Model of Rousselier in great deformations

Summary

One presents here an alternative of the model of Rousselier which makes it possible to describe the plastic growth of cavities in a steel. The relation of behavior is elastoplastic with isotropic work hardening, allows the changes of plastic volume and is written in great deformations. These last are based on the theory suggested by Simo and Miehe, modified to facilitate the digital integration of the law of behavior and to replace the model within the framework of generalized standard materials.
This model is available in the order `STAT_NON_LINE` via the keyword `RELATION: 'ROUSSELIER'` under the keyword factor `BEHAVIOR` and with the keyword `DEFORMATION: 'SIMO_MIEHE'`.
This model is established for three-dimensional modelings (`3D`), axisymmetric (`AXIS`) and in plane deformations (`D_PLAN`).

One presents the writing and the digital processing of this model.

Contents

Contents

1 Introduction

The mechanisms at the origin of the ductile rupture of steels are associated with the development of cavities within material. Three phases are generally distinguished:

- germination: it is the initiation of the cavities, into cubes sites which correspond preferentially to the defects of material,
- growth: it is the phase which corresponds to the development itself of the cavities, controlled primarily by the plastic flow of the metal matrix which surrounds these cavities,
- coalescence: it is the phase which corresponds to the interaction of the cavities between them to create macroscopic cracks.

In what follows, we treat only the phases of growth and coalescence. Rousselier [bib1] proposed a law of capable behavior to give an account of these two phases. Compared to this formulation of origin, Lorentz and al. [bib2] introduced several modifications relating primarily to the treatment of the great deformations (multiplicative decomposition), the evolution of porosity (function of the total deflection) and the expression of the law of flow at the singular point of surface threshold.

More precisely, the model is based on assumptions which introduce a microstructure made up of a cavity and a plastic rigid matrix thus isochoric. In this case, porosity f, definite like the relationship between the volume of the cavity V^C and total volume V representative ground volume, is directly connected to the macroscopic deformation by:

$$J=\det \mathbf{F}=\frac{1-f_0}{1-f} \quad \text{avec} \quad f=\frac{V^c}{V} \Leftrightarrow \dot{f}=(1-f)\,\mathrm{tr}\,\mathbf{D} \qquad \text{éq 1-1}$$

where f_0 indicate initial porosity, $\mathbf{F}$ the tensor gradient of the transformation, J variation of volume and $\mathbf{D}$ the rate of deformation.

To build the law of growth of the cavities, Rousselier took as a starting point a phenomenologic analysis which leads it to the following ingredients:

- great deformations figure,
- irreversible changes of volume,
- isotropic work hardening.

These considerations lead it to write the criterion of plasticity F in the following form:

$$F(\boldsymbol{\tau},\ R)=\tau_{eq}+\sigma_1 D f \exp\left(\frac{\tau_H}{\sigma_1}\right)-R(p)-\sigma_y \qquad \text{éq 1-2}$$

where $\boldsymbol{\tau}$ is the constraint of Kirchhoff, R isotropic work hardening function of the cumulated plastic deformation p and σ_1, D and σ_y parameters of material. The presence in the criterion of plasticity of the hydrostatic constraint τ_H authorize the changes of plastic volume. One also notices that this model does not comprise a specific variable of damage because only microstructural information selected is porosity, directly related to the macroscopic deformation by the equation [éq 1-1].

As for the treatment of the great deformations, one adopts the theory of Simo and Miehe but in a slightly modified formulation. The approximations brought make it possible to make easier the digital integration of the law of behavior but also to replace the theory of Simo and Miehe within the framework of generalized standard materials.

Thereafter, one briefly gives some concepts of mechanics in great deformations, then one points out the theory of Simo and Miehe as well as the made modifications. One presents finally the relations of behavior of the model of Rousselier and his digital integration.

2 Notations

One will note by:

$\mathbf{Id}$	matrix identity
$\mathrm{tr}\,A$	trace of the tensor **With**
$\mathbf{A}^{\mathbf{T}}$	transposed of the tensor **With**
$\det\mathbf{A}$	determinant of **With**
$\tilde{\mathbf{A}}$	deviatoric part of the tensor **With** defined by $\tilde{\mathbf{A}}=\mathbf{A}-(\frac{1}{3}\mathrm{tr}\,\mathbf{A})\mathbf{Id}$
A_H	hydrostatic part of the tensor **With** defined by $A_H=\frac{\mathrm{tr}\,\mathbf{A}}{3}$
$:$	doubly contracted product: $\mathbf{A}:\mathbf{B}=\sum_{i,j} A_{ij}B_{ij}=\mathrm{tr}(\mathbf{A}\mathbf{B}^{\boldsymbol{T}})$
$\otimes$	tensorial product: $(\mathbf{A}\otimes\mathbf{B})_{ijkl}=A_{ij}B_{kl}$
A_{eq}	equivalent value of Von Mises defined by $A_{eq}=\sqrt{\frac{3}{2}\tilde{A}:\tilde{A}}$
$\nabla_{\mathbf{X}}\mathbf{A}$	gradient : $\nabla_X\mathbf{A}=\frac{\partial\mathbf{A}}{\partial\mathbf{X}}$
λ,μ,E,ν,K	moduli of the isotropic elasticity
σ_y	elastic limit
α	thermal dilation coefficient
T	temperature
T_{ref}	temperature of reference

In addition, within the framework of a discretization in time, all the quantities evaluated at the previous moment are subscripted by $^-$, quantities evaluated at the moment $t+\Delta t$ are not subscripted and the increments are indicated by Δ . One has as follows:

$$\Delta Q=Q-Q^-$$

3 Theory of Simo and Miehe

3.1 Introduction

We point out here specificities of the formulation suggested by SIMO J.C and MIEHE C. [bib3] to treat the great deformations. This formulation was already used for models of thermo-élasto behavior (visco) - plastic with isotropic work hardening and criterion of Von Mises, [R5.03.21] for a model without effect of the metallurgical transformations and [R4.04.03] for a model with effect of the metallurgical transformations.
The kinematics choices make it possible to treat great displacements and great deformations but also of great rotations in an exact way.
Specificities of these models are the following ones:

- just like in small deformations, one supposes the existence of a slackened configuration, i.e. locally free of constraint, which makes it possible to break up the total deflection into a thermoelastic part and a plastic part,
- the decomposition of this thermoelastic deformation into cubes parts and plastic is not additive any more as in small deformations (or for the models great deformations written in rate of deformation with for example a derivative of Jaumann) but multiplicative,
- the elastic strain are measured in the current configuration (deformed) while the plastic deformations are measured in the initial configuration,
- as in small deformations, the constraints depend only on the thermoelastic deformations,
- if the criterion of plasticity depends only on the deviatoric constraint, then the plastic deformations are done with constant volume. The variation of volume is then only due to the thermoelastic deformations,
- this model led during its digital integration to a model incrémentalement objective (cf [§3.2.3]) what makes it possible to obtain the exact solution in the presence of great rotations.

Thereafter, one briefly points out some concepts of mechanics in great deformations.

3.2 General information on the great deformations

3.2.1 Kinematics

Let us consider a solid subjected to great deformations. That is to say Ω_0 the field occupied by the solid before deformation and $\Omega(t)$ the field occupied at the moment T by the deformed solid.

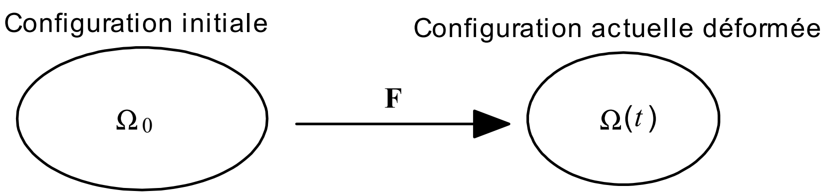

Figure 3.2.1-a: Representation of the initial and deformed configuration

In the initial configuration Ω_0 , the position of any particle of the solid is indicated by $\mathbf{X}$ (Lagrangian description). After deformation, the position at the moment t particle which occupied the position $\mathbf{X}$ before deformation is given by the variable $\mathbf{x}$ (description eulérienne).
The total movement of the solid is defined, with $\mathbf{u}$ displacement, by:

$$\mathbf{x}=\hat{\mathbf{x}}(\mathbf{X},t)=\mathbf{X}+\mathbf{u} \quad \text{éq 3.2.1-1}$$

To define the change of metric in the vicinity of a point, the tensor gradient of the transformation is introduced $\mathbf{F}$:

$$\mathbf{F}=\frac{\partial \hat{\mathbf{x}}}{\partial \mathbf{X}}=\mathbf{Id}+\nabla_{\mathbf{X}} \mathbf{u}$$ éq 3.2.1-2

The transformations of the element of volume and the density are worth:

$$d\Omega = J d\Omega_o \text{ with } J=\det F=\frac{\rho_o}{\rho}$$ éq 3.2.1-3

where ρ_o and ρ are respectively the density in the configurations initial and current.

Various tensors of deformations can be obtained by eliminating rotation in the local transformation. For example, by directly calculating the variations length and angle (variation of the scalar product), one obtains:

$$\mathbf{E}=\frac{1}{2}(\mathbf{C}-\mathbf{Id}) \text{ with } \mathbf{C}=\mathbf{F}^{\mathrm{T}}\mathbf{F}$$ éq 3.2.1-4

$$\mathbf{A}=\frac{1}{2}(\mathbf{Id}-\mathbf{b}^{-1}) \text{ with } \mathbf{b}=\mathbf{F}\mathbf{F}^{T}$$ éq 3.2.1-5

$\mathbf{E}$ and $\mathbf{A}$ are respectively the tensors of deformation of Green-Lagrange and Euler-Almansi and $\mathbf{C}$ and $\mathbf{b}$, tensors of right and left Cauchy-Green respectively.

In Lagrangian description, one will describe the deformation by the tensors $\mathbf{C}$ or $\mathbf{E}$ because they are quantities defined on Ω_0 , and of description eulérienne by the tensors $\mathbf{b}$ or $\mathbf{A}$ (definite on Ω).

