Law of behavior ENDO_HETEROGENE

Summary:

This documentation presents the theoretical writing and the digital integration of the law of behavior ENDO_HETEROGENE who takes into account the effect of the heterogeneity of the rock on his probability of rupture. This model was integrated like a law of nonlocal damage (with gradient of constraint) based on a model into two thresholds.

Contents

1 Introduction – Scope of application....3
2 Some general information on the probabilistic models of formation of networks of cracks....3
3 Development of law ENDO_HETEROGENE....4
3.1 Crack initiation....5
3.2 Propagation of the cracks....6
3.3 Coordination of the thresholds....8
4 Digital integration....9
4.1 Initialization....9
4.2 Resolution of the total problem....10
4.3 Implementation of the law of behavior....12
4.4 Parameters materials necessary to the law....15
4.4.1 Keywords....15
4.4.2 The Councils of identification....15
4.5 Description of the internal variables....15
5 Features and checking....16
6 Bibliography....16
7 Description of the versions of the document....17

1 Introduction – Scope of application

The law of behavior ENDO_HETEROGENE aim at modelling the heterogeneous material damage and was developed specifically for the géomatériaux ones (for example mudstone). This law is based on a probabilistic model of fracturing of the rocks developed within the framework of a collaboration between the BRGM and LMT-Cachan around the thesis of Nicolas Guy [bib1]. This model takes into account the effect of the heterogeneity of the rock on its probability of rupture. The initial distribution of microphone-defects is indeed given according to this heterogeneity starting from a classical Fish-Weibull model. The model rests then on the consideration of two threshold: a threshold of starting and a threshold of propagation based on the stress intensity factor regularized.

The regularization of the constraints allows us:

- to get results independent of the space discretization
- to evaluate the two thresholds starting from the same size, i.e., the regularized constraint
- to calculate the factor of intensity of the constraints “regularized” without refinement of the grid and with a good precision.

Finally, this model provides a way of cracking explicitly and takes account of the interaction of the cracks the ones compared to the others.

2 Some general information on the probabilistic models of formation of networks of cracks

As an introduction, we specified here the framework in which this model is developed and point out some basic concepts. In order to explain the formation of cracks, it is supposed that they start all on preexistent defects, modelled by cracks having a given size. In other words, the beginning of the propagation is regarded as starting. The model of Weibull is largely used to study the degradation and the rupture of materials with fragile behavior [bib8] or quasi fragile [bib3]. During the rupture of a test-tube of rock, one can consider that it is the rupture of the most critical defect, the local rupture, which involves the rupture of the structure, i.e., the total rupture. In other words, the rupture of the weakest link involves the chain breakage (i.e., the structure). In the framework of the assumption of the weakest link, the probability of total rupture P_F of a field Ω is connected to the probability of rupture P_{F0} of each element Ω_0 of surface S_0, by:

$$P_F = 1 - \exp\left[\frac{1}{S_0}\int_\Omega \ln(1-P_{F0})\, d\Omega\right] \quad (2.1)$$

Cumulated probability of the local rupture P_{F0} can be written for example starting from a law of Weibull [bib9]:

$$\ln(1-P_{F0}) = -\left(\frac{\langle \sigma_1 - \sigma_u \rangle}{\sigma_0}\right)^m \quad (2.2)$$

This formalism of Weibull is largely developed in documentation [R7.02.06].

One just recalls that for a given crack of size a and in mode I, the expression of the factor of intensity of the constraints can be rewritten in the following way:

$$K = Y \sigma \sqrt{a} \quad (2.3)$$

If one looks at application of a rock mechanics type, one can show that a network of cracks can be formed in a geological medium (around an cell for example) following the application of an internal pressure, if a crack can start [bib5]. This is due to the reduction in the factor of intensity of the constraints when the size of a crack becomes important compared to the diameter of the cell. The calculation of factor of intensity of the constraints for a crack leading to the interior edge of a hollow roll subjected to an internal pressure p_{int} watch two tendencies different according to the size from the crack [bib1]. In the first part, the effect of the size of crack compared to the level of the pressure applied is dominating and K increase with celleci. On the other hand in the second part, the effect of the reduction in the level of pressure applied to the crack becomes more important than the effect of the increase in its size and K decrease according to the size of crack. A crack can start since K reached tenacity (K_{IC}). The crack is propagated by increasing the internal pressure. When the interior pressure arrives at its maximum value, the crack continues to be propagated and stops when the value of K becomes lower than K_{IC} by neglecting the dynamic effects.

These results show that an isolated crack is propagated until a given size for which the value of the factor of intensity of the constraints becomes lower than the tenacity of the rock. When a first crack stops, another can start on a second site of starting. The second crack will stop with the same length that the first, a third crack can start and so on. That means that network of cracks can be formed.

