Indicator of error in residue for modelings HM

Summary

In this document, one presents the indicators of error developed a posteriori for modelings HM. The estimate a posteriori concerned is of standard explicit residue. two types of problems are treated: the permanent version and the transitory version saturated modeling HM. One gives initially a framework of work for the study a posteriori of these problems. One introduces then the families of indicators of error for the two types of problem and one states the theoretical results of reliability and optimality which guarantee the validity of the indicators. The evidence of the announced results will not be given, the interested reader will be able for that to consult [1].

Contents

1 Tally of work

In this paragraph, one points out the equations which constitute the versions permanent and transitory problem HM. One proposes some then adimensionnement who will be useful a posteriori for the analysis of error. One advises with the reader as a preliminary to have consulted the documents [2,3,4] for more details on the establishment of the equations given in this section and on their resolution in Code_Aster.

1.1 The continuous model problem

The framework of work is that of the linear poroelasticity. One considers a porous environment Ω , of border Γ , saturated by liquid water, with a linear elastic mechanical behavior. The behavior of the fluid influences that of the skeleton and reciprocally. Space dimension is noted $\dim$. One solves a mechanical problem of balance (assessment of momentum for the skeleton and the fluid) and a problem of evolution in hydraulics (fluid weight breakdown). It is pointed out that the treatment of these two problems is completely coupled in Code_Aster. The unknown factors of the problem are displacement u and water pressure p .
Being given a time of simulation T , mechanical balance is formulated

$$-\nabla\cdot\sigma'(u)+b\nabla p=f \text{ in } [0,\mathrm{T}]\times\Omega \qquad \textbf{éq 1.1-1}$$

where one posed $f=\rho_{ref}F^m$ and

σ'	Tensor of the effective constraints
b	Coefficient of Biot
ρ_{ref}	Homogenized density
F^m	Force of gravity

Within the framework of isotropic linear elasticity, the tensor of the effective constraints admits like expression

$$\sigma'(u)=\lambda_1(\nabla\cdot u)Id+\lambda_2(\nabla u+\nabla u^t) \qquad \textbf{éq 1.1-2}$$

where λ_1 and λ_2 are the coefficients of Lamé. The hydraulic equation comes as for it from the writing from the conservation from the fluid mass in the course of time and the law from Darcy. The law of Darcy provides a relation of proportionality between hydraulic flow M_{lq} and the gradient of pressure. More precisely, she is written

$$\frac{M_{lq}}{\rho}=\kappa(-\nabla p+\rho F^m) \qquad \textbf{éq 1.1-3}$$

The hydraulic equation is written

$$\partial_t\left(\frac{1}{M}p+b\nabla\cdot u\right)-\nabla\cdot(\kappa\nabla p)=g \text{ in } s[0,\mathrm{T}]\times\Omega \qquad \textbf{éq 1.1-4}$$

where $1/M=(b-\varphi)/K_s$ and $g=-\rho\nabla\cdot(\kappa F^m)$.

φ	Lagrangian porosity
ρ	Hydraulic density
κ	Hydraulic conductivity

K_s	Compressibility of the solid matter constituents

One introduces the permanent version of the equation [1.1-4]

$$-\nabla\cdot(\kappa\nabla p)=g \text{ in } \Omega$$ **éq 1.1-5**

The equation [1.1-1] coupled with [1.1-4] constitutes the version **transient** problem HM.

The equation [1.1-1] coupled with [1.1-5] constitutes the version **permanent** problem HM.

The boundary conditions are of mixed type Dirichlet/Neumann on displacement and the pressure. One considers a partition of the border in the form $\Gamma=\Gamma_D^H\cup\Gamma_N^H=\Gamma_D^M\cup\Gamma_N^M$.

On Γ_D^H , one imposes boundary conditions of Dirichlet in hydraulics: $p=p_D$.
On Γ_N^H , one imposes boundary conditions of Neumann in hydraulics: $M_{lq}\cdot n=M_{lq,nor}$.

