Methods of piloting of the loading

Summary:

This document describes the methods of piloting of the loading available in *Code_Aster* (by a degree of freedom, length of arc, increment of deformation and elastic prediction). They introduce an additional unknown factor, the intensity on behalf controllable of the loading, and an additional equation, the constraint of piloting. These methods make it possible in particular to as well calculate the answer of a structure which would have instabilities, of origins geometrical (buckling) as material (softening).

1 Introduction

1.1 Non-linearity and failure of Newton

Method of Newton presented in [R5.03.01] fail on certain problems which display an answer of the structure not strictly monotonous according to displacements or imposed loadings. On the figure 1.1-a, one presents the case where, for a level of loading given, there exist several solutions displacements. This problem is thus not soluble with Newton if the problem in loading is controlled.

A second case (figure 1.1-b) relates to the geometrical non-linear problems (typically the buckling of the thin hulls and problems of “soft” snap-back). In this case, for a level of loading, there are several solutions in displacement and for a level of displacement there are several possible loadings. This problem is not soluble directly by the method of Newton, whether it is while controlling in loading or displacement.

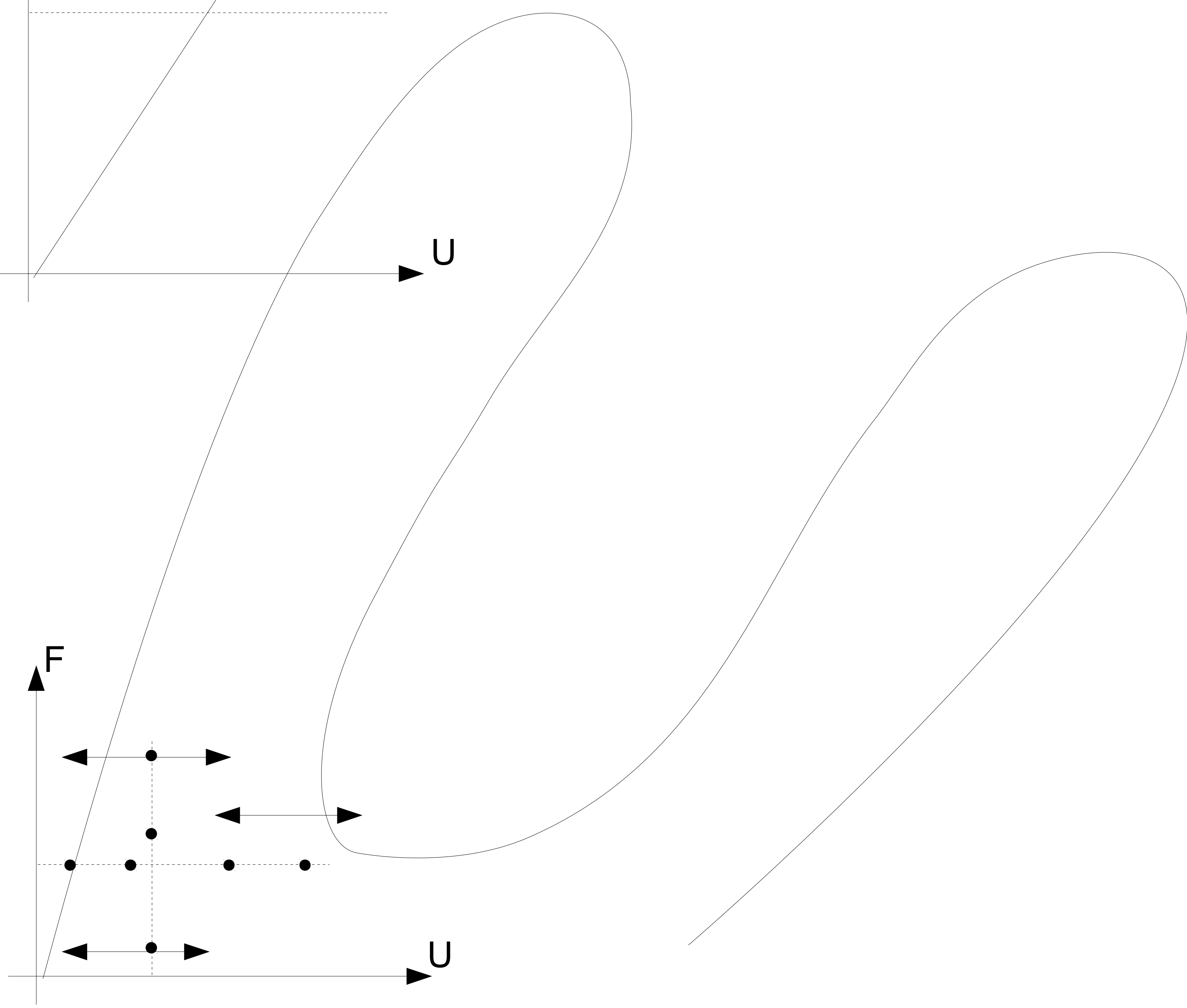


Figure 1.1-b: Not-monotonous response in displacement and loading

The third case (figure 1.1-c) described the problems of damages in which abrupt elastic returns (corresponding to the progressive damage of material) make the curve very irregular (snap-backs “abrupt”)

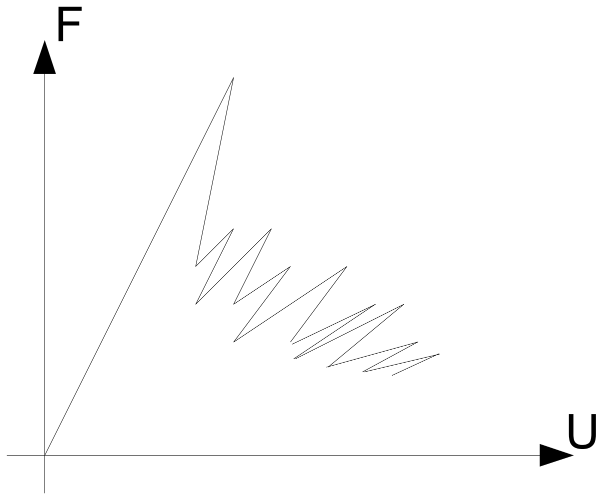
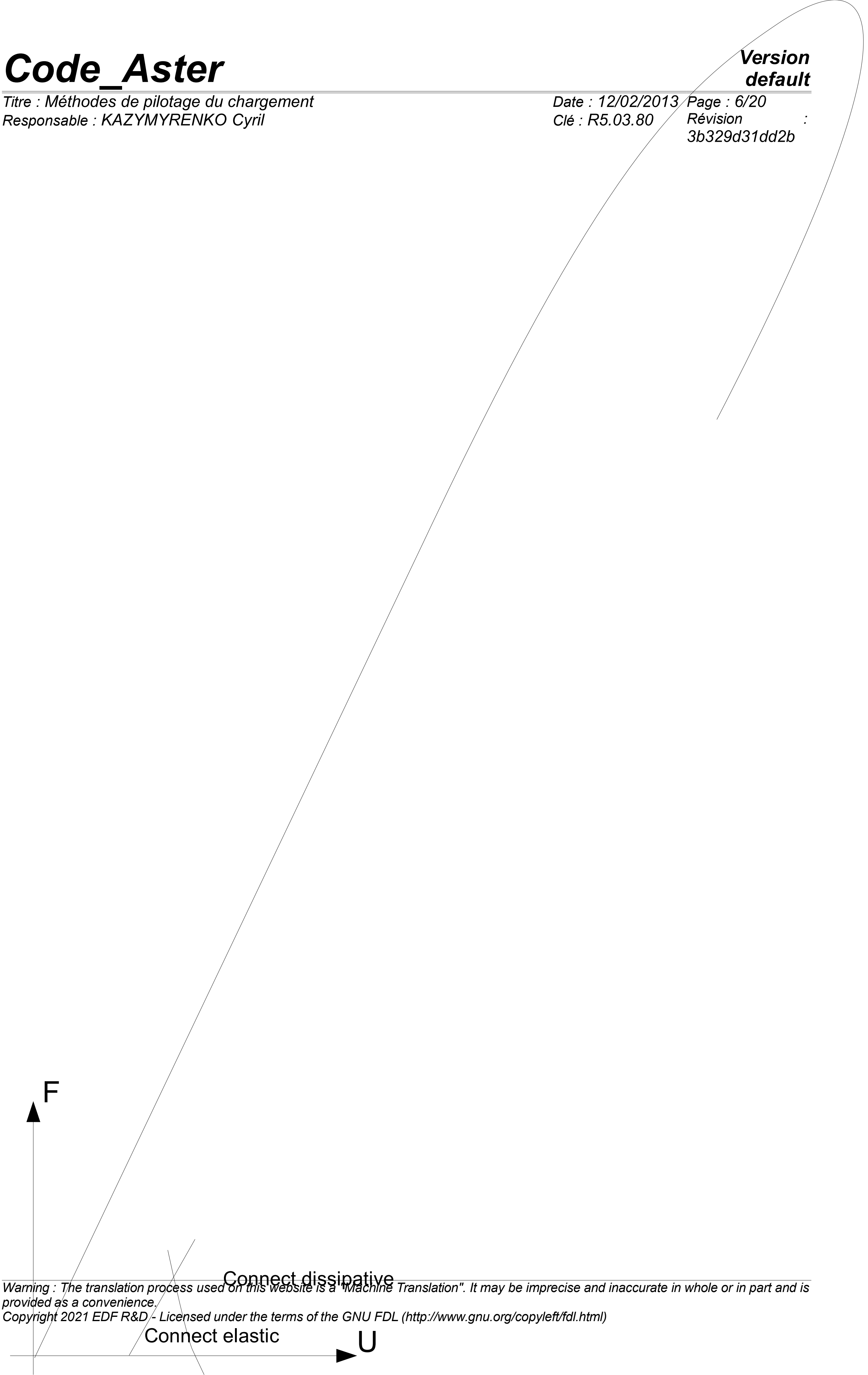


