Organization (S): EDF-CIH/GCED

Manuel de Référence
R7.01 booklet: Modelings for Génie Civil and the géomatériaux ones
R7.01.* document

beton_rag
integrated in the code into the finite elements Aster

P_G
σ_I

2.1
-

P_G
P_G
B^G.tr (ε)
$With_0$
AV^G
$With_0V^G$
P^G
P^G
AV^G
increase

With
T
Sr
C
C_{sat}
C
α_0
Ea
47000 J/mol/°K
R
8.31 J/mol
$T_{ref.}$
α_0
T
With
Sr^0
Sr
With
With

2.3-
(
α_0 = 0.0012
$T_{ref.}$ = 20 °C
Sr^0 = 0.2

)

1) VD^T

P_W
VEP^S
VEP^D

2.5-
2.5-
2.5-
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2.5-

$P^W = P^C < 0$

$P^W > 0$
K_2
.
spherical
m^T
σ^C
ε year
E ε
E ε
ε year
ε_{vdt}
ε_{vep}
D
I-D
I
D
V^{year}
D
R^0
R^D
D^T
m^T
σ^C
m^T
σ^C
J
K
D^N
R

2.6 -
.6 - 2

I_1
J_2
Y^C
P^G
With
With
V^{year}
m^W
P^{w0}
P^W
S^W
F^W
F^W
$K^W. S^W$
$P^C(S^W)$
V^{year}
K
T
T_0

Relation of behavior BETON_RAG

Summary:

This document presents the model of behavior BETON_RAG, used to consider the behavior long-term of the structures affected by the reaction alkali-aggregate.
One also details there the writing and the digital processing of the model.

Contents

1 Introduction

A certain number of civil engineer works of the park of production of EDF, mainly of the stoppings, present pathologies of expansion of concretes due to the reaction alkali-aggregates (RAG). In order to evaluate and the operation safety margins of these installations, and to control the costs of maintenance, EDF and the Laboratory Materials and Durability of Constructions (LMDC) of the university Paul Sabatier (Toulouse III) developed a digital model, [Grimal, 2007], allowing of to simulate the mechanical behavior of the structures affected by the RAG. The goal of this model is to evaluate the deformations and the anisotropic damage (cracking) of the works reached. Indeed, recent experimental research confirms that swelling due to the RAG becomes strongly anisotropic when the state of stress becomes deviatoric. Moreover, the kinetics and the amplitude of the reaction strongly depending on the water content and the temperature of the concrete, the model takes account of these environmental phenomena. In addition, creeps of compression and traction play a significant part in the behavior of the works, they were thus treated in a specific way.

In *code_aster*, the model is used under the name of `BETON_RAG`.

2 Description of the model [grimal, 2007]

2.1 Principle general

The reaction alkali-aggregates and its effects are modelled by using a phenomenologic approach. This approach keeps account of the various important phenomena evolving within the concrete and influencing the chemical reaction. The principal developments suggested in this model relate to the interactions between the chemical pressure of origin and, the deformations differed on the one hand and the anisotropic deformations induced by the presence from directed cracks on the other hand.
The effects of moisture on the development of the alkali-reaction as on the capillary pressure inducing the shrinking of the matrix are also considered. The dependence between the evolution of swellings and the state of stress is then a consequence of all these elementary phenomena; the mechanical effects of the alkali-reaction are thus the consequences of an internal loading "long run" due to the evolution of the chemical pressure (P_g in figure 2.1-a), combined with the external loading σ_i .

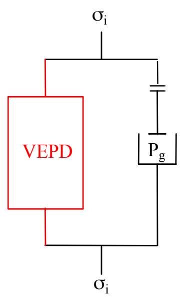


Figure 2.1-a : Principle of the model of behavior of the concrete subjected to a pressure of swelling.

P_g , in addition with the external constraint σ_i request the cementing matrix. The latter is regarded as a medium visco-élasto-plastic endommageable (module VEPD on figure 2.1-a). The next paragraph presents how the pressure due to the formation of gel is evaluated, in agreement with the environmental conditions and the state of deformation. Then, the mechanical model is exposed within the framework of the thermodynamics of the irreversible processes.

2.2 Law of evolution of the internal pressure

Pressure P_g had with the formation of gel in porosity is evaluated by making the assumption that the state of stress does not modify the chemical advance of the reaction alkali-aggregate.
In agreement with the various modelings available, studied in the bibliographical part [Ulm and al.; Lemarchand and al.; Coussy, 2002], the concrete is regarded as a porous environment made up of a solid matrix and freezing occupying part of connected porosity. This porosity is made of two types of

pores, those initially connected to the sites of reaction and those generated by the voluminal deformation modifying porosity ($b^g\, tr(\varepsilon)$ inequation (1).

The porosity connected to the sites of reaction is written in the form $\varphi_0 = A_0 V^g$, in which, A_0 is the advance from which initial connected porosity is filled. Thus, as long as the volume of freezing created (AV^g) is lower than $A_0 V^g$, freezing is placed in connected porosity and P_g remain worthless. P_g will increase only as from the moment when all porosity will be filled:

$$AV^g \geq \langle A_0 V^g + b^g\, tr(\varepsilon) \rangle$$

By taking account of these remarks, an expression connecting the pressure of freezing and the volume of freezing created (AV^g) is proposed:

$$Pg = M_g \langle A.V_g - \langle A_0.V_g + b_g\, tr(\varepsilon) \rangle_+ \rangle_+ \quad (1)$$

In this expression:

- V_g is the maximum volume of freezing which can be created by the chemical reaction.
- A is the advance of the chemical reaction, increasing 0 for the healthy concrete with 1 when the reaction is completed.
- M_g is comparable to a modulus of elasticity of freezing and b_g can be comparable to a coefficient of Biot for freezing.

The various positive parts allow:

- a taking into account of the influence of the voluminal deformation induced by an external loading on one variation connected porosity,
- an increase in the chemical pressure of origin if and only if freezing manages to fill connected porosity.

2.3 Advance of the reaction

The advance of the reaction A evoked previously is a function of the temperature and water content of the concrete.
The law of evolution used to evaluate chemical advance is inspired by work of S. Poyet [Poyet, 2003]. It shows that the freezing created during time (t) and its kinetics of creation are proportional to the degree of saturation (Sr) concrete, the degree of saturation being defined by:

$$Sr = \frac{C}{C_{sat}}$$

In this expression, C is the free water concentration contained per unit of volume of the concrete and C_{sat} is the value of C when the concrete is completely saturated.
In order to take into account the effect of the temperature, the law of Arrhenius is used to model the thermic action [Capra, 1997]. Finally, the following law is proposed:

$$\frac{\partial A}{\partial t} = \alpha_0 . \exp\left[\frac{E_a}{R}\left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right] \frac{\langle S_r - S_r^0 \rangle_+}{\left(1 - S_r^0\right)} \langle S_r - A \rangle_+ \quad (2)$$

In this law, α_0 is a parameter of kinetics, E_a is the energy of activation of the reaction alkali-aggregates (usually of a value close to $47000\, J/mol/{}^\circ K$ [Lombardi and al., 1995]), R is the constant of perfect gases ($8.31\, J/mol$), T_{ref} (in Kelvin) is the absolute temperature of the test allowing the identification of α_0 , T is the temperature of the material point. The term $\langle Sr - A \rangle$ mean that the chemical affinity of the reaction is conditioned by the degree of saturation of the concrete, Poyet having shown in its thesis that advance A (which is a standardized variable) cannot exceed a value limits very near to the degree of saturation, it explains that by the fact that only the fraction of aggregate in contact with water can lead to the reaction. Consequently the amplitude of the reaction is proportional to the degree of saturation S_r .
Same manner, the way traversed by the ions to reach silica reactive is all the more large as the degree of saturation is weak, which involves a reduction in the kinetics. This effect kinetics of the

degree of saturation is taken into account via the term $\frac{\langle Sr-S_r^0\rangle}{\left(1-S_r^0\right)}$, in which S_r^0 represent the threshold of saturation from which the evolution of the chemical reaction becomes possible.
figure 2.3-a present some examples of variations of advance A for various states of moisture. Pread S_r is important more A is large, A being maximum ($A=1$) when the concrete is saturated ($S_r=1$). If the concrete remains in a state unsaturated ($S_r<1$), the reaction is never complete ($A<1$). On the other hand, if the state of saturation changes and passes for example from 0.6 with 1.0 , the curve of advance is modified to join the state of maximum advance.

