Law of behavior of HOEK_BROWN modified

Summary

This document describes a law of behavior for the rocks. The criterion of plasticization is a parabolic criterion of type Hoek-Brown. He is written in principal constraints. The evolution of the plastic deformations of “nonassociated” type, is formulated starting from a criterion of Drucker-Prager whose angle of friction evolves with plasticization.

This law is usable in pure mechanics as in coupled modeling thermo hydro mechanics.

In modeling thermo-hydro-mechanics, it can be used in effective constraints or total constraints:

- In the first case, in fact the effective constraints are subjugated to respect the criterion of Hoek and Brown. The deformation figures are calculated by using the surface of Drucker and the angle of friction within the space of effective constraints
- In the second case, in fact the total constraints are subjugated to respect the criterion of Hoek and Brown. The deformation figures are calculated by using the surface of Drucker and the angle of friction within the space of total constraints

Contents

1 Introduction

This note presents a mechanical model of behavior for the rocks. To represent the rock mechanics behavior the user of Code_Aster can use the law of Drucker-Prager `DRUCK_PRAGER`, the law of `LAIGLE` or the law `HOEK_BROWN` presented here. The law of Drucker-Prager is simplest and furthest away from the real behavior of the rocks. The law of Laigle is most faithful to the physics of the phenomena. Thus the law presented in this note is intermediate between these two laws, in term of complexity as in term of representation of reality. It uses rather classical formulations in the medium of géomechanics.

This law is usable in modelings of mechanics alone or modelings of the type THM.
In modeling thermo-hydro-mechanics, it can be used in effective constraints or total constraints:

- In the first case, in fact the effective constraints are subjugated to respect the criterion of Hoek and Brown. The deformation figures are calculated by using the surface of Drucker and the angle of friction within the space of effective constraints

- In the second case, in fact the total constraints are subjugated to respect the criterion of Hoek and Brown. The deformation figures are calculated by using the surface of Drucker and the angle of friction within the space of total constraints

One will find in the reference [1] elements useful for the comprehension of this law
The formulation is of standard nonassociated plasticity:
- The field of elasticity is defined by a criterion of the type Hoek-Brown, whose parameters evolve with the parameter of work hardening
- The parameter of work hardening is a combination of the plastic deformation of shearing and voluminal plastic deformation
- The plastic deformations derive from a criterion of Drucker-Prager whose angle of friction evolves with plasticization.

1.1 Characteristics of the model

The model simulates the mechanical behavior in the short run rocks in 4 phases, described in the reference [1]:
1) Elastic phase characterized by a Young modulus and a constant Poisson's ratio
2) Elastoplastic phase with positive work hardening which simulates the initiation of a form of damage and its progression towards the rupture of the rock. This phase is modelled by a criterion of plasticity of the type Hoek-Brown. This criterion evolves according to the major unrecoverable deformation. The evolution of the unrecoverable deformation is determined by a plastic potential of flow expressed by a function of the Drucker-Prager type.
3) Elastoplastic phase with negative work hardening which represents the behavior post-rupture of the rocks. The criterion of rupture is of the type Hoek-Brown. The deformation is determined by a potential of plastic flow nonassociated with Drucker-Prager type.
4) Phase of residual resistance characterized by a function of the type Hoek-Brown modified.

2 Notations

2.1 General information

The deformations are told positive in extension and the constraints are positive for states of traction.

Notation	Description
$I_1 = tr(\boldsymbol{\sigma})$	First invariant of the constraints
$\mathbf{s} = \boldsymbol{\sigma} - \frac{tr(\boldsymbol{\sigma})}{3}\mathbf{I}$	Diverter of the constraints

$s_{II}=\sqrt{\mathbf{s}.\mathbf{s}}$	Second invariant of the tensor of the constraints déviatoires
$\mathbf{e}=\boldsymbol{\varepsilon}-\frac{tr(\boldsymbol{\varepsilon})}{3}\mathbf{I}$	Diverter of the deformations
$\boldsymbol{\varepsilon}_v=tr(\boldsymbol{\varepsilon})$	Trace of the deformations: voluminal deformation
$\boldsymbol{\varepsilon}^p$	Tensor of the plastic deformations
$\boldsymbol{\varepsilon}_v^p=tr(\boldsymbol{\varepsilon}^p)$	Plastic variation of volume
$\delta\gamma^p=\sqrt{\frac{2}{3}d\,\mathbf{e}^p:d\,\mathbf{e}^p}$	Cumulated plastic deformation of shearing
σ_1	Major principal constraint
σ_3	Minor principal constraint ($\sigma_1<\sigma_2<\sigma_3$)
$\mathbf{H}$	Matrix of Hooke
μ	Coefficient of Lamé

2.2 Parameters of the model

Notation	Description
γ	Parameter of work hardening (defined in the paragraph 3.2.3)
S	Represent the state of damage and fracturing of the rock
m	Parameter of smoothing of the model
σ_c	Resistance of solid rock without any damage
γ^{rup}	Parameter of work hardening corresponding to the rupture of material
γ^{res}	Parameter of work hardening corresponding to the beginning of residual resistance
$(S\sigma_c^2)^{rup}$	Value of the product $S\sigma_c^2$ with the rupture reached in γ^{rup}
$(S\sigma_c^2)^{end}$	Value of the product $S\sigma_c^2$ with the initiation of damage $(\gamma=0)$
$(m\sigma_c)^{rup}$	Value of the product $m\sigma_c$ with the rupture reached in γ^{rup}
$(m\sigma_c)^{end}$	Value of the product $m\sigma_c$ with the initiation of damage $(\gamma=0)$
E	Young modulus
ν	Poisson's ratio
β	Characterize residual resistance
ϕ^{end}	Angle of friction to the initiation of damage $(\gamma=0)$: optional parameter taken no by default
ϕ^{rup}	Angle of friction to the rupture reached in γ^{rup}
ϕ^{res}	Angle of friction to the residual resistance reached in γ^{res}
α	Parameter of the model characterizing the behavior post-rupture of material

3 Continuous model

We describe here the model independently owing to the fact that it is used in mechanics alone, or in calculations hydro-mechanical in total or effective constraint. Thus the notation σ used in the following paragraphs will have to be interpreted according to use.

3.1 Elastic behavior

The elastic behavior is given by a linear law. The two parameters characterizing this behavior are the module d'elasticity E and the Poisson's ratio ν .

3.2 Plastic behavior

The adopted formulation is resulting from the document [1].

3.2.1 Surface of load

$$F(\boldsymbol{\sigma},\gamma)=(\sigma_3-\sigma_1)-\sqrt{-\sigma_3 . m\sigma_c+S(\gamma)\sigma_c^2}-b(\gamma).\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right)$$

where:

- γ is the parameter of work hardening (defined in the paragraph 3.2.3)
- S characterize the state of damage and fracturing of the rock
- m is a parameter of smoothing of the model
- σ_c is the resistance of solid rock without any damage $\sigma_c>0$
- b is a function of the variable of work hardening to parabolic evolution which characterizes the behavior post-rupture
- σ_3^{b-d} is the intersection between the line $\sigma_1=\alpha\sigma_3$ (α being a parameter of the model) and the criterion at the instant of the failure ($\gamma=\gamma^{rup}$). $\sigma_3^{b-d}>0$
- σ_1 and σ_3 are the principal constraints major and minor: $\sigma_1<\sigma_2<\sigma_3$

3.2.2 Plastic potential of flow

The plastic potential of flow is given by a function resulting from the criterion of Drucker-Prager:

$G(\boldsymbol{\sigma},\gamma)=\eta(\gamma)I_1+\sqrt{\frac{3}{2}}s_{II}=\eta(\gamma)I_1+\sigma_{eq}$ with $\eta(\gamma)=\frac{2\sin\phi(\gamma)}{3+\sin\phi(\gamma)}$, $\sigma_{eq}=\sqrt{\frac{3}{2}}s_{II}$ and $\phi(\gamma)$ the angle of friction are equivalent.

3.2.3 Parameter of work hardening γ

The parameter of work hardening γ that one considers takes the following values:

$\gamma=0$ with the initiation of damage

$\gamma=\gamma^{rup}$ with the rupture

$\gamma=\gamma^{res}$ at the beginning of residual resistance

It is defined while being placed in triaxial compression by: $\gamma=\varepsilon_1^p$, and one can show whereas $\gamma^p=\left|\frac{\varepsilon_v^p}{3}-\gamma\right|$. One thus has $\gamma=\frac{\varepsilon_v^p}{3}\pm\gamma^p$ who must be positive.