Figure 1.1-c: Answer with losses of rigidity

Lastly, there exists finally a whole category of problems in which the loss of ellipticity of the problem creates junctions and thus branches of different solutions (see figure 1.1-d). In this case, which interests more the engineer it is generally the dissipative branch and not the elastic branch. We will see that there exist techniques to select “the good” branches.


F
Connect dissipative
Connect elastic
U

1.2 Problems with intensity of the unknown loading

Piloting can also be useful in the case or the problem displays, by its natural setting in data, an additional unknown factor which is the intensity of the loading applied. Piloting makes it possible to treat the case where only the direction and the point of application of the loading are known, intensity remaining an unknown factor of the problem. For example, on the figure 1.2-a, one sees the case of a cable tended between two pylons. It is known that it is necessary to apply a loading which draws the cable (one thus knows the point of application and the direction of the loading) but one is unaware of the intensity η to apply to obtain an arrow f data.

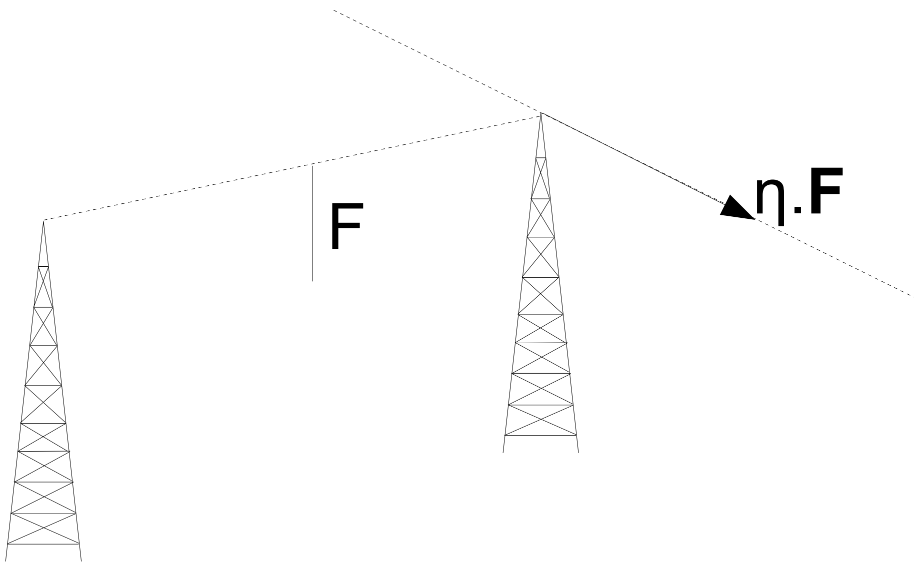


Figure 1.2-a: Example of partially unknown loading

2 Principle of the methods of piloting of the loading

In a general way, features of piloting available in *Code_Aster* allow to determine *intensity of part of the loading* to satisfy a constraint relating to displacements. Their employment is limited to simulations for which time does not play of physical role, which excludes *a priori* dynamic or viscous problems. It is also incompatible with the problems expressing a condition of unilaterality (contact and friction). One can distinguish several ranges from use which answer as many methods of piloting (keyword factor `PILOTING`):

- Physical control of the forces by the displacement of a point of the structure: piloting per degree of freedom imposed (`TYPE = ' DDL_IMPO'`);
- Follow-up of geometrical instabilities (buckling), the answer of the structure being able to display "soft" snap-back: piloting by length of arc (`TYPE = ' LONG_ARC'`);
- Follow-up of instabilities materials (in the presence of lenitive laws of behavior), the answer of the structure being able to display "brutal" snap-back: piloting by the elastic prediction (`TYPE = 'PRED_ELAS'`) or more generally by the increment of deformation, (`TYPE = ' DEFORMATION'`);
- Calculation of the limiting loads of the structures (`TYPE=' ANA_LIM'`), cf [R7.07.01].

More precisely, methods of piloting available in *Code_Aster* rest on the two following ideas.

On the one hand, one considers that the loading (external forces and displacements given) breaks up additivement into two terms, one known and imposed by the user ($\mathbf{L}_{\text{impo}}^{\text{méca}}$ and $\mathbf{u}_{\text{impo}}^{d}$) and the other ($\mathbf{L}_{\text{pilo}}^{\text{méca}}$ and $\mathbf{u}_{\text{pilo}}^{d}$) whose only direction is known, its intensity η becoming a new unknown factor of the problem:

$$\begin{cases} \mathbf{L}^{\text{méca}} = \mathbf{L}_{\text{impo}}^{\text{méca}} + \eta . \mathbf{L}_{\text{pilo}}^{\text{méca}} \\ \mathbf{u}^{d} = \mathbf{u}_{\text{impo}}^{d} + \eta . \mathbf{u}_{\text{pilo}}^{d} \end{cases} \quad (1)$$

In addition, in order to be able to solve the problem, one associates a new equation to him who relates to displacements and which depends on the increment of time: it is the constraint of piloting, which is expressed by:

$$P(\Delta \mathbf{U}) = \Delta \tau \quad \text{avec} \quad P(0) = 0 \quad (2)$$

where $\Delta \tau$ is indirectly an user datum which is expressed via the step of current time Δt and a coefficient of piloting (`COEF_MULT`) such as:

$$\Delta \tau = \frac{\Delta t}{\texttt{COEF\_MULT}} \quad (3)$$

The condition $P(0)=0$ is necessary in order to obtain an increment of all the more small displacement as the step of time is small. Finally, the unknown factors of the problem become displacements $\mathbf{u}$, multipliers of Lagrange $\boldsymbol{\lambda}$ associated with the boundary conditions and the intensity of the controlled loading η, baptized `ETA_PILOTAGE`. The nonlinear system to solve is written henceforth:

$$\begin{cases} \mathbf{L}^{\text{int}}(\mathbf{u}) + \mathbf{B}^{T} . \boldsymbol{\lambda} & = \mathbf{L}_{\text{impo}}^{\text{méca}} + \eta . \mathbf{L}_{\text{pilo}}^{\text{méca}} \\ \mathbf{B} . \mathbf{u} & = \mathbf{u}_{\text{impo}}^{d} + \eta . \mathbf{u}_{\text{pilo}}^{d} \\ P(\Delta \mathbf{u}) & = \Delta \tau \end{cases} \quad (4)$$

Note:

At the present time, following loadings (i.e which depends on displacements) and conditions of Dirichlet of the type " `DIDI` " are not controllable.