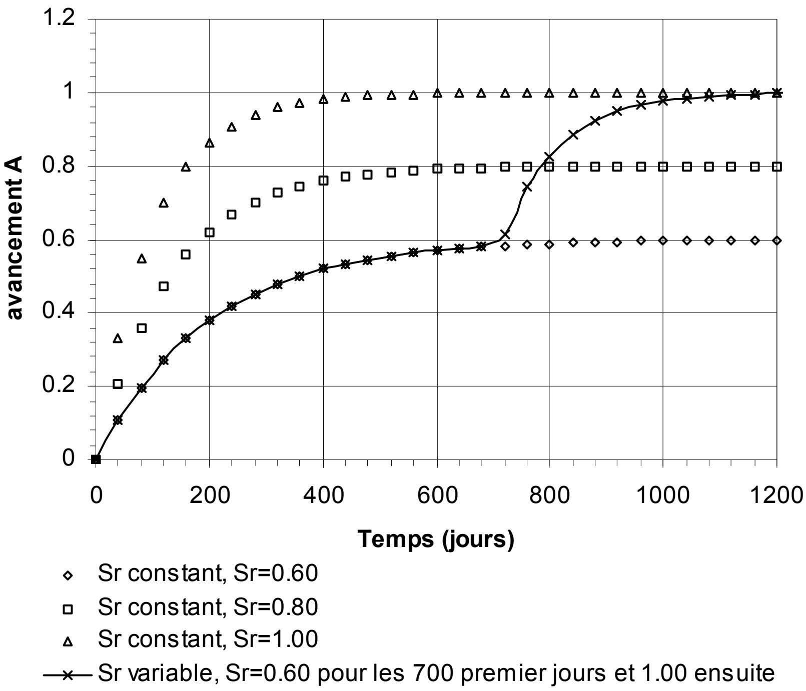


Figure 2.3-a : Evolution advance of the RAG under various degrees of saturation for $\alpha_0=0.0012$, $T_{ref}=20°C$, $Sr^0=0.2$

2.4 Dependence enters the damage and swelling

The alkali-reaction produces important swellings which should be accompanied by an important damage of the structure. But, various experiments [Larive, 1997; Multon, 2003; Gravel, 2001] showed that the reduction in the mechanical properties remained weak compared to the deformations reached.
A reduction in 20% mechanical characteristics is observed for a voluminal swelling of 0.1% (Figure 2.4-b). A test of direct traction leading to a state of comparable deformation would produce a complete damage of the cementing matrix. The particular behavior of the concrete reached by the reaction alkali-aggregates can be explained by two complementary phenomena: initially a phenomenon of cracking located around the reactive aggregate and then a viscoplastic adaptation for the long run of the cement paste. Cracking leads to great deformations because of the cumulative effect of the opening of the microscopic cracks. The viscoplastic behavior of the paste cement (in particular thanks to the HSC, to see [Grimal 2007]) can limit the stress concentration and thus the propagation of microscopic cracks and the associated damage is also limited.

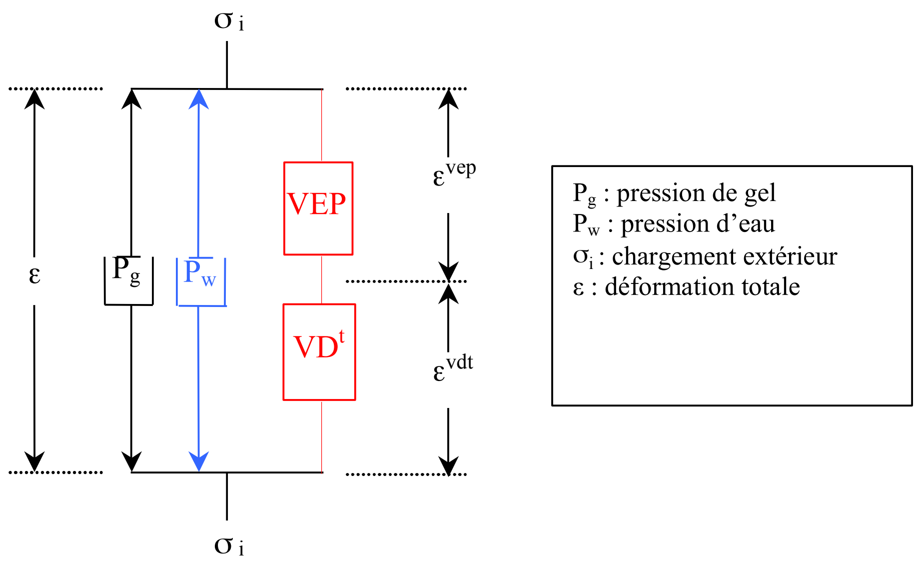


Figure 2.4-a : Model viscoelastoplastic

Compatibility between important swelling due to the RAG and the associated moderate damage is modelled by using a plastic deformation related to the damage of traction [Grimal and al., 2005; 2006]. This plastic deformation limits the damage by allowing a stress relaxation caused by the pressure of freezing on the cement paste. Although necessary to model the swelling of alkali-reaction, this one is naturally insufficient to explain other long-term deformations such as the multiaxial creep of the concrete not damaged. Thus, in order to obtain reliable forecasts of the deformations induced in the long run by the pressure due to the presence of freezing and the effects of loading, a deformation of creep was integrated into the model.
Consequently, the module visco-élasto-plastic endommageable (VEPD on Figure 2.1-a) was divided into two complementary levels (Figure 2.4-a):

- a module (VD^t), dedicated to the modeling of the deformation ε^{vdt} (Figure 2.4-a), respecting the empirical relation existing between swelling due to the RAG and damage (Figure 2.4-b).
- a module visco-élasto-plastic (VEP), corresponding to the deformation ε^{vep} on figure 2.4-a, allowing to model other aspects of the behavior of the concrete such as elasticity and creep.

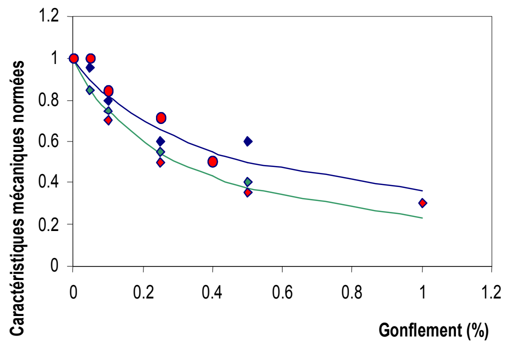


Figure 2.4-b : Evolution of the mechanical characteristics in traction and compression [Saddler, 1999]

2.5 Modeling of the unelastic behavior of the concrete

We now will describe successively the modules VEP and VD^T.

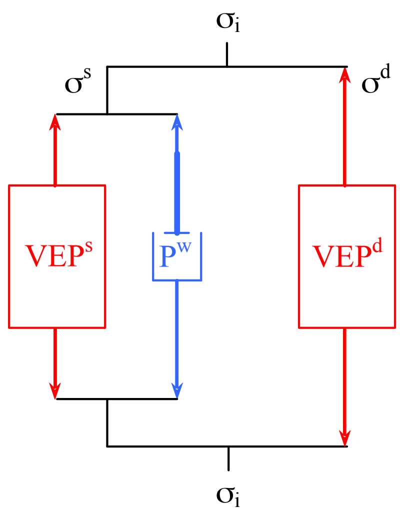
2.5.1 Rheological module simulating the creep and the shrinking of concrete (VEP)

Acker [Acker, 2003] proposes to explain the origins of creep by the particular behavior of the HSC, only component to have a viscous behavior. According to him, taking into account the particular structure of the HSC, two mechanisms of deformations are possible:

- Slips between the layers of the HSC
- Collapses in the stacking of the layers and a consolidation of the HSC.

The first mechanism is done with constant volume and suggests a behavior nonasymptotic long run. The second mechanism implies a water departure of the HSC towards capillary porosity, with an in the long run asymptotic behavior.

In order to model the difference in behavior, the module VEP is divided into two parts, a "spherical" part and a "deviatoric" part. The spherical part (noted VEP^s on Figure 2.5.1-a) represented the evolution of the structure of the HSC subjected to a hydrostatic constraint related to the water beginning like to viscoplastic compressing by "random" slips overall isotropic. The deviatoric part translates the slip of the layers subjected to a shear stress. Pressure P^w pfeels on Figure 2.5.1-a corresponds to the hydrous pressure. It generates a withdrawal if it is negative and a swelling if it is positive.



P_w : pressure of water will intra porous
VEP^s : spherical part of module VEP
VEP^d : deviatoric part of module VEP

Figure 2.5.1-a : Decomposition in parts spherical and deviatoric of module VEP.

Taking into account the heterogeneous character of the porous distribution, we propose to model the phenomenon of creep like a hydraulic problem of consolidation in the following way: when a loading is applied to representative ground volume (WORM), of interstitial overpressures appear in the hydrous network, the first overpressures to disappear are those present in the water of the connected macroporosity, the loading initially taken again by these overpressures is transferred towards the solid skeleton (figure 2.5.1-b). This part thus concerns the classical hydromechanics. The solid skeleton itself is consisted, if one observes it on a finer scale, of a connected microporosity and a solid skeleton which will be overloaded in its turn when overpressures present in its microporosity are evacuated towards the macroporosity. Creep then seems a succession of consolidation on increasingly fine scales, thus calling on transfers of less and less free water. This interpretation of the phenomenon of creep reveals a "fractal" character of the mechanism of consolidation, since the transfer of the constraints towards the solid skeleton is made in a way similar to increasingly fine scales (figure 2.5.1.1-a), this hypothesis is also put forth by Acker [Acker, 2003]. For the finest scale, an irreversible character of creep can be present; the interfoliaceous water molecules are driven out in an irreversible way by compressing of the hydrates.

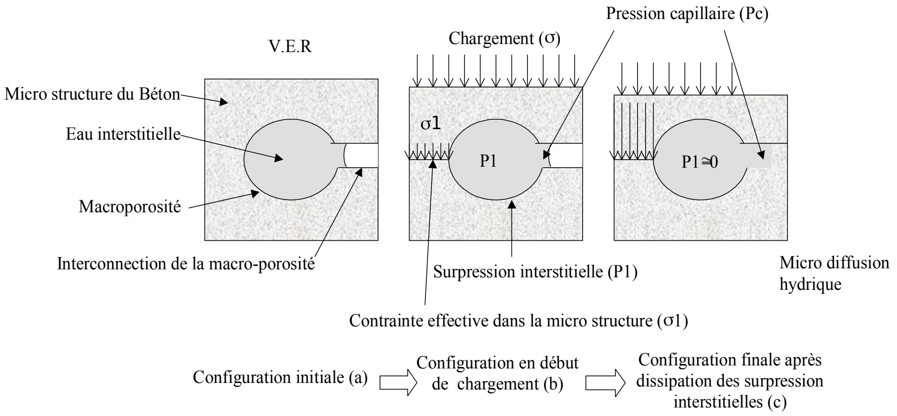


Figure 2.5.1-b : Viscoelastic mechanisms associated with the macro porosity

2.5.1.1 Spherical creep

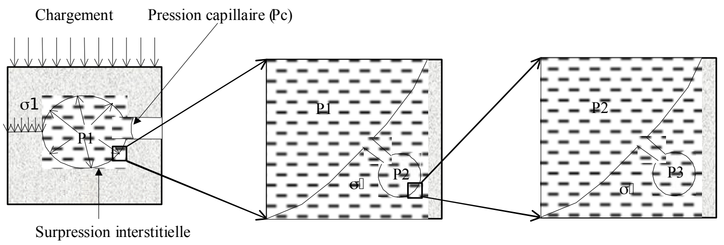


Figure 2.5.1.1-a : “Fractal” decomposition of the spherical viscoelastic mechanisms

One proposes to model these mechanisms of spherical creep by three levels (figure 2.5.1.1-a), each level representing a clean behavior.