On Γ_D^M , one imposes boundary conditions of Dirichlet in mechanics: $u=u_D$.
On Γ_N^M , one imposes boundary conditions of Neumann in mechanics: $(\sigma'(u)-bp\chi_S Id)\cdot n=\sigma_{nor}$, where one noted $\Gamma_S=\Gamma_N^M\cap\Gamma_N^H$, of characteristic function χ_S , the portion of border where the constraints and hydraulic flow are imposed.

1.2 Adimensionnement

The model problem utilizes two unknown factors, displacements and the pressure of the liquid from which the orders of magnitude can be very different. One thus proposes a setting at the level of the equations constituting the model problem, which will be used as starting point with the analysis of error a posteriori. One draws attention to the fact that this scaling is not used in the digital stage of resolution. The estimates of error a posteriori of section 2 will thus be obtained on the adimensionné problem, then redimensionnées for their use in Code_Aster.

For any variable a , one notes a^* the corresponding adimensionnée variable and A corresponding characteristic size. For example for the water pressure, there is the relation

$$p=Pp^*,$$

where P indicate a characteristic pressure. One uses same symbolism for the mathematical operators. Thus, one a: $\nabla^*\cdot u^*=\frac{L}{U}\nabla\cdot u$, $\varepsilon^*(u^*)=\frac{L}{U}\varepsilon(u)$, $\nabla^* p^*=\frac{L}{P}\nabla p$.

1.2.1 Transitory case

The mechanical equation is rewritten

$$-\nabla^*\cdot\sigma^*(u^*)+b\nabla^* p^*=f^*$$ **éq 1.2-1**

with $f^*=\frac{L}{P}f$. While noting ν the Poisson's ratio, $\sigma^*(u^*)$ is a adimensionnée version of the tensor of the constraints given by

$$\sigma^*(u^*)=\frac{\nu}{(1-2\nu)(1+2\nu)}(\nabla^*\cdot u^*)Id+\frac{1}{1+\nu}\varepsilon^*(u^*)$$ **éq 1.2-2**

One proposes the strategy of scaling following. The user chooses two quantities: the scale length L and the scale of pressure P. The scale length L is determined by the geometry of the model. The scale of pressure is determined by the limiting conditions of Dirichlet imposed on the pressure or possibly by the initial conditions. For the mechanical part, one imposes

$$U=\frac{LP}{E}$$ **éq 1.2-3**

In addition, the hydraulic equation is written, by taking account of [1.2-3],

$$\partial_t^*\left(\frac{E}{M}p^*+b\nabla^*\cdot u^*\right)-\frac{E}{M}\Delta^* p^*=g^*$$ **éq 1.2-4**

with $g^*=\frac{E}{M}\frac{L^2}{\kappa P}g$.

The adimensionnée version of transitory problem HM is thus:

$$\begin{cases}-\nabla^*\cdot\sigma^*(u^*)+b\nabla^* p^*=f^* & \text{dans } [0,T^*]\times\Omega^* \\ \partial_t^*\left(\frac{E}{M}p^*+b\nabla^*\cdot u^*\right)-\frac{E}{M}\Delta^* p^*=g^* & \text{dans } [0,T^*]\times\Omega^*\end{cases}$$ **éq 1.2-5**

1.2.2 Permanent case

One proceeds like above: the user chooses L and P then evaluates U according to [1.2-3]. The mechanical equation is treated as in the preceding paragraph. One thus has the equation [1.2-1]. The hydraulic equation is written

$$-\frac{E}{M}\Delta^* p^*=g^*$$ **éq 1.2-6**

where g^* is defined like above. Multiplication of the hydraulic equation by the factor E/M because the parameter can seem a little artificial M does not intervene in a permanent modeling. It is introduced so that the system [1.2-1] - [1.2-6] provides well the permanent version of [1.2-1] - [1.2-5].

