Indicators of discharge and loss of proportionality of the loading in elastoplasticity

Summary

One presents a set of scalar parameters called indicators, allowing to appreciate a discharge or a loss of proportionality of a loading during his history. Two types of indicators are proposed: indicators being appeared as scalar fields allowing to detect the zones of the structure undergoing of the discharges or the nonradial loadings, and the total indicators integrated on a zone of the structure chosen by the user. The latter are more especially intended for the evaluation of the validity of the rate of refund of energy in nonlinear breaking process.

The indicators described in this document are available:

- for the local indicators under the order CALC_CHAMP, options DERA_ELGA and DERA_ELNO ;
- for the total indicators, under the order POST_ELEM , options INDIC_SEUIL and INDIC_ENER.

Contents

1 Introduction

1.1 Definition of a loading proportional

Considering a structure subjected to a thermomechanical loading in the time interval $[0,t]$ it will be said that this loading is proportional (or radial) with the material point p if the stress field represented in this point by the tensor $\boldsymbol{\sigma}$ is proportional to a tensor independent of the moment considered, the proportionality factor being a monotonous function of time. Formally, that will be expressed by:

$\forall \tau [0,t], \boldsymbol{\sigma}(P,\tau)=\alpha(\tau)\boldsymbol{\sigma}_0(P), \alpha(\tau)>0$ monotonous function in $[0,t]$.

This definition implies, in particular, that the principal directions of the constraints remain constant, at the point considered, throughout the way of loading (these directions can be of course variable of a point with another).

1.2 Importance of the loading proportional and utility of indicators

For plastic materials, the mechanical fields depend on all the history run out during the way of loading. The laws of flow are thus incremental and their integration depends on each case of loading. A notable exception precisely relates to the loading proportional for which the law of flow can be integrated once and for all. For example, the law of plasticity of Prandtl-Reuss based on the criterion of Von Mises can be replaced by a nonlinear elastic law (called law of Hencky-Put). The cases of loading strictly proportional are rather rare. Indeed, it is necessary to meet a large number of conditions to carry out such a case [bib1] and these last are seldom checked for the industrial structures. One can even say that for structures presenting of the geometrical defects such as cracks, these conditions never are strictly checked.

When the loadings are multiaxial, cyclic, or thermomechanical transients, certain sections of the way of loading can be strongly nonproportional. It is then useful to locate these sections and to evaluate the importance of the loss of proportionality, so for example adjusting the discretization in time of the elastoplastic problem for the section considered, or measuring the validity of certain postprocessings (in breaking process for example).

1.3 Various types of indicators of loss of proportionality

It seems difficult to define a single and simple size which could detect at the same time space zones of loss of proportionality and sections of way of loading (temporal zones) in a material point. This is why we propose scalar sizes having each one their specificity: two, defined by fields measuring in each point the discharge and the deviation of the constraints between two steps of time (indicating buildings), two others of more total nature, characterizing in a given zone of the structure a history of loading nonproportional.

Note:

> *These indicators are closely related to the discretization in time of the problem. In particular, if this discretization is too coarse, one can not detect very well the discharge or the loss of radiality occurring during the increment of time.*

1.4 indicators of elastoplastic discharge

Another application of the indicators of discharge, consists in alerting the user, if, in the event of important discharge, the choice of a kinematic work hardening would provide a solution very different from isotropic work hardening used (cf CR-AMA-11.035 [3]).

This has a practical interest: in several studies, the behavior chosen very often “by default” (because one has very often only one traction diagram) is `VMIS_ISOT_TRAC`. However so of the local discharges are possible, this can lead typically to over-estimate the constraints (with imposed deformation) or to underestimate the deformations (with imposed constraint). It thus seems relevant to inform the user if, when it used laws of Von Mises with isotropic work hardening, it is likely to get false results when the discharge becomes too important (thus does not remain in the initial field of elasticity). That can indicate to him that it is necessary to use laws of behaviour with kinematic work hardening (`VMIS_CINE_LINE, VMIS_ECMI*, VMIS_CIN1_CHAB`, etc.).

2 Local indicators

The goal of these indicators is to determine the zones of the structure where, at one particular moment, occurs is a discharge or a loss of radiality of the stress field. They are produced in post - treatment of a static or dynamic calculation, 2D or 3D, using an elastic law of behavior or not. They are appeared as fields of scalars whose examination can be carried out by tracing their isovaleurs by a graphic post-processor.