3.2.4 Other parameters

Parameters $S\sigma_c^2, m\sigma_c, \phi, b$ vary in the following way according to the parameter of work hardening γ :

1) $$(m\sigma_c)(\gamma)=\begin{cases}\gamma\dfrac{(m\sigma_c)^{rup}-(m\sigma_c)^{end}}{\gamma^{rup}}+(m\sigma_c)^{end}=p_{m\sigma}\gamma+(m\sigma_c)^{end} & si \quad \gamma\leq\gamma^{rup}\\ (m\sigma_c)^{rup} & si \quad \gamma\geq\gamma^{rup}\end{cases}$$

2) $$(S\sigma_c^2)(\gamma)=\begin{cases}\gamma\dfrac{(S\sigma_c^2)^{rup}-(S\sigma_c^2)^{end}}{\gamma^{rup}}+(S\sigma_c^2)^{end}=p_{S\sigma^2}\gamma+(S\sigma_c^2)^{end} & si\quad \gamma\leq\gamma^{rup}\\ (S\sigma_c^2)^{rup} & si\quad \gamma\geq\gamma^{rup}\end{cases}$$

3) $$\phi(\gamma)=\begin{cases}\dfrac{\phi^{rup}-\varphi^{end}}{\gamma^{rup}}\gamma+\phi^{end} & \text{si } \gamma\leq\gamma^{rup}\\ \dfrac{\phi^{res}-\phi^{rup}}{\gamma^{res}-\gamma^{rup}}\gamma+\dfrac{\phi^{rup}\gamma^{res}-\phi^{res}\gamma^{rup}}{\gamma^{res}-\gamma^{rup}} & \text{si } \gamma^{rup}\leq\gamma\leq\gamma^{res}\\ \phi^{res} & \text{sinon}\end{cases}$$

4) $$b(\gamma)=\begin{cases}0 & \text{si } \gamma\leq\gamma^{rup}\\ a\gamma^2+d\gamma+c & \text{si } \gamma^{rup}\leq\gamma\leq\gamma^{res}\\ b^{res} & \text{si } \gamma\geq\gamma^{res}\end{cases}$$ where $a=-\dfrac{b^{res}}{(\gamma^{rup}-\gamma^{res})^2}$,

$$d=\frac{2b^{res}\gamma^{res}}{(\gamma^{rup}-\gamma^{res})^2}\text{ , } c=\frac{b^{res}\gamma^{rup}(\gamma^{rup}-2\gamma^{res})}{(\gamma^{rup}-\gamma^{res})^2}$$ and $b^{res}=\beta-\sqrt{(S\sigma_c^2)^{rup}}$

5) $$\sigma_3^{b-d}=\frac{-(m\sigma_c)^{rup}-\sqrt{((m\sigma_c)^{rup})^2+4(1-\alpha)^2(S\sigma_c^2)^{rup}}}{2(1-\alpha)^2}$$

6) coefficients $\alpha,\beta,(S\sigma_c^2)^{rup},(S\sigma_c^2)^{end},(m\sigma_c)^{rup},(m\sigma_c)^{end},\phi^{end},\phi^{rup},\phi^{res}$ are given.

4 Incremental form

One places oneself here within the framework of finished increases. The index – indicates a component at the beginning of step of loading and the absence of index a component at the end of the step of loading. The operator Δ indicate the increase in a component.

In pure mechanics or when the law is used in modeling THM in effective constraints, the equations translating the elastic behavior are written:

$$\mathbf{s}=\mathbf{s}^-+2\mu(\Delta\mathbf{e}-\Delta\mathbf{e}^p)=\mathbf{s}^e-2\mu\Delta\mathbf{e}^p\quad où\quad \mathbf{s}^e=\mathbf{s}^-+2\mu\Delta\mathbf{e}$$
$$I_1=I_{1^-}+3K(\Delta\varepsilon_v-\Delta\varepsilon_v^p)=I_1^e-3K\Delta\varepsilon_v^p\quad où\quad I_1^e=I_1^-+3K\Delta\varepsilon_v$$

When the law is used in modeling THM in total constraints, the tensor of the constraints and the equations translating the elastic behavior are written as follows:

$$\boldsymbol{\sigma}=\mathbf{H}(\varepsilon-\varepsilon^p)+\sigma_p\mathbf{I}$$
$$\boldsymbol{\sigma}^e=\boldsymbol{\sigma}^-+\boldsymbol{H}\Delta\varepsilon+\Delta\sigma_p\mathbf{I}=\sigma'^-+\boldsymbol{H}\Delta\varepsilon+\sigma_p I$$
$$\mathbf{s}=\mathbf{s}^e-2\mu\boldsymbol{\Delta}\mathbf{e}^p\quad \text{où}\quad \mathbf{s}^e=\mathbf{s}^-+2\mu\boldsymbol{\Delta}\mathbf{e}$$
$$I_1=I_1^e-3K\Delta\varepsilon_v^p\quad \text{où}\quad I_1^e=I_1^-+3K\Delta\varepsilon_v+3\Delta\sigma_p$$
$$\Delta\sigma_p=-b\left(S\Delta p_{lq}+(1-S)\Delta p_{gz}\right)=b\left(S\Delta p_c-\Delta p_{gz}\right)$$

In addition, the rule of flow is written:

$$d\varepsilon_{ij}^p=d\lambda.\frac{\partial G}{\partial\sigma_{ij}}(\sigma,\gamma)=d\lambda\left(\eta(\gamma)\delta_{ij}+\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}\right)$$

Like $\mathbf{e}^p=\boldsymbol{\varepsilon}^p-\frac{tr(\boldsymbol{\varepsilon}^p)}{3}\mathbf{I}$, $\Delta\boldsymbol{\varepsilon}^p=\Delta\mathbf{e}^p+\frac{\Delta\varepsilon_v^p}{3}\mathbf{I}$. One from of deduced that $\begin{cases}\Delta\mathbf{e}^p=\frac{3}{2}\frac{\mathbf{s}}{\sigma_{eq}}\Delta\lambda\\ \Delta\varepsilon_v^p=3\eta(\gamma)\Delta\lambda\end{cases}$, and consequently, like $\sigma_{eq}^e\mathbf{s}=\sigma_{eq}\mathbf{s}^e$, the following relations are identical for the two aspects of the law:

$$\begin{cases}\mathbf{s}=\mathbf{s}^e-3\mu\frac{\mathbf{s}}{\sigma_{eq}}\Delta\lambda=\mathbf{s}^e\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)\\ I_1=I_1^e-9K\eta(\gamma)\Delta\lambda\\ \sigma_{eq}=\sigma_{eq}^e-3\mu\Delta\lambda\end{cases}$$

Lastly, $\Delta\gamma^p=\sqrt{\frac{2}{3}\Delta\mathbf{e}^p:\Delta\mathbf{e}^p}=\sqrt{\frac{2}{3}\Delta\lambda^2\left(\frac{3}{2}\frac{\mathbf{s}}{\sigma_{eq}}\right):\left(\frac{3}{2}\frac{\mathbf{s}}{\sigma_{eq}}\right)}=\Delta\lambda$. Thus, by taking again the result got in the paragraph 3.2.3, it is seen that $\Delta\gamma=\Delta\lambda[\eta(\gamma)\pm1]$ and like $\Delta\gamma,\Delta\lambda\geq0$, one necessarily has $\boxed{\Delta\gamma=\Delta\lambda[\eta(\gamma)+1]\geq0.}$

<u>Notice</u> : The calculation of $\Delta\lambda$ is the same one in all the formulations.