The loading does not depend directly any more on time but results from the resolution of all the nonlinear system (4). That implies that the controlled share of the loading should not depend on physical time, but corresponds to an effort which one adjusts to satisfy an additional kinematic constraint.

3 Resolution of the total system

The introduction of a new equation does not disturb in addition to measurement the method of resolution of the nonlinear system. Indeed, one proceeds as in [R5.03.01], i.e. the resolution is incremental. The step of time is noted i in index and the iteration of Newton n while exposing. The non-linear problem is then solved in two times:

- A phase of prediction which gives a first estimate of displacements and multipliers of Lagrange noted $(\Delta\,\mathbf{u}_i^0, \Delta\boldsymbol{\lambda}_i^0)$;
- A phase of correction of Newton $(\delta\,\mathbf{u}_i^n, \delta\,\boldsymbol{\lambda}_i^n)$ who comes to correct this first estimate;

With a number n_{CV} sufficient from iterations of Newton, one gets a converged result[1]:

$$\begin{cases} \mathbf{u}_i^{\text{convergé}} = \mathbf{u}_{i-1} + \Delta\,\mathbf{u}_i^0 + \sum_{j=1}^{n=n_{CV}} \delta\,\mathbf{u}_i^j \\ \boldsymbol{\lambda}_i^{\text{convergé}} = \boldsymbol{\lambda}_{i-1} + \Delta\,\boldsymbol{\lambda}_i^0 + \sum_{j=1}^{n=n_{CV}} \delta\,\boldsymbol{\lambda}_i^j \end{cases} \tag{5}$$

The principle is thus to linearize the system (4) that one writes with the step time i :

$$\begin{cases} \mathbf{L}_i^{\text{int}} + \mathbf{B}^T.\boldsymbol{\lambda}_i & = \mathbf{L}_{\text{impo},i}^{\text{méca}} + \eta_i.\,\mathbf{L}_{\text{pilo},i}^{\text{méca}} \\ \mathbf{B}.\mathbf{u}_i & = \mathbf{u}_{\text{impo},i}^d + \eta_i.\mathbf{u}_{\text{pilo},i}^d \\ P(\Delta\,\mathbf{u}_i) & = \Delta\,\tau_i \end{cases} \tag{6}$$

There will be no linearization compared to the variable of piloting η_i . This way, one preserves all the methodology of reactualization of the tangent operator already put in work for calculations without piloting. Moreover, the structure “bandages” tangent matrix is preserved. The diagram of resolution is not thus strictly any more a method of Newton (what is not awkward because we will see that the equation of piloting is, at most, of degree two). In prediction, if one linearizes the system (6) without the equation of piloting, compared to time around $(\mathbf{u}_{i-1}, \boldsymbol{\lambda}_{i-1})$, one obtains (see all the development in [R5.03.01]):

$$\begin{cases} \mathbf{K}_{i-1}.\Delta\,\mathbf{u}_i^0 + \mathbf{B}^T.\Delta\,\boldsymbol{\lambda}_i^0 & = \mathbf{L}_{\text{impo},i}^{\text{méca}} + \eta_i.\mathbf{L}_{\text{pilo},i}^{\text{méca}} - \mathbf{Q}_{i-1}^T.\boldsymbol{\sigma}_{i-1} + \Delta\,\mathbf{L}_i^{\text{varc}} \\ \mathbf{B}.\Delta\,\mathbf{u}_i^0 & = \mathbf{u}_{\text{impo},i}^d + \eta_i.\mathbf{u}_{\text{pilo},i}^d - \mathbf{B}.\mathbf{u}_{i-1} \end{cases} \tag{7}$$

In correction, one linearizes the system (6) always without the equation of piloting, compared to time, but around $(\mathbf{u}_i^n, \boldsymbol{\lambda}_i^n)$, one a:

$$\begin{cases} \mathbf{K}_i^{n-1}.\delta\,\mathbf{u}_i^n + \mathbf{B}^T.\delta\,\boldsymbol{\lambda}_i^n & = \mathbf{L}_{\text{impo},i}^{\text{méca}} + \eta_i.\mathbf{L}_{\text{pilo},i}^{\text{méca}} - \mathbf{L}_i^{\text{int},n-1} - \mathbf{B}^T.\boldsymbol{\lambda}_i^{n-1} \\ \mathbf{B}.\delta\,\mathbf{u}_i^n & = \eta_i.\mathbf{u}_{\text{pilo},i}^d \end{cases} \tag{8}$$

One will join together the two systems in a common writing, in order to simplify the talk. The system to be solved is written finally:

$$\begin{bmatrix} \mathbf{K}_i^{n-1} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} . \begin{Bmatrix} \delta\,\mathbf{u}_i^n \\ \delta\,\boldsymbol{\lambda}_i^n \end{Bmatrix} = \begin{Bmatrix} \mathbf{L}_{\text{impo},i}^{n-1} \\ \mathbf{u}_{\text{impo},i} \end{Bmatrix} + \eta_i . \begin{Bmatrix} \mathbf{L}_{\text{pilo},i} \\ \mathbf{u}_{\text{pilo},i} \end{Bmatrix} \tag{9}$$

One can pass several note:

- The matrix $\mathbf{K}_i^{n-1}$ depends at the same time on the step of current time and possibly on the preceding iteration of Newton. The various manners of building it (quasi-Newton, rubber band, secant, coherent, tangent of speed, etc) are described in [R5.03.01];
- It was supposed that the external loadings were linear (they depend only on the step of time). One thus does not consider the following loadings such as the pressure or the centrifugal force although from the theoretical point of view, that does not raise difficulties. On the other hand, the material can be described with a non-linear behavior, which implies that $\mathbf{L}_i^{\text{int},n-1}$ depends on the iteration of Newton (result of the linearization of the interior efforts).

1 The concept of “converged” result is more amply detailed in [R5.03.01].

- The limiting conditions of Dirichlet are always linear, which allows the matrix $\mathbf{B}$ to be constant on all the transient.
- With the good choice of the tangent matrix and the second member, formally, there is equivalence enters $(\delta\,\mathbf{u}_i^{n=0},\delta\,\boldsymbol{\lambda}_i^{n=0})$ and the increment in prediction $(\Delta\,\mathbf{u}_i^0,\Delta\,\boldsymbol{\lambda}_i^0)$.

One can now express the corrections of displacements $\delta\,\mathbf{u}_i^n$ and of multipliers of Lagrange $\delta\,\boldsymbol{\lambda}_i^n$ according to η_i with the help of the resolution of the linear system (9) compared to each of the two second members. I.e. one separates the two solutions:

$$\begin{Bmatrix}\delta\,\mathbf{u}_i^n\\ \delta\,\boldsymbol{\lambda}_i^n\end{Bmatrix}=\begin{Bmatrix}\delta\,\mathbf{u}_{\text{impo},i}^n\\ \delta\,\boldsymbol{\lambda}_{\text{impo},i}^n\end{Bmatrix}+\eta_i.\begin{Bmatrix}\delta\,\mathbf{u}_{\text{pilo},i}^n\\ \delta\,\boldsymbol{\lambda}_{\text{pilo},i}^n\end{Bmatrix} \tag{10}$$

These two solutions correspond to the decoupling of the two loadings:

$$\begin{Bmatrix}\mathbf{L}_i^{n-1}\\ \mathbf{u}_i\end{Bmatrix}=\begin{Bmatrix}\mathbf{L}_{\text{impo},i}^{n-1}\\ \mathbf{u}_{\text{impo},i}\end{Bmatrix}+\eta_i.\begin{Bmatrix}\mathbf{L}_{\text{pilo},i}\\ \mathbf{u}_{\text{pilo},i}\end{Bmatrix} \tag{11}$$