Let us suppose that all the cracks start starting from initial defects distributed in a random way in the solid mass. A specific process of Fish is considered to describe the distribution of the sites of starting. Among all these potential sites of starting of crack, only a fraction will give rise to cracks. When the value of the factor of intensity of the constraints for a defect (a crack) exceeds a value criticizes (K_{IC}), the crack starts to be propagated. When a crack is initiated, the presence of this crack creates a stress relaxation zone around this one. To explain the starting of a crack, it is necessary to model the interaction of the zones (i.e., volume, surface or the length) affected by the reduction in constraint and the other sites which can create cracks. If the site is in a zone of stress relaxation, the microscopic constraint is lower than the macroscopic constraint applied. In this case, this site cannot give rise to a new crack.

In this approach, the zone of obscuration (i.e., stress relaxation) is the key quantity to describe the formation of a network of cracks. The zone of obscuration is defined as a zone in which the level of constraint is lower than the pressure applied. The form and the size of this zone can be defined using the digital simulations. It is said that the potential defects are écrantés by the close cracks.

3 Development of law ENDO_HETEROGENE

We made the choice in *Code_Aster* to represent the formation and the propagation of the cracks by an isotropic model of integrated damage. The mixing rate damage-elasticity being given classically by:

$$\sigma=(1-d)\underline{\underline{C}}\,\varepsilon \tag{3.1}$$

With σ the tensor of the constraints, ε the tensor of deformation, $\underline{\underline{C}}$ the isotropic tensor of elasticity and d the variable of damage.

One places oneself within the framework recalled in the section 2. The presence of cracks in the structure is modelled by lines of broken elements (i.e for which the damage D is such as $d=1$). For the healthy elements, $d=0$. Rupture of the elements being able to be caused by two phenomena here distinct; namely starting of a new crack starting from the element considered or the propagation through the element considered of a crack having started in another element. These two types of rupture are described by two types of functions different thresholds in this approach.

In addition, for laws of behavior corresponding to fragile materials the introduction of a heterogeneity of material projected on the grid causes to introduce a dependence with the size zone (and this same with the use of classical models not-rooms [bib1]). Moreover, one local law of damage cannot describe a fragile behavior of the standard weakest link.
In order to cure this problem, it is proposed here of:

1) to amend the local law to represent a brittle fracture of the standard weakest link in a heterogeneous material

2) to introduce an operator of regularization based on the gradient of the constraints.

It is shown that a not-local description with gradient of the constraints is on the one hand, very near to the models not-rooms to gradient of deformations and on the other hand allows us to correctly calculate the factor of intensity of the constraints which is the key parameter to describe the propagation of the macro-cracks. One introduces a constraint regularized such as

$$\bar{\underline{\sigma}}-l_c^2\Delta\bar{\underline{\sigma}}=\underline{\sigma} \quad (3.2)$$

With $\bar{\underline{\sigma}}$ the tensor of the regularized constraints and l_c the characteristic length. One will consider the boundary conditions of Neumann known as natural to know

$$(\nabla\bar{\underline{\sigma}}).\boldsymbol{n}=0 \quad (3.3)$$

with $\boldsymbol{n}$ the outgoing normal at the edge considered. One considers a perfectly fragile law of behavior of the type:

$$d=H\left(\langle\bar{\sigma}_1-\sigma_y(el)\rangle\right),\dot{d}\geq 0 \quad (3.4)$$

Where σ_y is a constraint associated with the elastic limit.

The two thresholds will thus be written according to a constraint regularized over a characteristic length l_c who will be tall in front of the size of the elements. One will endeavour in the present part to justify the choice of the two thresholds as well as the way in which they coordinate each other.

3.1 Crack initiation

Like known as previously, the starting of new macro-cracks is represented by a model of Weibull. In order to describe in a relevant way the step of cracking, a dispersion on the parameters of the material (module of Weibull, parameter of scale) specific to each finite element is introduced. By carrying out as a preliminary a lottery on a uniform distribution for each finite element in order to draw a value from it from probability of rupture of each element $P(el)\in[0;1]$, one can describe the constraint for which a crack will start in a finite element by the following equation:

$$\sigma_a(el)=\frac{\sigma_0}{\left(\lambda_0 Z_{el}\right)^{1/m}}.\left[-\ln\left(1-P(el)\right)\right]^{\frac{1}{m}} \quad (3.1.1)$$

With Z_{el} size (volume or surface according to dimension) of the finite element, m the modume of Weibull and $\frac{\sigma_0^m}{\lambda_0}$ the parameter of scale.