4.1 Calculation of $\Delta\lambda$

One notes $\mathbf{P}$ the matrix of passage such as:

$$\tilde{\mathbf{P}}.\mathbf{s}^e.\mathbf{P}=\bar{\mathbf{s}}^e \quad \text{avec}\ \bar{\mathbf{s}}^e=diag(s_1^e,s_2^e,s_3^e)\text{ , or}$$

$$\tilde{\mathbf{P}}.\boldsymbol{\sigma}^e.\mathbf{P}=\bar{\boldsymbol{\sigma}}^e \quad \text{avec}\ \bar{\boldsymbol{\sigma}}^e=diag(\sigma_1^e,\sigma_2^e,\sigma_3^e)\ \text{ et }\ \sigma_i^e=s_i^e+\frac{I_1^e}{3}$$

Then:

$$\begin{aligned}\tilde{\mathbf{P}}.\boldsymbol{\sigma}.\mathbf{P}&=\tilde{\mathbf{P}}.\left(\mathbf{s}+\frac{I_1}{3}\mathbf{I}\right).\mathbf{P}=\tilde{\mathbf{P}}.\left(\mathbf{s}^e\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)+\frac{1}{3}\left(I_1^e-9K\eta(\gamma)\Delta\lambda\right)\mathbf{I}\right).\mathbf{P}\\&=\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)\bar{\mathbf{s}}^e+\frac{1}{3}\left(I_1^e-9K\eta(\gamma)\Delta\lambda\right)\mathbf{I}\\&=diag\left(\sigma_1,\sigma_2,\sigma_3\right)\end{aligned}$$

$$\text{avec}\ \sigma_i=\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)s_i^e+\frac{1}{3}\left(I_1^e-9K\eta(\gamma)\Delta\lambda\right)=\sigma_i^e-3\Delta\lambda\left(\frac{\mu s_i^e}{\sigma_{eq}^e}+K\eta(\gamma)\right)$$

Thus, if $\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)>0$, i.e. $s_{II}>0$, them σ_i are ordered like s_i^e . If $\left(1-3\mu\frac{\Delta\lambda}{\sigma_{eq}^e}\right)=0$, i.e. $s_{II}=0$, them σ_i are all equal to $\frac{1}{3}\left(I_1^e-9K\eta\Delta\lambda\right)=\frac{1}{3}\left(I_1^e-3K\eta\frac{\sigma_{eq}^e}{\mu}\right)$. One can thus write the criterion F according to $\Delta\lambda$ ou $\Delta\gamma$ only, them s_i^e being given and being ordered, while taking:

$$\sigma_3 - \sigma_1 = \left(1 - 3\mu \frac{\Delta\lambda}{\sigma_{eq}^e}\right)(s_3^e - s_1^e)$$

$$\sigma_3 = s_3^e\left(1 - 3\mu \frac{\Delta\lambda}{\sigma_{eq}^e}\right) + \frac{1}{3}\left(I_1^e - 9K\eta(\gamma)\Delta\lambda\right)$$

A nonlinear function then is obtained $F(\Delta\lambda)$ solved by an algorithm of Newton, given by:

$$F(\Delta\gamma) = 0 = (s_3^e - s_1^e)\left[1 - \frac{3\mu}{\sigma_{eq}^e} h(\Delta\gamma)\Delta\gamma\right] - b(\Delta\gamma)\left[1 - \frac{1}{\sigma_3^{b-d}}\left(s_3^e + \frac{I_1^e}{3} - g(\Delta\gamma)\Delta\gamma\right)\right] - \left(S(\Delta\gamma)\sigma_c^2(\Delta\gamma) - m(\Delta\gamma)\sigma_c(\Delta\gamma)\left[s_3^e + \frac{I_1^e}{3} - g(\Delta\gamma)\Delta\gamma\right]\right)^{\frac{1}{2}} \quad \textbf{Eq 3.1}$$

where one noted:

¿

$$h(\Delta\gamma) = \frac{1}{\eta + 1} = \frac{3 + \sin\phi(\gamma^- + \Delta\gamma)}{3(1 + \sin\phi(\gamma^- + \Delta\gamma))} \quad \text{et} \quad g(\Delta\gamma) = \frac{3 + \sin\phi(\gamma^- + \Delta\gamma)}{3(1 + \sin\phi(\gamma^- + \Delta\gamma))}\left(\frac{6K\sin\phi(\gamma^- + \Delta\gamma)}{3 + \sin\phi(\gamma^- + \Delta\gamma)} + \frac{3\mu s_3^e}{\sigma_{eq}^e}\right)$$

In practice, one will apply the algorithm of Newton to the function

$$\bar{F}(\Delta\gamma) = \left[(\sigma_3 - \sigma_1) - b\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)\right]^2 - \left(S\sigma_c^2 - m\sigma_c\sigma_3\right)$$

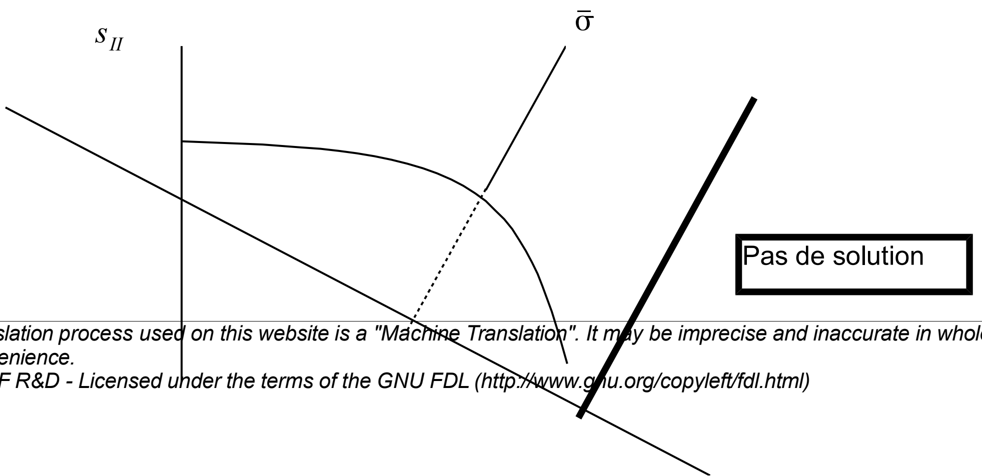
$$= \left[(s_3^e - s_1^e)\left[1 - \frac{3\mu}{\sigma_{eq}^e} h(\Delta\gamma)\Delta\gamma\right] - b(\Delta\gamma)\left[1 - \frac{1}{\sigma_3^{b-d}}\left(s_3^e + \frac{I_1^e}{3} - g(\Delta\gamma)\Delta\gamma\right)\right]\right]^2$$

$$- S(\Delta\gamma)\sigma_c^2(\Delta\gamma) + m(\Delta\gamma)\sigma_c(\Delta\gamma)\left[s_3^e + \frac{I_1^e}{3} - g(\Delta\gamma)\Delta\gamma\right]$$

in order to free itself from the difficulties related to the sign of the element under the root at the time of the iterations. The derivative of $\bar{F}$ compared to $\Delta\gamma$ is given in Appendix 2.

4.2 Existence of the solution

The principle of the analytical resolution consists in determining the point $(I_1, \mathbf{s})$ like the projection of the point $(I_1^e, \mathbf{s}^e)$ on the surface of load compared to the plastic potential of flow:



$\bar{\sigma}$

However, the solution must observe the condition $s_{II}>0$, i.e. $F(\bar{\sigma},\gamma)=(\bar{\sigma}_3-\bar{\sigma}_1)-\sqrt{-\bar{\sigma}_3.m\,\sigma_c+S\,\sigma_c^2}-b.\left(1-\frac{\bar{\sigma}_3}{\sigma_3^{b-d}}\right)$: it is thus seen that there exists a zone in which the problem does not admit a solution, which corresponds to $\frac{\partial F}{\partial\bar{\sigma}_i}=\delta_{i3}$-.

5 Calculation of the derivative

5.1 Derived from the criterion compared to the constraints

One a: $\frac{\partial F}{\partial\boldsymbol{\sigma}}=\frac{\partial F}{\partial\bar{\boldsymbol{\sigma}}}\frac{\partial\bar{\boldsymbol{\sigma}}}{\partial\boldsymbol{\sigma}}$ where $\bar{\boldsymbol{\sigma}}$ is the tensor of the constraints expressed in the base of the clean vectors. If the eigenvalues are ordered in $\bar{\boldsymbol{\sigma}}$, the criterion F will be written:

$$P(\sigma)=\begin{pmatrix}X_1(\sigma) & X_2(\sigma) & X_3(\sigma)\end{pmatrix}\quad F(\bar{\boldsymbol{\sigma}},\gamma)=(\bar{\boldsymbol{\sigma}}_3-\bar{\boldsymbol{\sigma}}_1)-\sqrt{-\bar{\boldsymbol{\sigma}}_3.m\,\boldsymbol{\sigma}_c+S\,\boldsymbol{\sigma}_c^2}-b.\left(1-\frac{\bar{\boldsymbol{\sigma}}_3}{\boldsymbol{\sigma}_3^{\bar{b}-d}}\right)$$

and $X_2(\sigma)\neq X_3(\sigma)$ $\frac{\partial F}{\partial\bar{\boldsymbol{\sigma}}_i}=\delta_{i3}-\delta_{i1}+\frac{1}{2}\delta_{i3}\boldsymbol{\sigma}_c m[-\bar{\boldsymbol{\sigma}}_3.m\,\boldsymbol{\sigma}_c+S\,\boldsymbol{\sigma}_c^2]^{\frac{1}{2}}+b\frac{\delta_{i3}}{\boldsymbol{\sigma}_3^{b-d}}$.