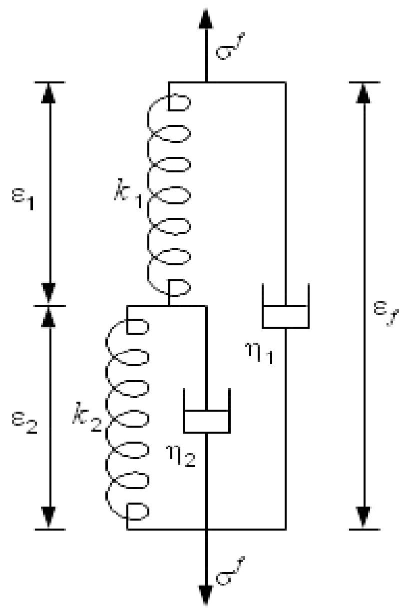


Figure 2.5.1.1-b : Spherical decomposition visco-élasto-plastic modelling spherical creep VEP

- Level 0: CE level is associated with the macroporosity subjected to the capillary pressure ($P_w = P_c < 0$) if the medium is not saturated, or with the pore water pressure ($P_w > 0$) if the model is used in a classical hydraulic approach. σ_0^s is thus the effective constraint on the solid skeleton. This level utilizes a purely elastic component of the concrete. It can thus be schematized and put in equation in the following way:

$$\sigma_0^s = k_0 \varepsilon_0^s \quad (3)$$

- Level 1: CE level corresponds to part of the microporosity slightly connected to the macroporosity open and prone to the reversible hydrous movements. Associated viscoelasticity is modelled by a solid of Kelvin (figure 2.5.1.1-b):

$$\sigma_1^s = k_1 \varepsilon_1^s + \eta_1 \dot{\varepsilon}_1^s \quad (4)$$

- Level 2:

$$\sigma_2^s = k_2 \varepsilon_2^s + \eta_2 \dot{\varepsilon}_2^s \quad (5)$$

Level 2 corresponds here to the interfoliaceous nanoporosity of the HSC. This level ensures the irreversible character of the viscous deformations of the HSC. In this last expression, k_2 is a kinematic module of work hardening and ε_2^s is a deformation irréversible. The rheological diagram makes it possible to establish the system of equation according to:

$$\dot{\varepsilon}_1^s = \left[k_2\left(\varepsilon_f^s - \varepsilon_1^s\right) - k_1 \varepsilon_1^s\right] / \eta_2 + \dot{\varepsilon}_f^s \text{ and } \sigma_f^s = \eta_1 \dot{\varepsilon}_f^s + k_1 \varepsilon_1^s \quad (6)$$

2.5.1.2 Creep deviatoric

In accordance with the observations of Acker and al. [Acker and al., 2003], Acker [Acker, 2001], Bernard and al. [Bernard and al., 2003], the viscoelastic aspect of the behavior deviatoric is allotted to the interfoliaceous shearing of the layers of HSC. This shearing occurs on two scales:

- A viscoelastic nanoscopic scale corresponding to water slightly related to the layers,
- A microscopic scale corresponding to free water slightly adsorbed between the layers and prone to desiccation.

Lastly, the part of the cement paste made up of better crystallized hydrates (portlandite and of hydrated calcium aluminate) has a quasi elastic behavior.
Deviatoric viscosity thus keeps a character multi scales, one then proposes to represent it in a way similar to the spherical branch, which moreover simplifies the implementation of the model (rheological equations similar to those of the deviatoric branch).
The Système managing the deformations of the rheological model deviatoric is written:

$$\dot{\varepsilon}_{1_{i,j}}^d = \left[k_2\left(\varepsilon_{f_{i,j}}^d - \varepsilon_{1_{i,j}}^d\right) - k_1 \varepsilon_{1_{i,j}}^d\right] / \eta_2 + \dot{\varepsilon}_{f_{i,j}}^s \text{ and } \sigma_{f_{i,j}}^d = \eta_1 \dot{\varepsilon}_{f_{i,j}}^d + k_1 \varepsilon_{1_{i,j}}^d \quad (7)$$

One can specify at this stage of the modeling which the systems of rheological equations admit an analytical solution for a loading in linear total deflection according to time. We will draw part of this solution in the numeric work implementation.

2.5.2 Module dedicated to the modeling of the anisotropic swelling of RAG (VD^T)

The module of VD^T (figure 2.4-a) model an increase in the unelastic deformation when the concrete is subjected to a tensile stress induced by RAG leading to the damage.
We know that in the presence of the RAG, the relation deformation – damage following must be respected (figure 2.4-a):

$$\varepsilon_i^{vdt} = \varepsilon_0 \frac{d_i^{RO}}{1 - d_i^{R0}} \quad (8)$$

In this equation, ε_0 is a parameter identified on a large number of tests and of the concretes of various natures [Saddler, 1999]. Usually, its value is close to 0.35%.

d_i^{R0} is a principal value of the tensor of the damages due to the tensile stresses induced by the RAG. Here, it will be evaluated in the following way:

$$d_i^{R0}=min\left\{d_i^t;1-\exp\left(-\frac{1}{m^t}\left(\frac{b_g.P_g}{\sigma^{ut}}\right)^{m^t}\right)\right\}$$

In this expression, d_i^t is a principal value of the tensor of the damages of traction. m^t and σ^{ut} are parameters of the law of evolution of the damage. We will reconsider the choice of the variables of damages and their law of evolution in the following chapter.

2.5.3 Assessment relating to the partition of the deformations

The modeling suggested for creep was tested on multiaxial tests. The confrontation of simulations with the experimental results watch capacity of the model of creep to reproduce the characteristics of the long-term behavior of the concrete. The next stage of modeling, consists in setting up a model of damage able to represent the behavior of the fissured concrete. This model will be then coupled with the clean model of creep and the model of swelling by RAG described up to now.

2.6 Model of damage

After having given the great principles of the model, we initially will point out the concepts thermodynamics on which we must support our formulation. In the second time, we propose the free potential energy and the complementary laws defining the model. This presentation makes it possible to check the conditions of dissipation imposed by the thermodynamics of the irreversible processes necessary to convergence of calculations.

2.6.1 Principle of modeling

The reaction alkali-aggregates causes swellings of the structure. These swellings can be very different according to the loadings from the damaged structures. The expansion considered isotropic in swelling not prevented becomes a strongly anisotropic expansion as soon as the concrete is subjected in a state of deviatoric stress [Multon, 2003; 2006]. The microfissuring generated by the intra-porous pressure is then prevented in the direction of loading and swelling appears systematically associated with the directions where the compressive stresses are lowest. This phenomenon is particularly visible in the beams where the cracks are directed the reinforcements parallel to. A mechanism of macro-cracking, due to the opening of the cracks in the direction where the material is damaged the most, amplifies this anisotropy. The microphone and generated macro-crackings lead to a fall of the elastic module of the concrete, this fall is usually modelled by the theory of the damage. We present below the principles of the model of damage which we will apply to the constraints resulting from the rheological diagram presented in the preceding chapter.
In order to simulate the behavior of the concrete degraded by the reaction alkali-aggregates, an anisotropic model of damage, based on a realistic criterion of cracking, must be used. This model of damage must be formulated within the framework of thermodynamics [Lemaitre, 2001]. The thermodynamic framework makes it possible to avoid possible inconsistencies of formulation which would lead to problems of physical representation and digital convergence. For this purpose, a thermodynamic potential of free energy from which derive the laws of behavior of material must be proposed. If mechanical dissipation can be uncoupled from other dissipations (thermics, hydrous or chemical), the condition of dissipation is given by the equation of Clausius-Duhem (equation Error: Reference source not found). This relation expresses the fact that the power of the “external forces” is at every moment higher than the restorable power stored by material; part of the power of the “external forces” being dissipated mechanically by decoherence, friction, etc. The equation of Clausius-Duhem for an isothermal mechanical transformation is written classically:

$$\sigma:\dot{\varepsilon}-\rho\,\dot{\psi}\geq 0 \qquad (9)$$

In this expression, σ is the tensor of the constraints, ε that of the total deflections and $\rho\,\psi$ voluminal free potential energy. This last is selected so that dissipation is worthless in the case of the perfectly elastic loading.