The adimensionnée version of permanent problem HM is thus:

$$\begin{cases}-\nabla^*\cdot\sigma^{'*}(u^*)+b\nabla^* p^*=f^* & \text{dans } \Omega^* \\ -\frac{E}{M}\Delta^* p^*=g^* & \text{dans } \Omega^*\end{cases}$$

1.3 Discretization finite elements and notations

Problem HM is discretized by a method finite elements which is based on a grid $(T_h)_{h>0}$. It is pointed out that displacements are discretized by polynomials of degree 2 and the pressure by polynomials of degree 1. Discrete displacements are noted u_h and discrete pressure p_h .

One indicates by F_h^i the whole of the interior faces of the grid and by F_h^∂ the whole of the faces located on the edge of the field Ω . For a mesh $K \in T_h$ data, one notes F_K the whole of the faces of K and one poses

$$F_K^i = F_h^i \cap F_K, F_K^\partial = F_h^\partial \cap F_K$$

That is to say $F \in F_h^i$, i.e. such as there exist two meshs K_1 and K_2 in T_h with $F = K_1 \cap K_2$. One indicates by n_{K_1} and n_{K_2} normals external with K_1 and K_2 , respectively (see figure 1). One notes, for almost all $x \in F$,

$$\left[\Phi\left(u_h\right)\cdot n\right]\left(x\right)=\left(\Phi\left(u_h\right)\right)_{|K_1}\left(x\right)\cdot n_{K_1}+\left(\Phi\left(u_h\right)\right)_{|K_2}\left(x\right)\cdot n_{K_2}$$

the jump of the normal component of $\Phi\left(u_h\right)$ through F .

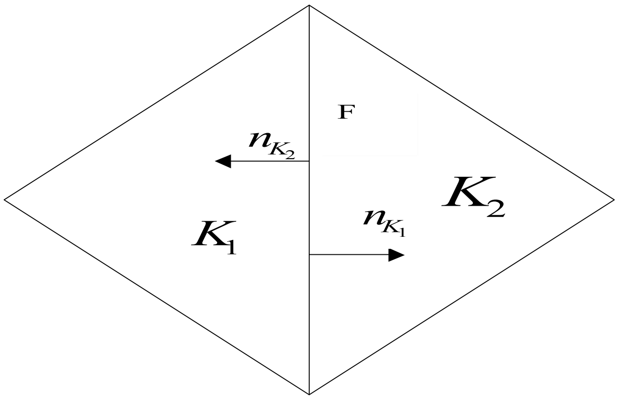

Figure 1: Example of interior face of grid.

One observes that like the vectors n_{K_1} and n_{K_2} are opposed, the quantity above defines this jump well.
For any mesh K , one indicates by Δ_K the whole of the meshs dividing an edge with K , including the mesh K , as illustrated on figure 2.

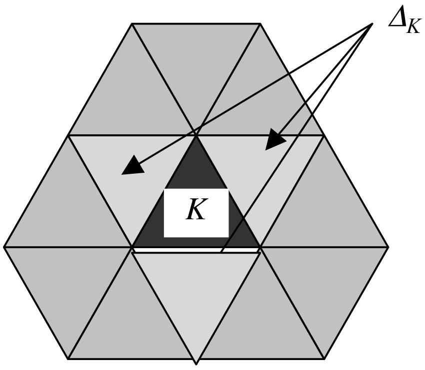

Figure 2: Example of mesh K and of macronutrient Δ_K

For the temporal discretization of the transitory problem, one considers a diagram of implicit Euler and one notes $\left\{t_i\right\}_{i=1}^N$ a discretization of the interval $\left[0,T\right]$. One notes $u_{h\tau}$ (respectively $p_{h\tau}$) the