2.1 Indicators of discharge

2.1.1 Local indicator of total discharge

This indicator measures at the point M and between the moment t and $t+\Delta t$, relative variation of the standard of the constraints within the meaning of Von Misès. He is written formally: $I_1=\frac{\|\boldsymbol{\sigma}(M,t+\Delta t)\|-\|\boldsymbol{\sigma}(M,t)\|}{\|\boldsymbol{\sigma}(M,t+\Delta t)\|}$. This quantity is negative in the event of local discharge at the point Mr. the standard $\|\boldsymbol{\sigma}(M,t)\|$ can be written in four ways different according to the choice from the modelisator:

1) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}\boldsymbol{\sigma}^D.\boldsymbol{\sigma}^D}$, where $\boldsymbol{\sigma}^D$ is the deviatoric part of the tensor of the constraints (this standard is useful in plasticity with isotropic work hardening).
2) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}\boldsymbol{\sigma}.\boldsymbol{\sigma}}$, where one considers the totality of the tensor of the constraints in order to detect for example the pressure decreases hydrostatic.
3) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}(\boldsymbol{\sigma}^D-\boldsymbol{X}).(\boldsymbol{\sigma}^D-\boldsymbol{X})}$, with $\boldsymbol{X}$ the tensor of the constraint of recall in the case of an elastoplastic law with a kinematic work hardening.
4) $\|\boldsymbol{\sigma}(M,t)\|=\sqrt{\frac{3}{2}(\boldsymbol{\sigma}-\boldsymbol{X}).(\boldsymbol{\sigma}-\boldsymbol{X})}$

2.1.2 Local indicator of elastoplastic discharge

This indicator allows to know if the discharge remains elastic or if there would be a risk of plasticization if a pure kinematic work hardening were used. It is an “extreme” indicator, knowing that, for metals, isotropic work hardenings and kinematics are both present.

The option `DERA_ELGA` thus calculate (besides the components `DCHA_V` and `DCHA_T`) components:

`I_decha=IND_DCHA:`
- `IND_DCHA`=0 unconstrained initial value.
- `IND_DCHA`=1 if elastic load
- `IND_DCHA`=2 if plastic load
- `IND_DCHA`=-1 if elastic discharge sells by auction (some that is to say the type of work hardening)
- `IND_DCHA`=-2 if abusive discharge (one would have plasticized with a kinematic work hardening).

`VAL_DCHA:` indicate the proportion of exit of the criterion (see further).

Operation is the following: for the laws VMIS_ISOT* only in each point of integration, at every moment t, starting from the tensor of the constraints $\boldsymbol{\sigma}(t)$, cumulated equivalent plastic deformation $\boldsymbol{p}(t)$, and of the isotropic curve of work hardening $\boldsymbol{R}(\boldsymbol{p}(t))$,

- Initialization: IND_DCHA=0, VAL_DCHA=0.
- as long as $p(t)=0$, IND_DCHA=1 (elasticity),
- if IND_DCHA=-2 (criterion of abusive discharge reached), one does nothing any more
- if $\Delta p(t)>0$:
  - if one is in load, therefore if the angle between the tensor increase in forced constraints and the tensor total is "small": , $\frac{\tilde{\boldsymbol{\sigma}}(t-\Delta t)\cdot\Delta\tilde{\boldsymbol{\sigma}}}{\|\tilde{\boldsymbol{\sigma}}(t-\Delta t)\|\|\Delta\tilde{\boldsymbol{\sigma}}\|}>0$ IND_DCHA=2 ; (points between A and B on the figure 2.1.2-a).
  - if one is in discharge, IND_DCHA=-2 (rare case: that means that in step of time, one crosses the surface of load in a direction distant from that of the preceding step)
- if $\Delta p(t)=0$:
  - if IND_DCHA>-1, the tensor is calculated $\boldsymbol{X}$ (center of the field of elasticity if one were in kinematics pure, therefore if the surface of initial load, represented by a circle in the deviative plan, had been relocated up to the point running) by:
  $$\boldsymbol{X}=\boldsymbol{\sigma}(t)\frac{R(p)-R(0)}{R(p)}$$
  - if IND_DCHA=-1, the tensor is used $\boldsymbol{X}$ already calculated.

