Quadratic law of damage regularized ENDO_CARRE

Summary:

This document describes the model of elastic behavior fragile ENDO_CARRE available only for nonlocal modeling with gradient of nodal damage GVNO. This law presents a quadratic rigidity in damage and a linear dissipation. The criterion is thus linear and thus admits a single solution in damage with fixed displacements, which makes it possible to get fast and easily comparable results with the analytical one. The other interest of this law is the distinction of behaviours in traction and compression. It allows a damage in shearing in the cases of compression and avoids the interpenetration.

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1 Scope of application

The law ENDO_CARRE aim at modelling a fragile elastic behavior in nonlocal version (GVNO [R5.04.04]). This law was built with an aim of providing a stable law of damage (a quadratic loss of rigidity decreases the presence of snap-back digital), of simple formulation (advantageous for the analytical studies) and which has the advantage of being quadratic in damage and displacements, which makes it possible to work with an algorithm of minimization alternated by using the secant matrix of GVNO. The modelled material is elastic isotropic. Its rigidity can decrease in an irreversible way when the deformation energy becomes important. The distinction between behaviours in compression and traction is carried out on the spherical part of the tensor of the deformations with which one associates a rigidity healthy in compression and damaged in traction. The width of the bands of localization is controlled by a parameter material, indicated in the operator DEFI_MATERIAU under the keyword C_GRAD_VARI keyword factor NON_LOCAL [U4.43.01].

2 Variational formulation of the problem of damage

2.1 Case of a generic law (clarified here in traction)

Two equivalent approaches can be used to describe the process of damage of a fragile isotropic material. On a side it is possible to derive the law from damage within the framework of generalized standard materials. In this case, it is necessary to define the free energy of the system as well as a potential of dissipation. The rule of flow establishes the evolution of the internal variables then.
As for the description of damage one needs only a scalar variable. Preceding description is simplified and been able to be brought back to a variational problem under constraint of increase in damage [bib2].

To define a law of behavior in gradient of damage, it is thus enough to express the density of total free energy (elastique+dissipation) according to the tensor to the deformations $\boldsymbol{\varepsilon}$ and of the variable of damage $0 \leqslant a \leqslant 1$. The space distribution of the damage is given then by a field $a(x)$. Density of free energy presents itself in general in the following form:

$$\Phi(\boldsymbol{\varepsilon}, a) = A(a) w(\boldsymbol{\varepsilon}) + \omega(a) + c/2 (\nabla a)^2 \qquad \textbf{éq 2.1-1}$$

Here c is the parameter of nonlocality (C_GRAD_VARI) $w(\boldsymbol{\varepsilon})$ the elastic deformation energy, $\omega(a)$ the energy of dissipation and $A(a)$ the function of rigidity. $a=0$ corresponds to healthy material and $a=1$ corresponds to material completely damaged: $A(1)=0, A(0)=1$. The problem of evolution is from now on a simple problem of minimization of free energy of Helmholtz $F \equiv \int \Phi(\boldsymbol{\varepsilon}, a) d\Omega$ under constraint $\dot{a} \geq 0$ [1].

$$min_{(\boldsymbol{\varepsilon}, a)} F(\boldsymbol{\varepsilon}, a), \text{ où } \quad F(\boldsymbol{\varepsilon}, a) = \int [\Phi(\boldsymbol{\varepsilon}, a)] d\Omega$$

where one replaces $w(\boldsymbol{\varepsilon}) = \boldsymbol{\varepsilon} : \boldsymbol{E} : \boldsymbol{\varepsilon}/2$ by using the definition of the tensor of Hooke. Two equations derive from the variational problem of minimization: $\delta F(\boldsymbol{\varepsilon}, a)/\delta \boldsymbol{\varepsilon} = 0$ [2] and $\delta F(\boldsymbol{\varepsilon}, a)/\delta a \geq 0$. The inequality in the second equation is related to the presence of imposed constraints. These two equations must be satisfied everywhere in the field with integration Ω . They are supplemented by an equation of coherence of Kuhn-Tucker $\dot{a} \cdot \delta F(\boldsymbol{\varepsilon}, a)/\delta a = 0$. On the edges $\partial \Omega$ we obtain an

1 One notes by ∇a the space derivative of the field of damage and by $\dot{a}$ that related to the temporal evolution
2 $\delta F(\boldsymbol{\varepsilon}, a)/\delta \boldsymbol{\varepsilon}$ is the variational derivative partial according to the direction of the space field $\boldsymbol{\varepsilon}(x)$, the field $a(x)$ remaining fixed.