This restriction implies that the constraints σ derive from the free potential energy by the elastic strain ε^e , that is to say:

$$\sigma=\rho\frac{\partial\phi}{\partial\varepsilon^e} \qquad (10)$$

With the usual assumption of partition of the total deflection:

$$\varepsilon^e=\varepsilon-\varepsilon^{an} \qquad (11)$$

Where ε^{an} is the deformation unelastic. Two internal variables, bound by this equation of partition, are thus defined: unelastic deformation ε^{an} and thermoelastic deformation ε^e . In our model, ε^e is the deformation on level 0 of the rheological diagram; ε^{an} contains the deformations ε_{vdt} and ε_{vep} presented on figure 2.4-a as well as the thermal deformation if it is necessary.
The microphone and macro crackings generated by the constraints lead to a fall of the elastic module of the concrete, this fall is usually modelled by the theory of the damage. We present below the principles of a model of damage
The model suggested is based on a tensorial representation of the damage. This representation is formulated so that the unilateral aspect of the behavior can be treated simply and independently in each of the three principal directions of the effective constraints.
The tensor of the effective constraints is estimated starting from the principle of equivalence in elastic strain, on level 0 of the rheological diagram (figure 2.4-a):

$$\tilde{\sigma}=R^0:\varepsilon^e \qquad (12)$$

With R^0 the tensor of elasticity of fourth order of the healthy concrete, calculated starting from Young and the Poisson's ratio modulus of the concrete not fissured.
The apparent constraints are connected to the effective constraints by the tensor of the damages D :

$$\sigma=(I-D):\tilde{\sigma} \qquad (13)$$

In this expression, ($I-D$) a tensor of integrity, also of fourth order, made up on the one hand matrix identity (I) and in addition tensor of damage.
By treating the phenomenon of cracking starting from the tensor of damage of fourth order D , it free potential energy takes the following general shape:

$$\rho\psi=\frac{1}{2}(I-D):R^0:\varepsilon^e:\varepsilon^e \qquad (14)$$

In indicielle notation this last relation is written:

$$\rho\psi=\frac{1}{2}(I-D)_{ijkl}R^0_{klmn}\varepsilon_{mn}\varepsilon_{ij} \qquad (15)$$

Thus, them equations precedents result in writing the relation of Clausius-Duhem in the following form:

$$\sigma:\dot{\varepsilon}^{an}-\rho\frac{\partial\psi}{\partial D}:\dot{D}-\rho\frac{\partial\psi}{\partial V^{an}}:\dot{V}^{an}\geq 0 \qquad (16)$$

Where them V^{an} are the internal variables associated with the unelastic deformations. That is to say still with the indicielle notation:

$$\underbrace{\sigma_{ij}\dot{\varepsilon}^{an}_{ij}-\rho\frac{\partial\psi}{\partial V^{an}_{ij}}\dot{V}^{an}_{ij}}_{\phi^{an}}\underbrace{-\underbrace{\rho\left(\frac{\partial\psi}{\partial D}\right)_{ijkl}}_{Y_{ijkl}}\dot{D}_{ijkl}}_{\phi^{D}}\geq 0 \qquad (17)$$

This last relation makes it possible to define dissipation by unelastic deformation ϕ^{an} as well as dissipation by mechanical damage ϕ^D . It also makes it possible to define the thermodynamic forces Y_{ijkl} called rate of refund of energy.

A simple way to ensure the positive character of dissipation whatever the state of material and the loading is to apply the decoupling of dissipations by unelastic deformations and by damage, one must then check separately $\phi^{an}\geq 0$ and $\phi^{D}\geq 0$ that is to say still:

$$\begin{cases}\sigma_{ij}\dot{\varepsilon}_{ij}^{an}-\rho\dfrac{\partial\Psi}{\partial V_{ij}^{an}}\dot{V}_{ij}^{an}\geq 0\\ -Y_{ijkl}\dot{D}_{ijkl}\geq 0\end{cases} \tag{18}$$

In viscoelasticity, the condition of dissipation by unelastic deformation led to choose, for the laws of viscosity, of the plus coefficients.
In plasticity, one applies the existence of a potential pseudonym of dissipation (ϕ^{*an}). This last being a positive within the space of constraints and worthless convex function in the beginning, so that the positivity of dissipation is automatically checked. The law of flow of the unelastic deformations can be written in the form:

$$\dot{\varepsilon}_{ij}^{an}=\dot{\lambda}\frac{\partial\phi^{*an}}{\partial\sigma_{ij}} \tag{19}$$

Where $\dot{\lambda}$ is a positive scalar called plastic multiplier. This last is calculated so that the increase in the unelastic deformation leads to a relaxation of the constraints, this making it possible to check the criterion of plasticity, also expressed within the space of constraints.
Remain to check the condition of positivity of dissipation by damage. One can for that take as a starting point the formalism briefly described previously for plasticity. I.e. to give a potential pseudonym of convex and null dissipation at the origin within the space of rate of refund of energy from which the increments derive from the components of the tensor of damage. One has then the law of evolution of the damage following:

$$\dot{D}_{ijkl}=-\dot{\lambda}\frac{\partial\phi^{*D}}{\partial Y_{ijkl}} \tag{20}$$

The "multiplier of damage" $\dot{\lambda}$ in front of being calculated in order to check the criterion of damage. The construction of the potential pseudonym being difficultE ; the use of a more rational means, making it possible to ensure the coherent shape of the tensor of damage with the physical phenomena at the origin of the damage, is regularly used. For that, one calls on the theory of the homogenisation. The tensor of damage D there is estimated starting from the law of behavior of healthy material (R^{0}) and of that of a ground volume representative (WORM) of fissured material (R^{D}), it comes then:

$$D_{ijkl}=\delta_{ik}\delta_{jl}-R_{ijmn}^{0}R_{mnkl}^{D^{-1}} \tag{21}$$

The representativeness of the law of behavior, on damaged equivalent material will depend on the quality of the homogenisation of the WORM. The unilateral character of the behavior of the concrete is induced by the restoration of stiffness related to Re-closings of the cracks. In the case of a cyclic uniaxial loading with change of the sign of the constraint, one can note in experiments the creation of tension cracks perpendicular to the axis of loading and cracks of compression parallel with the axis of loading. The losses of rigidity in traction and compression are thus allotted to at least two orthogonal networks of cracks. When the constraint passes from traction to compression, the tension cracks are closed and the cracks of compression are activated. When the constraint passes from compression to traction, the tension cracks open and are activated in their turn.
To model "physiquemenT" the unilateral behavior of the concrete, it is convenient to reason starting from a tensorial description of the network of cracks. The losses of rigidity in traction or compression can thus be treated in each direction of space according to the sign of the constraints and the orientation of the cracks: if a principal constraint is positive, the most active cracks will be those whose orientation is overall perpendicular to the constraint, whereas if the principal constraint is negative, the most active cracks will be parallel to the constraint.

2.6.1.1 Anisotropic damage by traction

The various reasons briefly stated previously lead us to propose a model of damage based on a representation of the cracking of traction by a tensor of the second order estimated starting from the tensor of the effective constraints of traction $\tilde{\sigma}^t$ and not on the extensions. They is here the effective constraints within the meaning of the damage, they should not be confused with the effective constraints within the meaning of the mechanics of the porous environments which affect all the solid skeleton whereas the effective constraints within the meaning of the damage affect only the not damaged part of the solid skeleton.
The tensor of the effective constraints of traction is obtained starting from the effective constraints principal, themselves resulting from the application of the principle of equivalence in deformation to the rheological model used for the solid skeleton:

$$\tilde{\sigma}^t = \langle \tilde{\sigma}_i \rangle . \left(\vec{e}^i \otimes \vec{e}^i \right) \quad (22)$$

Where $\vec{e}^i$ is the clean vector associated with $\langle \tilde{\sigma}_i \rangle$ who is the positive part of the effective principal constraint $\tilde{\sigma}_i$.
By using the principle of equivalence in deformation stated by Lemaître, the effective constraints of traction make it possible to define the tensor of the associated elastic strain ε^{et} :

$$\varepsilon^{et} = \frac{1+\nu^0}{E^0} . \tilde{\sigma}^t - \frac{\nu^0}{E^0} tr\left(\tilde{\sigma}^t\right) . Id \quad (23)$$

The criterion of cracking retained for the damage of traction is that of Rankine, the tensor of the constraints thresholds is noted σ^R , the criterion translated in each principal direction $\vec{e}^i$ tensor of the effective constraints nonthe going beyond the normal component of the constraint threshold:

$$f_i = \langle \tilde{\sigma}_i \rangle - \sigma^R : \left(\vec{e}^i \otimes \vec{e}^i \right) \quad (24)$$

The actualization of the tensor of the constraints thresholds is in conformity with the condition of consistency which is written simply:

$$\sigma^R_{ii} = \sup\left(\langle \tilde{\sigma}_i \rangle_+ , \sigma^R_{ii} \right) \quad (25)$$

where them σ^R_{ii} are the terms of the diagonal of the tensor σ^R expressed in the principal base of the effective constraints.
By admitting that the “equivalent” cracks, and thus the damages have the same principal directions that the constraints thresholds, one can define the tensor of the damages in the base of the principal constraints of σ^R :

$$d^t = d^t_i . \left(\vec{v}^i \otimes \vec{v}^i \right) \quad (26)$$

The damage d^t represent a set of three orthogonal networks of plane cracks coexistent within the same representative ground volume. Terms $\vec{v}^i$ are the clean vectors of σ^R and d^t_i eigenvalues of the tensor of damage estimated starting from the eigenvalues σ^R_i of σ^R , and of the law of following evolution inspired by the law of Weibull [Saddler, 1995].