3.2.2 Constraints

The tensor of the constraints used in the theory of Simo and Miehe is the tensor of Kirchhoff $\boldsymbol{\tau}$ defined by:

$$J\boldsymbol{\sigma}=\boldsymbol{\tau}$$ éq 3.2.2-1

where $\boldsymbol{\sigma}$ is the tensor eulérien of Cauchy. The tensor $\boldsymbol{\tau}$ thus result from a “scaling” by the variation of volume of the tensor of Cauchy $\boldsymbol{\sigma}$.

3.2.3 Objectivity

When a law of behavior in great deformations is written, one must check that this law is objective, i.e. invariant by any change of space reference frame of the form:

$$\mathbf{x}^{*}=\mathbf{c}(t)+\mathbf{Q}(t)\mathbf{x}$$ éq 3.2.3-1

where $\mathbf{Q}$ is an orthogonal tensor which represents the rotation of the reference frame and $\mathbf{c}$ a vector which represents the translation.
More concretely, if one carries out a tensile test in the direction $\mathbf{e_1}$, for example, followed by a rotation of 90° around $\mathbf{e_3}$, which amounts carrying out a tensile test according to $\mathbf{e_2}$, then the danger with a nonobjective law of behavior is not to find a uniaxial tensor of the constraints in the direction $\mathbf{e_2}$ (what is in particular the case with kinematics `PETIT_REAC`).

3.3 Formulation of Simo and Miehe

Thereafter, one will note $\mathbf{F}$ the tensor gradient which makes pass from the initial configuration Ω_0 with the current configuration $\Omega(t)$, $\mathbf{F}^{\mathbf{p}}$ the tensor gradient which makes pass from the configuration Ω_0 with the slackened configuration Ω^r , and $\mathbf{F}^{\mathbf{e}}$ configuration Ω^r with $\Omega(t)$. The index p refers to the plastic part, the index e with the elastic part.

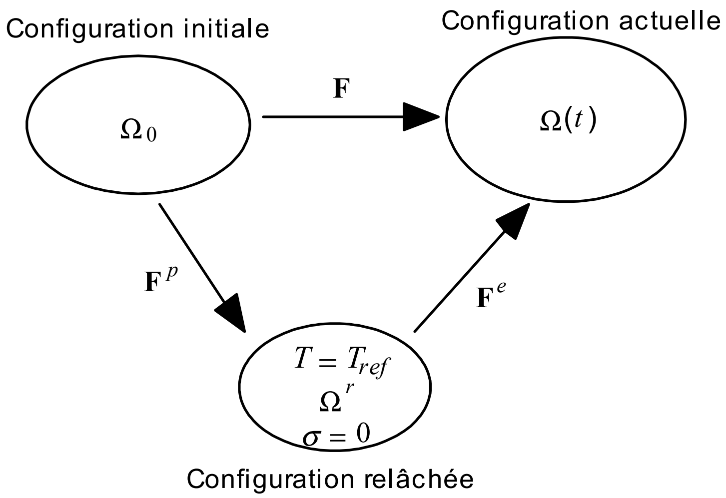

Figure 3.3-a: Decomposition of the tensor gradient $\mathbf{F}$ in an elastic part $\mathbf{F}^{\mathbf{e}}$ and plastic $\mathbf{F}^{\mathbf{p}}$

By composition of the movements, one obtains the following multiplicative decomposition:

$$\mathbf{F}=\mathbf{F}^{\mathbf{e}}\mathbf{F}^{\mathbf{p}} \qquad \text{éq 3.3-1}$$

The elastic strain are measured in the current configuration with the left tensor eulérien of Cauchy-Green $\mathbf{b}^{\mathbf{e}}$ and plastic deformations in the initial configuration by the tensor $\mathbf{G}^{\mathbf{p}}$ (Lagrangian description). These two tensors are defined by:

$$\mathbf{b}^{\mathbf{e}}=\mathbf{F}^{\mathbf{e}}\mathbf{F}^{eT} \ , \ \mathbf{G}^{\mathbf{p}}=(\mathbf{F}^{pT}\mathbf{F}^{\mathbf{p}})^{-1} \ \text{ from where } \ \mathbf{b}^{\mathbf{e}}=\mathbf{F}\mathbf{G}^{p}\mathbf{F}^{\mathbf{T}} \qquad \text{éq 3.3-2}$$

However, one will employ alternatively another measurement of the elastic strain $\mathbf{e}$ who coincides with the opposite of the linearized deformations when the elastic strain are small:

$$\mathbf{e}=\frac{1}{2}\left(\mathbf{Id}-\mathbf{b}^{\mathbf{e}}\right) \qquad \text{éq 3.3-3}$$

In the case of an isotropic material, one can show that the potential free energy depends only on the left tensor of Cauchy-Green $\mathbf{b}^{\mathbf{e}}$ (where in our case of the tensor $\mathbf{e}$) and in plasticity of the variable p dependent on isotropic work hardening. Moreover, one supposes that the voluminal free energy breaks up, just like in small deformations, in a hyperelastic part which depends only on the elastic strain and another related to the mechanism on work hardening:

$$\Phi\left(\mathbf{e},\,p\right)=\Phi^{el}\left(\mathbf{e}\right)+\Phi^{bl}\left(\mathbf{p}\right) \qquad \text{éq 3.3-4}$$

So instead of using the constraint of Cauchy $\boldsymbol{\sigma}$, one uses the constraint of Kirchhoff $\boldsymbol{\tau}$, the inequality of Clausius-Duhem is written (one forgets the thermal part):

$$\boldsymbol{\tau}:\mathbf{D}-\dot{\Phi}\geq 0 \quad \text{éq 3.3-5}$$

expression in which $\mathbf{D}$ represent the rate of deformation eulérien.

Under the preceding assumptions, dissipation is still written:

$$\left(\boldsymbol{\tau}+\frac{\partial\Phi}{\partial\mathbf{e}}\mathbf{b}^{e}\right):\mathbf{D}+\frac{1}{2}\frac{\partial\Phi}{\partial\mathbf{e}}:\left(\mathbf{F}\dot{\mathbf{G}}^{p}\mathbf{F}^{\mathrm{T}}\right)-\frac{\partial\Phi}{\partial p}\dot{p}\geq 0 \quad \text{éq 3.3-6}$$

The second principle of thermodynamics then requires the following expression for the relation stress-strain:

$$\boldsymbol{\tau}=-\frac{\partial\Phi}{\partial\mathbf{e}}\mathbf{b}^{e} \quad \text{éq 3.3-7}$$

One defines finally the thermodynamic forces associated with the elastic strain and the plastic deformation cumulated in accordance with the framework with generalized standard materials:

$$\mathbf{s}=-\frac{\partial\Phi}{\partial\mathbf{e}} \quad \text{soit} \quad \boldsymbol{\tau}=\mathbf{s}\;\mathbf{b}^{e} \quad \text{éq 3.3-8}$$

$$A=-\frac{\partial\Phi}{\partial p} \quad \text{éq 3.3-9}$$

where the thermodynamic force With is the opposite of the isotropic variable of work hardening R
.

It remains then for dissipation:

$$\boldsymbol{\tau}:(-\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^{p}\mathbf{F}^{\mathrm{T}}\mathbf{b}^{e-1})+A\dot{p}\;=\;\mathbf{s}:(-\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^{p}\mathbf{F}^{\mathrm{T}})+A\dot{p}\geq 0 \quad \text{éq 3.3-10}$$

3.3.1 Original formulation

The principle of maximum dissipation applied starting from the threshold of elasticity F, function of the constraint of Kirchhoff $\boldsymbol{\tau}$ and of the thermodynamic force A allows to deduce the laws of evolution from them from the plastic deformation and cumulated plastic deformation, is:

$$-\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^{p}\mathbf{F}^{\mathrm{T}}\mathbf{b}^{e\text{-}1}=\dot{\lambda}\;\frac{\partial F}{\partial\boldsymbol{\tau}} \quad \text{éq 3.3.1-1}$$

$$\dot{p}=\dot{\lambda}\frac{\partial F}{\partial A} \quad \text{éq 3.3.1-2}$$

$$\dot{\lambda}\geq 0 \quad F\leq 0 \quad F\dot{\lambda}=0 \quad \text{éq 3.3.1-3}$$

3.3.2 Modified formulation

The approximation introduced here on the original formulation of Simo and Miehe relates to the expression of the law of flow, all the more reduced approximation as the elastic strain are small, since $\boldsymbol{\tau}=\mathbf{s}\,\mathbf{b}^{\mathbf{e}}$. Indeed, one henceforth expresses the threshold of elasticity like a function of the thermodynamic forces and either of the constraints $F(\mathbf{s},\ A)\le 0$, and it is compared to these variables that one applies the principle of maximum dissipation, which leads to the following laws of flow:

$$-\frac{1}{2}\mathbf{F}\,\dot{\mathbf{G}}^{p}\,\mathbf{F}^{\mathbf{T}}=\dot{\lambda}\,\frac{\partial F}{\partial \mathbf{s}}$$ éq 3.3.2-1

$$\dot{p}=\dot{\lambda}\,\frac{\partial F}{\partial A}$$ éq 3.3.2-2

$$\dot{\lambda}\ge 0 \quad F\le 0 \quad F\dot{\lambda}=0$$ éq 3.3.2-3

3.3.3 Consequences of the approximation

By replacing the constraint $\boldsymbol{\tau}$ by the thermodynamic force $\mathbf{s}$ associated with the elastic strain in the expression of the criterion of plasticity, one introduces in fact a disturbance of the border of the field of reversibility of about size of $2\|\mathbf{e}\|$. Compared to the initial formulation, it results from it obviously an influence on the elastic limit observed but also on the direction from flow: in particular, the derivative compared to the time of the plastic variation of volume is written then:

$$\dot{J}^{p}=\dot{\lambda}\,J^{p}\,\mathbf{b}^{\mathbf{e-1}}:\frac{\partial F}{\partial \mathbf{s}}$$ éq 3.3.3-1

so that if the criterion F depends only on the diverter of the tensor of the constraints $\mathbf{s}$, one does not find $J^{p}=1$: the isochoric character of the plastic deformation is not preserved perfectly any more. We will then be brought to introduce a correction of volume a posteriori.