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additional condition of normality $\nabla a \cdot \boldsymbol{n} = 0$, where $\boldsymbol{n}$ is vector-normal. Finally the variable of damage and its gradient must be continuous at interior of the field of integration to carry out the minimum of functional calculus in question (see [bib2,4] for more details).

2.2 Relations of behavior

The link between the variational formulation and the usual laws of evolution is direct. The state of material is characterized by the deformation $\boldsymbol{\varepsilon}$ and the damage a , ranging between 0 and 1. The relation stress-strain is defined, which remains elastic, and rigidity is affected by the damage:

$$\boldsymbol{\sigma} = \delta F(\boldsymbol{\varepsilon}, a) / \delta \boldsymbol{\varepsilon} = A(a) \boldsymbol{E} : \boldsymbol{\varepsilon}$$ **éq 2.2-1**

with $\boldsymbol{E}$ the tensor of Hooke. The evolution of the damage, always increasing, is controlled by the following function threshold:

$$f(\boldsymbol{\varepsilon}, a) = -\delta \Phi(\varepsilon, a) / \delta a = -\frac{1}{2} A'(a) \boldsymbol{\varepsilon} : \boldsymbol{E} : \boldsymbol{\varepsilon} - \omega'(a) + c \Delta a$$ **éq 2.2-2**

The condition of coherence takes its usual form then:

$$f(\boldsymbol{\varepsilon}, a) \le 0 \quad \dot{a} \ge 0 \quad \dot{a} f(\boldsymbol{\varepsilon}, a) = 0$$ **éq 2.2-3**

Two characteristics of this formulation are noted. Firstly, the function threshold is not-local because of the presence of the Laplacian of damage. Then, the absence of condition of flow is justified by the double role of the damage a , on a side it is presented in the form of an internal variable of evolution, other side it fulfills the mission of the parameter of Lagrange $\lambda \equiv a$.
One sees also the advantage of presentation of the laws of damage in their variational form. It is enough to describe the density of total free energy (éq.2.1-1), which includes dissipation, to define the law of evolution completely.

2.3 Identification of the law ENDO_CARRE

In the law `ENDO_CARRE` the functions of rigidity and dissipation are selected as follows:

$$A(a) = (1-a)^2, \quad \omega(a) = ka$$

The mechanical parameters of the law are three. On the one hand, the Young modulus E and the Poisson's ratio ν who determine the tensor of Hooke by:

$$\mathbf{E}^{-1} \cdot \boldsymbol{\sigma} = \frac{1+\nu}{E} \boldsymbol{\sigma} - \frac{\nu}{E} (\mathrm{tr}\, \boldsymbol{\sigma}) \, \mathbf{Id}$$ **éq 2.3-1**

like k , which defines the lenitive behavior and the width characteristic of the band of damage. While noting σ_y the constraint with the peak and l the length interns characteristic of the size of the damaged zone, one establishes the following relations:

$$k = \frac{\sigma_y^2}{E}, \quad c = E l^2$$

Parameters of the model are directly given under the keywords factors `ECRO_LINE (SY)` and `NON_LOCAL (C_GRAD_VARI)` of the operator `DEFI_MATERIAU`. As for E and ν , they are given classically under the keyword factor `ELAS` or `ELAS_FO`. It should be noted that the parameter `D_SIGM_EPSI` is not used here although it is necessary to give him a value in the command file,

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because it is in obligatory statute. The keyword was used `ECRO_LINE` to avoid duplication in `DEFI_MATERIAU` objects making all reference to the ultimate stress.