The use of a model of Weibull to describe the starting of new cracks has an unquestionable advantage in order to get results independent of the grid. Let us consider two partitions different from the field Z_{do} each one constituted by under-fields Z_{di}^1 and Z_{di}^2. One can write:

$$\sum_{i=1}^{n1} Z_{di}^1=Z_{do}=\sum_{i=1}^{n2} Z_{di}^2 \quad (3.1.2)$$

where $n1$ and $n2$ the numbers of the under-fields in both cases mean. Probability of starting of the first crack of a under-field for a request σ is written:

$$P_{asdi}=1-\exp\left[-Z_{sd}\lambda_0\left(\frac{\sigma}{\sigma_0}\right)^m\right] \quad (3.1.3)$$

where Z_{sd} is the size (volume) of a under-field. Probability of starting the first crack on the whole of the under-fields P_{aesd1} for a request σ can be deduced about it:

$$P_{aesd1}=1-\prod_{i=1}^{n_{sd}}\left[1-P_{asdi1}\right]=1-\exp\left[-\sum_{i=1}^{n_1} Z_{di}^1\lambda_0\left(\frac{\sigma}{\sigma_0}\right)^m\right]=1-\exp\left[-\sum_{i=1}^{n_2} Z_{di}^2\lambda_0\left(\frac{\sigma}{\sigma_0}\right)^m\right]=P_{aesd2} \quad (3.1.4)$$

Because:

$$\sum_{i=1}^{n_1} Z_{di}^1=\sum_{i=1}^{n_2} Z_{di}^2=Z_{do} \quad (3.1.5)$$

The equation (3.1.1) we allows to find a probability of starting of the first crack on a field of size Z_{do} independent of cutting under-fields.

Finally one can write the law of evolution of the damage D related to the starting of a new crack in an element:

$$d=H\left(\langle\bar{\sigma}_1-\sigma_a(el)\rangle\right),\dot{d}\geq 0 \quad (3.1.6)$$

where H is the function level of Heaviside and $\bar{\sigma}_1$ the maximum principal regularized constraint.

3.2 Propagation of the cracks

We will show in this part, that the not-local approach suggested enables us to correctly calculate the factor of intensity of the constraints for cracks modelled like a series of broken elements. Indeed, it is possible to bind a criterion in factor of intensity of the constraints, resulting from the breaking process to the regularized constraint. One can consider that the asymptotic solution of stress field at a peak of crack is a good approximation of the fields which one will be able to find to the forefront of a crack in propagation. The stress field given by the solution of Westergaard can be written in a reference mark at a peak of crack as presented on the Figure 3.2-a like a composition of elastic fields (for a problem plan):

$$\underline{\underline{\sigma}}(r,\theta)=K_I\underline{\underline{f_I}}(r,\theta)+K_{II}\underline{\underline{f_{II}}}(r,\theta) \quad (3.2.1)$$

With K_I, K_{II} stress intensity factors in mode I and II, and $\underline{\underline{f_I, f_{II}}}$ associated functions weight (equation 3.2.2 and 3.2.3) in mode I and II.

$$\underline{\underline{f_I}}(r,\theta)=\frac{1}{\sqrt{2\pi r}}\begin{bmatrix}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) & \sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2} & 0\\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2} & \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) & 0\\ 0 & 0 & 0\end{bmatrix} \quad (3.2.2)$$

$$\underline{\underline{f_{II}}}(r,\theta)=\frac{1}{\sqrt{2\pi r}}\begin{bmatrix}-\sin\frac{\theta}{2}\left(2-\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) & \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) & 0\\ \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) & \sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2} & 0\\ 0 & 0 & 0\end{bmatrix} \quad (3.2.3)$$

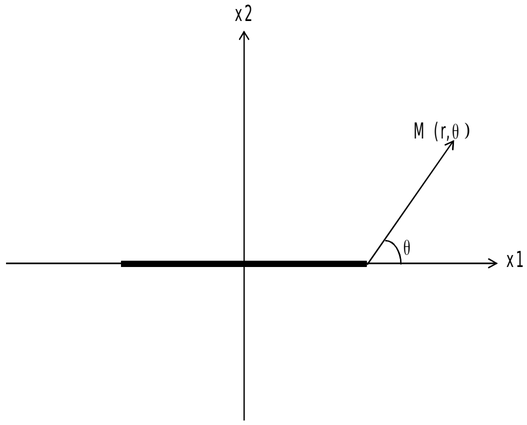

Figure 3.2-a: Definition of a reference mark at a peak of crack

The constraint regularized at a peak of crack can be estimated by

$$\underline{\underline{\bar{\sigma}_p}}(K_I,K_{II})=\sum_{i=1}^{II}\left[\frac{K_i}{\pi l_c^2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_0^{\%l_c}\underline{\underline{f_i}}(r,\theta)\,r\,\partial r\,\partial\theta\right] \quad (3.2.4)$$

Who can be finally written in the following form:

$$\underline{\underline{\bar{\sigma}_p}}(K_I,K_{II})=\frac{\Gamma^2(3/4)}{5\pi\sqrt{\pi lc}}\begin{bmatrix}4K_I & 4K_{II} & 0\\ 4K_{II} & 6K_I & 0\\ 0 & 0 & \bar{\sigma}_{33p}\end{bmatrix} \quad (3.2.5)$$

where $\bar{\sigma}_{33p}=-\nu(\bar{\sigma}_{11p}+\bar{\sigma}_{22p})$ in plane deformation and $\bar{\sigma}_{33p}=0$ in plane constraints. In addition it is pointed out that Γ is the function Gamma, such as $\Gamma(a)=\gamma(a,\infty)$ and $\gamma(a,b)=\int_0^b t^{a-1}\exp(-t)\,dt$ incomplete function gamma. Concretely $\Gamma(3/4)=1{,}225416$.