Equation 2.6.1.1-1 : Laws of evolution of the damage

$$d^t_i = 1 - \exp\left(\underbrace{ -\frac{1}{m^t} \left(\frac{\sigma^R_i}{\sigma^{ut}} \right)^{m^t} }_{\beta_i} \right)$$

In this expression m^t is a all the more large parameter as the material is fragile, σ^{ut} is also a parameter “material”, it is comparable to a cohesion, in practice m^t can be identified starting from the experimental damage measured starting from the peak of the law of behavior, σ^{ut} is directly connected to resistance. We will give the relations between the parameters of the model and the sizes measurable in the following paragraphs. By introducing the index of cracking defined by:

$$\beta_i = \frac{1}{m^t} \left(\frac{\sigma_i^R}{\sigma^{ut}} \right)^{m^t} \quad (27)$$

IL comes the Relation betweendamage and index of cracking:

$$\frac{1}{1-d_i^t} = e^{\beta_i} \quad (28)$$

This last relation will be practical for the construction of the free potential energy. The internal variables introduced on this level will also make it possible to memorize the state of cracking. It is also necessary, accordingly, to note that the damage is an increasing and continuous function of the index of cracking which is itself an increasing function of the constraint threshold. As the latter can only increase, the principal values of the tensor of damage can only grow.
With regard to the relations normal deformation-constraints, the passage of the tensor of damage with the law of behavior of damaged material must translate on the one hand a reduction in the Young modulus in the normal direction with the crack and on the other hand an attenuation of the effect Fish between the orthogonal direction with the plan of the crack and the directions contained in its plan. To observe these conditions, we propose to use the following law of behavior:

$$\varepsilon_{ii}^e = \frac{\sigma_{ii}^t}{E^0 (1-d_i^t)} - \frac{\nu^0}{E^0} (\sigma_{jj}^t + \sigma_{kk}^t) \quad (29)$$

That is to say still according to the indices of cracking:

$$\varepsilon_{ii}^e = \frac{\sigma_{ii}^t}{E^0} e^{\beta_i} - \frac{\nu^0}{E^0} (\sigma_{jj}^t + \sigma_{kk}^t) \quad (30)$$

In this expression E^0 is the Young modulus of healthy material and ν^0 its Poisson's ratio. The relation is written in the principal base of the damages. It should be noted that this law is expressed here according to the apparent constraints and not effective constraints, it is this particular writing which makes it possible to profit simply from an attenuation of the effect Fish according to cracking in the directions j and k as well as symmetry of the tensor of flexibility of the law of behavior. The relations slip-constraints of shearing are also written in the principal base of damage, the relation suggested is the following one:

$$\varepsilon_{ij}^e = \frac{\sigma_{ij}^t}{2\mu^0 (1-d_i^t)(1-d_j^t)} \quad (31)$$

That one can also express according to the indices of cracking:

$$\varepsilon_{ij}^e = \frac{\sigma_{ij}^t}{2\mu^0} e^{\beta_i + \beta_j} \text{ with } 2\mu^0 = \frac{E^0}{(1+\nu^0)} \quad (32)$$

The relation slip-constraint of shearing utilizes the damages on the two principal directions of cracking requested in shearing, the combination of the damages is of the standard weakest link. In addition to the interaction between damages which this writing gets, one can allot a statistical direction to him by considering that the damages are comparable to probabilities of meeting surface discontinuities on the facets [Saddler, 1995]. On this assumption, the theory of the weakest link indicates that the probability so that a shear stress forwards on two orthogonal facets is equal to the probability of having simultaneously two not broken orthogonal surfaces. By neglecting the statistical dependences between the two plans of cracking, this probability is equal to the product of the probabilities of having on each one of these two facets a continuity of material, one from of deduced the denominator fromequation (31). Lastly, the last reason for which we adopted this form for the damage of the coefficient of shearing is that the resulting law of behavior presents the answer expected to the test of rotation of the principal directions (known as test of Willam [Willam and al., 1987]).
To express the free potential energy according to the elastic strain, it is necessary to make sure of the inversibility of the relations stress-strains suggested previously. For that, let us express the apparent normal constraints according to the elastic strain:

$$\sigma_{ii}^{t}=\frac{E^{0}}{D^{n}}\left[\left(e^{\beta_j+\beta_k}-\nu^{0^2}\right)\varepsilon_{ii}^{et}+\nu^{0}\varepsilon_{jj}^{et}\left(\nu+e^{\beta_k}\right)+\nu^{0}\varepsilon_{kk}^{et}\left(\nu+e^{\beta_j}\right)\right] \quad (33)$$

with

$$D^{n}=e^{\beta_i+\beta_j+\beta_k}-\nu^{0^2}\left(e^{\beta_i}+e^{\beta_j}+e^{\beta_k}\right)-2\nu^{0^3}$$

It is shown that D^n is always positive if the two following conditions are checkedS:

$$\begin{cases}\beta_i\geq 0\ \forall\, i\in\{1,2,3\}\\ \nu\leq 0.5\end{cases} \quad (34)$$

The condition on the Poisson's ratio is checked, it is the same for the condition on β_i, taking into account the law of evolution of the damages proposed previously. Inversibility of the relation normal deformation - normal constraint is thus assured some is the level of damage.
With regard to the relations between the rates of shearing and shear stresses, the inversion is immediate and it comes:

$$\sigma_{ij}^{t}=\frac{E^{0}\varepsilon_{ij}^{et}}{\left(1+\nu^{0}\right)e^{\beta_i+\beta_j}} \quad (35)$$

As the denominator of this expression is understood in the interval $[1,+\infty[$ the inversibility is there too assured.
By integrating the relations stress-strains compared to the elastic strain associated with the effective constraints of traction, one obtains the elastic free potential energy associated with the effective constraints of traction. This noted potential $\rho\Psi^t$ thereafter is expressed here in the principal base of the damages:

$$\rho\Psi^{t}=\rho\Psi^{t(n)}+\rho\Psi^{t(s)} \quad (36)$$

With $\rho\psi^{t(n)}$ potential of the free energies associated with the extensions:

$$\rho\psi^{t(n)}=\frac{1}{2}\frac{E^{0}}{D^{n}}\left(\begin{array}{c}\left(e^{\beta_j+\beta_k}-\upsilon^{0^2}\right)\varepsilon_{ii}^{et^2}+\left(e^{\beta_i+\beta_k}-\upsilon^{0^2}\right)\varepsilon_{jj}^{et^2}+\left(e^{\beta_j+\beta_i}-\upsilon^{0^2}\right)\varepsilon_{kk}^{et^2}\\+2\upsilon^{0}\left(\varepsilon_{jj}^{et}\varepsilon_{ii}^{et}\left(\upsilon^{0}+e^{\beta_k}\right)+\varepsilon_{kk}^{et}\varepsilon_{ii}^{et}\left(\upsilon^{0}+e^{\beta_j}\right)+\varepsilon_{jj}^{et}\varepsilon_{kk}^{et}\left(\upsilon^{0}+e^{\beta_i}\right)\right)\end{array}\right) \quad (37)$$

And $\rho\psi^{t(s)}$ that associated with the elastic strain of shearing:

$$\rho\psi^{t(s)}=\frac{1}{2}G^{0}\left(e^{-(\beta_i+\beta_j)}\varepsilon_{ij}^{et^2}+e^{-(\beta_i+\beta_k)}\varepsilon_{ik}^{et^2}e^{-(\beta_j+\beta_k)}\varepsilon_{jk}^{et^2}\right) \quad (38)$$

The potential can be also expressed simply according to the apparent constraints σ^t associated with the effective constraints of traction. Maybe with regard to the potential associated with the extensions:

$$\rho\psi^{t(n)}=\frac{1}{2}\left(\frac{\sigma_{ii}^{t^2}}{E_i}+\frac{\sigma_{jj}^{t^2}}{E_j}+\frac{\sigma_{ii}^{t^2}}{E_i}+2\frac{\upsilon^{0}}{E^{0}}\left(\sigma_{ii}^{t}\sigma_{jj}^{t}+\sigma_{ii}^{t}\sigma_{kk}^{t}+\sigma_{kk}^{t}\sigma_{jj}^{t}\right)\right) \quad (39)$$

$E_i=E^0(1-d_i^t)$ being Young moduli of damaged material. And for the potential associated with shearings:

$$\rho\psi^{t(s)}=\left(1+\upsilon^{0}\right)E^{0}\left(\frac{\sigma_{ij}^{t^2}}{E_iE_j}+\frac{\sigma_{ik}^{t^2}}{E_iE_k}+\frac{\sigma_{kj}^{t^2}}{E_kE_j}\right) \quad (40)$$

Indices of cracking β_i being able to be connected in a bijective way to the principal damages of traction d_i^t, the expression of Clausius-Duhem can be expressed according to the indices of cracking. For more clearness we disregard here temporarily role of the unelastic deformations, by supposing an elastic transformation endommageable utilizing only the effective constraints of traction:

$$\sigma:\dot{\varepsilon}_{ij}^{et}-\left(\underbrace{\rho\frac{\partial\psi^{t(n)}}{\partial\beta_i}}_{-\phi^{Dt(n)}}\dot{\beta}_i+\rho\frac{\partial\psi^{t(n)}}{\partial R}:\dot{R}+\underbrace{\rho\frac{\partial\psi^{t(s)}}{\partial\beta_i}}_{-\phi^{Ft(s)}}\dot{\beta}_i+\frac{\partial\psi^{t(s)}}{\partial R}:\dot{R}+\rho\frac{\partial\psi^{t}}{\partial\varepsilon_I^{et}}\varepsilon_I^{et}\right)\geq 0 \quad (41)$$

In this expression, ε_I^{et} is a principal value of the tensor of the deformations associated with the effective constraints with traction, $\phi^{Dt(n)}$ is dissipation associated with the normal deformations and $\phi^{Dt(s)}$ that associated with shearings. R is the matrix of passage of the principal base of the elastic strain at the principal base of the damages, $\dot{R}$ can be interpreted as a rate of rotation of the tensor of damage if the principal directions of deformation do not turn. The taking into account of the form of the free potential energy associated with shearings then makes it possible to calculate dissipation due to the damage associated with shearings. It comes:

$$\phi^{Dt(s)\dot{\beta}}=\dot{\beta}_i\underbrace{\left(\frac{1}{2}G^0\left(e^{-(\beta_i+\beta_j)}\varepsilon_{ij}^{et^2}+e^{-(\beta_i+\beta_k)}\varepsilon_{ik}^{et^2}\right)\right)}_{\geq 0} \quad (42)$$