One solves independently[2]:

$$\begin{bmatrix}\mathbf{K}_i^{n-1} & \mathbf{B}^T\\ \mathbf{B} & \mathbf{0}\end{bmatrix}.\begin{Bmatrix}\delta\,\mathbf{u}_{\text{impo},i}^n\\ \delta\,\boldsymbol{\lambda}_{\text{impo},i}^n\end{Bmatrix}=\begin{Bmatrix}\mathbf{L}_{\text{cst},i}^{n-1}\\ \mathbf{u}_{\text{cst},i}\end{Bmatrix} \tag{12}$$

And:

$$\begin{bmatrix}\mathbf{K}_i^{n-1} & \mathbf{B}^T\\ \mathbf{B} & \mathbf{0}\end{bmatrix}.\begin{Bmatrix}\delta\,\mathbf{u}_{\text{pilo},i}^n\\ \delta\,\boldsymbol{\lambda}_{\text{pilo},i}^n\end{Bmatrix}=\begin{Bmatrix}\mathbf{L}_{\text{pilo},i}^{n-1}\\ \mathbf{u}_{\text{pilo},i}\end{Bmatrix} \tag{13}$$

One can now substitute the correction of displacement $\Delta\,\mathbf{u}_i^n$ in the equation of control of the piloting of the system; it results a scalar equation from it in η_i :

$$\tilde{P}(\eta_i)\underset{\text{déf.}}{=}P\left(\Delta\,\mathbf{u}_i^{n-1}+\delta\,\mathbf{u}_{\text{impo},i}^n+\eta_i.\,\delta\,\mathbf{u}_{\text{pilo},i}^n\right)=\Delta\,\tau_i \tag{14}$$

The method of solution of this equation depends on nature on control on piloting adopted cf [§ 10]. Finally, it any more but does not remain to reactualize the unknown factors displacements and multipliers of Lagrange:

$$\begin{cases}\Delta\,\mathbf{u}_i^n=\Delta\,\mathbf{u}_i^{n-1}+\delta\,\mathbf{u}_{\text{impo},i}^n+\eta_i.\,\delta\,\mathbf{u}_{\text{pilo},i}^n\\ \boldsymbol{\lambda}_i^n=\boldsymbol{\lambda}_i^{n-1}+\delta\,\boldsymbol{\lambda}_{\text{impo},i}^n+\eta_i.\,\delta\,\boldsymbol{\lambda}_{\text{pilo},i}^n\end{cases} \tag{15}$$

4 Equation of control of piloting

4.1 Piloting by control of a degree of freedom of displacements: DDL_IMPO

For this first type of piloting, the function P restricts itself to extract a degree of freedom from the increment of displacement. In particular, it is thus about a linear function:

$$P\left(\Delta\,\mathbf{u}_i^n\right)=\langle\mathbf{S}\rangle.\{\Delta\,\mathbf{u}_i^n\}=\Delta\,\tau_i \tag{16}$$

Where the nodal vector $\langle\mathbf{S}\rangle$ is a vector of selection which is null everywhere except for the degree of freedom being extracted where it is worth one:

$$\langle\mathbf{S}\rangle=\Big\langle 0\ldots0\ldots\underset{\text{noeud n, ddl i}}{1}\ldots0\ldots0\Big\rangle \tag{17}$$

The equation (14) is reduced then also to a linear equation:

2 In practice, the factorization of the matrix is made only once and one solves simultaneously the two systems.

$$\eta_i = \frac{\Delta\tau_i - \langle \mathbf{S} \rangle . \{\Delta \mathbf{u}_i^{n-1}\} - \langle \mathbf{S} \rangle . \{\delta \mathbf{u}_{\text{impo},i}^n\}}{\langle \mathbf{S} \rangle . \{\delta \mathbf{u}_{\text{pilo},i}^n\}} \quad (18)$$

It will be noted that there is no solution when the correction of controlled displacement $\delta \mathbf{u}_{\text{pilo},i}^n$ does not allow to adjust the degree of required freedom, which can arrive if, by error, the degree of freedom in question is blocked. In this case, the code will stop in `FAILURE OF PILOTING`.

4.2 Piloting in modeling X-FEM by control of the jump of displacement according to a direction: `SAUT_IMPO`

In the case of one classical modeling FEM, one controls the increment of displacement of only one node project on only one direction, both being specified by the user. With the finite element method wide (XFEM), one can control the increment of jump of displacement through the interface. This jump is controlled on average, for a certain number of points of intersection of the interface with the edges of the grid and project on only one direction $\boldsymbol{n}_a$. He is written using the degrees of freedom enriched $\boldsymbol{b}$ detailed in [R7.02.12]. For an intersected edge a, with the notations defined on the figure (4.2-a), the function of piloting is:

$$P(\Delta \mathbf{u}_i^n) = P([\![\Delta \mathbf{u}]\!]_i^n) = \frac{1}{N} \sum_{aretes\,a}^{N} \boldsymbol{n}_a \cdot \left(\alpha_a \Delta \boldsymbol{b}_i^n(N_a^-) + (1-\alpha_a)\Delta \boldsymbol{b}_i^n(N_a^+)\right) = \Delta\tau_i \quad (19)$$

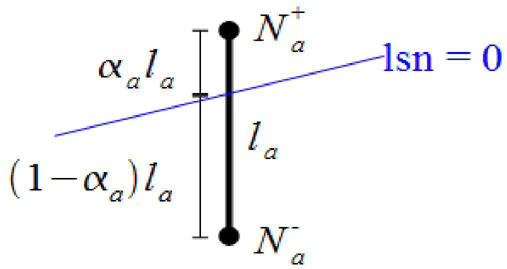


Figure 4.2-a: Edge intersected by the interface

To take again the preceding notations, the nodal vector $\langle \mathbf{S} \rangle$ is thus written:

$$\langle \mathbf{S} \rangle = \frac{1}{N} \left\langle 0 \ldots \underset{\substack{noeud\, N_1^+ \\ ddl\, h_x}}{\alpha_1 n_x} \quad \underset{ddl\, h_y}{\alpha_1 n_y} \ldots \underset{noeud\, N_1^-}{(1-\alpha_1) n_x} \quad (1-\alpha_1) n_y \quad \ldots \quad \underset{noeud\, N_N^+}{\alpha_N n_x} \quad \alpha_N n_y \ldots 0 \right\rangle \quad (20)$$

Edges a are selected like pertaining to a set of independent edges, i.e. that they have no joint node, and which it is impossible to add some such as this property (figure is preserved 4.2-b). Such a set of edges controlled satisfied condition LBB for the treatment with the contact (see [R7.02.12]) and thus for any imposition of condition of control on the interface. Nevertheless, the condition of the type `SAUT_IMPO` being imposed on average, a piloting on the whole of the degrees of freedom should be possible, since this one takes place in fine degree of freedom per degree of freedom. The choice of a set of independent edges is justified all the same because it largely facilitates the data-processing storage of the coefficients since one has only one information to store per degree of freedom in the vector $\langle \mathbf{S} \rangle$ (see formula 20). In addition, the re-use of algorithms of selection explained in [R7.02.12] makes it possible to build such a unit easily.

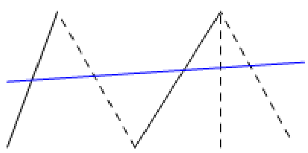


Figure 4.2-b: Selection of independent edges

4.3 Piloting by length of arc: LONG_ARC

Another form of piloting very largely used consists in controlling the standard of the increment of displacement (compared to certain nodes and certain components): one speaks then about piloting by cylindrical length of arc, to see Bonnet and Wood [bib1]. More precisely, the function P express yourself by:

$$P\left(\Delta \mathbf{u}_i^n\right)=\|\Delta \mathbf{u}_i^n\|_{<S>}=\sqrt{\left(\langle \mathbf{S}\rangle.\{\Delta \mathbf{u}_i^n\}\right).\left(\langle \mathbf{S}\rangle.\{\Delta \mathbf{u}_i^n\}\right)}=\Delta \tau_i \quad (21)$$