Then, while being based on the expression of the constraint regularized at a peak of crack, it is possible to propose a law of evolution associated with the propagation with a macro-crack relating to the maximum principal constraint with the type:

$$d=H\left(\langle\bar{\sigma}_I-\sigma_p(el)\rangle\right),\ \dot{d}\geq 0 \quad (3.2.6)$$

With:

$$\sigma_p(el)=\frac{6\Gamma^2(3/4)}{5\pi}\frac{K_{Ic}}{\sqrt{\pi l_c}} \quad (3.2.7)$$

$\sigma_p(el)$ is the threshold of rupture of an element by propagation of a crack on a macroscopic scale and K_{Ic} the tenacity of material. For a request in pure mode I, the threshold suggested corresponds to a threshold of the standard stress intensity factor resulting from the breaking process. The threshold suggested has in particular the advantage of being independent owing to the fact that one places oneself in plane constraints or deformation plane for a problem 2D and it approaches to a criterion in factor of intensity the constraints in mode I for a 3D problem. It is noted that in practice the criterion presented is a criterion in mixed mode, indeed, if one observes the threshold with rupture by propagation of a crack on a macroscopic scale under a request made up for example of modes I and II in plane constraints one obtains at a peak of crack

$$\bar{\sigma}_{Ip}=\frac{6\Gamma^2(3/4)}{5\pi}\frac{1}{\sqrt{\pi l_c}}\left[5K_I+\sqrt{K_I^2+16K_{II}^2}\right] \quad (3.2.8)$$

who also depends on the level of request in mode II .

3.3 Coordination of the thresholds

The model of Weibull with two parameters describes the crack initiation with a very conservative character. Indeed, to use a model of Weibull with two parameters is equivalent supposing the existence of a probability of starting of nonworthless crack as soon as the maximum principal constraint becomes positive. Of another with dimensions, the use of a criterion of propagation of the cracks in factor of intensity of the constraints on a crack on a microscopic scale is equivalent supposing the existence of a threshold of starting not no one. For example, let us consider the case of a zone which has a dimension about l_c and which is understood in a uniformly requested larger zone. If one supposes the existence of a threshold of starting of Weibull σ_a , one can associate to him a length of crack by a criterion of breaking process:

$$\sigma_a(el)=\frac{\sigma_0}{(\lambda_0 Z_{el})^{1/m}}[-\ln(1-P(el))]^{1/m}=\frac{K_{Ic}}{\sqrt{\pi a(el)}} \quad (3.3.1)$$

With $a(el)$ the half-length of the largest crack in the zone considered. By preoccupation with a coherence with the threshold of propagation σ_p , it is advisable to pay attention so that the use of a model of Weibull to describe the starting of crack does not imply to suppose the presence in a zone considered of a crack larger than the size of the zone considered. Moreover it appears reasonable to suppose that a given element will be more easily broken by the propagation of a macro-crack than by starting of a new crack. In such manner, we will also guarantee that the sites of starting which are in the vicinity of the point of a crack will be ecrantés by this one. This assumption amounts posing the following inequality:

$$\sigma_a(el)\geq\sigma_p(el) \quad (3.3.2)$$

who is equivalent to:

$$a(el)\leq\left(\frac{5\pi}{6\Gamma(3/4)}\right)^2 l_c \quad (3.3.3)$$

to define a value criticizes of size of initial defect. One can interpret the preceding inequality as follows: one cannot suppose that an initial crack of size comparable on a macroscopic scale is contained in a zone of size comparable on a microscopic scale. In practice, if there exist initial cracks of half-length higher than approximately l_c they could not be taken into account by the model of starting suggested, one will be able on the other hand to represent them by a line of broken finite elements. The model of Weibull will thus be used to represent the starting of cracks starting from initial defects of size lower than $2l_c$. The characteristic length l_c seems a scale of transition between the microscopic scale from starting from cracks and the macroscopic scale from the propagation from cracks. The characteristic length can then be interpreted as being of about size of the largest cracks initially present at the microscopic scale.

In addition it is noticed that if each finite element does not contain an initial defect larger than him even and than the characteristic length l_c is larger than the size of the finite elements, then the inequality 3.3.2 is automatically checked.

To carry out a coordination of the thresholds, UN location of the elements is necessary. With this intention, a bearing criterion on the vicinity of a finite element was developed. Within this framework, the criterion of propagation will be checked only on elements which are in direct vicinity of an already started crack. This model required thus the knowledge of the neighbors of an element. The details of this location are explained in the following chapter.