5.2 Derived from the tensor of the constraints compared to the principal constraints

One can show (see Appendix 1) that:

$$\left|\begin{array}{l}\text{Si } \tilde{\boldsymbol{P}}(\sigma).\sigma.P(\sigma)=\bar{\sigma}\\ \quad\text{où } P(\sigma)=\text{matrice de passage (matrice des vecteurs propres)}\\ \quad\text{et } \bar{\boldsymbol{\sigma}}=\text{ matrice diagonale des valeurs propres de } \sigma\\ \text{alors } \frac{\partial\bar{\sigma}_k}{\partial\sigma_{ij}}=P_{ik}P_{jk}\text{ (sans sommation sur les indices)}\end{array}\right.$$

5.2.1 Typical case of multiple eigenvalues

In the typical case where several of the principal constraints are equal, for example $\sigma_2=\sigma_3$, the preceding result will apply to the fields $\sigma_2<\sigma_3$ and $\sigma_2>\sigma_3$. One will thus have, in the first field, $\mathbf{P}(\boldsymbol{\sigma})=\begin{pmatrix}\mathbf{X}_1(\boldsymbol{\sigma}) & \mathbf{X}_2(\boldsymbol{\sigma}) & \mathbf{X}_3(\boldsymbol{\sigma})\end{pmatrix}$ where $\mathbf{X}_2\neq\mathbf{X}_3$ and, in the second field, $\mathbf{P}(\boldsymbol{\sigma})=\begin{pmatrix}\bar{\mathbf{X}}_1(\boldsymbol{\sigma}) & \bar{\mathbf{X}}_2(\boldsymbol{\sigma}) & \bar{\mathbf{X}}_3(\boldsymbol{\sigma})\end{pmatrix}$. Thus, when $\sigma_2-\sigma_3\rightarrow 0^-$ (resp. $\sigma_2-\sigma_3\rightarrow 0^+$), the matrix of passage will tend towards $\mathbf{P}^-=\begin{pmatrix}\mathbf{X}_1 & \mathbf{X}_2 & \mathbf{X}_3\end{pmatrix}$ (resp. towards $\mathbf{P}^+=\begin{pmatrix}\mathbf{X}_1 & \mathbf{X}_3 & \mathbf{X}_2\end{pmatrix}$) with $\mathbf{X}_2\neq\mathbf{X}_3$, vectors $(\mathbf{X}_2,\mathbf{X}_3)$ defining the clean subspace associated with $\sigma_2=\sigma_3$. It is thus seen that the tensor $\frac{\partial\bar{\boldsymbol{\sigma}}}{\partial\boldsymbol{\sigma}}$ is not defined in a single way in this point.

Moreover, the vector $\frac{\partial \sigma_3}{\partial \sigma_{ij}} = \frac{\partial \sigma_2}{\partial \sigma_{ij}}$ is defined only from one of the two vectors $\mathbf{X}_2$ or $\mathbf{X}_3$ (it is equal to $\mathbf{P}_{i3}\mathbf{P}_{j3}$ or $\mathbf{P}_{i2}\mathbf{P}_{j2}$), and thus makes some only with only one of both derivative corresponds directional. This remark applies in the same way to $\frac{\partial s_3^e}{\partial \sigma_{ij}}$ for the calculation of $\frac{\partial \sigma}{\partial \varepsilon}$ $\frac{\partial \lambda}{\partial \varepsilon_{ij}}$ in the coherent tangent matrix (see paragraph 6).

5.3 Derived from the criterion compared to the variable of work hardening

$$\frac{\partial F}{\partial \gamma} = -\frac{1}{2}\left(-\frac{\partial (m\sigma_c)}{\partial \gamma}\sigma_3 + \frac{\partial (S\sigma_c^2)}{\partial \gamma}\right)\left[-\sigma_3 . m\sigma_c + S\sigma_c^2\right]^{-\frac{1}{2}} - \frac{\partial b}{\partial \gamma}.\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)$$

$$= \begin{cases} -\frac{1}{2}\left(-p_{m\sigma}\sigma_3 + p_{S\sigma^2}\right)\left[-\sigma_3 . m\sigma_c + S\sigma_c^2\right]^{-\frac{1}{2}} & \text{si } \gamma < \gamma^{rup} \\ -(2a\gamma + d).\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right) & \text{si } \gamma^{rup} < \gamma <= \gamma^{res} \\ 0 & \text{si } \gamma >= \gamma^{res} \end{cases} \qquad \frac{\partial \sigma'}{\partial \varepsilon}$$

5.4 Derived from the parameters compared to the variable of work hardening

[1] $$\frac{\partial (m\sigma_c)}{\partial \gamma}(\gamma) = \begin{cases} \frac{(m\sigma_c)^{rup} - (m\sigma_c)^{end}}{\gamma^{rup}} = p_{m\sigma} & si \quad \gamma < \gamma^{rup} \\ 0 & si \quad \gamma > \gamma^{rup} \end{cases}$$

[2] $$\frac{\partial (S\sigma_c^2)}{\partial \gamma}(\gamma) = \begin{cases} \frac{(S\sigma_c^2)^{rup} - (S\sigma_c^2)^{end}}{\gamma^{rup}} = p_{S\sigma^2} & si \quad \gamma < \gamma^{rup} \\ 0 & si \quad \gamma > \gamma^{rup} \end{cases}$$

[3] $$\frac{\partial \varphi}{\partial \gamma}(\gamma) = \begin{cases} \frac{\phi^{rup} - \phi^{end}}{\gamma^{rup}} & \text{si } \gamma \le \gamma^{rup} \\ \frac{\phi^{res} - \phi^{rup}}{\gamma^{res} - \gamma^{rup}} & \text{si } \gamma^{rup} \le \gamma \le \gamma^{res} \\ 0 & \text{sinon} \end{cases} \qquad \frac{\partial \sigma}{\partial p_g}$$

[4] $$\frac{\partial b}{\partial \gamma}(\gamma) = \begin{cases} 0 & \text{si } \gamma < \gamma^{rup} \\ 2a\gamma + d & \text{si } \gamma^{rup} < \gamma \le \gamma^{res} \\ 0 & \text{si } \gamma \ge \gamma^{res} \end{cases} \qquad \frac{\partial \sigma}{\partial p_c}$$

6 Calculation of the coherent tangent operator

In pure mechanics or modeling THM with the law Hoek-Brown used according to its first aspect, the tensor of the constraints represents the tensor of the effective constraints which depends only on the tensor of the deformations. The mechanical law of behavior provides thus only the derivative $\frac{\partial \sigma}{\partial \varepsilon}$ in pure mechanics noted $\frac{\partial \sigma'}{\partial \varepsilon}$ in modeling thermo-hydro-mechanics.

The calculation of the quantity $\frac{\partial \sigma}{\partial \varepsilon}$ $\frac{\partial \sigma}{\partial p_c}=\frac{\partial \sigma}{\partial \sigma_p}\frac{\partial \sigma_p}{\partial p_c}$ **coherent tangent matrix is the same one for the two aspects of the law.** The detail of this derivative is presented under paragraph 6.1.

In modeling THM with the law Hoek-Brown used according to its second aspect, the tensor of the constraints representing the tensor of the total constraints, the variation of the tensor of the constraints depends on the variation of the tensor of the deformations $\frac{\partial \sigma}{\partial \varepsilon}$, variation of the gas pressure $\frac{\partial \boldsymbol{\sigma}}{\partial p_g}$ as well as variation of the capillary pressure $\frac{\partial \boldsymbol{\sigma}}{\partial p_c}$.

The gas pressure and the capillary pressure always do not intervene in calculation but according to the case of simulation. In the code, one will thus calculate rather $\frac{\partial \boldsymbol{\sigma}}{\partial p_p}$ then $\frac{\partial \sigma}{\partial p_g}=\frac{\partial \sigma}{\partial \sigma_p}\frac{\partial \sigma_p}{\partial p_g}$ and.

$\frac{\partial \sigma_p}{\partial p_c}$ and $\frac{\partial \sigma_p}{\partial p_g}$ depend on hydraulic simulation, and are independent of the mechanical law (11).