Insofar as the elastic strain remain small, the results got with this modified model do not deviate significantly from those obtained with the old formulation (cf [bib4]), while digital integration will be simplified by it. Indeed, one will see thereafter whom this model follows the same diagram of integration as that of the models written in small deformations.

Note:

This new formulation of the great deformations makes it possible to replace the theory of Simo and Miehe within the framework of generalized standard materials. From a digital point of view, this results in to express the resolution of the law of behavior like a problem of minimization compared to the internal increments of variables.

Indeed, one recalls that within the framework of generalized standard materials, the data of the two potentials the free energy $\Phi(\boldsymbol{\varepsilon},a)$ and potential of dissipation $D(\dot{a})$, function of the tensor of deformation $\boldsymbol{\varepsilon}$ and of a certain number of internal variables a , allows to define the law of behavior completely (one places oneself in the case as of materials independent of time).

$$\boldsymbol{\sigma}=\frac{\partial \Phi}{\partial \boldsymbol{\varepsilon}}\ ,\ A=-\frac{\partial \Phi}{\partial a}\in\partial D(\dot{a})$$ *éq 3.3.3-2*

where $\partial D(\dot{a})$ is under differential of the potential of dissipation D .

The laws of generalized behavior of the standard type which do not depend on time are characterized by a potential of dissipation positively homogeneous of degree 1, which results in the following property:

$$\forall \dot{a} \quad \forall \lambda > 0 \quad D(\lambda \dot{a}) = \lambda D(\dot{a}) \quad \Rightarrow \quad \partial D(\lambda \dot{a}) = \partial D(\dot{a}) \qquad \text{éq 3.3.3-3}$$

Now if one writes the problem [éq 3.3.3-2] in form discretized in time and if one uses the property of under differentials [éq 3.3.3-3], one obtains the following discretized problem:

$$\boldsymbol{\sigma} = \frac{\partial \Phi}{\partial \boldsymbol{\varepsilon}} \;,\; A = -\frac{\partial \Phi}{\partial a} \in \partial D(\Delta a) \qquad \text{éq 3.3.3-4}$$

One can show that the equation [éq 3.3.3-4] is equivalent (cf [bib5]) to solve the problem of minimization compared to the increments of internal variables Δa *according to:*

$$-\frac{\partial \Phi}{\partial a} \in \partial D(\Delta a) \Leftrightarrow \Delta a = \operatorname{Arg}\underset{\Delta a^*}{\operatorname{Min}}\left[\Phi(a^- + \Delta a^*) + D(\Delta a^*)\right] \qquad \text{éq 3.3.3-5}$$

The application of the equation [éq 3.3.3-5] to the model of Rousselier in great modified deformations is written:

$$\underbrace{\Phi(\mathbf{e}, p) \text{ et } D(\mathbf{D}^{\mathbf{p}}, \dot{p})}_{\text{énergie continue}} \quad \underset{\text{discrétisation}}{=>} \quad \underbrace{\Phi(\mathbf{e}^{Tr} + \Delta \mathbf{e}, p^- + \Delta p) \text{ et } D(\Delta \mathbf{e}, \Delta p)}_{\text{énergie discrétisée}} \qquad \text{éq 3.3.3-6}$$

$$A = -\frac{\partial \Phi}{\partial a} = \begin{cases} \mathbf{s} = -\dfrac{\partial \Phi}{\partial \mathbf{e}} \\ -R = -\dfrac{\partial \Phi}{\partial p} \end{cases} \in \partial D(\Delta \mathbf{e}, \Delta p)$$
$$\Leftrightarrow \underset{\Delta \mathbf{e}, \Delta p}{\operatorname{Min}}\left[\Phi(\mathbf{e}^{Tr} + \Delta \mathbf{e}, p^- + \Delta p) + D(\Delta \mathbf{e}, \Delta p)\right] \qquad \text{éq. 3.3.3-7}$$

One will find in the paragraph [§4], the relation which binds the rate of plastic deformation $\mathbf{D}^{\mathbf{p}}$ *once discretized and the increment of elastic strain* $\Delta \mathbf{e}$ *, as well as the definition of* $\mathbf{e}^{Tr}$ *.*

One sees well here whom if one takes the initial formulation of Simo and Miehe, one cannot write any more the problem of minimization [éq 3.3.3-7] with the constraint of Kirchhoff $\boldsymbol{\tau}$ *because of term in* $\mathbf{b}^{\mathbf{e}}$ *in the expression:*

$$\boldsymbol{\tau} = -\frac{\partial \Phi}{\partial e} \mathbf{b}^{\mathbf{e}} \qquad \text{éq 3.3.3-8}$$

4 Model of Rousselier

We now describe the application of the great deformations to the model of Rousselier presented in introduction.

4.1 Equations of the model

To describe a thermoelastoplastic model with isotropic work hardening (the equivalent in small deformations with the model with isotropic work hardening and criterion of Von Mises), Simo and Miehe propose an elastic potential polyconvexe. By reason of simplicity, one chooses here the potential of Coming Saint who is written:

$$\Phi(\mathbf{e},\ p)=\frac{1}{2}\left[K(\operatorname{tr}\mathbf{e})^2+2\mu\,\tilde{\mathbf{e}}:\tilde{\mathbf{e}}+6\mathrm{K}\alpha\Delta T\operatorname{tr}\mathbf{e}\right]+\int_0^p R(u)\,du \qquad \text{éq 4.1-1}$$

In accordance with the equations [éq 3.3-8] and [éq 3.3-9], the laws of state which derive from the elastic potential above write then:

$$\mathbf{s}=-\left[K\operatorname{tr}\mathbf{e}\ \ \mathbf{Id}+2\mu\,\tilde{\mathbf{e}}+3\mathrm{K}\alpha\Delta T\,\mathbf{Id}\right] \qquad \text{éq 4.1-2}$$

$$A=-R(p) \qquad \text{éq 4.1-3}$$

The threshold of elasticity is given by:

$$F(\mathbf{s},R)=s_{eq}+\sigma_1 Df\exp\left(\frac{s_H}{\sigma_1}\right)-R-\sigma_y \qquad \text{éq 4.1-4}$$

According to the equations [éq 3.3.2-1] and [éq 3.3.2-2], the laws of flow are defined by:

$$-\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^p\mathbf{F}^{\mathbf{T}}=\dot{\lambda}\left[\frac{3\tilde{\mathbf{s}}}{2s_{eq}}+\frac{Df}{3}\exp\left(\frac{s_H}{\sigma_1}\right)\mathbf{Id}\right] \qquad \text{éq 4.1-5}$$

$$\dot{p}=\dot{\lambda} \qquad \text{éq 4.1-6}$$

$$\dot{\lambda}\geq 0\quad F\leq 0\quad F\dot{\lambda}=0 \qquad \text{éq 4.1-7}$$

4.2 Treatment of the singular points

In fact, the equation of flow [éq 4.1-5] translated the membership of the direction of flow to the normal cone on the surface of the field of elasticity. It is valid only at the regular points, characterized by:

$$s_{eq}\neq 0 \qquad \text{éq 4.2-1}$$

It thus remains to characterize the normal cone at the singular points, i.e. checking:

$$\tilde{\mathbf{s}}=0\quad \text{et}\quad \sigma_1 D\,f\exp\left(\frac{s_H}{\sigma_1}\right)-R=\sigma_y \qquad \text{éq 4.2-2}$$

The normal cone with convex of elasticity in such a point is the whole of the directions of flow which carry out the problem of maximization according to:

$$\Delta^*(\mathbf{s},\mathrm{R})=\sup_{\mathbf{D^p},\,\dot{p}}\left[\mathbf{s}:\mathbf{D^p}-R\,\dot{p}-\Delta\left(\mathbf{D^p},\dot{p}\right)\right] \qquad \text{éq 4.2-3}$$

where Δ^* is the indicating function of the convex one F and $\Delta(\mathbf{D^p},\dot{p})$ potential of dissipation obtained by transform of Legendre-Fenchel of the indicating function of F :

$$\Delta(\mathbf{D^p},\dot{p})=\underset{\substack{\mathbf{s},R\\ F(\mathbf{s},R)\leq 0}}{\mathrm{Sup}}\left[\mathbf{s}:\mathbf{D^p}-R\,\dot{p}\right] \qquad \text{éq 4.2-4}$$

After some calculations, one obtains:

$$\Delta(\mathbf{D^p},\dot{p})=\sigma_y\,\dot{p}+\sigma_1\,\mathrm{tr}\,\mathbf{D^p}\left(\ln\frac{\mathrm{tr}\,\mathbf{D^p}}{D\,f\,\dot{p}}-1\right)+I_{\mathbb{R}^+}(\mathrm{tr}\,\mathbf{D^p})+I_{\mathbb{R}^+}\left(\dot{p}-\frac{2}{3}D^p_{eq}\right) \qquad \text{éq 4.2-5}$$

with

$$I_{\mathbb{R}^+}(x)=\begin{cases}0 & \text{si} \quad x\geq 0\\ +\infty & \text{sinon}\end{cases} \qquad \text{éq 4.2-6}$$

For $\tilde{\mathbf{s}}=0$, Δ^* is worth:

$$\Delta^*(\mathbf{s},R)=\underset{\substack{\mathbf{D^p},\,\dot{p}\\ \mathrm{tr}\,\mathbf{D^p}\geq 0\\ \dot{p}-\frac{2}{3}D^p_{eq}\geq 0}}{\mathrm{Sup}}\left[\underbrace{s_H\,\mathrm{tr}\,\mathbf{D^p}-\sigma_1\,\mathrm{tr}\,\mathbf{D^p}\left(\ln\frac{\mathrm{tr}\,\mathbf{D^p}}{D\,f\,\dot{p}}-1\right)}_{G(\mathrm{tr}\,\mathbf{D^p})}-R\,\dot{p}-\sigma_y\,\dot{p}\right] \qquad \text{éq 4.2-7}$$