4 Digital integration

4.1 Initialization

In order to carry out a calculation using the two thresholds of damage, it is necessary to define a set of parameters beforehand. The parameters characterizing material are the following: the Young modulus E , the Poisson's ratio ν , tenacity K_{Ic} , the module of Weibull m , the characteristic length l_c and a nominal constraint of starting at the level of the characteristic length. The nominal constraint of starting at the level of the characteristic length is introduced instead of scale factor. One can express it by:

$$\sigma_{l_c}=\frac{\sigma_0^m}{\left(\lambda_0 l_c^3\right)^{1/m}} \quad (4.1.1)$$

With σ_0^m/λ_0 the m and scale factor the module of Weibull. The nominal constraint of starting at the level of the characteristic length σ_{l_c} we gives an idea of about size of the constraint of starting of a field of a size of l_c^3 who possibly consists of several finite elements during a calculation. More precisely probability of starting corresponding to a request of σ_{l_c} is approximately 0.63. Besides the parameters characterizing material, one specifies the thickness of the modelled structure E_{ps} . It is possible to use an alternative of the surface model of Weibull instead of voluminal, for which the unit of the scale factor is $MPa^m m^2$ instead of $MPa^m m^3$. In this case, it should be specified that the structure has a thickness of $E_{ps}=l_c$ During the phase of initialization one calculates for each element the value of the threshold of starting (stored in like internal variable). As shown on Figure 4.1-a, the calculation of the threshold of starting σ_a is realized for each finite element and consists of several stages.

- For each finite element, one starts by calculating the size of the elements Z_{el} according to the position nodes and thickness E_{ps} . Then, one calculates the threshold of propagation by the equation 3.2.7.
- The threshold of starting (also variable intern) is then calculated (equation 3.2.1) with $P(el)\in[0;1]$ a drawn figure with the fate on a density of probability uniform. If the condition of coherence of the two thresholds (equation 3.3.2) is not checked, one starts again

after having drawn with the fate another value for $P(el)$. In order to make the two thresholds coherent, one carries out here a truncation of the distribution of Weibull. Of another choice could have been considered, one carries out here a truncation by preoccupation with a simplicity. One notices the choice to carry out a truncation on the distribution of Weibull obliges to choose a realistic characteristic length. Indeed, if the selected characteristic length is incoherent (*i.e.,* weak) with the parameters of Weibull used, one can truncate the major part of the distribution of Weibull. In this case, the description of starting is not faithful to the parameters of Weibull considered.

One will refer to the section 4.4.2 for the choice of the parameters.

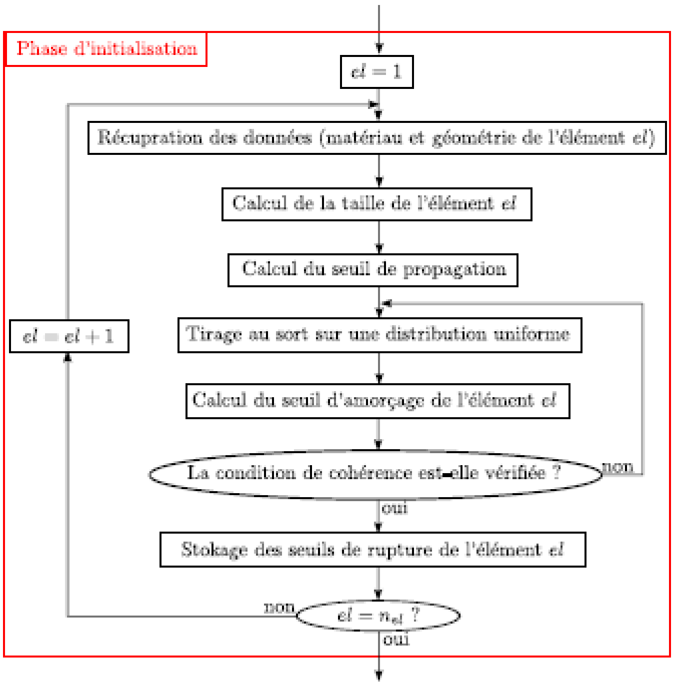

Figure 4.1-a: Diagram of initialization of calculation

4.2 Resolution of the total problem

The total problem relates to the calculation of displacements and the regularized constraints. The law of behavior which makes it possible to calculate with each iteration of Newton the constraint and the damage is implemented according to an implicit scheme. The problem is discretized using the new particular finite elements introduced into Code_Aster as described on the figure 4.2-a.