6.1 Calculation of $\frac{\partial \sigma}{\partial \varepsilon}$

One seeks to calculate the coherent matrix: $\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \mathbf{s}}{\partial \boldsymbol{\varepsilon}}+\frac{1}{3}\frac{\partial I_1}{\partial \boldsymbol{\varepsilon}}\mathbf{I}$ $\frac{\partial s_i^e}{\partial \varepsilon_{pq}}$. However:

$$\frac{\partial \mathbf{s}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \mathbf{s}^e}{\partial \boldsymbol{\varepsilon}}\left(1-\frac{3\mu}{\sigma_{eq}^e}\Delta\lambda\right)+\frac{3\mu\Delta\lambda}{(\sigma_{eq}^e)^2}\mathbf{s}^e\frac{\partial \sigma_{eq}^e}{\partial \boldsymbol{\varepsilon}}-\frac{3\mu}{\sigma_{eq}^e}\mathbf{s}^e\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}$$

$$\frac{\partial I_1}{\partial \boldsymbol{\varepsilon}}=\frac{\partial I_1^e}{\partial \boldsymbol{\varepsilon}}-9\mathrm{K}\left(\frac{\partial \eta}{\partial \Delta\lambda}\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}\Delta\lambda+\eta\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}}\right)$$

$$\tilde{P}.s^e.P=\bar{s}^e$$

$$\frac{\partial s_{ij}^e}{\partial \varepsilon_{pq}}=2\mu\left(\delta_{ip}\delta_{jq}-\frac{1}{3}\delta_{ij}\delta_{pq}\right)$$

$$\frac{\partial I_1^e}{\partial \varepsilon_{pq}}=3\mathrm{K}\,\delta_{pq}$$

with:

$$\frac{\partial \sigma_{eq}^e}{\partial \varepsilon_{pq}}=\sqrt{\frac{3}{2}}\frac{\partial s_{II}^e}{\partial \varepsilon_{pq}}=\frac{3}{2\sigma_{eq}^e}\sum_{i,j}s_{ij}^e\frac{\partial s_{ij}^e}{\partial \varepsilon_{pq}}=\frac{3}{2\sigma_{eq}^e}2\mu\sum_{i,j}s_{ij}^e\left(\delta_{ip}\delta_{jq}-\frac{1}{3}\delta_{ij}\delta_{pq}\right)=\frac{3\mu}{\sigma_{eq}^e}s_{pq}^e$$

$$\frac{\partial \eta}{\partial \Delta\lambda}=\frac{\partial \eta}{\partial \Delta\gamma}\frac{\partial \Delta\gamma}{\partial \Delta\lambda}=(\eta+1)\frac{\partial \eta}{\partial \Delta\gamma}$$

$$\bar{s}^e=diag(s_1^e,s_2^e,s_3^e)$$

from where:

$$\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}=\frac{\partial\mathbf{s}^e}{\partial\boldsymbol{\varepsilon}}\left(1-\frac{3\mu}{\sigma^e_{eq}}\Delta\lambda\right)+\frac{9\mu^2\Delta\lambda}{(\sigma^e_{eq})^3}\mathbf{s}^e.\mathbf{s}^e-\frac{3\mu}{\sigma^e_{eq}}\mathbf{s}^e.\frac{\partial\Delta\lambda}{\partial\boldsymbol{\varepsilon}}+\frac{1}{3}\frac{\partial I^e_1}{\partial\boldsymbol{\varepsilon}}-3K\left(\frac{\partial\eta}{\partial\Delta\lambda}\frac{\partial\Delta\lambda}{\partial\boldsymbol{\varepsilon}}\Delta\lambda+\eta\frac{\partial\Delta\lambda}{\partial\boldsymbol{\varepsilon}}\right)$$

P

To calculate $\frac{\partial\Delta\lambda}{\partial\varepsilon_{pq}}$, the equation is used $\dot{F}(\Delta\lambda)=0$ $\frac{\partial s^e}{\partial\varepsilon_{pq}}=P.\frac{\partial\bar{s}^e}{\partial\varepsilon_{pq}}.\tilde{P}$. One obtains:

$$\frac{\partial\Delta\lambda}{\partial\varepsilon_{pq}}\left|\begin{array}{l}
-(s^e_3-s^e_1)\frac{3\mu}{\sigma^e_{eq}}-\frac{\partial b}{\Delta\lambda}\left[1+\frac{1}{\sigma_3^{b-d}}\left[s^e_3+\frac{I^e_1}{3}-\left(3K\eta+s^e_3\frac{3\mu}{\sigma^e_{eq}}\right)\Delta\lambda\right]\right]\\
+\frac{b}{\sigma_3^{b-d}}\left(\frac{\partial\eta}{\partial\Delta\gamma}3K(\eta+1)\Delta\lambda+\left(3K\eta+s^e_3\frac{3\mu}{\sigma^e_{eq}}\right)\right)\\
-\frac{1}{2}\left(\begin{array}{l}\frac{\partial(S\sigma_c^2)}{\partial\Delta\lambda}-\frac{\partial(m\sigma_c)}{\partial\Delta\lambda}\left(s^e_3+\frac{I^e_1}{3}-\left(3K\eta+s^e_3\frac{3\mu}{\sigma^e_{eq}}\right)\Delta\lambda\right)\\
+\sigma_c m\left(\frac{\partial\eta}{\partial\Delta\gamma}3K(\eta+1)\Delta\lambda+\left(3K\eta+s^e_3\frac{3\mu}{\sigma^e_{eq}}\right)\right)\end{array}\right)\times\\
\left(S\sigma_c^2-m\sigma_c\left(s^e_3+\frac{I^e_1}{3}-\left(3K\eta=s^e_3\frac{3\mu}{\sigma^e_{eq}}\Delta\lambda\right)\right)^{-\frac{1}{2}}\right)\\
=-\left(\frac{\partial s^e_3}{\partial\varepsilon_{pq}}-\frac{\partial s^e_1}{s^e_3}\right)\left[1-\frac{3\mu}{\sigma^e_{eq}}\Delta\lambda\right]-(s^e_3-s^e_1)\frac{3\mu}{(\sigma^e_{eq})^2}\frac{\partial\sigma^e_{eq}}{\partial\varepsilon_{pq}}\Delta\lambda\\
+\left(\frac{\partial s^e_3}{\partial\varepsilon_{pq}}+\frac{1}{3}\frac{\partial I^e_1}{\partial\varepsilon_{pq}}-\left(-\frac{1}{\sigma^e_{eq}}\frac{\partial\sigma^e_{eq}}{\partial\varepsilon_{pq}}s^e_3+\frac{\partial s^e_3}{\partial\varepsilon_{pq}}\right)\frac{3\mu}{\sigma^e_{eq}}\right)\times\\
\left(\frac{b}{\sigma_3^{b-d}}-\frac{1}{2}\sigma_c m\left(S\sigma_c^2-m\sigma_c\left(s^e_3+\frac{I^e_1}{3}-\left(3K\eta+s^e_3\frac{3\mu}{\sigma^e_{eq}}\right)\Delta\lambda\right)\right)^{-\frac{1}{2}}\right)
\end{array}\right|$$

It thus remains us to calculate $\frac{\partial s^e_i}{\partial\varepsilon_{pq}}$. By taking again the notations of the paragraph 4.1, one a: $\tilde{\mathbf{P}}.\mathbf{s}^e.\mathbf{P}=\bar{\mathbf{s}}^e$ where $\bar{\mathbf{s}}^e=diag(s^e_1,s^e_2,s^e_3)$ and $\mathbf{P}$ is the matrix of the associated clean vectors. Consequently, $\frac{\partial\mathbf{s}^e}{\partial\varepsilon_{pq}}=\tilde{\mathbf{P}}.\frac{\partial\bar{s}^e}{\partial\varepsilon_{pq}}.\mathbf{P}+\frac{\partial\tilde{\mathbf{P}}}{\partial\varepsilon_{pq}}.\bar{s}^e.\mathbf{P}+\tilde{\mathbf{P}}.\bar{s}^e.\frac{\partial\mathbf{P}}{\partial\varepsilon_{pq}}$.

By taking again the same reasoning that in Appendix 1, one can show that $\frac{\partial\mathbf{s}^e}{\partial\varepsilon_{pq}}=\mathbf{P}.\frac{\partial\bar{\mathbf{s}}^e}{\partial\varepsilon_{pq}}.\tilde{\mathbf{P}}$ and finally: . $\tilde{\mathbf{P}}.\frac{\partial\mathbf{s}^e}{\partial\varepsilon_{pq}}.\mathbf{P}=\frac{\partial\bar{\mathbf{s}}^e}{\partial\varepsilon_{pq}}$

6.2 Typical case of multiple eigenvalues for $\mathbf{s}^e$

If the matrix $\mathbf{s}^e$ have multiple eigenvalues, the remarks passed in the paragraph 5.2.1 apply. In the case $\mathbf{s}_3^e = \mathbf{s}_2^e$ for example, the vector $\dfrac{\partial s_3^e}{\partial \varepsilon_{pq}}$ thus into account only one of both derivative takes directional of $\mathbf{s}_3^e = \mathbf{s}_2^e$. From where the idea to write $s_2^e = s_3^e = \dfrac{1}{2}(s_2^e + s_3^e)$ and thus $\dfrac{\partial s_2^e}{\partial \varepsilon_{ij}} = \dfrac{\partial s_3^e}{\partial \varepsilon_{ij}} = \dfrac{1}{2}\left(\dfrac{\partial s_2^e}{\partial \varepsilon_{ij}} + \dfrac{\partial s_3^e}{\partial \varepsilon_{ij}}\right)$.