By noticing that for $\mathrm{tr}\,\mathbf{D^p}\geq 0$, the function $G(\mathrm{tr}\,\mathbf{D^p})$ is concave, the suprémum compared to the trace of the rate of plastic deformation $\mathbf{D^p}$ is obtained for:

$$G'(\mathrm{tr}\,\mathbf{D^p})=0 \quad \text{d'où} \quad \mathrm{tr}\,\mathbf{D^p}=Df\,\dot{p}\,\exp\left(\frac{s_H}{\sigma_1}\right) \qquad \text{éq 4.2-8}$$

Note:

One finds well then for the indicating function of the threshold of elasticity F *.*

$$D^*(\mathbf{s},\mathrm{R})=\underset{\substack{\dot{p}\\ \dot{p}\geq\frac{2}{3}D^p_{eq}\geq 0}}{\mathrm{Sup}}\left[F\,\dot{p}\right]=\begin{cases}0 & \text{si}\,F\leq 0\\ +\infty & \text{sinon}\end{cases} \qquad \text{éq 4.2-9}$$

In a singular point, the normal cone, together of the acceptable directions of flow, is thus characterized by:

$$\mathrm{tr}\,\mathbf{D^p}=D\;f\;\dot{p}\exp\left(\frac{s_H}{\sigma_1}\right) \qquad \text{éq 4.2-10}$$

$$\dot{p}\geq\frac{2}{3}D^p_{\mathrm{eq}}\geq 0 \qquad \text{éq 4.2-11}$$

4.3 Expression of porosity

One saw in introduction that the microscopic inspiration of the model of Rousselier is based on a microstructure made up of a cavity and a plastic rigid matrix, therefore isochoric. In this case, porosity is directly connected to the macroscopic deformation by the relation eq. 1-1.

In this expression, the change of elastic volume of origin is neglected. Without special precaution, this approximation can prove penalizing in the presence of elastic compression, even reasonable, because it leads to a possibly negative porosity.

One thus prefers the following equivalent expression to him, together with an explicit decrease by initial porosity:

$$f=\max\left(f_{0,}1-\frac{1-f_0}{J}\right) \qquad \text{éq 4.3-1}$$

Rousselier proposes as for him to express porosity while basing himself on the rate of plastic deformation $\mathbf{D^p}$. The relation is written in incremental form:

$$\dot{f}=(1-f)\,\mathrm{tr}\,\mathbf{D^p} \qquad \text{éq 4.3-2}$$

That means that variable porosity employed to parameterize the criterion of plasticity F depends only on the plastic deformation. In fact, the rate of plastic deformation is a quantity evaluated in the slackened configuration. Its transport in the current configuration (like $\mathbf{D}$) still express yourself:

$$\mathbf{F^e}\,\mathbf{D^p}\,\mathbf{F^{e^T}}=-\frac{1}{2}\mathbf{F}\,\dot{\mathbf{G}}^p\,\mathbf{F^T} \qquad \text{éq 4.3-3}$$

In this case, the law of evolution of porosity is expressed:

$$\dot{f}=(1-f)\,\mathrm{tr}\left(-\frac{1}{2}\mathbf{F}\,\dot{\mathbf{G}}^p\,\mathbf{F^T}\right) \qquad \text{éq 4.3-4}$$

To avoid that the integration of porosity does not interfere with that of plasticity (since the two variables are coupled), it is necessary to separate integration from the law of behavior in two times: integration of plasticity with porosity fixed at its value at the beginning of the step of time, then integration of porosity by means of the equation 4.3-4 where the plastic evolution is that calculated with the preceding phase.

Notice important:

There are thus two possible versions of the model (PORO_TYPE = 1 or 2, cf U4.43.01), according to whether one respectively chooses the total deflection or the plastic deformation in the evolution of porosity. It was noticed that for an initial porosity f0 low, the behavior at the beginning of evolution strongly changes according to this parameter. Indeed, the choice seems to have an impact determining on the answer at the level of the structure, since it privileges or not the junctions of the zone where the deformations are located. Thus, this strong sensitivity must lead the user to a greatest caution in the use of this model. Research is in hand to understand this sensitivity and to discriminate the two alternatives for the evolution of porosity.

4.4 Relation ‘ROUSSELIER‘

This relation of behavior is available via the argument ‘ROUSSELIER‘keyword BEHAVIOR under the operator STAT_NON_LINE, with the argument ‘SIMO_MIEHE‘keyword factor DEFORMATION.

The whole of the parameters of the model is provided under the keywords factors ‘ROUSSELIER‘or ‘ROUSSELIER_FO‘and ‘TRACTION‘(to define the traction diagram) order DEFI_MATERIAU ([U4.43.01]).

Notice :

The user must make sure well that the “experimental” traction diagram used, either directly, or to deduce the slope from it from work hardening is well given in the plan forced rational $\sigma=F/S$ *- deformation logarithmic curve* $\ln(1+\Delta l/l_0)$ where l_0 *is the initial length of the useful part of the test-tube,* Δl *variation length after deformation,* F *the force applied and* S *current surface*.

4.5 Internal constraints and variables

The constraints are the constraints of Cauchy, $\boldsymbol{\sigma}$ thus calculated on the current configuration (six components in 3D, four in 2D).

Internal variables produced in *Code_Aster* are:

- V1, cumulated plastic deformation p ,
- V2, porosity f ,
- V3, the indicator of plasticity (0 if the last calculated increment is elastic, 1 if regular plastic solution, 2 if singular plastic solution),
- V4 with V9, the tensor of elastic strain $\mathbf{e}$.

Notice :

If the user wants to possibly recover deformations in postprocessing of his calculation, it is necessary to trace the deformations of Green-Lagrange $\mathbf{E}$ *, which represents a measurement of the deformations in great deformations (option* EPSG_ELGA *or* EPSG_ELNO *CALC_CHAMP). Linearized deformations* $\boldsymbol{\varepsilon}$ *classics measure deformations under the assumption of the small deformations and do not have a direction in great deformations.*

5 Digital formulation

For the variational formulation, it is same as that given in the note [R5.03.21] and which refers to the law of behavior with isotropic work hardening and criterion of Von Mises in great deformations. We recall only that it is about a eulérienne formulation, with reactualization of the geometry to each increment and each iteration, and that one takes account of the rigidity of behavior and geometrical rigidity.
We now present the digital integration of the law of behavior and give the form of the tangent matrix (options FULL_MECA and RIGI_MECA_TANG).

5.1 Expression of the discretized model

Knowing the constraint $\boldsymbol{\sigma}^-$, cumulated plastic deformation p^- , elastic strain $\mathbf{e}^-$, displacements $\mathbf{u}^-$ and $\Delta\mathbf{u}$, one seeks to determine $(\boldsymbol{\sigma}, p, \mathbf{e})$.
Displacements being known, gradients of the transformation of Ω_0 with Ω^- , noted $\mathbf{F}^-$, and of Ω^- with Ω , noted $\Delta\mathbf{F}$, are known.

To integrate this model of behavior, one employs a diagram of implicit Euler, porosity being an explicit function of the deformation via equation 4.3-1, therefore known during the integration of the behavior.

Once discretized, the following system then is obtained:

$$\mathbf{F}=\Delta\,\mathbf{F}\,\mathbf{F}^{-}$$ éq 5.1-1

$$J=\det\mathbf{F}$$ éq 5.1-2

$$J\,\boldsymbol{\sigma}=\boldsymbol{\tau}$$ éq 5.1-3

$$\boldsymbol{\tau}=\mathbf{s}\;\mathbf{b}^{e}$$ éq 5.1-4

$$\mathbf{b}^{e}=\mathbf{Id}-2\,\mathbf{e}$$ éq 5.1-5

•Equations of state:

$$\mathbf{s}=-\left[2\,\mu\,\tilde{\mathbf{e}}+K\,\mathrm{tr}\,\mathbf{e}\;\mathbf{Id}+3\mathrm{K}\,\alpha\,\Delta\,T\,\mathbf{Id}\right]$$ éq. 5.1-6

$$A\;=-R(p)$$ éq 5.1-7

Thereafter, one expresses the laws of flow and the criterion of plasticity directly according to the tensor of the elastic strain $\mathbf{e}$.