  - one calculates the "kinematic" criterion $(\boldsymbol{\sigma}(t)-\boldsymbol{X})_{eq}\leq R(0)$
  - If this criterion is checked, then the discharge would be elastic also with a kinematic work hardening, it is thus "licit": IND_DCHA=-1 (any point between B and E on figure 2). To apply the criterion to the next moment, one stores $\boldsymbol{X}$ (6 components)
    - if not, IND_DCHA=-2 and this value keeps (because the continuation of calculation would be modified if work hardening were kinematic). (points between E and F, or C and This on the following figure). VAL_DCHA $=\frac{\|\boldsymbol{\sigma}-\boldsymbol{X}\|}{R_0}$

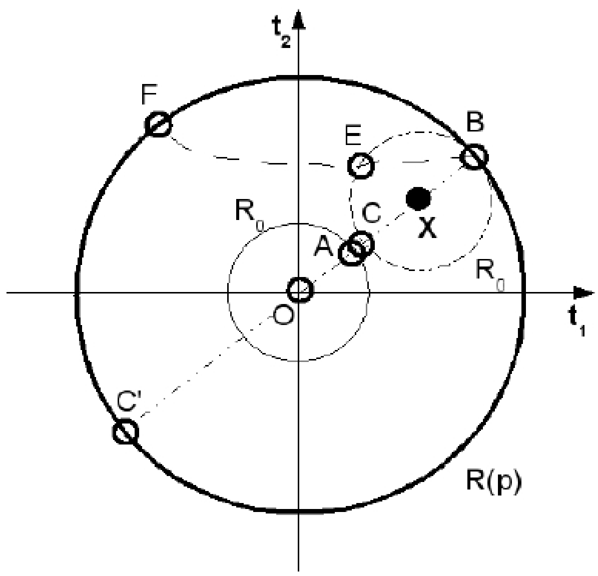

Figure 2.1.2-a : elastoplastic indicator of discharge

- **Comments** :

- The disadvantage of this method comes from what it is not systematic: it is necessary that the user "thinks" of calling the option DERA_ELGA.
- The criterion obtained is relatively severe: it supposes that one replaces pure isotropic work hardening by a pure kinematic work hardening. An upgrading capability would consist in defining a ray of surface threshold larger than R_0 .

2.2 Indicators of loss of radiality

2.2.1 Local indicator of radiality of the loading

This indicator measures at the point M and between the moment t and $t+\Delta t$, variation of the direction of the constraints. He is written:

$I_2=1-\frac{|\sigma(M,t).\Delta\sigma|}{\|\sigma(M,t)\|\|\Delta\sigma\|}$, where the scalar product "." is associated with the one of the four preceding standards. This quantity is worthless when the radiality is preserved during the increment of time. This criterion can also be interpreted like quantity $1-\cos(\theta)$, where θ is the angle enters σ and $\Delta\sigma$. This indicator with actual value evolves between 0 for the radial loadings and 1. It is useful in particular to determine the validity of an elastoplastic solution in breaking process,

2.2.2 Indicator of error due to the temporal discretization

It provides a m esure of the error η had with the discretization in time, directly connected to the rotation of the normal on the surface of load. One calculates the angle enters n^- , the normal with the criterion of plasticity at the beginning of the step of time (urgent T), and n^+ , the normal with the criterion of plasticity calculated at the end of the step of time in the following way: $I_\eta=\frac{1}{2}\|\Delta n\|=\frac{1}{2}\|n^+-n^-\|=\left|\sin(\frac{\alpha}{2})\right|$. This indicator is directly related to the standard of the variation of the normal on convex of plasticity (this can spread easily with any elastoplastic law with normal flow), and it can be also interpreted like the sine of half of the angle the two normals. That provides a measurement of the error (see [4]). This criterion is operational for the elastoplastic behaviors of Von Mises with work hardening isotropic, kinematic linear and mixed. It can be used to control the automatic subdivision of the step of time.

3 Total indicators

These indicators are intended to detect if, during the history of the structure and until the current moment t , and for a zone of the structure chosen by the modelisator, there was loss of proportionality of the loading (these indicators thus leave a trace of the history contrary to the local indicators which are instantaneous). They are only usable within the framework of an elastoplastic behaviour with isotropic work hardening (in 2D or 3D).