6.3 Calculation of $\dfrac{\partial \sigma}{\partial \sigma_p}$

1) <u>Elasticity:</u>

One a: $\boldsymbol{\sigma} = \boldsymbol{\sigma}^e = \mathbf{H}\,\boldsymbol{\varepsilon} + \sigma_p \mathbf{I}$ and consequently $\dfrac{\partial \boldsymbol{\sigma}}{\partial \sigma_p} = \mathbf{I}$.

2) <u>Plasticity:</u>

One a: $\dfrac{\partial \boldsymbol{\sigma}}{\partial \sigma_p} = \dfrac{\partial \mathbf{s}}{\partial \sigma_p} + \dfrac{1}{3}\dfrac{\partial I_1}{\partial \sigma_p}\mathbf{I}$ with:

$$\frac{\partial \mathbf{s}}{\partial \sigma_p} = \frac{\partial \mathbf{s}^e}{\partial \sigma_p}\left(1 - \frac{3\mu}{\sigma_{eq}^e}\Delta\lambda\right) + \frac{3\mu\Delta\lambda}{(\sigma_{eq}^e)^2}\mathbf{s}^e\frac{\partial \sigma_{eq}^e}{\partial \sigma_p} - \frac{3\mu}{\sigma_{eq}^e}\mathbf{s}^e\frac{\partial \Delta\lambda}{\partial \sigma_p}$$

$$\frac{\partial I_1}{\partial \sigma_p} = \frac{\partial I_1^e}{\partial \sigma_p} - 9\mathrm{K}\left(\frac{\partial \eta}{\partial \Delta\lambda}\frac{\partial \Delta\lambda}{\partial \sigma_p}\Delta\lambda + \eta\frac{\partial \Delta\lambda}{\partial \sigma_p}\right)$$

However:

$$\frac{\partial \mathbf{s}^e}{\partial \sigma_p} = 2\mu\frac{\partial \Delta\mathbf{e}}{\partial \sigma_p} = 0$$

$$\frac{\partial I_1^e}{\partial \sigma_p} = 3\mathrm{K}\frac{\partial \Delta\varepsilon_v}{\partial \sigma_p} + 3 = 3$$

$$\frac{\partial \sigma_{eq}^e}{\partial \varepsilon_{pq}} = \sqrt{\frac{3}{2}}\frac{\partial s_{II}^e}{\partial \varepsilon_{pq}} = 0$$

$$\frac{\partial \eta}{\partial \Delta\lambda} = \frac{\partial \eta}{\partial \Delta\gamma}\frac{\partial \Delta\gamma}{\partial \Delta\lambda} = (\eta + 1)\frac{\partial \eta}{\partial \Delta\gamma}$$

from where: $\dfrac{\partial \boldsymbol{\sigma}}{\partial \sigma_p} = -\dfrac{3\mu}{\sigma_{eq}^e}\mathbf{s}^e.\dfrac{\partial \Delta\lambda}{\partial \sigma_p} + \mathbf{I} - 3\mathrm{K}\left(\dfrac{\partial \eta}{\partial \Delta\lambda}\dfrac{\partial \Delta\lambda}{\partial \sigma_p}\Delta\lambda + \eta\dfrac{\partial \Delta\lambda}{\partial \sigma_p}\right)\mathbf{I}$

To calculate $\frac{\partial \Delta\lambda}{\partial \sigma_p}$, the equation is used $\dot{F}(\Delta\lambda)=0$. One obtains:

$$\frac{\partial \Delta\lambda}{\partial \sigma_p}\left[\begin{array}{c}\frac{3\mu}{\sigma_{eq}^e}(s_3^e - s_1^e)+\frac{\partial B}{\partial \Delta\lambda}\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right)+3\frac{B}{\sigma_3^{b-d}}\left(\frac{\mu s_3^e}{\sigma_{eq}^e}+K\eta+K\frac{\partial \eta}{\partial \Delta\lambda}\Delta\lambda\right)\\ +\left(\frac{\partial S\sigma_c^2}{\partial \Delta\lambda}-\frac{\partial m\sigma_c}{\partial \Delta\lambda}\sigma_3+3m\sigma_c\left[\frac{\mu s_3^e}{\sigma_{eq}^e}+K\eta+K\frac{\partial \eta}{\partial \Delta\lambda}\Delta\lambda\right]\right)\times\left(S\sigma_c^2-\sigma_3.m\sigma_c\right)^{-1/2}\end{array}\right]$$

$$=\frac{B}{\sigma_3^{b-d}}+\frac{m\sigma_c}{2\sqrt{S\sigma_c^2-\sigma_3.m\sigma_c}}$$

3) In the tangent operator it is the derivative $\frac{\partial \boldsymbol{\sigma}'}{\partial \sigma_p}=\frac{\partial \boldsymbol{\sigma}}{\partial \sigma_p}-\mathbf{I}$ who must be returned.

7 Calculation of the tangent operator of speed

The tangent operator of speed is given as an indication. In the programming, one calculates the operator corresponding to `RIGI_MECA` by the same formulas as those providing `FULL_MECA` in which one poses: $\Delta\lambda=0$, H and $\gamma^+=\gamma^-$.

<u>If the tensor of constraints represents the effective constraint</u> :

The condition $\dot{F}=0$ is written: $\dot{F}=\frac{\partial F}{\partial \boldsymbol{\sigma}}\dot{\boldsymbol{\sigma}}+\frac{\partial F}{\partial \gamma}\dot{\gamma}=0$. This thus gives us:

$$\dot{\gamma}=-\frac{\sum_{ij}\frac{\partial F}{\partial \sigma_{ij}}\dot{\sigma}_{ij}}{\frac{\partial F}{\partial \gamma}}$$

However, one has $\dot{\boldsymbol{\sigma}}=\mathbf{H}(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^p)$ where $\mathbf{H}$ is the matrix of Hooke. Moreover, $\dot{\boldsymbol{\varepsilon}}^p=\dot{\lambda}\frac{\partial G}{\partial \boldsymbol{\sigma}}=\frac{\dot{\gamma}}{(\eta(\gamma)+1)}\frac{\partial G}{\partial \boldsymbol{\sigma}}$, from where

$$\dot{\gamma}=-\frac{\sum_{ijkl}\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\dot{\varepsilon}_{kl}}{\frac{\partial F}{\partial \gamma}-\frac{1}{\eta(\gamma)+1}\sum_{ijkl}\frac{\partial F}{\partial \sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial \sigma_{kl}}} \quad \text{avec} \quad \frac{\partial G}{\partial \sigma_{ij}}=\eta(\gamma)\delta_{ij}+\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}},$$

and finally

$$\dot{\sigma}_{ab}=H_{abcd}\left(\dot{\varepsilon}_{cd}-\dot{\lambda}\frac{\partial G}{\partial\sigma_{cd}}\right)=\sum_{cd}D_{abcd}\dot{\varepsilon}_{cd}$$
$$\text{où}\quad D_{abcd}=H_{abcd}+\frac{\left(\sum_{kl}H_{abkl}\frac{\partial G}{\partial\sigma_{kl}}\right)\left(\sum_{ij}\frac{\partial F}{\partial\sigma_{ij}}H_{ijcd}\right)}{(\eta(\gamma)+1)\frac{\partial F}{\partial\gamma}-\sum_{pqmn}\frac{\partial F}{\partial\sigma_{pq}}H_{pqmn}\frac{\partial G}{\partial\sigma_{mn}}}.$$

If the tensor of constraints represents the total constraint :

One has $\dot{\boldsymbol{\sigma}}=H(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^p)+\dot{\sigma}_p\mathbf{I}$ where $\mathbf{H}$ is the matrix of Hooke. In subscripted notation, one can write:

$$\dot{\sigma}_{ij}=H_{ijkl}\left(\dot{\varepsilon}_{kl}-\dot{\varepsilon}_{kl}^p\right)+\dot{\sigma}_p\delta_{ij}$$

Moreover, $\dot{\boldsymbol{\varepsilon}}^p=\dot{\lambda}\frac{\partial G}{\partial\boldsymbol{\sigma}}=\frac{\dot{\gamma}}{(\eta(\gamma)+1)}\frac{\partial G}{\partial\boldsymbol{\sigma}}$,