•Laws of flow

$$
\begin{aligned}
\mathbf{D^p} &\simeq -\frac{1}{2}\mathbf{F}\dot{\mathbf{G}}^p\mathbf{F^T} = -\frac{1}{2\Delta t}\left[\underbrace{\mathbf{FG}^p F^T}_{\mathbf{b^e}} - \Delta\mathbf{F}\underbrace{\mathbf{F^-}\mathbf{G}^{p-}\mathbf{F^{-T}}}_{\mathbf{b^{e-}}}\Delta\mathbf{F^T}\right] \\
&= -\frac{1}{2\Delta t}\left[\mathbf{Id}-2\mathbf{e}-\Delta F\left(\mathbf{Id}-2\mathbf{e}^-\right)\Delta\mathbf{F^T}\right] \\
&= (\mathbf{e}-\underbrace{\frac{1}{2}\left[\mathbf{Id}-\Delta\mathbf{F}\left(\mathbf{Id}-2\mathbf{e}^-\right)\Delta\mathbf{F^T}\right]}_{\mathbf{e^{Tr}}})/\Delta t = (\mathbf{e}-\mathbf{e}^{Tr})/\Delta t
\end{aligned}
\qquad \text{éq 5.1-8}
$$

By taking the parts traces and deviatoric of the equation [éq 4.1-5], one obtains:

$$
\mathrm{tr}\,\mathbf{e}-\mathrm{tr}\,\mathbf{e}^{Tr} = \Delta p\, D\, f\exp\left(\frac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(\frac{-K\,\mathrm{tr}\,\mathbf{e}}{\sigma_1}\right) \qquad \text{éq 5.1-9}
$$

$$
\tilde{\mathbf{e}} = \begin{cases} \tilde{\mathbf{e}}^{Tr} - \dfrac{3}{2}\Delta p\dfrac{\tilde{\mathbf{e}}}{e_{eq}} & \text{si solution régulière} \\ 0 \quad \text{et} \quad \Delta p \geq \dfrac{2}{3}(\tilde{\mathbf{e}}-\tilde{\mathbf{e}}^{Tr})_{eq} & \text{si solution singulière} \end{cases} \qquad \text{éq 5.1-10}
$$

•Conditions of coherence

$$
F = \begin{cases} 2\mu e_{eq}+\sigma_1 D f\exp\left(\dfrac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(\dfrac{-K\,tr\,\mathbf{e}}{\sigma_1}\right)-R-\sigma_y & \text{si solution régulière} \\ \sigma_1 D f\exp\left(\dfrac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(\dfrac{-K\,tr\,\mathbf{e}}{\sigma_1}\right)-R-\sigma_y & \text{si solution singulière} \end{cases} \qquad \text{éq 5.1-11}
$$

avec $F \leq 0 \quad \Delta p \geq 0 \quad F\Delta p = 0$

5.2 Resolution of the nonlinear system

The integration of the law of behavior is thus summarized to solve the system [éq 5.1-9], [éq 5.1-10] and [éq 5.1-11]. We will see that this resolution is brought back to that of only one scalar equation, of which the unknown factor x is the increment of the trace of the elastic strain:

$$x=\mathrm{tr}\,\mathbf{e}-\mathrm{tr}\,\mathbf{e}^{Tr} \quad \text{éq 5.2-1}$$

Thanks to this choice, that the solution is elastic or plastic, attack in a singular point or not, the equation [éq 5.1-9] bearing on the trail of the elastic increment is always valid and makes it possible to express the increment of cumulated plastic deformation:

$$\mathrm{tr}\,\mathbf{e}-\mathrm{tr}\,\mathbf{e}^{Tr}=\underbrace{\Delta p\, D\, f \exp\left(\frac{-K\,\mathrm{tr}\,\mathbf{e}^{Tr}}{\sigma_1}\right)\exp\left(\frac{-3K\alpha\Delta T}{\sigma_1}\right)}_{G}\exp\left(\frac{-K(\mathrm{tr}\,\mathbf{e}-\mathrm{tr}\,\mathbf{e}^{Tr})}{\sigma_1}\right)$$
$$\Rightarrow \Delta p(x)=\frac{1}{G}x\exp\frac{Kx}{\sigma_1} \quad \text{éq 5.2-2}$$

This equation shows us that one can seek $x\geq 0$ to guarantee a positive cumulated plastic deformation and that the elastic solution is obtained for $x=0$. One also notices that the increment of cumulated plastic deformation is a function continuous and strictly increasing of x . With the help of these remarks, if one notes by S the term [éq 5.2-3] in the criterion of plasticity, it acts then, there too, of a continuous and strictly increasing function of x :

$$F=2\mu e_{eq}-S(x) \quad \text{avec} \quad S(x)=-\sigma_1 \mathrm{G}\exp\left(-\frac{Kx}{\sigma_1}\right)+R(p(x))+\sigma_y \quad \text{éq 5.2-3}$$

This stage, the approach of resolution breaks up into two times.

5.2.1 Examination of the singular points

Such a singular point is characterized by [éq 5.1-10] (low) and [éq 5.1-11] (low), therefore in particular by $S(x)=0$. Because of the properties of S , this equation admits with more the one positive solution, say x^S who exists if and only if $S(0)\leq 0$. The knowledge of x^S allows to deduce the tensor from it from elastic strain $\mathbf{e}$, cumulated plastic deformation p as well as the thermodynamic forces $\mathbf{s}$ and R .

Finally, this singular point will be solution if the inequality in [éq 5.1-11] (low) is checked, i.e. if:

$$\Delta p^s \geq \frac{2}{3}(\tilde{\mathbf{e}}^s-\tilde{\mathbf{e}}^{\mathrm{Tr}})_{\mathrm{eq}} \quad \text{éq 5.2.1-1}$$

5.2.2 Regular solution

The equation of flow [éq 5.1-10] (high) which determines the deviatoric part of the tensor of elastic strain makes it possible to deduce a scalar equation from it function from the increment of cumulated plastic deformation:

$$\tilde{\mathbf{e}}-\tilde{\mathbf{e}}^{\mathrm{Tr}}=-\frac{3}{2}\Delta p\frac{\tilde{\mathbf{e}}}{e_{\mathrm{eq}}} \quad \Rightarrow \quad \begin{cases} e_{\mathrm{eq}}=e_{\mathrm{eq}}^{\mathrm{Tr}}-\frac{3}{2}\Delta p \\ \tilde{\mathbf{e}}=\frac{e_{\mathrm{eq}}}{e_{\mathrm{eq}}^{\mathrm{Tr}}}\tilde{\mathbf{e}}^{\mathrm{Tr}} \end{cases} \qquad \text{éq 5.2.2-1}$$

One notes that because of positivity of e_{eq} , the value sells by auction Δp is limited:

$$\Delta p\leq\frac{2}{3}e_{\mathrm{eq}}^{\mathrm{Tr}} \qquad \text{éq 5.2.2-2}$$

The condition of coherence determines now x :

$$F=2\mu e_{eq}^{Tr}-S(x)-3\mu\Delta p \leq 0 \qquad \text{éq 5.2.2-3}$$

Being given this expression, the increase of the value sells by auction Δp is reduced to the only condition $S(x)\geq 0$ or, in an equivalent way, with $x\geq x^S$.

The elastic solution is obtained for x = 0. It is the solution of the problem if and only if:

$$F(0)=2\mu e_{eq}^{Tr}-S(0)<0 \qquad \text{éq 5.2.2-4}$$

In the contrary case, one must then solve:

$$F(x)=2\mu e_{eq}^{Tr}-S(x)-\frac{3\mu}{G}x\exp\left(\frac{Kx}{\sigma_1}\right)=0 \quad \text{avec} \quad \begin{cases} x>x^s & \text{si } x^s \text{ existe} \\ x>0 & \text{sinon} \end{cases} \qquad \text{éq 5.2.2-5}$$

This function is continuous and strictly decreasing and tends towards $-\infty$ with x . She thus admits with more the one solution. The demonstration of the existence of this solution is immediate. Indeed, it is enough to prove that F is positive on the lower limit of the interval of research.
When x^S do not exist, $F(0)>0$ since the solution is not elastic.

When x^S exist, the function is worth:

$$F(x^s)=2\mu e_{\mathrm{eq}}^{\mathrm{Tr}}-3\mu\Delta p^s >0\Leftrightarrow\Delta p^s<\frac{2}{3}e_{\mathrm{eq}}^{\mathrm{Tr}} \qquad \text{éq 5.2.2-6}$$

This condition is checked since the singular solution was rejected.

5.3 Course of calculation

The approach to solve the whole of the equations of the model is the following one:

1) One searches if the solution is elastic
   •calculation of $F(0)$
   •if $F(0)<0$, the solution of the problem is the elastic solution $x^{Sol}=0$
   •if not one passes in 2)
2) If $S(0)>0$, the solution is plastic and regular
   •one passes in 4)
3) If $S(0)<0$, one seeks if the solution is singular
   •one solves $S(x^s)=0$
   •if x^s check the inequality $\Delta p^s \geq \frac{2}{3}(\tilde{\mathbf{e}}^s - \tilde{\mathbf{e}}^{\mathrm{Tr}})_{\mathrm{eq}}$, then the solution is singular $x^{Sol}=x^s$
   •if not, x^s is a lower limit to solve $F(x)=0$, one passes in 4)
4) The solution is plastic and regular
   •one solves $F(x)=0$

5.4 Resolution

To solve the two equations $S(x)=0$ and $F(x)=0$, one employs a method of Newton with controlled terminals coupled to dichothomy when Newton gives a solution apart from the interval of the two terminals. One now presents the determination of the terminals for each preceding case (items 2) 3) and 4) preceding paragraph).

5.4.1 Hight delimiters and lower if the function S is strictly positive in the beginning

One solves:

$$\begin{cases} F(x)=0 \\ F(0)>0 \end{cases} \Leftrightarrow \begin{cases} \underbrace{2\mu e_{eq}^{Tr} - S(x)}_{f_1} = \underbrace{\frac{3\mu}{G} x \exp\left(\frac{Kx}{\sigma_1}\right)}_{3\mu\,\Delta p} \\ f_1(0)>0 \end{cases} \qquad \text{éq 5.4.1-1}$$

where the function $\Delta p(x)$ is continuous, strictly increasing and worthless at the origin and the function $f_1(x)$ is continuous, strictly decreasing and strictly positive at the origin (see [Figure 5.4.1-a]).

One poses:

$$f_1 = \underbrace{2\mu e_{eq}^{Tr} - R(x) - \sigma_y}_{f_2} + \sigma_1 \mathrm{G} \exp\left(-\frac{Kx}{\sigma_1}\right) \quad \text{alors} \quad f_2(x) < f_1(x) \quad \forall x \geq 0 \qquad \text{éq 5.4.1-2}$$

where the function $f_2(x)$ is continuous, strictly decreasing. In this case, the resolution of the equations:

$$f_2(\Delta p^{inf})=3\mu\Delta p^{inf} \text{ and } x^{inf}\exp(\frac{Kx^{inf}}{\sigma_1})=G\Delta p^{inf} \quad \text{éq 5.4.1-3}$$

to deduce some successively Δp then x give a lower limit x^{Inf} who corresponds to the solution of the model to isotropic work hardening and criterion of Von Mises. If $f_2(0)<0$, the lower limit is taken equalizes to zero: $x^{inf}=0$.
The upper limit x^{Sup} is such as:

$$x^{\mathrm{Sup}}\exp(\frac{\mathrm{Kx}^{\mathrm{Sup}}}{\sigma_1})=\frac{G}{3\mu}f_1(x^{\mathrm{Inf}}) \quad \text{éq 5.4.1-4}$$

The equation of the type $x\exp(\frac{Kx}{\sigma_1})=constante$ is solved by a method of Newton.