Cette expression is always positive since $\dot{\beta}_i$ is positive, the positivity of dissipation associated with shearings is thus checked.
With regard to dissipation associated with the normal deformations, one can notice that by using the expressions in constraint of $\psi^{t(n)}$ and of ε^{et} in the equation of Clausius-Duhem, it comes:

$$\sigma:\dot{\varepsilon}^{et}-\rho\dot{\psi}^{t(n)}=\left(\underbrace{\sigma:\frac{\partial\varepsilon^{et}}{\partial E_i}}_{-\frac{\sigma_i^{t^2}}{E_{i^2}}}-\underbrace{\rho\frac{\partial spi^{t(n)}}{\partial E_i}}_{-\frac{1}{2}\frac{\sigma_i^{t^2}}{E_{i^2}}}\right)\dot{E}_i=\underbrace{-\frac{1}{2}\frac{\sigma_i^{t^2}}{E_{i^2}}}_{\leq 0}\cdot\dot{E}_i\geq 0 \quad (43)$$

What implies that $\dot{E}_i\leq 0$. OR $\dot{E}_i=-E^0\dot{d}_i^t$ and $\dot{d}_i^t$ being positive, the second principle is also checked for the damage associated with the normal deformations.
In the case of the nonradial loading, the relation of Clausius-Duhem contains the terms due to the evolutions of the principal damages, which are always positive as we have just seen it, as well as terms due to rotations of the directions of damage. The latter translate a bringing together of the directions of orthotropism of material and those of the loading. From an analytical point of view, this second transformation is similar to a rotation of the principal directions of deformation without damage. It is thus done without dissipation, the following relation must thus be respected at every moment:

$$\left(\sigma:\frac{\partial\varepsilon^{et}}{\partial R}-\rho\frac{\partial\Psi^t}{\partial R}\right)\dot{R}=0 \quad (44)$$

CE which implies:

$$\sigma:\frac{\partial\varepsilon^{et}}{\partial R}=\rho\frac{\partial\Psi^t}{\partial\varepsilon^{et}}:\frac{\partial\varepsilon^{et}}{\partial R} \quad (45)$$

However, by construction, $\sigma=\rho\frac{\partial\Psi^t}{\partial\varepsilon^{et}}$, therefore this condition is identically observed and the rotation of the directions of damage does not involve complementary but induced dissipation a variation of the free potential energy compatible with the evolution of the loading.

We have just presented the principles of an orthotropic model of damage in effective constraint, this last enters within the thermodynamic framework and profits moreover from a method of direct digital resolution conformément (not of under-iterationS withX notS Gauss). Indeed, the calculation of the damage according to the effective constraint is direct, like that of the apparent constraint according to the effective constraint and of the damage.

2.6.1.2 Isotropic damage by compression

Under request of traction, the criterion usually used is that of Rankine (constraint principal of traction), this criterion gives an account of a propagation of cracks in mode I. In compression, the criteria of Mazars [Mazars, 1994] or of Drücker-Prager are usually employed; the criterion of Mazars utilizes the concept of extension of the WORM, the criterion of Drücker-Prager account of the unfavourable effect of the diverter of the constraints returns. In practice, the rupture in compression is induced by elastic heterogeneities leading to the appearance of local fields of auto--balanced constraints of which the positive part involves the starting and the propagation of cracks (figure 2.6.1.2-a). From their origins, these induced positive constraints can exist only in the orthogonal directions with the compressive stresses which generate them. When these auto-contraintes reaches the tensile strength a crack starts and the damage of compression appears. In the ordinary concretes, this damage appears in a progressive way and it is possible to observe multiple vertical cracks distributed on broad band localised and inclined of shearing (figure 2.6.1.2-b).

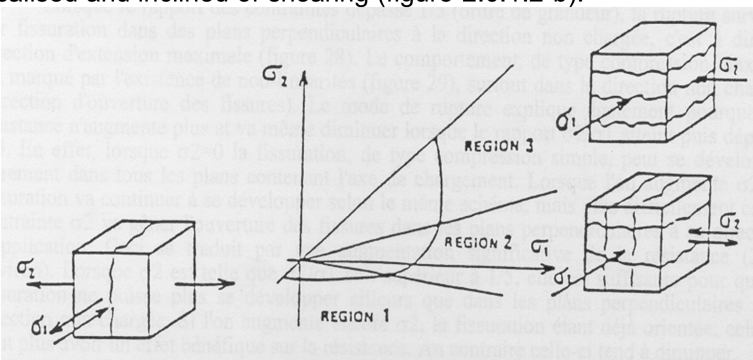


Figure 2.6.1.2-a : Facies of cracking according to the way of loading [Torrenti 1987]

The cracks parallel with the free edges are started in the vicinity of the aggregates by induced autocontraintes, they fissure the concrete forming of small "columns" which perish by side instability as shows it figure 2.6.1.2-b.

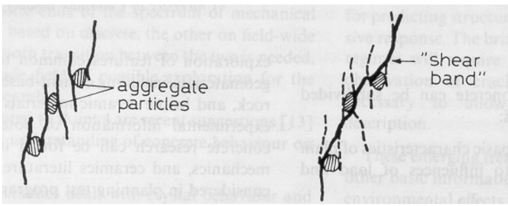


Figure 2.6.1.2-b: Appearance of "columns" in the vicinity of the aggregates [Torrenti 1987]

Thus, the propagation of the cracks of compression can be comparable to a phenomenon of multiple instabilities in compression along a band of shearing. However phenomena of instability mechanics are very closely related to the conditions of side maintenance of the compressed element. The instability always occurring in the direction of weaker maintenance, this explains the cracks parallel with the edges noncharged on figure 2.6.1.2-a.

In addition to the mode of rupture describes above, it is important to recall how much the compressive strength is sensitive to containment as the various biaxial and triaxial tests of the literature show it. Taking into account a share of the complexity of the mode of rupture in compression and in addition of the sensitivity to containment, it is usual to model the damage of the concrete in compression to adopt an isotropic macroscopic criterion of Drücker-Prager if one reasons in terms of constraints, or a criterion of Mazars if one reasons in term of deformations. We chose here to adopt the criterion of Drücker-Prager to be coherent with the formalism in effective constraints introduces at the time of the presentation of the model of damage of traction. The constraints at the origin of the damage of compression are thus supposed to be the negative effective constraints $\tilde{\sigma}^c$, these last are defined in the principal base of the effective constraints by:

$$\tilde{\sigma}^c = \tilde{\sigma} - \tilde{\sigma}^t \quad (46)$$

Where $\tilde{\sigma}$ is the effective constraint resulting from the rheological model and $\tilde{\sigma}^t$ the effective constraint of traction defined byequation (22). The equivalent constraint of Drücker-Prager is written then:

$$\sigma^{ceq}=\sqrt{J_2}+\delta_{\mathrm{hom}}\frac{I_1}{3} \tag{47}$$

Where I_1 and J_2 are them two first invariants of the tensor of the effective constraints of compression:

$$\begin{cases} J_2=\frac{1}{6}\left(\left(\tilde{\sigma}_1^c-\tilde{\sigma}_2^c\right)^2+\left(\tilde{\sigma}_1^c-\tilde{\sigma}_3^c\right)^2+\left(\tilde{\sigma}_2^c-\tilde{\sigma}_3^c\right)^2\right) \\ I_1=\tilde{\sigma}_1^c+\tilde{\sigma}_2^c+\tilde{\sigma}_3^c \end{cases} \tag{48}$$

The flow is carried out by considering that the constraint threshold of damage memorizes the maximum value of the constraint of Drücker-Prager:

$$\sigma^{DP}=\sup\left(\sigma^{DP},\sigma^{ceq}\right) \tag{49}$$

The damage evolves then not linearly according to the constraint threshold:

$$d^c=1-\exp\left(-\frac{1}{m^c}\left(\frac{\sigma^{DP}}{\sigma^{uc}}\right)^{m^c}\right) \tag{50}$$

The free potential energy associated with the effective constraints of compression is written very simply because of assumption of isotropy of the damage of compression:

$$\rho\,\psi^c=\left(1-d^c\right)\frac{1}{2}\left[\lambda^0\left(tr\left(\varepsilon^{ec}\right)\right)^2+2\mu^0 tr\left(\left(\varepsilon^{ec}\right)^2\right)\right] \tag{51}$$

Lhas dissipation associated with the damage with compression is written:

$$\phi^{dc}=\underbrace{-\rho\frac{\partial\psi^c}{\partial d^c}}_{Y^c}\dot{d}^c \tag{52}$$

The rate of refund of energy Y^c being negative and the damage strictly growing, the positivity of dissipation by damage of compression is assured.

2.6.1.3 Coupling of the damages of traction and compression

The isotropic character prevailing of the damage of compression is related to the complexity “quasi random and thus statistically isotropic” of the ways of cracking generated during the process of degradation. It is obvious that the cracks created will affect behaviour in traction. A way simple to give an account of the reduction in the elastic modules in traction according to the damage of compression is to consider that the isotropic damage of compression also comes to reduce the free potential energy of traction which becomes:

$$\rho\,\psi^{t*}=\left(\left(1-d^c\right)\underbrace{\left(\rho\,\psi^{t(n)}+\rho\,\psi^{t(n)}\right)}_{\rho\,\psi^t}\right. \tag{53}$$

The term $\rho\,\psi^t$ being positive, the positivity of dissipation by damage of compression remains assured:

$$-\rho\left(\frac{\partial\psi^c}{\partial d^c}+\frac{\partial\psi^{t*}}{\partial d^c}\right)\dot{d}^c\geqslant 0 \tag{54}$$

And the term $\left(1-d^c\right)$ being also positive, dissipation by damage of traction remains positive.