Where, again, the nodal vector $\langle \mathbf{S}\rangle$ allows to select the degrees of freedom employed for the calculation of the standard (it is worth for the degrees of freedom selected and zero elsewhere). In this case there one will notice that $\langle \mathbf{S}\rangle.\{\Delta \mathbf{u}_i^n\}$ is not a scalar, but a vector corresponding to the product of the components of $\langle \mathbf{S}\rangle$ by those of $\{\Delta \mathbf{u}_i^n\}$. In this case, the equation of piloting is reduced to a quadratic equation:

$$\begin{aligned}&\eta_i^2.\left[\left(\langle \mathbf{S}\rangle.\{\delta \mathbf{u}_{\text{pilo},i}^n\}\right).\left(\langle \mathbf{S}\rangle.\{\delta \mathbf{u}_{\text{pilo},i}^n\}\right)\right]+\\&2.\eta_i.\left[\left(\langle \mathbf{S}\rangle.\{\Delta \mathbf{u}_i^{n-1}+\delta \mathbf{u}_{\text{impo},i}^n\}\right).\left(\langle \mathbf{S}\rangle.\{\delta \mathbf{u}_{\text{pilo},i}^n\}\right)\right]+\\&\left[\left(\langle \mathbf{S}\rangle.\{\Delta \mathbf{u}_i^{n-1}+\delta \mathbf{u}_{\text{impo},i}^n\}\right).\left(\langle \mathbf{S}\rangle.\{\Delta \mathbf{u}_i^{n-1}+\delta \mathbf{u}_{\text{impo},i}^n\}\right)-\Delta \tau_i^2\right]=0\end{aligned} \quad (22)$$

This equation can not admit a solution. In this case, the value is chosen η_i who minimizes the polynomial (22). One checks then well $\tilde{P}\left(\eta_i\right)>\Delta \tau_i$. In the contrary case, she admits two roots (or a double root). One chooses that of both which minimizes:

- that is to say the standard of the increment of displacement $\|\Delta \mathbf{u}_i^n\|$;
- that is to say the residue of balance $\mathbf{R}(\mathbf{u}_i, t_i)=\mathbf{L}_i^{\text{int}}-\mathbf{L}_i^{\text{méca}}$;
- that is to say the angle formed by $\Delta \mathbf{u}_{i-1}$ and $\Delta \mathbf{u}_i^n$ (where $\Delta \mathbf{u}_{i-1}$ is the increment of solution displacement of the step of previous time), i.e. that which maximizes the cosine of this angle whose expression is:

$$\cos\left(\Delta \mathbf{u}_{i-1}, \Delta \mathbf{u}_i^n\right)=\frac{\langle \Delta \mathbf{u}_i^{n-1}+\delta \mathbf{u}_{\text{impo},i}^n+\eta_i.\delta \mathbf{u}_{\text{pilo},i}^n\rangle.\{\Delta \mathbf{u}_{i-1}\}}{\|\Delta \mathbf{u}_i^{n-1}+\delta \mathbf{u}_{\text{impo},i}^n+\eta_i.\delta \mathbf{u}_{\text{pilo},i}^n\|.\|\Delta \mathbf{u}_{i-1}\|} \quad (23)$$

To calculate the angle, one uses only the unknown factors corresponding to displacements, other than all the others (in particular rotations in the elements of structures).

It is noticed that this last angular criterion selects the bad solution when one passes from a phase of load to a phase of discharge or vice versa, i.e. when the coefficient of piloting C_i change sign. The value C_{i-1} is thus put in memory and is brought up to date with each step of converged time, its initialization for each call to `STAT_NON_LINE` being transparent for the user: C_{i-1} is thus initialized during the reading of the initial state, C_i having been included in the parameters of calculation to file in the concept result. If $C_i C_{i-1}>0$, one maximizes the criterion (23) . In the contrary case, one minimizes it in order to select the increment of displacement in opposite direction of the preceding increment.

4.4 Piloting in modeling X-FEM by control of the standard of the jump of displacement : SAUT_LONG_ARC

In the case of a modeling XFEM, piloting by length of arc consists in controlling the standard of the increment of the jump of displacement for a certain number of points of intersection of the grid with the interface, which are selected on a set of independent edges in the same way that in 4.2. The function of piloting is written then:

$$P\left(\Delta \mathbf{u}_i^n\right)=P\left(\llbracket\Delta \mathbf{u}\rrbracket_i^n\right)=\sqrt{\frac{1}{N}\sum_{aretes\,a}^{N}\left[\alpha_a\Delta \boldsymbol{b}_i^n(N_a^-)+(1-\alpha_a)\Delta \boldsymbol{b}_i^n(N_a^+)\right]^2}=\Delta\tau_i \quad (24)$$

If we develop this last equation, we have:

$$P\left(\llbracket\Delta \mathbf{u}\rrbracket_i^n\right)^2= \underbrace{\frac{1}{N}\sum_{aretes\,a}^{N}\left(\alpha_a\Delta \boldsymbol{b}_i^n(N_a^-)\right)^2+\left((1-\alpha_a)\Delta \boldsymbol{b}_i^n(N_a^+)\right)^2}_{N\|\langle\mathbf{S}\rangle.\{\Delta u_i^n\}\|^2}+\underbrace{\frac{1}{N}\sum_{aretes\,a}^{N}2\alpha_a\left(1-\alpha_a\right)\Delta \boldsymbol{b}_i^n(N_a^+)\Delta \boldsymbol{b}_i^n(N_a^-)}_{N\left([M]\langle\mathbf{S}\rangle.\{\Delta u_i^n\}\right)\cdot\left(\langle\mathbf{S}\rangle.\{\Delta u_i^n\}\right)} \quad (25)$$

where $\langle\mathbf{S}\rangle$ is the nodal vector defined by the formula 20 (it will be noticed that $\langle\mathbf{S}\rangle.\left\{\Delta \mathbf{u}_i^n\right\}$ is not a scalar, but a vector corresponding to the product of the components of $\langle\mathbf{S}\rangle$ by those of $\left\{\Delta \mathbf{u}_i^n\right\}$), and $[M]$ the matrix which with the degrees of freedom of a node makes correspond the degrees of freedom of the node located on the same edge independent, at the other end, i.e.:

$$\begin{aligned}&\forall\,\text{arête}\,a,\forall\,\text{direction}\,i,[M]\left\langle\mathbf{S}^{\mathbf{i}}(N_a^+)\right\rangle=\left\langle\mathbf{S}^{\mathbf{i}}(N_a^-)\right\rangle \quad et \quad [M]\left\langle\mathbf{S}^{\mathbf{i}}(N_a^-)\right\rangle=\left\langle\mathbf{S}^{\mathbf{i}}(N_a^+)\right\rangle\\&\text{où}\left\langle\mathbf{S}^{\mathbf{i}}(N)\right\rangle=\langle 0\ldots\underset{\substack{\text{noeud}\,N\\\text{direction}\,i}}{1}\ldots 0\rangle\end{aligned} \quad (26)$$

Finally, the notation in terms of vectors is:

$$P\left(\Delta \mathbf{u}_i^n\right)=\sqrt{N\|\langle\mathbf{S}\rangle.\{\Delta u_i^n\}\|^2+N\left([M]\langle\mathbf{S}\rangle.\{\Delta u_i^n\}\right)\cdot\left(\langle\mathbf{S}\rangle.\{\Delta u_i^n\}\right)}=\Delta\tau_i \quad (27)$$

At the time of the initialization of piloting, it is thus necessary to initialize besides the nodal vector $\langle\mathbf{S}\rangle$ a structure $[M]$ who locates which points belong to the same edge. While breaking up $\left\{\Delta \mathbf{u}_i^n\right\}$, one finds an equation similar to (22) who takes account of the additional term.

For modeling XFEM, the methods of selection of the solution remain strictly identical, put except for the replacement in the expressions of the criteria of standard and angle of displacement by the jump of displacement through the interface. Thus, the angular criterion amounts for example maximizing $\cos\left(\llbracket\Delta \mathbf{u}\rrbracket_{i-1},\llbracket\Delta \mathbf{u}\rrbracket_i^n\right)$.

4.5 Piloting by the increment of deformation: `DEFORMATION`

Piloting by increment of deformation consists in requiring that the increment of deformation of the step running remain close in the direction of the deformation at the beginning to the step of time, and this for at least a point of Gauss of the structure. It is required thus qualitatively that **has minimum** a point of the structure preserves the mode of deformation which it had as a preliminary (for example, traction in a given direction).