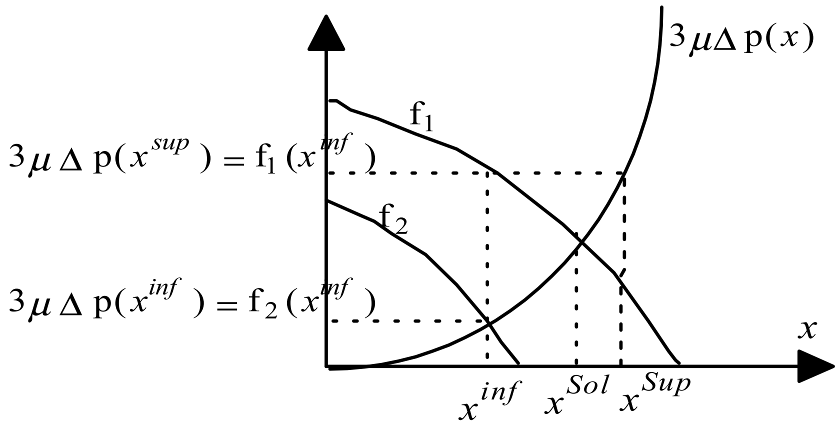

Figure 5.4.1-a: chart of the hight delimiters and lower

5.4.2 Hight delimiters and lower if the function S is negative or worthless in the beginning

The system to be solved is the following:

$$\begin{cases} S(x)=0 \\ S(0)<0 \end{cases} \Leftrightarrow \begin{cases} R\left(p^- + \frac{x}{G}\exp\left(\frac{Kx}{\sigma_1}\right)\right)+\sigma_y = \sigma_1 \mathrm{G}\exp\left(-\frac{Kx}{\sigma_1}\right) \\ R\left(p^-\right)+\sigma_y < \sigma_1 G \end{cases} \quad \text{éq 5.4.2-1}$$

The part of left is a continuous function, strictly increasing of x and strictly positive in the beginning, the part of right-hand side is a continuous function, strictly decreasing of x and strictly positive at the origin. Using the properties of these two functions, a chart (cf [Figure 5.4.2-a]) of these functions shows that the upper limit x^{Sup} is such as:

$$\sigma_1 \mathrm{G}\ \exp\left(-\frac{Kx^{Sup}}{\sigma_1}\right) = R\left(p^-\right)+\sigma_y \quad \Leftrightarrow \quad x^{Sup} = \frac{\sigma_1}{K}\log\left(\frac{\sigma_1 G}{R\left(p^-\right)+\sigma_y}\right) \quad \text{éq 5.4.2-2}$$

The lower limit x^{Inf} is such as:

$$\sigma_1 \mathrm{G}\exp\left(-\frac{Kx^{Inf}}{\sigma_1}\right) = R\left(p^- + \frac{x^{Sup}}{G}\exp\left(\frac{Kx^{Sup}}{\sigma_1}\right)\right)+\sigma_y$$
$$\Leftrightarrow x^{Inf} = \left\langle \frac{\sigma_1}{K}\log\left(\frac{\sigma_1 G}{R\left(p^- + \frac{x^{Sup}}{G}\exp\left(\frac{Kx^{Sup}}{\sigma_1}\right)\right)+\sigma_y}\right)\right\rangle^+ \quad \text{éq 5.4.2-3}$$

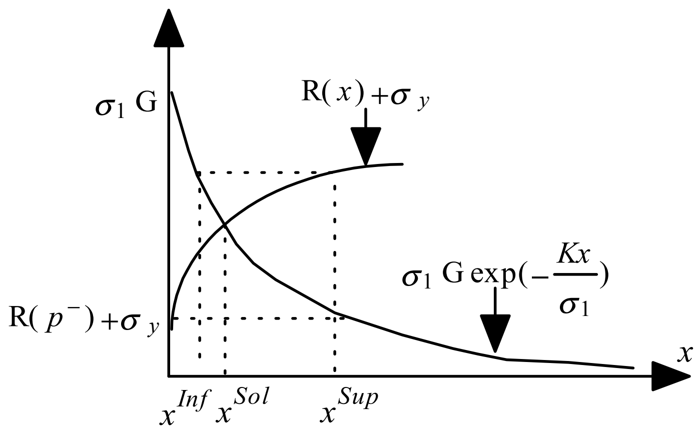

Figure 5.4.2-a: chart of the hight delimiters and lower

5.4.3 Hight delimiters and lower if the function S is strictly negative in the beginning and x^s not solution

The following system is solved:

$$\begin{cases} F(x)=0 \\ S(0)<0 \\ S(x^s)=0 \end{cases} \Leftrightarrow \begin{cases} \underbrace{2\mu e_{eq}^{Tr}-S(x)}_{f_1}=\underbrace{\frac{3\mu}{G}x\exp\left(\frac{Kx}{\sigma_1}\right)}_{3\mu\Delta p} \\ f_1(0)>0 \\ 2\mu e_{eq}^{Tr}=\frac{3\mu}{G}x^s\exp\left(\frac{Kx^s}{\sigma_1}\right) \end{cases} \quad \text{éq 5.4.3-1}$$

The solution x^{Sol} is such as $S(x^{Sol})>0$.
For the lower limit, one takes $x^{Inf}=x^s$. Being given properties of the functions f_1 (strictly decreasing) and $3\mu\Delta p(x)$ (strictly increasing), the upper limit x^{Sup} is such as (cf [Figure 5.4.3-a]):

$$x^{Sup}\exp\left(\frac{Kx^{Sup}}{\sigma_1}\right)=\frac{2G}{3}e_{eq}^{Tr} \quad \text{éq 5.4.3-2}$$

This equation is solved by a method of Newton.

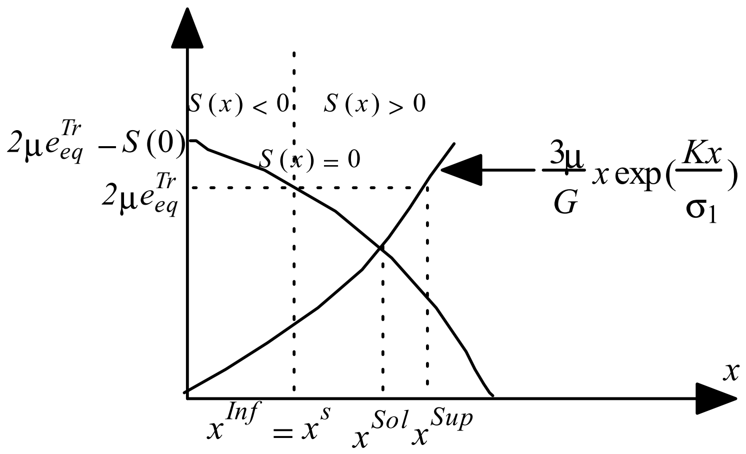

Figure 5.4.3-a: chart of the hight delimiters and lower

5.5 Correction of volume a posteriori

In the case of the model of Rousselier, the change of volume plays a crucial role, so that the mistake made by the approximation $\mathbf{s}=\boldsymbol{\tau}$ in the equations of flow can lead to an evolution can precise of porosity, cf [Bib2]. While following the proposal for this reference, one will correct a posteriori (i.e after having calculated all the quantities) the trace of the elastic strain.

Indeed, in the absence of approximation, the hydrostatic part of the equation of flow leads to:

$$\frac{d}{dt}\left(\ln J^p\right)=\dot{p}\,D\,f\,\exp\left(\frac{\mathrm{tr}\,\boldsymbol{\tau}}{3\,\sigma_1}\right) \quad \text{(éq. 5.5-1)}$$

After discretization in time, one then obtains a proposal corrected for the changes of plastic and elastic volume:

$$J^p_{corr}=J^{p-}\exp\left(\mathrm{tr}\,\mathbf{e}-\mathrm{tr}\,\mathbf{e}^{Tr}\right) \quad ; \quad J^e_{corr}=\frac{J}{J^p_{corr}} \quad \text{(éq. 5.5-2)}$$

One then will seek a new trace of elastic strain such as the elastic change of volume corresponds to the value corrected above:

$$\mathbf{e_{corr}}=\tilde{\mathbf{e}}+t\,\mathbf{Id} \quad \text{où } t \text{ tel que : } \det\left(\mathbf{Id}-2\,\mathbf{e_{corr}}\right)={J^e_{corr}}^2 \quad \text{(éq. 5.5-3)}$$

That led to a polynomial of degree 3 in t, of which one will choose the solution nearest to $\mathbf{e}$.

5.6 Form of the tangent matrix of the behavior

One gives the form of the tangent matrix here (option FULL_MECA during iterations of Newton, option RIGI_MECA_TANG for the first iteration).
For the option FULL_MECA, this one is obtained by linearizing the system of equations which governs the law of behavior. We give hereafter the broad outlines of this linearization.
For the option RIGI_MECA_TANG, it is the same expressions as those given for FULL_MECA but with $\Delta\,p=0$. In particular, one will have $\Delta\,\mathbf{F}=\mathbf{Id}$.