Free potential energy associated with the macroscopic elastic strain

The partition of the deformations according to the sign of the effective constraints being made equations (22) and (23), the free potential energy is the sum of the free energies associated with the effective constraints of traction on the one hand and the effective constraints of compression on the other hand:

$$\rho\,\psi=\left(1-d^c\right)\underbrace{\left(\rho\,\psi^{t(n)}+\rho\,\psi^{t(s)}\right)}_{\rho\,\psi^t}+\rho\,\psi^c \quad (55)$$

Free potential energy associated with an expansive internal chemical reaction

If the model integrates a pressure (P_g) created by the expansive chemical reaction of advance A , the free potential energy, at imposed temperature and a given moment, becomes a function of A :

$$\rho\,\psi\left(\varepsilon^{ec},\varepsilon^{et},A,d^t,d^c,V^{an}\right) \quad (56)$$

In this expression V^{an} represent the internal variables associated with the unelastic phenomena. The inequality of Clausius-Duhem thus becomes:

$$\sigma:(\dot{\varepsilon}^e+\dot{\varepsilon}^{an})+P_g(\dot{A}\,V^g)-\rho\frac{\partial\psi}{\partial\varepsilon^e}\dot{\varepsilon}^e-\rho\frac{\partial\psi}{\partial d^t}:\dot{d}^t-\rho\frac{\partial\psi}{\partial d^c}\dot{d}^c-\rho\frac{\partial\psi}{\partial A}\dot{A}-\rho\frac{\partial\psi}{\partial V^{an}}\dot{V}^{an}\geq 0 \quad (57)$$

In this expression, the term $P_g\left(\dot{A}\,V^g\right)$ is the mechanical power brought to the solid skeleton by the chemical reaction. If one considers a physical transformation of the WORM without unelastic damage nor deformation, therefore without mechanical dissipation, this energy must be completely regained in the free potential energy, which is written:

$$P_g(\dot{A}\,V^g)=\rho\frac{\partial\Psi}{\partial A}\dot{A} \quad (58)$$

On the other hand the physical considerations lead us to express the pressure of freezing according to the voluminal fraction of matter in excess compared to the volume of the vacuums connected to the reactive site on the one hand and of the total deflections in addition, equation (1).
The respect of the two preceding relations results in proposing for the share of the potential due to freezing the following form:

$$\Psi^g=\frac{1}{2}M^g b^{g^2}\left(\frac{A-A^0}{b^g}V^g-tr(\varepsilon^e+\varepsilon^{an})\right)^2 \quad (59)$$

This form of potential is close to that proposed by Ulm et al. [Ulm and al., 2002]. The derivative of this potential compared to the elastic strain gives the share of the macroscopic constraint induced by the internal pressure of freezing in the solid skeleton:

$$\pi^g=-b^g P_g=\rho\frac{\partial\Psi^g}{\partial\varepsilon^e} \quad (60)$$

2.6.1.4 Free potential energy associated with the hydrous pressures

In saturated medium, the pressures will intra porous act on the solid skeleton by deforming the walls of the porous network; the equation of Clausius-Duhem is written, in the absence of damage and of unelastic phenomena:

$$-\left(P_w-P_{w0}\right)\dot{\varphi}^w-\rho\frac{\partial\Psi}{\partial\varphi^w}\dot{\varphi}^w=0 \quad (61)$$

In this expression $\varphi^w=\dfrac{m^w}{\rho^{w0}}$ is contribution of the standardized water mass, m^w being the contribution of mass and ρ^{w0} density of the water defined in the pressure of reference P_{w0} . In addition the physical considerations result in expressing the water pressure in the form [Coussy, 2002]:

$$P_w-P_{w0}=M^w\left(\varphi^w-b^w tr\left(\varepsilon\right)\right) \quad (62)$$

An acceptable form for the free potential energy associated with these equations is then:

$$\rho\,\psi^{w}=\frac{1}{2}M^{w}b^{w^2}\left(\frac{\varphi^{w}}{b^{w}}-tr(\varepsilon)\right)^2 \quad (63)$$

The derivative of this potential compared to the elastic strain gives the share of the macroscopic constraint induced by the pressure of water:

$$\pi^{w}=\rho\frac{\partial\psi^{w}}{\partial\varepsilon^{e}}=-b^{w}\left(P_{w}-P_{w0}\right) \quad (64)$$

If the state of reference is defined for material saturated with the atmospheric pressure $\left(P_{w0}=0\right)$, then the preceding expression is valid only if the fluid contribution of mass is positive. In the contrary case, the concrete die-is saturated and the hydrous pressure is not managed any more by relative compressibilities of the various phases but by the capillary phenomena. Average hydrous pressure P_{w} is thus a function of the capillary pressure and degree of saturation S^{w}. In addition to the variation of pressure in the water induced by the capillary phenomena, it is also necessary to consider the action of the interfaces of the fluid with the gaseous medium and the solid. Indeed, these interfaces are prone to the phenomena of capillary tension which act they-also directly on the solid skeleton. If the gas pressure intra porous is in balance with the atmospheric pressure [Coussy, 2002], the share of the macroscopic constraint induced by these two phenomena can be put in the following general form:

$$\pi^{w}=b^{w}f^{w(S^{w})}P^{c} \quad (65)$$

Where f^{w} taking of account on the one hand the reduction in the volume of water is a function and on the other hand the increase amongst solid sites subjected to the surface tension of interface. In present modeling, we are interested in the works located in partially saturated zone, in this case we propose to use a function f^{w} linear of the form $k^{w}\cdot S^{w}$, the contribution of the hydrous effects in the macroscopic constraint has the following form then:

$$\pi^{w}=-b^{w}S^{w}k^{w}P^{c}=\rho\frac{\partial\psi^{w}}{\partial\varepsilon^{e}}\ \text{ with }\ S^{w}=\frac{\varphi\,\rho^{w0}+m^{w}}{\rho^{w0}}=\varphi+\varphi^{w} \quad (66)$$

By considering that variation of porosity φ had with the deformation remains small compared to φ^{w}, one can admit that for the medium unsaturated $\varphi\approx\varphi^{0}$. If in addition, it is admitted that the capillary curve of pressure $P^{c}(S^{w})$ is almost insensitive with the state of deformation of the solid skeleton, then:

$$\pi^{w}\approx b^{w}\left(\varphi^{0}+\varphi^{w}\right)k^{w}P^{c(\varphi^{0}+\varphi^{w})}=\rho\frac{\partial\psi^{w}}{\partial\varepsilon^{e}} \quad (67)$$

If the capillary curve of pressure admits an equation of the type “Van Genuchten then” π^{w} takes the following shape:

$$\pi^{w}=\text{-}b^{w}\left(\varphi^{0}+\varphi^{w}\right)\left(a\left(\left(\varphi^{0}+\varphi^{w}\right)^{-b}-1\right)^{\left(1-\frac{1}{b}\right)}\right) \quad (68)$$

Where a and b are two constants of chock.

The free potential energy can then be put in the form:

$$\rho\,\psi^{w}=-b^{w}\left(\varphi^{0}+\varphi^{w}\right)\left(a\left(\left(\varphi^{0}+\varphi^{w}\right)^{-b}-1\right)^{\left(1-\frac{1}{b}\right)}\right)tr(\varepsilon) \quad (69)$$

The thermodynamic force associated with the variation of the water mass is obtained by derivation of this potential compared to φ^{w}.

2.6.1.5 Dissipation due to the unelastic deformations

The equation of Clausius-Duhem reveals the unelastic deformations ε^{an} in the term $\sigma:\dot{\varepsilon}^{an}$. It is the power of the efforts external of the unelastic field of deformation, the evolution of these deformations induces an evolution of the associated internal variables V^{an} possibly causing a variation of the free potential energy:

$$\rho\frac{\partial\phi}{\partial V^{an}}\dot{V}^{an} \tag{70}$$

This last term represents the elastic energy blocked in the solid skeleton by the unelastic deformation. In our model the unelastic deformations have a viscoelastic or viscoplastic origin and can directly be used as internal variablesN, is $V^{an}=\varepsilon^{an}$.
Thus, in the typical case of a purely unelastic transformation, dissipation is summarized with:

$$\phi^{an}=\sigma:\dot{\varepsilon}^{an}-\rho\frac{\partial\Psi}{\partial\varepsilon^{an}}\dot{\varepsilon}^{an}\geq 0 \tag{71}$$

In the model suggested here we adopted the principle of partition of the partly elastic deflection total and unelastic part, which makes it possible to express the elastic strain according to the unelastic deformations and to conclude that the thermodynamic force associated with the unelastic deformation is the constraint:

$$\rho\frac{\partial\Psi}{\partial\varepsilon^{e}}=-\rho\frac{\partial\Psi}{\partial\varepsilon^{an}}=\sigma \tag{72}$$

The second principle is thus checked if $\sigma:\dot{\varepsilon}^{an}\geq 0$. In the model studied here, the unelastic deformation associated with long-term creep or the expansive chemical reaction is proportional to the constraint, which one can write in shortened form $\dot{\varepsilon}^{an}=k\,\sigma$, where k is a plus coefficient depend on the internal variables for each component of the tensor of the deformations. Thus unelastic dissipation due to the long-term deformations is a viscous dissipation of the form $k\,\sigma^2\geq 0$. Concerning the deformation of RAG, we authorize his evolution only if the dissipation calculated numerically on the step of time is indeed positive.