4.5.1 Case of the small deformations

For the small deformations, one can give an account of this requirement with the help of the choice of the following function of piloting:

$$P\left(\Delta \mathbf{u}_i^n\right)=\underset{g}{\mathrm{Max}}\frac{\boldsymbol{\varepsilon}_{g,i-1}}{\|\boldsymbol{\varepsilon}_{g,i-1}\|}.\Delta\boldsymbol{\varepsilon}_{g,i}=\Delta\tau_i \quad (28)$$

Where the index g sweep the points of Gauss of the structure (or only the meshs specified by the keyword `GROUP_MA` in `PILOTING`) and where the deformation in a point of Gauss results from the nodal vector from displacements via the symmetrical use of the matrices "left the gradient the functions form" $\mathbf{B}_g$ (not to be confused with the matrix of the conditions of Dirichlet):

$$\boldsymbol{\varepsilon}_{g,i-1}=\mathbf{B}_g\cdot\mathbf{u}_{i-1} \text{ and } \Delta\boldsymbol{\varepsilon}_{g,i}=\mathbf{B}_g\cdot\Delta\mathbf{u}_i^n \tag{29}$$

The control of piloting according to η_i is written then:

$$P\left(\Delta\mathbf{u}_i^n\right)=\underset{g}{\mathrm{Max}}\left(A_g^{(0)}+\eta_i\cdot A_g^{(1)}\right)=\underset{g}{\mathrm{Max}}\left(L_g\left(\eta_i\right)\right)=\Delta\tau_i \tag{30}$$

With the two terms:

$$A_g^{(0)}=\frac{\boldsymbol{\varepsilon}_{g,i-1}}{\left\|\boldsymbol{\varepsilon}_{g,i-1}\right\|}\cdot\mathbf{B}_g\left(\Delta\mathbf{u}_i^{n-1}+\delta\mathbf{u}_{\mathrm{impo},i}^n\right) \tag{31}$$

And:

$$A_g^{(1)}=\frac{\boldsymbol{\varepsilon}_{g,i-1}}{\left\|\boldsymbol{\varepsilon}_{g,i-1}\right\|}\cdot\mathbf{B}_g\left(\delta\mathbf{u}_{\mathrm{pilo},i}^n\right) \tag{32}$$

This piloting does not depend on the law of behavior, provided which it uses the tensor of the small deformations (option `DEFORMATION=' PETIT'` or `DEFORMATION=' PETIT_REAC'`).

4.5.2 Case of the great deformations

In the presence of great deformations, one can generalize the function of piloting (28) by employing deformations of Green-Lagrange (Lagrangian measurement of the deformations in the initial configuration):

$$P\left(\Delta\mathbf{u}_i^n\right)=\underset{g}{\mathrm{Max}}\frac{\mathbf{E}_{g,i-1}}{\left\|\mathbf{E}_{g,i-1}\right\|}.\Delta\mathbf{E}_{g,i} \tag{33}$$

The measurement of the deformations $\mathbf{E}$ is written according to the tensor gradient of transformation $\mathbf{F}$:

$$\mathbf{E}=\frac{1}{2}\cdot\left(\mathbf{F}^T.\mathbf{F}-\mathbf{I}\right) \text{ with } \mathbf{F}=\mathbf{I}+\nabla\mathbf{u} \tag{34}$$

However, one would not lead any more like previously to a function closely connected per pieces. To cure it, one decides to carry out a linearization of $\Delta\mathbf{E}_{g,i}$ compared to $\Delta\mathbf{u}_i^{n-1}$. P an expression similar has then to (30) with:

$$A_g^{(0)}=\frac{\mathbf{E}_{g,i-1}}{\left\|\mathbf{E}_{g,i-1}\right\|}\cdot\mathrm{sym}\left[\mathbf{F}_{g,i}^T.\nabla_g\left(\Delta\mathbf{u}_i^{n-1}+\delta\mathbf{u}_{\mathrm{impo},i}^n\right)\right] \tag{35}$$

And:

$$A_g^{(1)}=\frac{\mathbf{E}_{g,i-1}}{\left\|\mathbf{E}_{g,i-1}\right\|}\cdot\mathrm{sym}\left[\mathbf{F}_{g,i}^T.\delta\mathbf{u}_{\mathrm{pilo},i}^n\right] \tag{36}$$

Where $\nabla_g(\mathbf{u})$ indicate the gradient (not symmetrized) of displacements $\mathbf{u}$ evaluated at the point of Gauss of index g . Just like in the preceding case, this piloting does not depend on the law of behavior, provided which it uses the tensor of the great deformations.

4.5.3 Resolution of the non-linear equation of piloting

The function $P\left(\Delta\mathbf{u}_i^n\right)$ is convex and linear per pieces. She generally admits no, one or two solutions, cf appears (4.6.4-a). When she does not admit solutions, one stops the algorithm of `STAT_NON_LINE` (failure of piloting): if the user uses the automatic subdivision of the step of time, this last will then be activated.
If she admits two solutions, three opportunities are given: one chooses that of both which minimizes:

- that is to say the standard of the increment of displacement $\left\|\Delta\mathbf{u}_i^n\right\|$;
- that is to say the residue of balance $\mathbf{R}(\mathbf{u}_i,t_i)=\mathbf{L}_i^{\mathrm{int}}-\mathbf{L}_i^{\mathrm{méca}}$;
- that is to say the angle formed by $\Delta\mathbf{u}_{i-1}$ and $\Delta\mathbf{u}_i^n$;

To solve the equation (30), one proposes the algorithm presented below. It is based on the construction of encased intervals: the terminals of the last of them are the solutions of the equation and, as announced previously, one chooses that which leads to $\tilde{\mathbf{u}}(\eta)$ nearest to $\mathbf{u}_{i-1}$. This algorithm, rapid, are based on the resolution of G linear scalar equations, where G indicate the full number of points of Gauss. The algorithm can end prematurely when one of the intervals is empty, which means that the equation (30) does not admit solutions.

(1)	Initialization of the interval	$I_0=]-\infty,+\infty[$
(2)	Buckle on the points of Gauss g	
(2.0)	The slope is worthless	Si $A_g^{(1)}=0$
(2.0.1)	If $A_g^{(0)}>\Delta\tau$ failure If not one continues	
(2.1)	Root of the active linear function	η_g tel que $L_g(\eta_g)=\Delta\tau$
(2.2)	Construction of the following interval	
(2.2.1)	If the active linear function is increasing	Si $A_g^{(1)}>0 \Rightarrow I_g=I_{g\text{-}1}\cap]-\infty,\eta_g]$
(2.2.2)	If the active linear function is decreasing	Si $A_g^{(1)}<0 \Rightarrow I_g=I_{g\text{-}1}\cap[\eta_g,+\infty[$
(2.3)	To stop if the interval is empty	
(3)	The solutions are the terminals of the interval[3]	$\eta\in\mathrm{Fr}(I_G)\Rightarrow\max_g L_g(\eta)=\Delta\tau$

Algorithm of resolution of the equation refines per pieces

4.6 Piloting by the elastic prediction: `PRED_ELAS`

4.6.1 Algorithm

If piloting by the increment of deformation proves to be sufficient to follow dissipative solutions in most instabilities materials, the existence of solutions nevertheless is not proven. One prefers a method of piloting then to him based on the elastic prediction for which the existence of solutions is shown but which, on the other hand, is specific to each law of behavior (only established for the laws `ENDO_SCALAIRE`, `ENDO_FRAGILE`, `ENDO_ISOT_BETON`, `ENDO_ORTH_BETON`, `BETON_DOUBLE_DP`, `VMIS_ISOT_*` and those relating to the elements of joints [R5.06.09]). More precisely, when the law of behavior is controlled by a threshold, one defines P like the maximum on all the points of Gauss of the value of the function threshold in the case of an elastic test (elastic incremental answer of material).
Thus, let us consider that the state of material is described by the deformation $\boldsymbol{\varepsilon}$ and a set of internal variables a. Let us call respectively $\boldsymbol{\sigma}(\boldsymbol{\varepsilon},a)$ and $A(\boldsymbol{\varepsilon},a)$ constraints and thermodynamic forces associated with a. Let us suppose moreover that the laws of evolution of a are controlled by a threshold $f(A,\boldsymbol{\varepsilon},a)$:

$$f(A,\boldsymbol{\varepsilon},a)\leq 0 \text{ et } \lambda\cdot f(A,\boldsymbol{\varepsilon},a)=0 \quad (37)$$