The law of behavior can be put in the following general form:

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}\left(\boldsymbol{\tau}\,,\Delta\,\mathbf{F}\right) \quad \text{éq 5.6-1}$$

$$\boldsymbol{\tau}=\boldsymbol{\tau}\left(\mathbf{e}\right) \quad \text{éq 5.6-2}$$

$$\mathbf{e}=\mathbf{e}\left(\mathbf{e^{Tr}},f\right) \quad \text{éq 5.6-3}$$

$$\mathbf{e^{Tr}}=\mathbf{e^{Tr}}\left(\Delta\,\mathbf{F}\right) \quad \text{éq 5.6-4}$$

The linearization of this system gives:

$$\delta\,\boldsymbol{\sigma}=\left(\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\tau}}:\frac{\partial\boldsymbol{\tau}}{\partial\mathbf{e}}:\left(\frac{\partial\mathbf{e}}{\partial\mathbf{e^{Tr}}}:\frac{\partial\mathbf{e^{Tr}}}{\partial\boldsymbol{\Delta}\,\mathbf{F}}+\frac{\partial\mathbf{e}}{\partial f}\frac{\partial f}{\partial J}\otimes\frac{\partial J}{\partial\boldsymbol{\Delta}\,\mathbf{F}}\right)+\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\Delta}\,\mathbf{F}}\right):\delta\,\Delta\,\mathbf{F}=\mathbf{H}:\delta\,\Delta\,\mathbf{F} \quad \text{éq 5.6-5}$$

where $\mathbf{H}$ is the tangent matrix. Thereafter, one separately determines the five terms of the preceding equation.
In the linearization of the system, one will often use the tensor $\mathbf{C}$ defined below and two following equations:

$$\delta a_{\text{ij}}=\frac{1}{2}\left(\delta_{\text{ik}}\delta_{\text{jl}}+\delta_{\text{jk}}\delta_{\text{il}}\right)\delta a_{\text{kl}} \quad \text{éq 5.6-6}$$

$$\delta\ a_{pp}=\delta_{\ kl}\delta\ a_{kl} \quad \text{éq 5.5-7}$$

$$C_{\text{ijkl}}=\frac{1}{2}\left(\delta_{\text{ik}}\delta_{\text{jl}}+\delta_{\text{jk}}\delta_{\text{il}}\right) \quad \text{éq 5.6-8}$$

•Calculation of $\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\tau}}$ and of $\frac{\partial\boldsymbol{\sigma}}{\partial\Delta\mathbf{F}}$

Linearization of the relation which binds the constraint of Cauchy $\boldsymbol{\sigma}$ and the constraint of Kirchhoff $\boldsymbol{\tau}$ give:

$$J\boldsymbol{\sigma}=\boldsymbol{\tau} \quad \Leftrightarrow \quad \delta\boldsymbol{\sigma}=\frac{1}{J}\delta\boldsymbol{\tau}-\left(\frac{\boldsymbol{\sigma}}{J}\otimes\frac{\partial J}{\partial\Delta F}\right):\delta\ \Delta\mathbf{F} \quad \text{éq 5.6-9}$$

By using the relation [éq 5.6-6], one obtains for $\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\tau}}$:

$$\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\tau}}=C \quad \text{éq 5.6-10}$$

and for $\frac{\partial\boldsymbol{\sigma}}{\partial\Delta\mathbf{F}}$:

$$\frac{\partial\boldsymbol{\sigma}}{\partial\Delta\mathbf{F}}=-\frac{\boldsymbol{\sigma}}{J}\otimes\frac{\partial J}{\partial\Delta\mathbf{F}} \quad \text{éq 5.6-11}$$

with

$$\begin{aligned}
&\frac{\partial J}{\partial F_{11}}=\Delta F_{22}\Delta F_{33}-\Delta F_{23}\Delta F_{32}\\
&\frac{\partial J}{\partial F_{22}}=\Delta F_{11}\Delta F_{33}-\Delta F_{13}\Delta F_{31}\\
&\frac{\partial J}{\partial F_{33}}=\Delta F_{11}\Delta F_{22}-\Delta F_{12}\Delta F_{21}\\
&\frac{\partial J}{\partial F_{12}}=\Delta F_{31}\Delta F_{23}-\Delta F_{33}\Delta F_{21} \qquad \frac{\partial J}{\partial F_{21}}=\Delta F_{13}\Delta F_{32}-\Delta F_{33}\Delta F_{12}\\
&\frac{\partial J}{\partial F_{13}}=\Delta F_{21}\Delta F_{32}-\Delta F_{22}\Delta F_{31} \qquad \frac{\partial J}{\partial F_{31}}=\Delta F_{12}\Delta F_{23}-\Delta F_{22}\Delta F_{13}\\
&\frac{\partial J}{\partial F_{23}}=\Delta F_{31}\Delta F_{12}-\Delta F_{11}\Delta F_{32} \qquad \frac{\partial J}{\partial F_{32}}=\Delta F_{13}\Delta F_{21}-\Delta F_{11}\Delta F_{23}
\end{aligned} \quad \text{éq 5.6-12}$$

•Calculation of $\frac{\partial \boldsymbol{\tau}}{\partial \mathbf{e}}$

The relation which binds the constraint of Kirchhoff $\boldsymbol{\tau}$ and the tensor of elastic strain $\mathbf{e}$ is given by:

$$\boldsymbol{\tau}=\mathbf{s}\,\mathbf{b}^{e}=-2\mu\ \mathbf{e}-\lambda\ \mathrm{Tr}\,\mathbf{e}\,\mathbf{Id}+4\mu\ \mathbf{e}\,\mathbf{e}+2\lambda\ (\mathrm{tr}\,\mathbf{e})\,e \\ -\ 3\mathrm{K}\,\alpha\Delta\,T\,\mathbf{Id}+6\mathrm{K}\,\alpha\Delta\,T\,\mathbf{e} \qquad \text{éq 5.6-13}$$

One obtains after linearization:

$$\delta\boldsymbol{\tau}=2\left(\lambda\ \mathrm{tr}\,\mathbf{e}-\mu+3\mathrm{K}\,\alpha\Delta\,T\right)\delta\,\mathbf{e}+\lambda\left(2e-\mathbf{Id}\right)\mathrm{Tr}\,\delta\,\mathbf{e}\ +4\mu\left(\delta\,\mathbf{e}\,\mathbf{e}+\mathbf{e}\,\delta\,\mathbf{e}\right) \qquad \text{éq 5.6-14}$$

from where

$$\frac{\partial\tau_{\mathrm{ij}}}{\partial e_{\mathrm{kl}}}=2\left(\lambda\,tr\,e-\mu+3\mathrm{K}\,\alpha\Delta\,T\right)C_{\mathrm{ijkl}}+\lambda\left(2e_{\mathrm{ij}}-\delta_{\mathrm{ij}}\right)\delta_{\mathrm{kl}}+2\mu\left(\delta_{\mathrm{ik}}e_{\mathrm{lj}}+\delta_{\mathrm{il}}e_{\mathrm{kj}}+e_{\mathrm{il}}\delta_{\mathrm{kj}}+e_{\mathrm{ik}}\delta_{\mathrm{jl}}\right) \qquad \text{éq 5.6-15}$$

•Calculation of $\frac{\partial \mathbf{e}^{Tr}}{\partial\Delta\,\mathbf{F}}$

The relation between the tensor of elastic strain $\mathbf{e}^{\mathbf{Tr}}$ and the increment of the gradient of the transformation $\Delta\,\mathbf{F}$ is written:

$$\mathbf{e}^{\mathbf{Tr}}=\frac{1}{2}\left(\mathbf{Id}-\boldsymbol{\Delta}\,\mathbf{F}\,\mathbf{b}^{e-}\,\boldsymbol{\Delta}\,\mathbf{F}^{\mathbf{T}}\right) \qquad \text{éq 5.6-16}$$

Its linearization gives:

$$\frac{\partial e_{\mathrm{ij}}^{\mathrm{Tr}}}{\partial\Delta\,F_{\mathrm{kl}}}=-\frac{1}{2}\left(\delta_{\mathrm{ik}}\Delta\,F_{\mathrm{jp}}b_{\mathrm{pl}}^{\mathrm{e-}}+\Delta\,F_{\mathrm{ip}}b_{\mathrm{pl}}^{\mathrm{e-}}\delta_{\mathrm{jk}}\right) \qquad \text{éq 5.6-17}$$

•Calculation of $\frac{\partial \mathbf{e}}{\partial \mathbf{e}^{\mathbf{Tr}}}$

Elastic case

In the elastic case, the calculation of $\frac{\partial \mathbf{e}}{\partial \mathbf{e}^{\mathbf{Tr}}}$ is immediate since $\delta\,\mathbf{e}=\delta\,\mathbf{e}^{\mathbf{Tr}}$ from where

$$\frac{\partial \mathbf{e}}{\partial \mathbf{e}^{\mathbf{Tr}}}=C \qquad \text{éq 5.6-18}$$

Plastic case – regular Solution

To determine $\dfrac{\partial \mathbf{e}}{\partial \mathbf{e}^{\mathbf{Tr}}}$, one operates in two stages. By the law of flow discretized, one calculates in first $\delta\,\mathbf{e}$ according to $\delta\,\mathbf{e}^{\mathbf{Tr}}$ and $\delta\,\Delta\,p$. Then the condition of coherence makes it possible to deduce some $\delta\,\Delta\,p$ according to $\delta\,\mathbf{e}^{\mathbf{Tr}}$. These two stages are thereafter detailed.
The deviatoric part of the law of flow discretized is written:

$$\tilde{\mathbf{e}}-\tilde{\mathbf{e}}^{\mathrm{Tr}}=-\frac{3}{2}\Delta\,p\frac{\tilde{\mathbf{e}}}{\mathbf{e}_{eq}} \qquad \text{éq 5.6-19}$$

One obtains after linearization:

$$\underbrace{\left(1+\frac{3}{2}\frac{\Delta\,p}{e_{\mathrm{eq}}}\right)}_{1/\alpha}\delta\,\tilde{\mathbf{e}}=\delta\,\tilde{\mathbf{e}}^{\mathrm{Tr}}-\frac{3}{2}\frac{\tilde{\mathbf{e}}}{e_{\mathrm{eq}}}\delta\,\Delta\,p+\frac{9}{4}\Delta\,p\frac{\tilde{\mathbf{e}}}{e_{\mathrm{eq}}^{3}}\left(\tilde{\mathbf{e}}:\delta\,\tilde{\mathbf{e}}\right) \qquad \text{éq 5.6-20}$$