The condition $\dot{F}=0$ is written: $\dot{F}=\frac{\partial F}{\partial\sigma}\dot{\sigma}+\frac{\partial F}{\partial\gamma}\dot{\gamma}=0$. This thus gives us:

$$\dot{\gamma}=-\frac{\frac{\partial F}{\partial\sigma_{ij}}\dot{\sigma}_{ij}}{\frac{\partial F}{\partial\gamma}}$$

while replacing $\dot{\sigma}_{ij}$ and $\dot{\varepsilon}_{kl}^p$ by their expressions one can deduce:

$$\dot{\gamma}\frac{\partial F}{\partial\gamma}=-\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\left(\dot{\varepsilon}_{kl}-\frac{\dot{\gamma}}{(\eta(\gamma)+1)}\frac{\partial G}{\partial\sigma_{kl}}\right)-\frac{\partial F}{\partial\sigma_{ij}}\dot{\sigma}_p\delta_{ij}$$

and then the expression of $\dot{\gamma}$:

$$\dot{\gamma}=-\frac{\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\dot{\varepsilon}_{kl}}{\frac{\partial F}{\partial\sigma_{ij}}-\frac{1}{(\eta(\gamma)+1)}\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial\sigma_{kl}}}-\frac{\frac{\partial F}{\partial\sigma_{ij}}\dot{\sigma}_p\delta_{ij}}{\frac{\partial F}{\partial\sigma_{ij}}-\frac{1}{(\eta(\gamma)+1)}\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial\sigma_{kl}}}$$

$$\text{avec}\quad\frac{\partial G}{\partial\sigma_{ij}}=\eta(\gamma)\delta_{ij}+\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}},$$

and finally

$$\dot{\sigma}_{ab}=H_{abcd}\left(\dot{\varepsilon}_{cd}-\dot{\lambda}\frac{\partial G}{\partial\sigma_{cd}}\right)+\dot{\sigma}_p\delta_{ab}=D_{abcd}\dot{\varepsilon}_{cd}+E_{ab}\dot{\sigma}_p$$

$$\dot{\sigma}_{ab}=H_{abcd}\left(\dot{\varepsilon}_{cd}+\frac{\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\dot{\varepsilon}_{kl}\frac{\partial G}{\partial\sigma_{cd}}}{(\eta(\gamma)+1)\frac{\partial F}{\partial\gamma}-\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial\sigma_{kl}}}+\frac{\frac{\partial F}{\partial\sigma_{ij}}\delta_{ij}\frac{\partial G}{\partial\sigma_{cd}}\dot{\sigma}_p}{(\eta(\gamma)+1)\frac{\partial F}{\partial\gamma}-\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial\sigma_{kl}}}\right)+\dot{\sigma}_p\delta_{ab}$$

où

$$D_{abcd}=H_{abcd}+\frac{H_{abkl}\frac{\partial G}{\partial\sigma_{kl}}\frac{\partial F}{\partial\sigma_{ij}}H_{ijcd}}{(\eta(\gamma)+1)\frac{\partial F}{\partial\gamma}-\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial\sigma_{kl}}}$$

et

$$E_{ab}=\frac{H_{abcd}\frac{\partial F}{\partial\sigma_{ij}}\delta_{ij}\frac{\partial G}{\partial\sigma_{cd}}}{(\eta(\gamma)+1)\frac{\partial F}{\partial\gamma}-\frac{\partial F}{\partial\sigma_{ij}}H_{ijkl}\frac{\partial G}{\partial\sigma_{kl}}}+\delta_{ab}$$

8 Algorithm

8.1 Internal variables

The law of behavior of Hoek-Brown modified is governed by the three following internal variables:

1) *the parameter of work hardening* γ corresponding to the major unrecoverable deformation.
2) *cumulated plastic voluminal deformation* ε_v^p whose law of evolution is given by $d\varepsilon_v^p=3\eta\,d\lambda=\frac{3\eta}{\eta+1}d\gamma$.
3) *the state of plasticization* ; it is worth 0 if the point of Gauss is in elastic load or discharge, and 1 if the point of Gauss is in plastic load.

8.2 Algorithm

One retains an implicit formulation compared to the criterion and the direction of flow. One places oneself in a material point and one supposes known with t^- :

- The tensor of increase in deformations $\Delta\boldsymbol{\varepsilon}$ from where one deduces $\Delta\mathbf{e}$, $\Delta\varepsilon_v$
- Constraints at the beginning of the step of time $\boldsymbol{\sigma}^-$ from which one deduces $\mathbf{s}^-, I_1^-$
- The value of the internal variables γ^- and ε_v^{p-} at the beginning of the step of times which give us $(S\sigma_c^2)^-, (m\sigma_c)^-, b^-, \phi^-$

The goal of the algorithm is then to calculate:

- Constraints at the end of the step of time σ
- Internal variables at the end of the step of time
- The tangent behavior at the end of the step of time: $\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}$ if the law is in effective constraints

- The tangent behavior at the end of the step of time: $\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}$ and $\frac{\partial \boldsymbol{\sigma}}{\partial \sigma_p}$ if the law is in total constraints.

Algorithm:

Calculation of the elastic solution :

If the law is in effective constraints:

$$\begin{cases} \mathbf{s}^e = \mathbf{s}^- + 2\mu \Delta \mathbf{e} \\ I_1^e = I_1^- + 3K \Delta \varepsilon_v \end{cases}$$

If the law is in total constraints:

$$\begin{cases} \mathbf{s}^e = \mathbf{s}^- + 2\mu \Delta \mathbf{e} \\ I_1^e = I_1^- + 3K \Delta \varepsilon_v + 3 \Delta \sigma_p \end{cases}$$

Calculation of the elastic criterion $F(\boldsymbol{\sigma}^e, \gamma^-)$. If $S\sigma_c^2 - \sigma_3^e . m \sigma_c < 0$, function F is not defined in the point $(\boldsymbol{\sigma}^e, \gamma^-)$. It is considered whereas one is in the plastic case.

Resolution : calculation of $\boldsymbol{\sigma}, \gamma$

If $F(\boldsymbol{\sigma}^e, \gamma^-) \leq 0$, then $\Delta \boldsymbol{\varepsilon} = \Delta \boldsymbol{\varepsilon}^e, \gamma^p = \gamma^{p^-}, \Delta \boldsymbol{\sigma} = \mathbf{H} \Delta \boldsymbol{\varepsilon}$.

If not, one seeks $\boldsymbol{\sigma}, \gamma$ such as $F(\boldsymbol{\sigma}, \gamma) \leq 0$, which amounts seeking $\Delta\gamma$ such as $F(\Delta\gamma) = \bar{F}(\Delta\gamma) = 0$. This problem is solved by using a method of Newton on $\bar{F}$.

Algorithm of Newton:

Initialization: $\Delta \gamma^0 = 0$

After each iteration:

- if $\Delta \gamma^{n+1} \leq 0$, there was not convergence: the step of time is subdivided
- if $\Delta \gamma^{n+1} \leq \frac{\sigma_{eq}^e}{3\mu[\eta(\Delta\gamma^{n+1})+1]}$, there is no solution (see paragraph 4.2): the step of time is subdivided

Update of the variables : internal constraints, variables

Calculation of the coherent tangent matrix $\frac{\partial \boldsymbol{\sigma}}{\partial \varepsilon}$ *if the law is in effective constraints and* $\frac{\partial \boldsymbol{\sigma}}{\partial \varepsilon}$ *and* $\frac{\partial \boldsymbol{\sigma}}{\partial \sigma_p}$ *if law is in total constraints for the option* `RIGI_MECA_TANG` or `FULL_MECA` .

9 Bibliography

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- Conceptual François Laigle Modèle for the development of laws of behavior adapted to the design of the underground works. Thesis Central School of Lyon

10 Features and checking

The law of behavior can be defined by the keyword `HOEK_BROWN` (order `STAT_NON_LINE`, keyword factor `BEHAVIOR`). It is associated with material `HOEK_BROWN` (order `DEFI_MATERIAU`).