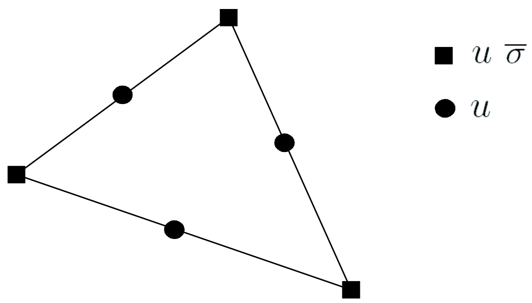

Figure 4.2-a: Nodal unknown factors of an element triangle

The finite elements used are quadratic in displacement and linear in constraints regularized, they thus comprise 21 degrees of freedom for an element triangle for a problem plan (28 for a quadrangle). Two sets of functions of forms and functions of derived forms are associated with displacements $(N_u\, et\, B_u)$ and with the regularized constraints $(N_{\bar{\sigma}}\, et\, B_{\bar{\sigma}})$.

Two equations control the total problem, one corresponds to classical balance; the other corresponds to the calculation of the regularized constraints (for more details on the development of the nonlocal problems with gradient, one will refer for example to [R5.04.01]). The integral formulation of the problem is:

$$\forall u^* \in V_{ad} \int_\Omega \underline{\sigma} : \underline{\epsilon}(u^*)\, d\Omega = \int_G u^* . T_G\, dG + \int_\Omega u^* . f_V\, d\Omega \tag{4.2.1}$$

With V_{ad} the space of acceptable displacements, T_G forces imposed on the edge G, f_V voluminal forces and:

$$\forall \underline{s}^* \in [H^1(\Omega)]^6 \int_\Omega (\underline{s}^* . \underline{\bar{\sigma}} + \nabla \underline{s}^* . l_c^2 \underline{\bar{\sigma}})\, d\Omega = \int_\Omega \underline{s}^* . \underline{\sigma}\, d\Omega \tag{4.2.2}$$

One will note in the part which follows $\sigma, \varepsilon\, et\, \bar{\sigma}$, the writing in the form of vectors of the tensors of the constraints, strains and stresses regularized; one notes U the vector displacement. The total resolution is summarized with the cancellation of a bearing residue on displacements which can be written for each element (for more details, one will refer to [R5.03.01]):

$$F^u = F_{int} + D^T \lambda - F_{ext} \tag{4.2.3}$$

With $F_{int} = \int_\Omega B_u^T \sigma\, d\Omega$, $F_{ext} = \int_\Gamma N_u^T T_g\, dG$ the external loading in efforts with T_G nodal reactions. D is such as $Du = u^d$ with u^d imposed displacements and λ multiplcateurs of Lagrange for boundary conditions of Dirichlet. One minimizes also a bearing residue on the regularized constraints:

$$F^{\bar{\sigma}} = K^{\bar{\sigma}\sigma} \bar{\sigma} - F^\sigma \tag{4.2.4}$$

With $F^\sigma = \int_\Omega N_{\bar{\sigma}}^T \sigma\, d\Omega\, et\, K^{\bar{\sigma}\sigma} = \int_\Omega (N_{\bar{\sigma}}^T N_{\bar{\sigma}} + l_c^2 B_{\bar{\sigma}}^T B_{\bar{\sigma}})\, d\Omega$. In order to minimize the residues presented the following tangent matrix is used:

$$K = \begin{bmatrix} \frac{\partial F^u}{\partial u} & \frac{\partial F^u}{\partial \bar{\sigma}} \\ \frac{\partial F^{\bar{\sigma}}}{\partial u} & \frac{\partial F^{\bar{\sigma}}}{\partial \bar{\sigma}} \end{bmatrix} \tag{4.2.5}$$

With:

$$\frac{\partial F^u}{\partial u} = \int_\Omega B_u^T \frac{\partial \sigma}{\partial \varepsilon} B_u\, d\Omega \tag{4.2.6}$$

$$\frac{\partial F^u}{\partial \bar{\sigma}} = \int_\Omega B_u^T \frac{\partial \sigma}{\partial \bar{\sigma}} N_{\bar{\sigma}}\, d\Omega \tag{4.2.7}$$

$$\frac{\partial F^{\bar{\sigma}}}{\partial \bar{\sigma}} = \int_\Omega (N_{\bar{\sigma}}^T N_{\bar{\sigma}} + l_c^2 B_{\bar{\sigma}}^T B_{\bar{\sigma}})\, d\Omega \tag{4.2.8}$$

$$\frac{\partial F^{\bar{\sigma}}}{\partial u}=\int_{\Omega}-N_{\bar{\sigma}}^{T}\frac{\partial\sigma}{\partial\varepsilon}B_{u}d\Omega \quad (4.2.9)$$

4.3 Implementation of the law of behavior

Healthy finite elements (*i.e.,* for which $d=0$) can be in various situations. Either they are to the forefront of a crack, in which case they are subjected to the threshold of propagation, or they are not to the forefront of a crack, in which case they are subjected to the threshold of starting. Thus the position of the points of cracks and their appointed elements are memorized during calculation. The Figure 4.3-a represent the evolution of the state of the finite elements during the starting and the propagation of a crack. Initially, if all the finite elements are healthy, they all are subjected to their threshold of rupture by starting. If there is starting of a crack in a given element, the element considered is broken (*i.e., d=1*) and it points if possible two close elements as shown on the Figure 4.3-a (A). To the following iteration of Newton, the pointed finite elements are not subjected any more to their threshold of rupture by starting but to the threshold of rupture by propagation.