function continues and closely connected per pieces in time such as for all $n\in\{0,\ldots,N\}$, $u_{h\tau}(t_n)=u_h^n$ (respectively $p_{h\tau}(t_n)=p_h^n$). One also needs to consider constant functions by pieces in time, namely $\pi^0 p_{h\tau}$ equal to p_h^n on $[t_{n-1},t_n]$. For all $n\in\{1,\ldots,N\}$, one poses $\tau_n=t_n-t_{n-1}$ and $I_n=[t_{n-1},t_n]$.
One also notes the "derivative" discrete of displacements and the pressure

$$\delta_t u_h^n=\tau_n^{-1}\left(u_h^n-u_h^{n-1}\right)$$
$$\delta_t p_h^n=\tau_n^{-1}\left(p_h^n-p_h^{n-1}\right)$$

The notation $x<y$ mean that there exists a constant $c>0$ independent of the grid such as $x\leq cy$. The notation $\|\cdot\|_V$ indicate a standard on a space V . His version is introduced $\|\cdot\|_{V,K}$, localised with the mesh K , by $\|\cdot\|_V^2=\sum_{K\in T_h}\|\cdot\|_{V,K}^2$.

2 Study of the permanent problem

In this paragraph, one introduces 2 families of indicators of error put in work in Code_Aster for the permanent problem. The first family is optimal for the estimate of error on the oscillations of pressure. Second is optimal for the estimate of error in energy on displacements.

One points out the expression of the problem in which one is interested. To find (u^*, p^*) such as

$$\begin{cases}-\nabla^*\cdot\sigma'^*(u^*)+b\nabla^* p^*=f^* & \text{dans } \Omega^*\\ -\dfrac{E}{M}\Delta^* p^*=g^* & \text{dans } \Omega^*\end{cases}$$

2.1 Estimate for the pressure

The estimators of error in displacement are defined

$$\begin{aligned}E_u&=\sum_{K\in T_h}E_{u,K}\\ &=\sum_{K\in T_h}\Big(h_K^2\frac{1}{P^2L^{\dim}}\|f+\nabla\cdot\sigma'(u_h)-b\nabla p_h\|_{0,K}^2+h_K\frac{1}{P^2L^{\dim}}\sum_{F\in F_K^i}\|[\sigma'(u_h)\cdot n]\|_{0,F}^2\\ &\quad+h_K\frac{1}{P^2L^{\dim}}\sum_{F\in F_K^{\partial}\cap\Gamma_N^M}\|\sigma_{nor}-\left(\sigma'(u_h)\cdot n-bp_h n\right)\|_{0,F}^2\Big)\end{aligned}$$

The estimators in pressure are defined

$$\begin{aligned}E_{p,0}&=\sum_{K\in T_h}E_{p,0,K}\\ &=\sum_{K\in T_h}\Big(h_K\frac{E^2}{P^2L^{\dim-2}M^2\kappa^2\rho^2}\sum_{F\in F_K^i}\|[M_{lq,h}\cdot n]\|_{0,F}^2\\ &\quad+h_K\frac{E^2}{P^2L^{\dim-2}M^2\kappa^2\rho^2}\sum_{F\in F_K^{\partial}\cap\Gamma_N^H}\|M_{lq,nor}-M_{lq,h}\cdot n\|_{0,F}^2\Big)\end{aligned}$$

One has the result of total increase in space of the error:

Theorem 1 (Reliability)

$$\|u-u_h\|_a^2+\|p-p_h\|_d^2 < E_u+E_{p,0}$$

where for all $(v,q)\in\left[H^1(\Omega)\right]^{\dim}\times H^1(\Omega)$, one posed:

$$\begin{cases}-\nabla^*\cdot\sigma'^*(u^*)+b\nabla^* p^*=f^* & \text{dans } [0,T^*]\times\Omega^*\\ \partial_t^*\left(\frac{E}{M}p^*+b\nabla^*\cdot u^*\right)-\frac{E}{M}\Delta^* p^*=g^* & \text{dans } [0,T^*]\times\Omega^*\end{cases}$$

$$\|v\|_a^2=\frac{E^2}{P^2L^{\dim}}\left(\int_\Omega \lambda_1(\nabla\cdot v)^2+2\int_\Omega \lambda_2\varepsilon(v)^2\right)$$

$$\|q\|_d^2=\frac{1}{P^2L^{\dim-2}}\frac{E}{M}\int_\Omega(\nabla q)^2$$

There are the following results of local decrease in space of the error

Theorem 2 (Optimality)