Example:	$E=30\,\mathrm{GPa},\ \nu=0.2$ $\sigma_y=3\,\mathrm{MPa}$	ELAS $(E=3.10^{10}, NU=0.2)$ ECRO_LINE$(SY=3.10^{6}, D_SIGM_EPSI=2.0)$ NON_LOCAL $(C\_GRAD\_VARI=3.10^{10} l^2)$

The answer below is presented $\sigma(\varepsilon)$ obtained into 1 point, starting from the parameters chosen in example:

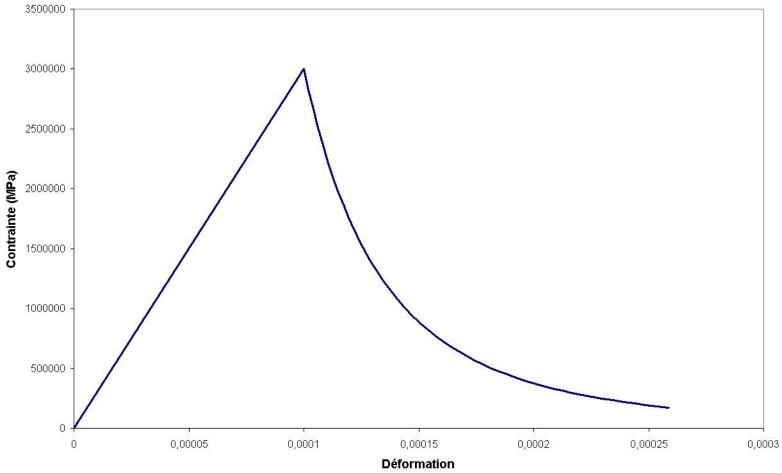

Figure 1 : Answer stress-strain

As long as one does not reach the critical stress σ_y , the answer is elastic linear and the damage is null. Once this value reached, one observes a decrease of the constraint according to the deformation in ε^{-3} . What is explained by the quadratic reduction in rigidity according to the damage.

2.4 Behaviour in compression

In the case of traction ("opening of crack"), one corrects the elastic energy of a quadratic factor corresponding to the loss of rigidity (see **éq 2.2.1**). In the case of compression ("closing"), one corrects only the part associated with the tensor deviatoric of the deformations. The stress field can then be written in the form:

$$\boldsymbol{\sigma}=(1-a)^2(k_2 tr(\boldsymbol{\varepsilon})Htr(\boldsymbol{\varepsilon})+2\mu\boldsymbol{\varepsilon_D})+k_2 tr(\boldsymbol{\varepsilon})Htr(-\boldsymbol{\varepsilon})$$

Where $\boldsymbol{\varepsilon_D}$ is the tensor deviatoric of the deformations, H the function of Heaviside and $k_2=\lambda+\frac{2\mu}{3}$, with λ and μ coefficients of Lamé.

The constraints having to be finished, the trace of the tensor of the deformations is controlled. That makes it possible to avoid the important deformations in the direction of the efforts, contrary to what is made for traction. Taking into account the loss of rigidity assigned to the deviatoric part, it is however possible to obtain jumps of slip in the direction of shearing. This behavior prohibits the interpenetration but thus allows the damage by shearing.

The criterion is also found some modified as follows:

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$$f(\boldsymbol{\varepsilon}, a) = -\frac{1}{2} A'(a)\left[2\mu\, \boldsymbol{\varepsilon}_{\boldsymbol{D}} : \boldsymbol{\varepsilon}_{\boldsymbol{D}} + H(tr(\boldsymbol{\varepsilon}))\, k_2 (tr(\boldsymbol{\varepsilon}))^2\right] - \omega'(a) + c\,\Delta a$$

This model is very quite representative of the behavior of a concrete structure when the damage is appeared as band. What is always the case in 2D. However, to have a good behaviour in compression in 3D, if the damage would not be presented any more in the form of band, it would be necessary that one makes no longer carry the damage on the diverter, but on the rotational one.

2.5 Description of the internal variables

The internal variables are two:

- $VI(1)$ damage a
- $VI(2)$ indicator χ

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4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.	J.BEAURAIN, K.KAZYMYRENKO EDF-R&D/AMA	Initial text

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