal deformations definite on a grid sensor and a grid of order. They can be obtained with the operator **PROJ_CHAMP** or with the operator **OBSERVATION**, which makes it possible, besides the precedent, to define for each node or node of the not measured directions groups, or to define local reference marks.

$\underline{\underline{Z}}$ is the matrix of impedance $diag\left(-\omega^2+2j\,\xi_i\,\omega_i\,\omega+\omega_i^2\right)_{1\le i\le N}$ associated with the base of the modal deformations $\underline{\underline{\Phi}}$. This base is in general defined on a model of good quality. A base of good quality must indeed have modal parameters close to reality. The deformations must also be close to reality, and being sufficiently regular. Thus, an experimental base of deformations is defined on a reduced number of sensors, and is thus not very regular (one is likely to have a resolution of the problem reverses problematic). One will prefer to him the base of deformations of a readjusted digital model, or, better still, an experimental base extended on digital model. One clarifies this point below.

<u>Notice on the modal expansion</u>

A modal base is had $\left(f_i,\xi_i,m_i,\underline{\varphi_{\exp i}}\right)$ experimental. The modal parameters are supposed to be identified with a good approximation, but the deformations, of good quality, are defined only on one restricted number of sensors. One thus tries to extend this base of deformations on a digital model which is rather representative of the structure (but inevitably not readjusted). The base of expansion can be the base of the modes of the digital model (obtained with `CALC_MODES`). To carry out a modal expansion thus means to find the vector of generalized parameters η minimizing:

$$\|\underline{\varphi_{\exp i}}-\underline{\underline{\Phi_{num}}}.\underline{\eta}\|^2$$

The mitre "correlation" of **CALC_ESSAI**, allows to carry out this expansion, thanks to the use of **PROJ_MESU_MODAL**. For more details on the principle of the expansion, one will refer to the documentation of **PROJ_MESU_MODAL** (U4.73.01).

2.1.1 Resolution of the opposite problem

The equation (1) of the direct problem, can, under certain conditions, being reversed:

$$\underline{\underline{S_{ff}}}=\underline{\underline{\Phi^T B}}^{\oplus}.\underline{\underline{Z}}.\underline{\underline{C\,\Phi}}^{\oplus}.\underline{\underline{S_{yy}}}.\underline{\underline{C\,\Phi}}^{H^{\oplus}}.\underline{\underline{Z}}.\underline{\underline{\Phi^T B}}^{H^{\oplus}}$$

Matrices $\underline{\underline{C\,\Phi}}$ and $\underline{\underline{\Phi^T B}}$ being rectangular, the sign $\oplus$ indicate their pseudo-opposite of Moore Penrose. This opposite pseudonym can be obtained by the use of an algorithm of SVD (Singular Been worth Decomposition, cf Doc. R6.03.01). In **CALC_ESSAI**, two possibilities of regularization are available:

1The concept of inter-spectrum, or DSP, makes it possible to describe random phenomena. For a deterministic phenomenon, $\underline{\underline{S_{yy}}}=\underline{y}.\underline{y}^H$, where H appoint the square operator. For a random signal, $\underline{\underline{S_{yy}}}$ contains, on its diagonal, the spectral powers (auto--spectra) and on its extra-diagonal terms, the correlations between the signals. For example, in the case of a fluid effort applied to a telegraphic structure, one imagines easily that the swirls created at the base tend to be propagated along the structure. The extra-diagonal terms thus describe this propagation with the phase (see R4.07.02 for more details).

1) truncation of the small singular values: one notes $\sigma_{\max}$, the greatest singular value. The SVD consists in fixing for all the singular values lower than $\varepsilon\sigma_{\max}$, for ε given, value 0. They are thus not taken into account in opposite calculation. Truncation eliminates from information on the matrices to reverse, but improves conditioning;
2) regularization of Tikhonov: the reverse of the matrix of the singular values is not worth $diag\left(\frac{1}{s_i}\right)_{1\le i\le N}$ but $diag\left(\frac{s_i}{s_{i^2}+\alpha}\right)_{1\le i\le N}$. The parameter α parameter of Tikhonov is called, and makes it possible to limit the divergence of the opposite solution.

In **CALC_ESSAI**, one calculates successively the inter-spectra:

- generalized displacements (inversion of $\underline{\underline{C\,\Phi}}$),
- reconstituted physical displacements (for checking of the quality of the inversion),
- generalized efforts (multiplication by the matrix of impedance),
- physical efforts (inversion of $\underline{\underline{\Phi^T B}}$),
- reconstituted generalized efforts (for checking)
- physical displacements by synthesis on modal basis.

A good quality standard of the results of the inversion can be the comparison between measured displacements, and those reconstituted on the same modal basis which was used for the inversion (matrix of impédence, and matrices of modal deformations).

3 References

[1] L. Perotin, R. Nhili, *Software MEIDEE version 2.1: note of principle* . EDF note/R & D HT-32/92/014/B.

[2] S. Granger, *Software MEIDEE Version 2.1: Data-processing documentation* . EDF note/R & D HT-32/92-15/A.

[3] C. Raynaud, *Identification of the turbulent fluid spectra of excitation on the model SNIPE with* to leave the tests pencil modes for configurations AFAG2G – 4 grids and ENUSA Westinghouse V5H – 4grilles. EDF note/R & D HI-86/03/030/A.

[4] C. Bodel, *Identification of fluid efforts applied to a tube of EPR, control rod model. Methodology and results* . EDF note/R & D H-T61-2007-02808-FR.

[5] A. Adobes, *Modeling of the turbulent excitations*. Documentation Code_Aster R4.07.02.

4 History of the versions of the document

Index Doc.	Version Aster	Author (S) or contributor (S), organization	Description of the modifications
With	9.4	C.BODEL EDF/R & D /MMN	Initial text