For all $K\in T_h$, one has

$$E_{u,K}<\sum_{K'\in\Delta_K}\left(h_K^2\|f-f_h\|_{0,K'}^2+\|u-u_h\|_{a,K'}^2+\|p-p_h\|_{0,K'}^2\right)$$

$$E_{p,0,K}<\sum_{K'\in\Delta_K}\left(h_K^2\|g-g_h\|_{0,K'}^2+\|p-p_h\|_{d,K'}^2\right)$$

The estimates obtained are optimal for the estimate of error on the pressure. Indeed, analysis of error a priori watch that in standard H^1 displacements converge with order 2 and the pressure with order 1. By combining the estimates of theorem 1 with those of theorem 2 and by using this result of analysis of error a priori, one has that the estimators a posteriori E_u and $E_{p,0}$ converge overall with order 1, which is optimal for the estimate of error on the pressure but not for the estimate on displacements. One "is bridled" by the convergence of $E_{p,0}$, which is only with order 1. That justifies the following paragraph.

2.2 Estimate for displacements

One defines a new estimator in pressure, which is a version only slightly modified of the preceding estimator (there is an additional order of convergence)

$$E_{p,1}=\sum_{K\in T_h} E_{p,1,K}$$
$$=\sum_{K\in T_h}\left(h_K^3\frac{E^2}{P^2 L^{\dim} M^2\kappa^2\rho^2}\sum_{F\in F_K^i}\|[M_{lq,h}\cdot n]\|_{0,F}^2\right.$$
$$\left.+h_K^3\frac{E^2}{P^2 L^{\dim} M^2\kappa^2\rho^2}\sum_{F\in F_K^\partial\cap\Gamma_N^H}\|M_{lq,nor}-M_{lq,h}\cdot n\|_{0,F}^2\right)$$

One has the following properties:

Theorem 1 (Reliability)

$$\|u-u_h\|_a^2<E_u+E_{p,1}$$

Theorem 2 (Optimality)
For all $K\in T_h$, one has

$$E_{p,1,K}<\sum_{K'\in\Delta_K}\left(h_K^4\|g-g_h\|_{0,K'}^2+h_K^2\|p-p_h\|_{d,K'}^2\right)$$

The estimates obtained are optimal for the estimate of error on displacement because the convergence of the estimators E_u and $E_{p,1}$ place with order 2 has.

3 Study of the transitory problem

One points out the expression of the continuous problem in which one is interested. To find (u^*,p^*) such as

$$\begin{cases}-\nabla^*\cdot\sigma'^*(u^*)+b\nabla^*p^*=f^* & \text{dans } [0,T^*]\times\Omega^*\\ \partial_t^*\left(\frac{E}{M}p^*+b\nabla^*\cdot u^*\right)-\frac{E}{M}\Delta^*p^*=g^* & \text{dans } [0,T^*]\times\Omega^*\end{cases}$$ **éq 3.1-1**

In the thesis [1], 2 families of indicators of error for this problem were proposed. Only was restored in Code_Aster, allowing to effectively evaluate the error on the pressure.

