Calculation of the thermal deformation

Summary

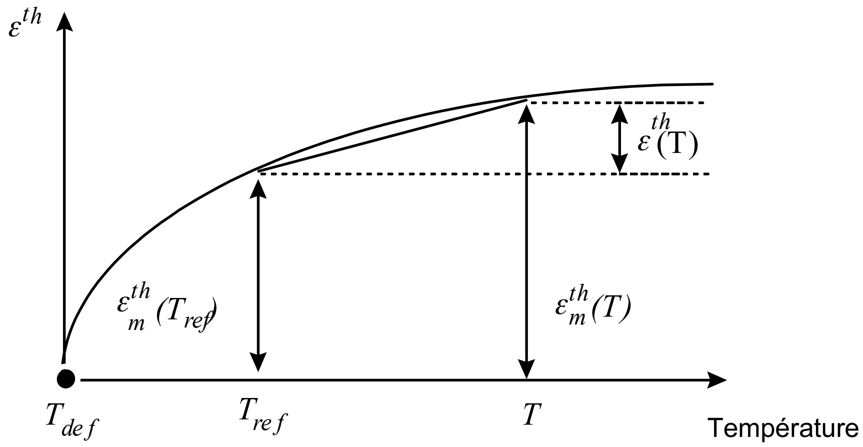
This document is devoted to the presentation of the calculation of the thermal deformation. One indicates to it the necessary information to the calculation of the thermal deformation and the various possibilities of definition of this information by the user.

Contents

1 Introduction

The values of the dilation coefficients are determined by tests of dilatometry which take place starting from the room temperature ($0°C$ or more generally $20°C$). So one in general has the values of the dilation coefficient defined compared to $20°C$ (temperature to which one supposes the worthless thermal deformation).

Certain studies require to take a temperature of reference different from the room temperature (worthless thermal deformation for another temperature that the room temperature). It is then necessary to carry out a change of reference mark in the calculation of the thermal deformation (equation 1 and appears below).



$$\varepsilon^{th}(T)=\varepsilon_m^{th}(T)-\varepsilon_m^{th}(T_{ref}) \quad (1)$$

Where ε_m^{th} is the measured thermal deformation (definite compared to the room temperature) and ε^{th} is the calculated thermal deformation (definite compared to a temperature of reference).

In C*ode_hasster*, the thermal deformation is calculated by the expression following:

$$\varepsilon^{th}(T)=\hat{\alpha}(T)(T-T_{ref}) \quad (2)$$

where $\hat{\alpha}(T)$ is the average dilation coefficient (with direction RCC_M) at the temperature T determined compared to the temperature T_{ref} (T_{ref} being the temperature to which one considers that $\varepsilon^{th}(T_{ref})=0$).
In THM, the calculation of the thermal deformation is different. The thermal deformation is evaluated by the following formula:

$$\varepsilon^{th}(T)=\alpha(T-T_{ini}) \quad (3)$$

The thermal dilation coefficient α is inevitably a constant. One cannot shift the curve. It is necessary thus that the thermal dilation coefficient was evaluated with the room temperature. Moreover, the temperature of reference is given by `THM_INIT/TEMP` in `DEFI_MATERIAU`.

2 Calculation of the thermal dilation coefficient

2.1 From the temperature of reference

The values of the thermal dilation coefficient were determined by tests of dilatometry carried out starting from the temperature T_{ref} . In this case, the keyword TEMP_DEF_ALPHA does not have to be specified in the order DEFI_MATERIAU [U4.23.01]. The equation 1 is simplified, since $\varepsilon_m^{th}(T_{ref})=0$. From where:

$$\varepsilon^{th}(T)=\hat{\alpha}(T)(T-T_{ref}) \text{ and } \varepsilon^{th}(T_{ref})=0 \tag{4}$$

Where values of the dilation coefficient $\hat{\alpha}(T)$ are well informed under the keyword ALPHA (or ALPHA_*) in DEFI_MATERIAU .

2.2 From a temperature different from the temperature of reference

The values of the thermal dilation coefficient were determined by tests of dilatometry which took place starting from a temperature T_{def} different from the temperature of reference T_{ref} .

Indeed, in general one has the values of the dilation coefficient defined compared to the room temperature, $0°C$ or more generally $20°C$, and certain studies require to take a temperature of reference different from the room temperature.