And a function of flow $G(A,\boldsymbol{\varepsilon},a)$:

$$\dot{a}=\lambda G\cdot(A,\boldsymbol{\varepsilon},a) \quad (38)$$

Such a formulation includes most models of behavior dissipative and independent the rate loading. The function threshold is worth then for an elastic test:

$$f^{\mathrm{el}}(\boldsymbol{\varepsilon})=f\left(A(\boldsymbol{\varepsilon},a_{i-1}),\boldsymbol{\varepsilon},a_{i-1}\right) \quad (39)$$

One simplifies the problem while linearizing f^{el} compared to $\boldsymbol{\varepsilon}$ in the vicinity of the point $\boldsymbol{\varepsilon}^*$ such as $f^{\mathrm{el}}=\Delta\tau$:

3 One notes $\eta\in\mathrm{Fr}(I_G)$ the fact that η belongs at the boundaries of the interval I_G

$$f_L^{\mathrm{el}}(\boldsymbol{\varepsilon}) \underset{\mathrm{def}}{=} f^{\mathrm{el}}(\boldsymbol{\varepsilon}^*) + \left(\frac{\partial f}{\partial A}\cdot\frac{\partial A}{\partial \boldsymbol{\varepsilon}} + \frac{\partial f}{\partial \boldsymbol{\varepsilon}}\right)\Bigg|_{\boldsymbol{\varepsilon}^*}\cdot(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^*) \quad (40)$$

The choice of $\boldsymbol{\varepsilon}^*$ allows to solve the exact problem, as shown in [bib3]. It thus requires to solve a local problem non-linear for each point of Gauss, then the total algorithm is based on lines, which ensures a great effectiveness to him. There exists zero, one or two solutions with the local problem $f^{\mathrm{el}}=\Delta\tau$. In the case of absence of solution to the local problem, the total problem cannot have solution: one stops. If not (case with one or two solutions), one linearizes around each solution.

Finally, the function of control of piloting is defined like the maximum of f_L^{el} compared to all the points of Gauss g , function which depends only on $\Delta\mathbf{U}$:

$$P(\Delta\mathbf{u}_i^n) = \underset{g}{\mathrm{Max}}\, f_L^{\mathrm{el}}(\boldsymbol{\varepsilon}_{g,i-1} + \mathbf{B}_g\cdot\Delta\mathbf{u}_i^n) = \Delta\tau_i \quad (41)$$

Finally, the equation of control of piloting is written:

$$P(\Delta\mathbf{u}_i^n) = \underset{g}{\mathrm{Max}}(A_g^{(0)} + \eta_i\cdot A_g^{(1)}) = \underset{g}{\mathrm{Max}}(L_g(\eta_i)) = \Delta\tau_i \quad (42)$$

With:

$$A^{(0)} = f^{el}(\boldsymbol{\varepsilon}^*) + \left(\frac{\partial f}{\partial A}\cdot\frac{\partial A}{\partial \boldsymbol{\varepsilon}} + \frac{\partial f}{\partial \boldsymbol{\varepsilon}}\right)\Bigg|_{\boldsymbol{\varepsilon}^*}\cdot\left(\mathbf{B}_g(\mathbf{u}_{i-1} + \Delta\mathbf{u}_i^{n-1} + \delta\mathbf{u}_{\mathrm{impo},i}^n) - \boldsymbol{\varepsilon}^*\right) \quad (43)$$

And:

$$A^{(1)} = \left(\frac{\partial f}{\partial A}\cdot\frac{\partial A}{\partial \boldsymbol{\varepsilon}} + \frac{\partial f}{\partial \boldsymbol{\varepsilon}}\right)\Bigg|_{\boldsymbol{\varepsilon}^*}\cdot(\mathbf{B}_g\cdot\delta\mathbf{u}_{\mathrm{pilo},i}^n) \quad (44)$$

4.6.2 Expressions according to the laws of behavior

One is thus brought back to a problem identical to that of piloting by the increment of the deformation. One of course employs the same algorithms of resolution as those presented in the preceding paragraph (see §14).

The characteristics of the equation of piloting depend on the law of behavior because the expression of the elastic function threshold f^{el} a non-linear fact of the case of behavior is itself:

- For the elements with internal discontinuity PLAN_ELDI and AXIS_ELDI, the law of behavior is of type CZM_EXP and the expression of the law of piloting will be in documentation [R7.02.14]. The elastic function threshold is written $f_{el}(\sigma_n,\sigma_t)=\sqrt{<\sigma_n>_+^2+\sigma_t^2}-R(\kappa)$;
- For the elements of joint *_JOINT, the laws of behavior are regularized laws CZM CZM_EXP_REG and CZM_LIN_REG (see [R7.02.11]), and for the elements of the type interfaces *_INTERFACE, the laws of behavior are laws CZM not-regularized like CZM_OUV_MIX and CZM_TAC_MIX. L'expression of the law of piloting will be in documentation [R7.02.11]. The elastic function threshold is written $f_{el} \simeq \underset{pt\,gauss}{max}\left\{\frac{\|\delta^0+\eta\,\delta^1\|-\kappa^-}{G_c/\sigma_c+\kappa^-}\right\}=\frac{\Delta\tau}{C}$;
- For the law ENDO_ISOT_BETON ([R7.01.04]), there is a characteristic. Instead of seeking a parameter of piloting η who makes leave the criterion a value $\Delta\tau$ with the damage resulting from the step of previous time, one seeks a parameter η who brings back for us on the criterion with a damage increased by $\Delta\tau$. The elastic function threshold is thus written $\tilde{f}^{el}(\eta,d^-)=\Delta\tau \;\Rightarrow\; \tilde{f}^{el}(\eta,d^-+\Delta\tau)=0$;
- For the law ENDO_ORTH_BETON, to see [R7.01.09];
- For the other laws of damage of the elastic type like ENDO_SCALAIRE or ENDO_FRAGILE (with local formulation or with gradients), just like for the law ENDO_ISOT_BETON, the elastic function threshold is selected so as to seek a parameter of piloting controls by the value of damage and not by the exit of the criterion. The elastic function threshold is written:

$$\tilde{f}^{\text{el}}(\eta)=\frac{1}{2}\left(\boldsymbol{e_0}+\eta\ \boldsymbol{e_1}\right)\cdot\boldsymbol{E}\cdot\left(\boldsymbol{e_0}+\eta\ \boldsymbol{e_1}\right)-s$$

$$\begin{cases} \text{loi locale} & \boldsymbol{e_0}=\boldsymbol{\epsilon_0} \quad e_1=\boldsymbol{\epsilon_1} \quad s=k\left(d^-\right) \\ \text{gradient d'endommagement} & \boldsymbol{e_0}=\boldsymbol{\epsilon_0} \quad \boldsymbol{e_1}=\boldsymbol{\epsilon_1} \quad s=k\left(d^-\right)-c\ \Delta\ d^- \\ \text{déformation régularisée} & \boldsymbol{e_0}=\bar{\boldsymbol{\epsilon}}_0 \quad \boldsymbol{e_1}=\bar{\boldsymbol{\epsilon}}_1 \quad s=k\left(d^-\right) \end{cases} \quad (45)$$

4.6.3 Illustration of the algorithm of maximization

In order to illustrate our matter and in particular to include why such a choice of $\boldsymbol{\varepsilon}^*$ allows to solve the exact problem, one takes the example of a cohesive law of behavior `CZM_EXP_REG` or `CZM_LIN_REG` (cf [R7.02.11]) implemented on an element of joint (cf [R3.06.09]). The jump of displacement $\boldsymbol{\delta}$ is the primal variable (noted generically $\boldsymbol{\epsilon}$ in what precedes), and κ the variable interns (noted generically A). The function threshold is written then:

$$f^{èl}(\boldsymbol{\delta})=\frac{\|\boldsymbol{\delta}\|_{èq}-\kappa_{i-1}}{\kappa_{REF}} \quad \text{avec} \quad \kappa_{i-1}=\max_{v\in[0,t_{i-1}]}\|\boldsymbol{\delta}(v)\|_{èq} \quad ; \quad \|\boldsymbol{\delta}\|_{èq}=\sqrt{\langle\delta_n\rangle_+^2+\delta_\tau^2} \quad (46)$$