2.6.1.6 Total potential of free energy and laws of state

The free potential energy of the solid skeleton is obtained by summoning the energy contributions of the various phenomena studied previously:

$$\psi=\psi^{t^*}+\psi^{c}+\psi^{g}+\psi^{w} \tag{73}$$

The laws of state are obtained by derivation of the total potential:

$$\sigma=\rho\frac{\partial\Psi}{\partial\varepsilon^{e}} \tag{74}$$

If one expresses the constraints in the principal base of the damages of traction, it comes for the diagonal terms from the tensor from the constraints:

$$\begin{aligned}\sigma_{ii} \quad = \quad & \left(1-d^{c}\right)\frac{E^{0}}{D^{n}}\left[\left(e^{\beta_j+\beta_k}-\nu^{0^2}\right)\varepsilon_{ii}^{et}+\nu^{0}\varepsilon_{jj}^{et}\left(\nu+e^{\beta_k}\right)+\nu^{0}\varepsilon_{kk}^{et}\left(\nu+e^{\beta_j}\right)\right]\\ & +\left(1-d^{c}\right)\left[\lambda^{0}\mathrm{tr}\left(\varepsilon^{ec}\right)+2\mu^{0}\varepsilon_{ii}^{ec}\right]-\left(1-d^{c}\right)\left(b^{g}P_{g}+b^{w}P_{w}\right)\end{aligned} \tag{75}$$

and for the terms except diagonal:

$$\sigma_{ij}=\left(1-d^{c}\right)2\mu^{0}\left(\frac{\varepsilon_{ij}^{et}}{e^{\beta_i+\beta_j}}+\varepsilon_{ij}^{ec}\right) \text{ with } i\neq j \tag{76}$$

Withvec $D^{n}=e^{\beta_i+\beta_j+\beta_k}-\nu^{0^2}\left(e^{\beta_i}+e^{\beta_j}+e^{\beta_k}+2\nu^{0}\right)$ and $e^{\beta_i}=\frac{1}{1-d_i^{t}}$

Let us recall that the elastic strain of traction ε^{et} and of compression ε^{ec} result from two successive decompositions, the partition of the total deflection on the one hand:

$$\varepsilon=\varepsilon^{e}+\varepsilon^{an}+\varepsilon^{th}+\varepsilon^{0} \quad (77)$$

and in addition of the partition function of the sign of the effective constraint within the meaning of the damage:

$$\varepsilon^{e}=\varepsilon^{et}+\varepsilon^{ec} \quad (78)$$

And the equation of Dthermal eformation:

$$\varepsilon^{th}=\alpha^{\theta}\left(T-T_{0}\right)I \quad (79)$$

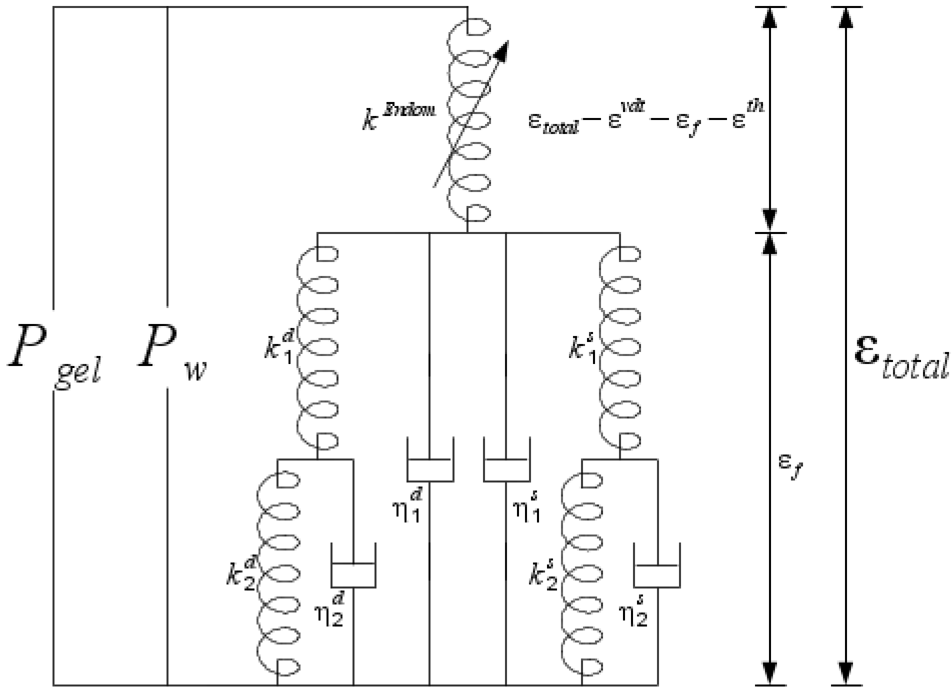
Inequation (77), ε^{th} is the deformation due to classically definite thermal dilation byequation (79) where α^{θ} is the thermal dilation coefficient, T the temperature and T_0 the temperature of reference. Let us note that inequation (77), a deformation of reference ε^{0} is also present, it acts of a deformation of nonworthless reference corresponding in a free state of stress. It can be associated only with one voluminal variation with the solid skeleton of chemical origin (chemical withdrawal dependent of the hydration for example). Indeed, the deformations of hydrous withdrawal or swelling by internal expansive products are manifestations of the pressures will intra porous appearing via the elastic strain and unelastic, they should not thus be confused with this “imposed chemical deformation of reference” which is not accompanied by internal pressures and cannot thus create damage in free deformation.
The elastic strain and unelastic are obtained by resolution of the system of the rheological equations presented to the preceding chapter. If the resolution of the system of equations of the rheological model is direct (without nonlinearity requiring of under digital iterations at the point of Gauss in the implementation finite elements), then the resolution of the whole of the model is direct for the data of the field of total deflection. Let us recall that this aspect of the model answers the objective of facility of implementation and reduction of the computing time that we initially fixed ourselves. table 2.6.1.6-1 below recapitulates the variables.

Table 2.6.1.6-1 : Thermodynamic variables.

Variables of state		Associated variables
Observable	**Interns**	
ε		σ
T		S
	ε^{e}	σ
	ε^{an}	$-\sigma$
	d^{t}	Y^{t}
	d^{c}	Y^{c}
	AV_{g}	P_{g}
	ϕ^{w}	P_{w}

2.6.2 Coupling of the model of damage and the rheological model

The rheological model and the model of damage having been described individually in the preceding paragraphs, they must now be connected. As we mentioned in the preceding paragraph, the effective constraints and the deformations to which it is refers in the chapter devoted to the model of damage result from the rheological diagram. The coupling between the viscoelastic phenomena and the damage thus consists in defining on the rheological diagram the effective constraints and the deformations to use to calculate the damage. Currently, the effective constraints used in the model of damage result directly from the rheological model for which the integration of equations (12) and (13) is realized with the viscoelastic characteristics of healthy material. The coupling between the damage and rheology is thus immediate. Indeed, if the damage increases during a creep test, then the effective constraints will increase in the rheological model, leading to an increase the speed of creep.



Béton_RAG : Phénomènes

3 Description of the internal variables

The following table gives the correspondence between the number of the internal variables accessible by code_aster and their description:

Constraints thresholds of damage (1 to 7)

- Tensor of the thresholds of damage in traction (6 components):
  BR_SUT11 BR_SUT22 BR_SUT33 BR_SUT12 BR_SUT13 BR_SUT23
- Threshold of damage compression (1 scalar)
  BR_SIGDP

Deformations of creep (8 to 14)

- Tensor deviatoric (6 components)
  BR_EFU11 BR_EFU22 BR_EFU33 BR_EFU12 BR_EFU13 BR_EFU23
- Spherical part (1 scalar)
  BR_EFUSP

Deformations of internal creep (15 to 21)

- Tensor deviatoric (6 components)
  BR_EP111 BR_EP122 BR_EP133 BR_EP112 BR_EP113 BR_EP123
- P artie spherical (1 scalar)
  BR_EP1SP

Chemical advance (22)

BR_AVCHI

Viscoplastic damage of the RAG (23 to 25)

BR_DVRAG1 BR_DVRAG2 BR_DVRAG3

Deformations viscoplasticS hadS with the RAG (26 to 31)

BR_EPV11 BR_EPV22 BR_EPV33 BR_EPV12 BR_EPV13 BR_EPV23

Pressure of freezing (32)

BR_PRGEL

Study carried out (33: phenomenon taken into account)

BR_ETUDE

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5 Features and checking

The law of behavior BETON_RAG (keyword COMPORTEMENT of STAT_NON_LINE) and its associated material (order DEFI_MATERIAU) is checked by the following tests :

C OMP003	Test of behaviors specific to the concretes. Simulation on a material point. Tests with Pa and MPa and rotation of the axes.	[V6.07.103]
SSNV400A	Tensile test in deformation imposed, on a free cube to become deformed in the direction perpendicular to the loading.	[V6.04.400]
SSNV400B	Essai of compression in deformation imposed, on a free cube to become deformed in the direction perpendicular to the loading.	[V6.04.400]
SSNV400D	Essai of traction in deformation imposed, on a confined cube (blocked transverse deformations)	[V6.04.400]
SSNV400E	Test of compression in deformation imposed on a confined cube (blocked transverse deformations)	[V6.04.400]
SSNV401A	Creep test in compression with a constraint forced on a free cube to become deformed in the direction perpendicular to the loading	[V6.04.401]
SSNV401B	Essai of creep in compression with an imposed deformation on a free cube to become deformed in the direction perpendicular to the loading.	[V6.04.401]
SSNV401D	Creep test in compression with a constraint imposed on a confined cube (blocked transverse deformations).	[V6.04.401]
SSNV401E	Creep test in compression with a deformation imposed on a confined cube (blocked transverse deformations).	[V6.04.401]