The estimators of error in space are defined:

- Estimator for the hydraulic equation: for all $m\in[1,N]$,

$$\tau_m E_{p,0}^m=\sum_{K\in T_h}\tau_m E_{p,0,K}^m$$
$$=\sum_{K\in T_h}\left(\tau_m h_K^2\frac{E^2}{P^2 L^{\dim}\kappa M}\left\|\frac{1}{M}\delta_t p_h^m+b\nabla\cdot\left(\delta_t u_h^m\right)\right\|_{0,K}^2+\tau_m h_K\frac{E^2}{P^2 L^{\dim}\kappa\rho^2 M}\sum_{F\in F_K^i}\|[M_{lq,h}^m\cdot n]\|_{0,F}^2\right.$$
$$\left.+\tau_m h_K\frac{E^2}{P^2 L^{\dim}\kappa\rho^2 M}\sum_{F\in F_K^\partial\cap\Gamma_N^H}\|M_{lq,nor}^m-M_{lq,h}^m\cdot n\|_{0,F}^2\right)$$

- Estimators for the mechanical equation: for all $m\in[1,N]$,

$$E_u^m = \sum_{K \in T_h} E_{u,K}^m$$

$$= \sum_{K \in T_h} \Big(h_K^2 \frac{1}{P^2 L^{\dim}} \| f^m + \nabla \cdot \sigma' \left(u_h^m \right) \|_{0,K}^2 + h_K \frac{1}{P^2 L^{\dim}} \sum_{F \in F_K^i} \| \left[\sigma' \left(u_h^m \right) \cdot n \right] \|_{0,F}^2$$

$$+ h_K \frac{1}{P^2 L^{\dim}} \sum_{F \in F_K^{\partial} \cap \Gamma_N^M} \| \sigma_{nor}^m - \left(\sigma' \left(u_h^m \right) \cdot n - b p_h^m n \right) \|_{0,F}^2 \Big)$$

$$E_u^m \left(\delta_t \right) = \sum_{K \in T_h} E_{u,K}^m \left(\delta_t \right)$$

$$\sum_{K \in T_h} \Big(h_K^2 \frac{1}{P^2 L^{\dim}} \| f^m - f^{m-1} + \nabla \cdot \sigma' \left(u_h^m - u_h^{m-1} \right) - b \nabla \left(p_h^m - p_h^{m-1} \right) \|_{0,K}^2$$

$$+ h_K \frac{1}{P^2 L^{\dim}} \sum_{F \in F_K^i} \| \left[\sigma' \left(u_h^m - u_h^{m-1} \right) \cdot n \right] \|_{0,F}^2$$

$$+ h_K \frac{1}{P^2 L^{\dim}} \sum_{F \in F_K^{\partial} \cap \Gamma_N^M} \| \sigma_{nor}^m - \sigma_{nor}^{m-1} - \left(\sigma' \left(u_h^m - u_h^{m-1} \right) \cdot n - b \left(p_h^m - p_h^{m-1} \right) n \right) \|_{0,F}^2$$

One defines the estimator in time

$$E_{tim}^m = \tau_m \frac{E}{P^2 L^{\dim} \rho^2 \kappa} \| M_{lq,h}^m - M_{lq,h}^{m-1} \|_{0,\Omega}^2$$

One has the following properties:

Theorem 1 (Reliability) For all $n \in [1, N]$,

$$\int_0^{t_n} \| \left(p - p_{h\tau} \right) (s) \|_d^2 + \int_0^{t_n} \| \left(p - \pi^0 p_{h\tau} \right) (s) \|_d^2 < \sum_{m=1}^{N} \tau_m E_{p,0}^m + \sup_{0 \le m \le N} E_u^m + \left(\sum_{m=1}^{N} \left(E_u^m \left(\delta_t \right) \right)^{1/2} \right)^2 + \sum_{m=1}^{N} E_{tim}^m$$

The operator π^0 appoint the operator of projection on the constant functions per pieces in time, namely $\pi^0 p_{h\tau}$ equal to p_h^n on I_n for all $n \in \{1, \cdots, N\}$.