To determine $\tilde{\mathbf{e}}:\delta\,\tilde{\mathbf{e}}$, one contracts the equation [éq 5.6-20] with $\tilde{\mathbf{e}}$ what gives:

$$\tilde{\mathbf{e}}:\delta\,\tilde{\mathbf{e}}=\tilde{\mathbf{e}}:\delta\,\tilde{\mathbf{e}}^{\mathrm{Tr}}-e_{\mathrm{eq}}\delta\,\Delta\,p \qquad \text{éq 5.6-21}$$

from where

$$\delta\,\tilde{\mathbf{e}}=\underbrace{\alpha\left[\frac{9\,\Delta\,p}{4\,e_{\mathrm{eq}}^{3}}\tilde{\mathbf{e}}\otimes\tilde{\mathbf{e}}+\mathbf{C}\right]}_{\mathbf{A}_1}:\delta\,\tilde{\mathbf{e}}^{\mathrm{Tr}}\underbrace{-\frac{3}{2}\frac{\tilde{\mathbf{e}}}{e_{\mathrm{eq}}}}_{\mathbf{A}_2}\delta\Delta p \qquad \text{éq 5.6-22}$$

For the part law of flow traces discretized, one a:

$$\mathrm{Tr}\,\mathbf{e}-\mathrm{Tr}\,\mathbf{e}^{Tr}=D\,f\,\Delta\,p\exp\left(\frac{3\,K\,\alpha\,\Delta\,T}{\sigma_1}\right)\exp\left(-\frac{K}{\sigma_1}\mathrm{Tr}\,\mathbf{e}\right) \qquad \text{éq 5.6-23}$$

what gives, while posing $k_1=1+\dfrac{D\,f\,K\,\Delta\,p}{\sigma_1}\exp\left(\dfrac{3\,K\,\alpha\,\Delta\,T}{\sigma_1}\right)\exp\left(-\dfrac{K}{\sigma_1}\mathrm{Tr}\,\mathbf{e}\right)$:

$$\mathrm{tr}\,\delta\,\mathbf{e}=\underbrace{\frac{1}{k_1}}_{\alpha_1}\mathrm{tr}\,\delta\,\mathbf{e}^{Tr}+\underbrace{\frac{D\,f\exp\left(\frac{-K}{\sigma_1}\mathrm{tr}\,\mathbf{e}\right)\exp\left(\frac{3\,K\,\alpha\,\Delta\,T}{\sigma_1}\right)}{k_1}}_{\alpha_2}\delta\,\Delta\,p+\underbrace{\frac{\Delta\,p\exp\left(\frac{-K}{\sigma_1}\mathrm{tr}\,\mathbf{e}\right)\exp\left(\frac{3\,K\,\alpha\,\Delta\,T}{\sigma_1}\right)}{k_1}}_{\beta_1}D\,\delta\,f \qquad \text{éq 5.6-24}$$

In the plastic case, the condition of coherence is worth:

$$2\mu\, e_{eq}+D\, f\,\sigma_1\exp\left(-\frac{3K\alpha\Delta T}{\sigma_1}\right)\exp\left(-\frac{K}{\sigma_1}\mathrm{Tr}\,\mathbf{e}\right)-R-\sigma_y=0 \qquad \text{éq 5.6-25}$$

from where

$$\frac{3\mu}{e_{eq}}\left(\tilde{\mathbf{e}}:\delta\tilde{\mathbf{e}}\right)+D\sigma_1\exp\left(\frac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(\frac{-K\,\mathrm{tr}\,\mathbf{e}}{\sigma_1}\right)\delta f$$
$$-D\, f\, K\exp\left(\frac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(\frac{-K\,\mathrm{tr}\,\mathbf{e}}{\sigma_1}\right)\mathrm{tr}\,\delta\,\mathbf{e}-h\,\delta\Delta p=0 \qquad \text{éq 5.6-26}$$

By injecting the relation [éq 5.6-21] in the equation above, one obtains then, while posing

$k_2=3\mu+h+\alpha_2 D\, f\, K\exp\left(\frac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(-\frac{K}{\sigma_1}\mathrm{Tr}\,\mathbf{e}\right)$:

$$\delta\Delta p=\underbrace{\frac{3\mu}{e_{eq}}\frac{1}{k_2}}_{\alpha_3}\tilde{\mathbf{e}}:\delta\tilde{\mathbf{e}}^{Tr}-\underbrace{\frac{\alpha_1 D\, f\, K\exp\left(\frac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(-\frac{K}{\sigma_1}\mathrm{Tr}\,\mathbf{e}\right)}{k_2}}_{\alpha_4}\mathrm{tr}\,\delta\,\mathbf{e}^{Tr}$$
$$+\underbrace{\frac{\exp\left(\frac{-3K\alpha\Delta T}{\sigma_1}\right)\exp\left(\frac{-K}{\sigma_1}\mathrm{Tr}\,\mathbf{e}\right)\left(\sigma_1-\beta_1 D\, f\, K\right)}{k_2}}_{\beta_2}D\delta f \qquad \text{éq 5.6-27}$$

While replacing $\delta\Delta p$ by his value obtained above in the equations [éq 5.6-22] and [éq 5.6-24], one obtains:

$$\delta\,\mathbf{e}=\underbrace{\left[\mathbf{A_1}+\left(\mathbf{A_2}+\frac{1}{3}\alpha_2\,\mathbf{Id}\right)\otimes\left(\frac{3\mu\alpha_3}{e_{eq}}\tilde{\mathbf{e}}\right)\right]}_{\mathbf{ddvetr}}:\delta\,\tilde{\mathbf{e}}^{Tr}+\underbrace{\left[\frac{1}{3}\alpha_1\,\mathbf{Id}+\alpha_4\left(\mathbf{A_2}+\frac{1}{3}\alpha_2\,\mathbf{Id}\right)\right]}_{\mathbf{dtretr}}\mathrm{tr}\,\delta\,\mathbf{e}^{Tr}$$
$$+\underbrace{\left[\frac{1}{3}\beta_1\,\mathbf{Id}+\beta_2\left(\mathbf{A_2}+\frac{1}{3}\alpha_2\,\mathbf{Id}\right)\right]}_{\mathbf{dedf}}D\,\delta f \qquad \text{éq 5.6-28}$$

from where

$$\frac{\partial\,\mathbf{e}}{\partial\,\mathbf{e}^{Tr}}=\mathbf{ddvetr}+\left(\mathbf{dtretr}-\frac{1}{3}\mathbf{ddvetr}:\mathbf{Id}\right)\otimes\mathbf{Id} \qquad \text{éq 5.6-29}$$

Plastic case – singular Solution

The approach is identical to that used previously.
One obtains for the law of flow discretized:

$$\tilde{\mathbf{e}}=0 \quad\Leftrightarrow\quad \delta\,\tilde{\mathbf{e}}=0 \qquad \text{éq 5.6-30}$$

for the deviatoric part and the part traces, the relation is identical to that found for the regular solution.

$$\mathrm{tr}\,\delta\,\mathbf{e}=\alpha_1\,\mathrm{tr}\,\delta\,\mathbf{e}^{\mathbf{Tr}}+\alpha_2\delta\,\Delta\,p+\beta_1\,D\,\delta\,f \qquad \text{éq 5.6-31}$$

where α_1 , α_2 and β_1 the same definitions as in the preceding paragraph have.

The condition of coherence then makes it possible to find $\delta\,\Delta\,p$ according to $\partial\,\mathbf{e}^{\mathbf{Tr}}$.

$$D\,f\,\sigma_1\exp\left(\frac{3\mathrm{K}\,\alpha\,\Delta\,T}{\sigma_1}\right)\exp\left(-\frac{K}{\sigma_1}\mathrm{Tr}\,e\right)-R-\sigma_y=0 \qquad \text{éq 5.6-32}$$

maybe after linearization:

$$\delta\,\Delta\,p=\alpha_4\,\mathrm{tr}\,\delta\,\mathbf{e}^{\mathbf{Tr}}+\beta_2\,D\,\delta\,f \qquad \text{éq 5.6-33}$$

that is to say finally:

$$\delta\,\mathbf{e}=\underbrace{\frac{1}{3}\left[\alpha_1+\alpha_4\,\alpha_2\right]\mathbf{Id}\;\mathrm{tr}\,\delta\,\mathbf{e}^{\mathbf{Tr}}}_{\mathbf{dtretr}}+\underbrace{\frac{1}{3}\left[\beta_1+\beta_2\,\alpha_2\right]\mathbf{Id}\,D\,\delta\,f}_{\mathbf{dedf}} \qquad \text{éq 5.6-34}$$

from where

$$\frac{\partial\,\mathbf{e}}{\partial\,\mathbf{e}^{Tr}}=\mathbf{dtretr}\otimes\mathbf{Id} \qquad \text{éq 5.6-35}$$

•Calculation of $\dfrac{\partial\,f}{\partial\,J}$

Taking into account relation 4.3-1, the derivative is written simply:

$$\begin{cases}\dfrac{\partial\,f}{\partial\,J}=D\dfrac{1-f_0}{J^2} & \mathrm{si}\;f>f_0\\[2mm] \dfrac{\partial\,f}{\partial\,J}=0 & \mathrm{si}\;f=f_0\end{cases} \qquad \text{éq 5.6-36}$$

Note:

The tangent matrix is not deteriorated by the correction of volume because this one is carried out a posteriori, i.e. after the calculation of the constraints.

6 Bibliography

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7 Description of the versions of the document

Version Aster	**Author (S) or contributor (S), organization**	**Description of the modifications**
6.0	V.Cano, E.Lorentz EDF/R & D /AMA	Initial text
9.4	E.Lorentz EDF/R & D /SINETICS	Modification of L" expression of porosity, function of change of volume J (what makes the model entirely implicit) and introduction of a correction on the elastic change of volume a posteriori.
12.1	J.M.Proix EDF/R & D /AMA	Card 19650: modification about the internal variables