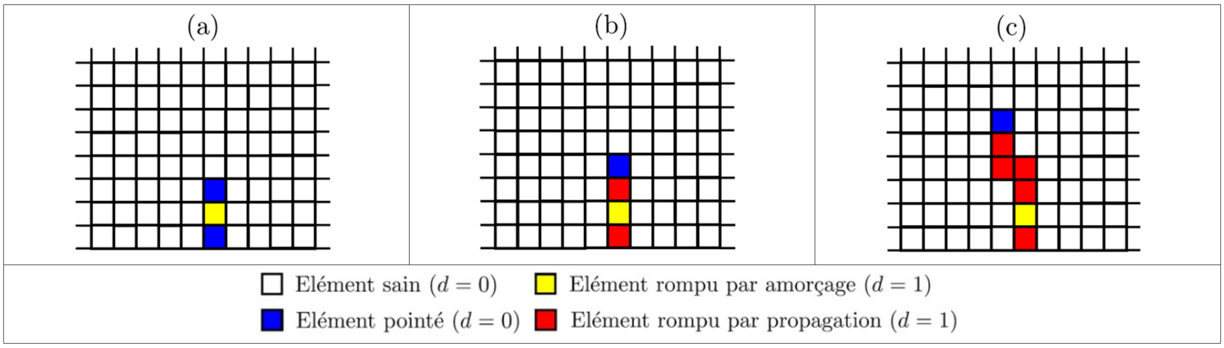

Figure 4.3-a: States of the finite elements

If the pointed finite elements undergo a request such as they break, each one of them then if possible will point a nearby finite element which will be then subjected to the threshold of rupture by propagation. As shown on the Figure 4.3-a B, if a finite element of edge does not find elements suitable neighbor to point, then nothing occurs. Then, the propagation of the started crack can continue according to the level and the orientation of the requests.

The figure 4.3-b more precisely the method of pointing of the elements close during the phase to starting (figure makes it possible to observe 4.3-b(A)) and during the phase of propagation (figure 4.3-b (b)). So in a structural analysis the level of request reached is sufficient to generate the starting of a crack in the finite element A_{el} , then the assumption is made that the crack considered starts in the center of the element and the damage of the finite element considered is worth $d=1$. Two points of cracks then are defined B_{pt} and C_{pt} who are the intersections of a line $\Delta_{A_{el}}$ and of the edges of the finite element considered. The line is defined $\Delta_{A_{el}}$ like passer by the point A_{pt} and being perpendicular to vector $\vec{n_{IA_{pt}}}$ who corresponds to the direction associated with the maximum principal regularized constraint with the point A_{pt} during the rupture $\sigma_{IA_{pt}}$. Each point of crack is then assumption of responsibility by a finite element close to the finite element A_{el} namely the elements B_{el} and C_{el} . Elements B_{el} and C_{el} being then subjected to the threshold of rupture by propagation.

As shown on the figure 4.3-b (b), if one reaches the threshold of propagation on the level of a point of crack D_{pt} assumption of responsibility by an element D_{el} , then the damage of the element D_{el} master key with $d=1$. One then moves the point of crack according to a line $\Delta_{D_{el}}$ defined as passer by D_{pt} and being perpendicular to $\vec{n_{ID_{pt}}}$ direction associated with the maximum principal constraint regularized at the point D_{pt} during the rupture $\sigma_{ID_{pt}}$. The point of crack is then positioned at the point E_{pt} and assumption of responsibility by the finite element E_{el}. The method of management of the vicinity developed makes it possible to manage several cracks simultaneously. It also makes it possible each definite crack to fork according to the problem of structure like according to possible interactions with other cracks of the same structure.

Integration in *Code_Aster* system described is summarized on the figure 4.3-c. In addition, the internal variables used are described in section 4.5.