3.1 Indicator on the parameters of plasticity

This quantity allows, in the case of the plasticity of Von Mises with isotropic work hardening, on the one hand of knowing (on average about a zone Ω_S field Ω) if the constraints and the plastic deformations have the same directions and if the plastic threshold is reached at the current moment, and in addition so during the history the plastic deformation changed direction. This quantity is written:

$$I_3=\frac{1}{\Omega_S}\int_{\Omega_s}\left(1-\frac{|\sigma.\varepsilon^p|}{(\sigma_Y+R(p))p}\right)d\Omega ,$$

where σ_Y is the initial plastic threshold, R extension of the surface of load related to work hardening and p cumulated plastic deformation. The scalar product is associated with the standard within the meaning of Von Misès. This indicator is standardized and has a value ranging between zero and one. It is null if the loading preserved its character proportional in each point of Ω_S throughout the past history.

Notice 1:

The indicator is not affected so during the history, there were discharges then elastic refills without change of management of the constraints when one reconsiders the threshold (cf [Figure 3.1-a]).

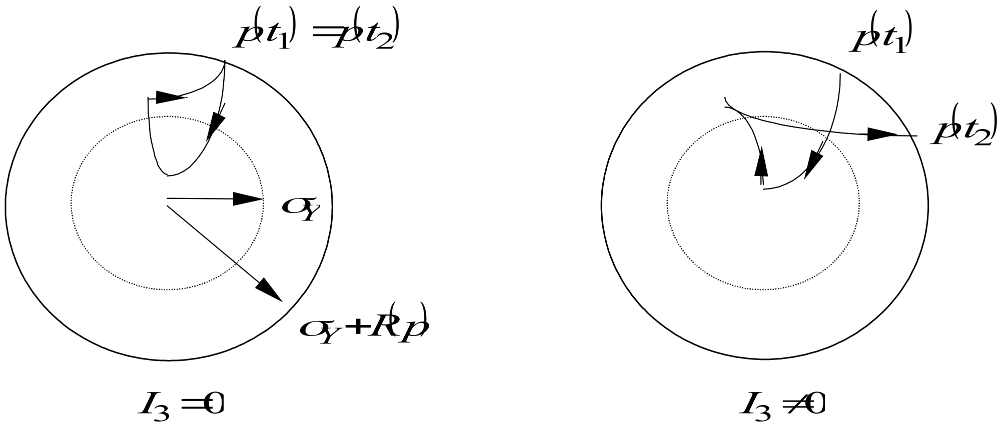

Figure 3.1-a : Way of loading enters t_1 and t_2 in the plan deviatoric of the constraints

Notice 2:

In the formulation of this indicator three ingredients related to plasticity intervene:

- *the variation enters the direction of the constraints and the current plastic deformations ($\sigma . \varepsilon ¿ p \neq \|\sigma\| \|\varepsilon^p\|$),*
- *the position of the constraints compared to the current threshold ($\|\sigma\| \leq (\sigma_Y + R(p))$),*
- *the variation enters the standard of the current plastic deformation and the cumulated plastic deformation ($\|\varepsilon^p\| \neq p$).*

A loss of proportionality could occur during the history without the indicator not detecting it via the first two ingredients (i.e. one can have at the end of the loading coincidence of the directions of the constraints and the plastic deformations and be on the plastic threshold). On the other hand, one will have, $\|\varepsilon^p\| \neq p$ and the indicator will be obligatorily higher than zero, consequently the user will be informed loss of proportionality.

Notice 3:

If the indicator detects obligatorily a loss of proportionality in a zone, in practice it is necessary that the latter contains sufficient material points with loading nonproportional. Indeed, if one chooses a very vast zone with few points concerned, the standardisation of the indicator carried out with division by the volume of the zone implies a certain "crushing" towards zero of the value of the quantity. Typically, for a structure containing a defect source of nonproportionality, one may find it beneficial to choose a zone of integration Ω_S surrounding the defect with a weak vicinity in order to obtain a significant value of the indicator.

3.2 Energy indicator

This indicator has the same function that the precedent, but is founded on the density of energy. He is written:

$$I_4=\frac{1}{\Omega_S}\int_{\Omega_S}(1-\frac{\Psi}{\omega})d\,\Omega\ ,$$

where ω is the density of deformation energy defined by: $\omega(t)=\int_0^t \boldsymbol{\sigma}.\dot{\boldsymbol{\varepsilon}}\,d\tau$, and ψ is the density of energy elastic associated with the traction diagram if nonlinear elastic material were considered. More precisely, this quantity is written:

$$\psi(\boldsymbol{\varepsilon}(t))=\frac{1}{2}Ktr^2(\boldsymbol{\varepsilon})+\frac{2\mu}{3}\|\boldsymbol{\varepsilon}\|^2,\ \text{si}\ \|\boldsymbol{\sigma}\|<(\sigma_Y+R(p)),$$
$$\psi(\boldsymbol{\varepsilon}(t))=\frac{1}{2}Ktr^2(\boldsymbol{\varepsilon})+\frac{\mathrm{R}^2(P)}{6\mu}+\int_0^P \mathrm{R}(q)\,dq,\ \text{si}\ \|\boldsymbol{\sigma}\|=(\sigma_Y+R(p)),$$

with K the module of compressibility, μ the coefficient of shearing of Lamé, R the threshold of the traction diagram associated with the standard with the plastic deformation $P=\|\boldsymbol{\varepsilon}^p\|$ (this one can be different from the true plastic threshold, because $P\neq p$ if the loading is nonproportional). This indicator is also standardized between 0 and 1. It is null for a loading having always kept its character proportional $(\psi\neq\omega)$.

4 Features and checking

The indicators presented here are usable in postprocessing of a mechanical calculation and are available for the finite elements of the continuous mediums in 2D (mode of plane strains, stresses plane or axisymmetric, triangular or quadrangular meshs) or 3D (hexahedral, tetrahedral, pentaedric meshs or pyramids). The telegraphic elements, beams, plates and hulls are excluded from this application.

4.1 Local indicators

These indicators are accessible after a static or dynamic calculation whatever the law from behavior of material. The operator `CALC_CHAMP` present the options `'DERA_ELGA'` and `'DERA_ELNO'` for the indicator of discharge I_1 and the indicator of loss of radiality I_2 evaluated with the nodes or the points of Gauss of the element [2]. In the case general, the first two standards described in the paragraph 2.1 are calculated, the two last being used only if one carried out as a preliminary an elastoplastic calculation with kinematic work hardening.

The components of the field thus calculated are:

- `DCHA_T` : the indicator of discharge I_1 calculated starting from the tensor of the constraints;
- `DCHA_V` : the indicator of discharge I_1 calculated starting from the diverter of the constraints;
- `IND_DCHA` : the indicator of discharge I_{decha}
    - `IND_DCHA`=0 unconstrained initial value
    - `IND_DCHA`=1 if elastic load
    - `IND_DCHA`=2 if plastic load
    - `IND_DCHA`=-1 if elastic discharge sells by auction (some that is to say the type of work hardening)
    - `IND_DCHA`=-2 if abusive discharge (one would have plasticized with a kinematic work hardening).
- `VAL_DCHA` : the indicator of the proportion of exit of the criterion
- `RADI_V` : the indicator of loss of radiality I_2 calculated starting from the diverter of the constraints.

- ERR_RADI: the indicator of error I_η had with the temporal discretization

These indicators are checked by the cases following tests:

SSNP111	Passage des points of Gauss to the nodes on quadratic elements	[V6.03.111]
SSNP14	Plate in traction-shearing - Von Mises (kinematic Work hardening)	[V6.03.014]
SSNP15	Plate in traction-shearing - Von Mises (isotropic work hardening)	[V6.03.015]

4.2 Total indicators

These indicators are accessible only after one elastoplastic calculation with isotropic work hardening. The operator POST_ELEM present the options 'INDIC_SEUIL' and 'INDIC_ENER' corresponding respectively to the total indicators I_3 and I_4. Those are evaluated on a group of mesh previously defined by the user (for example by the order DEFI_GROUP).

These indicators are checked by the cases following tests:

HSNV100i	Thermoplasticity in simple traction	[V7.22.100]
SSNP15	Plate in traction-shearing - Von Mises (isotropic work hardening)	[V6.03.015]

5 Bibliography

[1] J. LEMAITRE, J. - L.CHABOCHE, Mechanics of solid materials, Dunod 1985.

[2] Instruction manual of *Code_Aster*. Document [U4.61.02].

[3] CR-AMA-11.035: "Project LOCO - batch B2 - local Indicators of discharge and radiality" A.Foucault, 1/24/2011

[4] CR-AMA-11.30 "Study of a criterion of radiality" J.M.Proix

6 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
4	*G. DEBRUYNE (EDF/IMA/MMN)*	Initial text
11.1	J.M.PROIX EDF/R & D /AMA	Addition of the new indicators of discharge.
11.2	J.M.PROIX EDF/R & D /AMA	Addition of a new criterion of radiality.