The law `HOEK_BROWN` is checked by the cases following tests:

SSNA116	[V6.01.116]	Triaxial compression test with the model of Hoek-Brown modified into axisymmetric
SSNV184	[V6.04.184]	Triaxial compression test with the model of Hoek-Brown modified
WTNV128	[V7.31.128]	Triaxial compression test not drained with the model of Hoek-Brown modified in effective constraints
WTNV129	[V7.31.129]	Triaxial compression test not drained with the model of Hoek-Brown modified in total constraints

11 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
8,5	C.Chavant, J.El-Gharib EDF-R&D/AMA V.Gervais, CS	Initial text

Annexe 1Derivation of the principal constraints

That is to say $\boldsymbol{\sigma}$ a symmetrical tensor and $\boldsymbol{\sigma}_d$ this tensor in the base which it diagonalise. Let us indicate by $\boldsymbol{P}(\boldsymbol{\sigma})$ the matrix of passage which diagonalise the tensor σ : $\boldsymbol{\sigma}:\boldsymbol{\sigma}=\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma})$. In indicielle writing, we adopt following convention for the matric writings:

$$M^{i\leftarrow ligne}_{j\leftarrow colonne}$$

so that the matric product is written: $(\mathbf{A}.\mathbf{B})^i_j=A^i_m\mathbf{B}^m_j$ with the rule of summation of the repeated indices.
Then there is the relation:

$$\boxed{\frac{\partial\sigma_d}{\partial\sigma^i_j}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\sigma}{\partial\sigma^i_j}.\mathbf{P}(\boldsymbol{\sigma})}$$

$\dfrac{\partial\sigma_{d_k}}{\partial\sigma^i_j}=P^i_kP^j_k$ or in indicielle form **<u>without summation on the index</u>** k

Demonstration:
In what follows, we will note σ an unspecified component of the tensor $\boldsymbol{\sigma}$ without specifying the indices of them when they do not play any part.
One has $\boldsymbol{\sigma}=\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma})$, and consequently:

1) $$\frac{\partial\boldsymbol{\sigma}_d}{\partial\sigma}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\boldsymbol{\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\boldsymbol{\sigma}.\frac{\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}$$

2) .

By deferring in the last two terms the equality $\boldsymbol{\sigma}=\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma})$, one obtains:

1) $$\frac{\partial\boldsymbol{\sigma}_d}{\partial\sigma}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\mathbf{P}(\boldsymbol{\sigma}).\tilde{\mathbf{P}}(\boldsymbol{\sigma})+\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}$$

i.e.:

1) $$\frac{\partial\boldsymbol{\sigma}_d}{\partial\sigma}=\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}$$

In matric writing, this is written:

$$\frac{\partial\boldsymbol{\sigma}^i_d}{\partial\sigma}=\left(\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\boldsymbol{\sigma}}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma})+\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}\right)^i_i$$

Let us show that the sum of the last two terms of this expression is worthless:

- $$\left(\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}\right)^i_j=\partial\tilde{P}^i_m.P^m_l.\sigma_{d^l_j}+\sigma_{d^i_m}.\tilde{P}^m_l.\partial P^l_j$$

where ∂a indicate $\dfrac{\partial a}{\partial\sigma}$ in order to reduce the writing.

We write whereas $i=j$ and that only terms $\sigma^p_{d_p}$ are nonworthless.
One obtains:

$$\left(\frac{\partial\tilde{\mathbf{P}}(\boldsymbol{\sigma})}{\partial\sigma}.\mathbf{P}(\boldsymbol{\sigma}).\boldsymbol{\sigma}_d+\boldsymbol{\sigma}_d.\tilde{\mathbf{P}}(\boldsymbol{\sigma}).\frac{\partial\mathbf{P}(\boldsymbol{\sigma})}{\partial\sigma}\right)^i_i=\partial\tilde{P}^i_m.P^m_i.\sigma_{d^i_i}+\sigma_{d^i_i}.\tilde{P}^i_l.\partial P^l_i$$

$$=\left(\partial\tilde{P}^i_m.P^m_i+\tilde{P}^i_m.\partial P^m_i\right)\sigma_{d^i_i}\underline{\text{sans sommation sur l'indice } i}$$

who is clearly null since $\tilde{\mathbf{P}}.\mathbf{P}=\mathbf{I}$. From where finally $\dfrac{\partial\sigma_{d_k}}{\partial\sigma^i_j}=\dfrac{\partial\sigma_{d^k_k}}{\partial\sigma^i_j}=P^i_kP^j_k$.

Annexe 2 Derived from the function $\bar{F}(\Delta\lambda)$

$$\frac{\partial h}{\partial \Delta\gamma}=\frac{\partial}{\partial \Delta\gamma}\left(\frac{3+\sin\phi(\gamma^{-}+\Delta\gamma)}{3(1+\sin\phi(\gamma^{-}+\Delta\gamma))}\right)=\begin{cases}-\dfrac{2\dfrac{\partial\phi}{\partial\gamma}\cos\phi(\gamma)}{3(1+\sin\phi(\gamma))^{2}} & \text{si } \gamma<\gamma^{res}\\ 0 & \text{sinon}\end{cases}$$

$$\frac{\partial g}{\partial \Delta\gamma}=\frac{\partial}{\partial \Delta\gamma}\left[\frac{3+\sin\phi(\gamma^{-}+\Delta\gamma)}{3(1+\sin\phi(\gamma^{-}+\Delta\gamma))}\left(\frac{6\mathrm{K}\sin\phi(\gamma^{-}+\Delta\gamma)}{3+\sin\phi(\gamma^{-}+\Delta\gamma)}+\frac{3\,mus_{3}^{e}}{\sigma_{eq}^{e}}\right)\right]$$

$$=\frac{6\mathrm{K}\dfrac{\partial\phi}{\partial\gamma}\cos\phi(\gamma)}{(3+\sin\phi(\gamma))(1+\sin\phi(\gamma))}-\frac{2\dfrac{\partial\phi}{\partial\gamma}\cos\phi(\gamma)}{3(1+\sin\phi(\gamma))^{2}}\left(\frac{6\mathrm{K}\sin\phi(\gamma)}{3+\sin\phi(\gamma)}+\frac{3\,mus_{3}^{e}}{\sigma_{eq}^{e}}\right)$$

$$\frac{\partial\bar{F}}{\partial\Delta\gamma}=2\left[-(s_3^e-s_1^e)\frac{3\mu}{\sigma_{eq}^{e}}\left(\frac{\partial h}{\partial\Delta\gamma}\Delta\gamma+h\right)-\frac{\partial b}{\partial\Delta\gamma}\left(1-\frac{1}{\sigma_3^{b-d}}\left[s_3^e+\frac{I_1^e}{3}-g\Delta\gamma\right]\right)-\frac{b}{\sigma_3^{b-d}}\left(\frac{\partial g}{\partial\Delta\gamma}\Delta\gamma+g\right)\right]$$

$$\times\left[(s_3^e-s_1^e)\left[1-\frac{3\mu}{\sigma_{eq}^{e}}h\Delta\gamma\right]-b\left[1-\frac{1}{\sigma_3^{b-d}}\left(s_3^e+\frac{I_1^e}{3}-g\Delta\gamma\right)\right]\right]$$

$$-\left(\frac{\partial(S\sigma_c^2)}{\partial\Delta\gamma}-\frac{\partial(m\sigma_c)}{\partial\Delta\gamma}\left(s_3^e+\frac{I_1^e}{3}-g\Delta\gamma\right)+\sigma_c m\left(\frac{\partial g}{\partial\Delta\gamma}\Delta\gamma+g\right)\right)$$

Annexe 3Calculation of the derivative $\frac{\partial\sigma_p}{\partial p_g}$ and $\frac{\partial\sigma_p}{\partial p_c}$

- `LIQU_SATU (PRE1=` p_{lq} `):` $\frac{\partial\sigma_p}{\partial p_c}=-\frac{\partial\sigma_p}{\partial p_{lq}}=bS$
- `LIQU_GAZ_ATM (PRE1=-` p_{lq} `):` $\frac{\partial\sigma_p}{\partial p_c}=-\frac{\partial\sigma_p}{\partial p_{lq}}=bS$
- `GAS (PRE1=` p_g `) :` $\frac{\partial\sigma_p}{\partial p_g}=-b(1-S)$
- `LIQU_VAPE_GAZ (PRE1=` p_c `, PRE2=` p_g `):` $\frac{\partial\sigma_p}{\partial p_g}=-b$, $\frac{\partial\sigma_p}{\partial p_c}=bS$
- `LIQU_GAZ (PRE1=` p_c `, PRE2=` p_g `):` $\frac{\partial\sigma_p}{\partial p_g}=-b$, $\frac{\partial\sigma_p}{\partial p_c}=bS$
- `LIQU_VAPE (PRE1=` p_{lq} `):` $\frac{\partial\sigma_p}{\partial p_c}=-\frac{\partial\sigma_p}{\partial p_{lq}}=bS$
- `LIQU_AD_GAZ_VAPE (PRE1=` p_c `, PRE2=` p_g `) :` $\frac{\partial\sigma_p}{\partial p_g}=-b$, $\frac{\partial\sigma_p}{\partial p_c}=bS$