Without detailing the demonstration, one can show that $\|\boldsymbol{\delta}\|_{èq}$ check a triangular inequality. This enables us to prove the convexity of the function:

$$f^{èl}(\eta)=f^{èl}\left(\boldsymbol{\delta}(\eta)\right)=\frac{\|\boldsymbol{\delta}_{i-1}+\Delta\,\boldsymbol{\delta}_i^n+d\,\boldsymbol{\delta}^{cte}+\eta\,d\,\boldsymbol{\delta}^{pilo}\|_{èq}-\kappa_{i-1}}{\kappa_{REF}} \quad (47)$$

We traced in figure 4.6.3-a a chart **qualitative** functions $f^{èl}$ for the various points of Gauss of the elements of joint, as well as the solutions of the exact problem of maximization total on the whole of the points of Gauss:

$$\max_g f_g^{èl}(\eta)\overset{dèf}{=}\max_g f^{èl}\left(\boldsymbol{\delta}_g(\eta)\right)=\Delta\tau_i$$

It appears whereas the solutions η this exact problem are identical to the solutions obtained with the algorithm of resolution of the equation closely connected per pieces, i.e. the solutions of the equation (42) $\max_g A_1^{(g)}\eta+A_0^{(g)}=\Delta\tau_i$, with in our example:

$A_1^{(g)}=\frac{\partial f_g^{èl}}{\partial\eta}(\eta_g^*)$; $A_0^{(g)}=\Delta\tau-A_1^{(g)}\eta_g^*$ where η_g^* is solution of the local equation $f_g^{èl}(\eta)=\Delta\tau_i$

We once more return the reader to [3] for a rigorous demonstration.

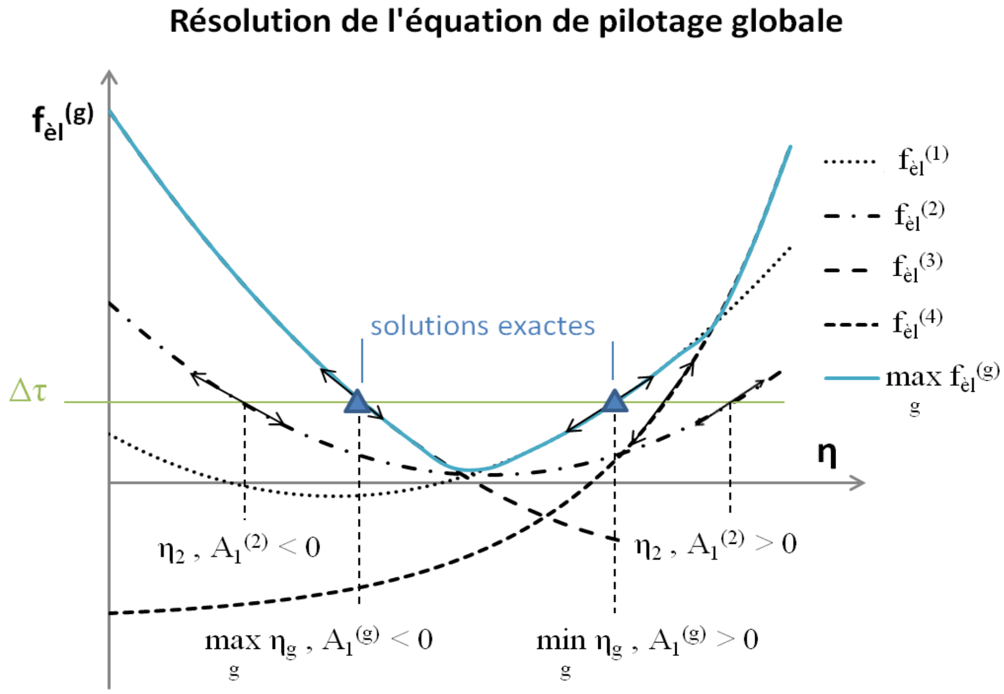


Figure 4.6.3-a: Resolution of the equation of piloting, qualitative chart

4.6.4 Choice of solution

If the total problem has two solutions, algorithms of choices described in 4.3 are re-used. In general, the selection by residue is retained because of its robustness. However, the fact of choosing the solution which minimizes the residue does not guarantee us the convergence of the method of Newton. Certain cases could be displayed for which the solution of maximum initial residue converges while it is not the case of the solution of minimal initial residue.

In general pour calculations controlled the step of times filled more the role of loading, but just a parameter of follow-up of the solution. Although the mechanical solutions are identical, one can thus obtain a significant difference with the same step in piloting for the problems with important instabilities, by changing for example the object computer. What is illustrated on the figure 4.6.4-a, or at the time of the snap-back in the total answer force-displacement one observes the shift of moment of calculation between the two curves.

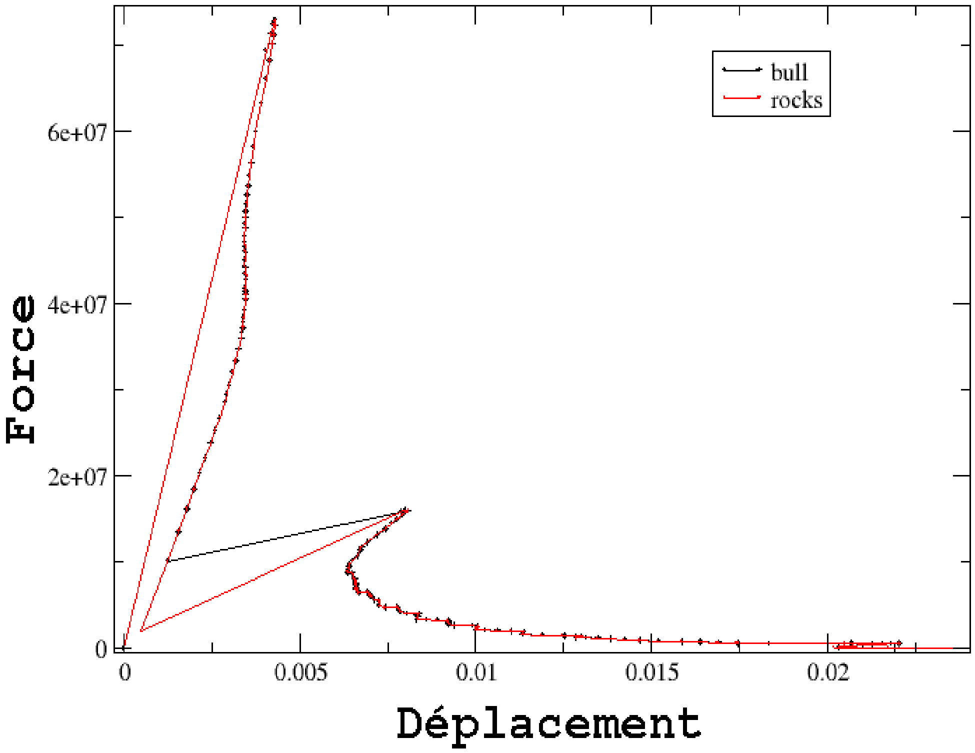


Figure 4.6.4-a: Illustration of shift of the moment of piloting on a curved force-displacement for a strongly unstable problem according to the precision machine

5 Bibliography

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6 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
7	E.Lorentz EDFR & D /AMA	Initial text
8.4	V.Godard, P.Badel, E.Lorentz EDF-R&D/AMA	
10.2	M.Abbas EDF-R&D/AMA	Setting in conformity of the notations with [R5.03.01], addition of an explanatory introduction of the utility of piloting
10.5	P.Massin, G. Ferte EDF-R&D/AMA	Introduction of piloting for XFEM
10.5	K.Kazymyrenko EDF-R&D/AMA	Correction of the card of anomaly Doc. 17120