Theorem 2 (Optimality of the indicator in time)

There is the following estimate

$$E_{tim} = \sum_{m=1}^{N} E_{tim}^m < \int_0^T \| \left(p - p_{h\tau} \right) (s) \|_d^2 ds + \int_0^T \| \left(p - \pi^0 p_{h\tau} \right) (s) \|_d^2 ds$$

Theorem 3 (Optimality of the indicators in space) For all $K \in T_h$, one has

$$E_{u,K}^m < \sum_{K' \in \Delta_K} \left[h_K^2 \| f^m - f_h^m \|_{0,K'} + \| u^m - u_h^m \|_{a,K'}^2 + \| p^m - p_h^m \|_{0,T'}^2 \right]$$

$$E_{u,K}^m(\delta_t) < \sum_{K' \in \Delta_K} \tau_m^2 \left[h_K^2 \| \delta_t f^m - \delta_t f_h^m \|_{0,K'} + \| \delta_t u^m - \delta_t u_h^m \|_{a,K'}^2 + \| \delta_t p^m - \delta_t p_h^m \|_{0,T'}^2 \right]$$

$$\tau_m E_{p,0,K}^m < \sum_{K' \in \Delta_K} h_K^2 \int_{I_m} [\| (g - \pi^0 g_{h\tau})(s) \|_{0,K'}^2 ds + \tau_m^2 \| \delta_t u^m - \delta_t u_h^m \|_{a,K'}$$

$$+ \tau_m^2 \| \delta_t p^m - \delta_t p_h^m \|_{0,K'}^2 + h_T^{-2} \| (p - \pi^0 p_{h\tau})(s) \|_{d,K'}^2] ds$$

4 Use in Code_Aster

The calculation of the temporal indicators is started in STAT_NON_LINE by the keyword ERRE_TEMPS_THM= ' OUI' in the keyword factor CRIT_QUALITE. It makes it possible to calculate the quantities ERRE_TPS_LOC , E_{tim}^m , and ERRE_TPS_GLOB , $\sum_{m=1}^{N} E_{tim}^m$.

The calculation of the indicators in space is started in CALC_ERREUR by the option 'ERME_ELEM' .

For the indicators in space into permanent, one has access to the parameters ERRE_MEC , ERRE_HYD_S and ERRE_HYD_D. They are respectively the noted quantities E_u , $E_{p,0}$ and $E_{p,1}$ in this document.

Example of extract of command file:

```
RESU [K] =CALC_ERREUR(reuse =RESU [K],
            RESULTAT=RESU [K],
            LIST_INST=LINST,
            OPTION= ('ERME_ELEM', 'ERME_ELNO',),);

    dictionary = RESU [K] .LISTE_PARA ()

    print dictionary ['ERRE_MEC']
```

The values of the indicators thus are recovered E_u list LINST.

For the indicators in space in transient, one has access to the following parameters:

ERRE_MEC_LOC	E_u^m
ERRE_MEC_LOC_D	$E_u^m(\delta_t)$
ERRE_MEC_GLOB	$\sup_{0 \le m \le N} \left(E_u^m\right)^{1/2}$
ERRE_MEC_GLOB_D	$\sum_{m=1}^{N} \left(E_u^m(\delta_t)\right)^{1/2}$
ERRE_HYD_LOC	$E_{p,0}^m$
ERRE_HYD_GLOB	$\left(\sum_{m=1}^{N} \tau_m E_{p,0}^m\right)^{1/2}$

5 Conclusion - Outline

The estimators for modelings HM come to supplement the consequent panoply estimators in existing space in Code_Aster. For the first time, indicators in time make it possible to quantify the error on the temporal discretization. The prospects for this work are several orders:

1) To extend the perimeter of use of estimators HM to nonlinear modelings;
2) To develop estimators for modelings THHM in general;
3) To set up a procedure of recutting of the steps of time starting from the estimators in time. For the moment, only the digital values of the estimators in time are provided in `STAT_NON_LINE`, without being connected on an adaptation mechanism of the temporal discretization.

6 Bibliography

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7 History of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
9.4	S.MEUNIER EDF-R&D/AMA	Initial text