It is then necessary to carry out a change of reference mark (by the equation 1). In this case, the user informs under the keyword TEMP_DEF_ALPHA order DEFI_MATERIAU, the value of the temperature T_{def} , and under the keyword ALPHA (or ALPHA_*) values of the dilation coefficient $\alpha(T)$ (definite compared to the temperature T_{def}). In the order AFFE_MATERIAU under the keyword TEMP_REF, it indicates the value of the temperature of reference T_{ref} . The calculation of $\varepsilon^{th}(T)$ is done by using the formula:

$$\varepsilon^{th}(T)=\alpha(T)(T-T_{def})-\alpha(T_{ref})(T_{ref}-T_{def})=\hat{\alpha}(T)(T-T_{ref}) \text{ and } \varepsilon^{th}(T_{ref})=0 \tag{5}$$

The calculation of $\varepsilon^{th}(T)$ require the preliminary calculation of the values of the function $\hat{\alpha}(T)$.

The function $\hat{\alpha}(T)$ remain defined (or well informed) for the same values of T that $\alpha(T)$ and keeps the same attributes (even standard of interpolation ('FLAX', 'LOG') and even type of prolongation ('CONSTANT', 'LINEAR', 'EXCLUDED').

2.2.1 Calculation into cubes temperatures different from the reference (except for a precision)

One obtains the expression of $\hat{\alpha}(T_i)$ by using the equation 5.

$$\hat{\alpha}(T_i)=\frac{\alpha(T_i)(T_i-T_{def})-\alpha(T_{ref})(T_{ref}-T_{def})}{(T_i-T_{ref})} \quad \forall i \text{ telque } |T_i-T_{ref}| \geq Prec \tag{6}$$

The value of the precision is:

- that is to say specified by the user under the keyword PRECISION keyword factor ELAS_FO (order DEFI_MATERIAU [U4.23.01]),
- that is to say equalizes with $1.$: value by default.

2.2.2 Calculation into cubes temperatures close relations of the reference (except for a precision)

One cannot use the equation 5 directly. The equation is derived 5 compared to the temperature:

$$\frac{\partial \varepsilon^{th}}{\partial T}=\alpha'(T)(T-T_{def})+\alpha(T)=\hat{\alpha}'(T)(T-T_{ref})+\hat{\alpha}(T) \tag{7}$$

ON takes the value of the derivative at the temperature T_{ref} , one obtains:

$$\hat{\alpha}(T_{ref})=\alpha'(T_{ref})(T_{ref}-T_{def})+\alpha(T_{ref}) \tag{8}$$

It is considered that:

$$\hat{\alpha}(T_i)=\hat{\alpha}(T_{ref}) \quad \forall i \ \text{ tel que } |T_i-T_{ref}| \ \geq \ Prec \tag{9}$$

The value of the precision is:

- that is to say specified by the user under the keyword `PRECISION` keyword factor `ELAS_FO` (order `DEFI_MATERIAU` [U4.23.01]),
- that is to say equalizes with $1.$: value by default.

Also, to calculate $\hat{\alpha}(T_i)$ it is necessary as a preliminary to calculate $\alpha'(T_{ref})$.

2.2.3 Calculation of the derivative of the thermal dilation coefficient

The calculation of the derivative of the thermal dilation coefficient is done by an algorithm which treats three cases.

First case:

$$\begin{gathered}\forall i \ \text{ tel que } |T_i-T_{ref}| \ < \ Prec \text{ avec } i\neq 1, i\neq N \\ \alpha'(T_{ref})=\frac{1}{2}\left(\frac{\alpha(T_{i+1})-\alpha(T_{ref})}{T_{i+1}-T_{ref}}+\frac{\alpha(T_{ref})-\alpha(T_{i-1})}{T_{ref}-T_{i-1}}\right)\end{gathered} \tag{10}$$

Second case:

$$\begin{gathered}\forall i \ \text{ tel que } |T_i-T_{ref}| \ < \ Prec \text{ avec } i=N \\ \alpha'(T_{ref})=\frac{\alpha(T_{ref})-\alpha(T_{i-1})}{T_{ref}-T_{i-1}}\end{gathered} \tag{11}$$

Third case:

$$\begin{gathered}\forall i \ \text{ tel que } |T_i-T_{ref}| \ < \ Prec \text{ avec } i=1 \\ \alpha'(T_{ref})=\frac{\alpha(T_{i+1})-\alpha(T_{ref})}{T_{i+1}-T_{ref}}\end{gathered} \tag{12}$$