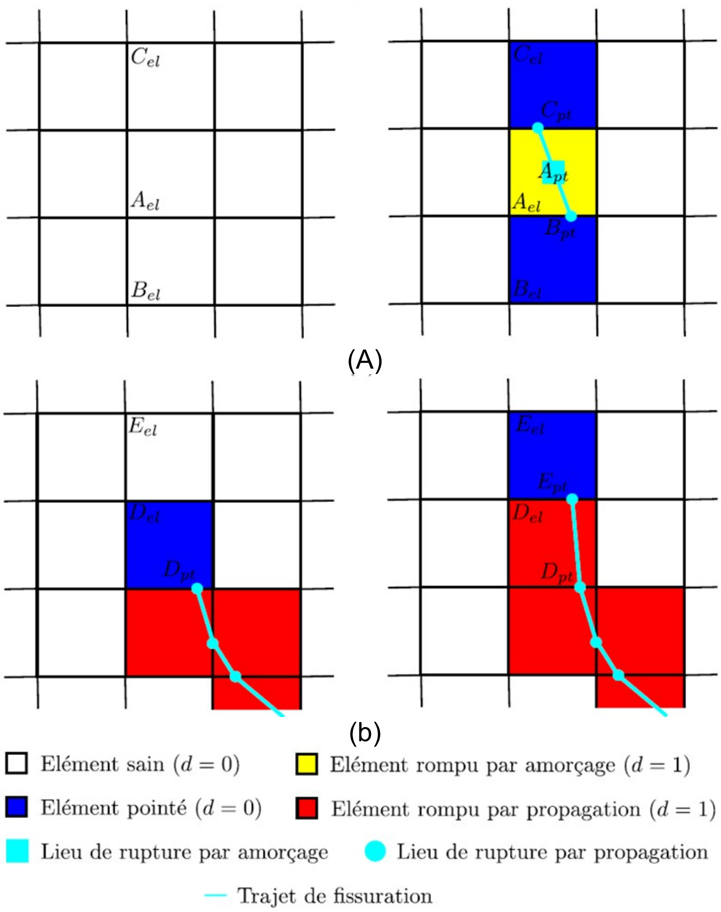

Figure 4.3-b: Location of the close elements

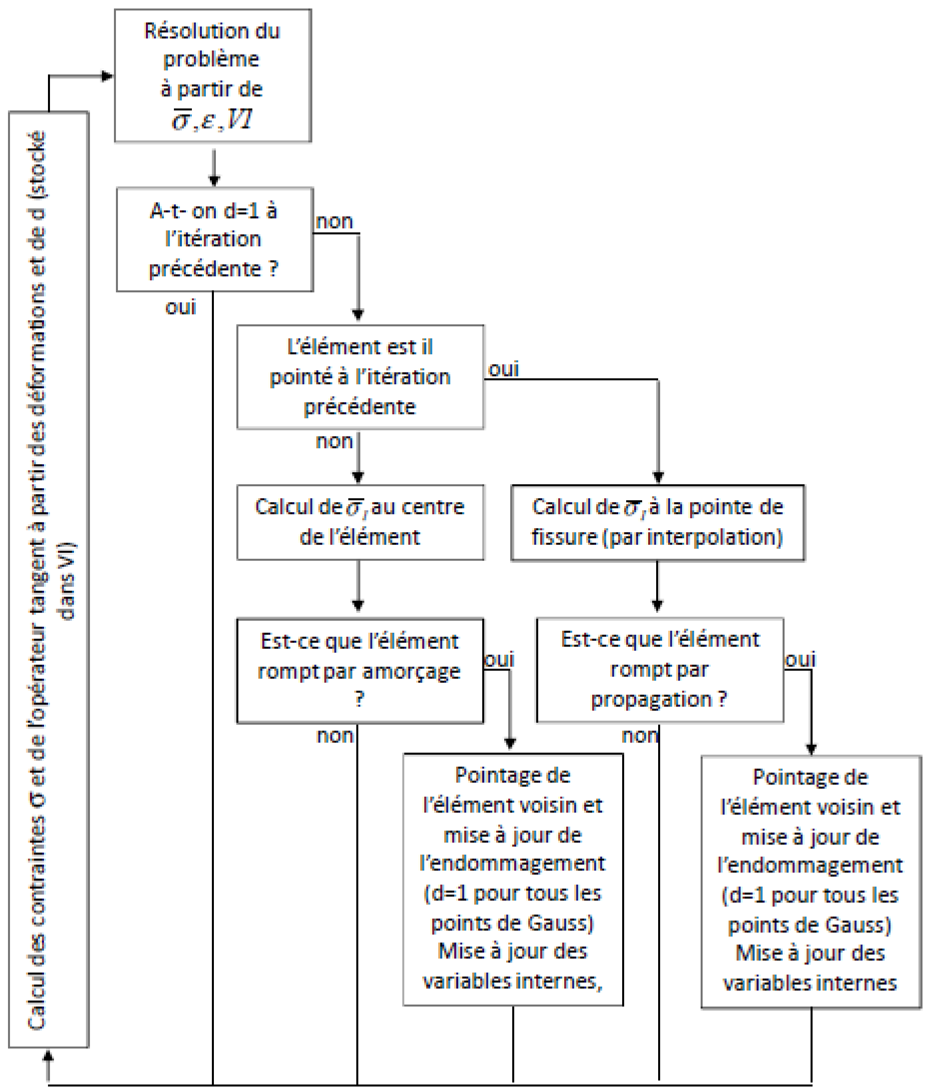

Figure 4.3-c: Integration of the thresholds of starting and propagation in *Code_Aster*

The method of integration of the management system of the vicinity presented considers only one finite element as being pointed that if it were pointed during the preceding iteration of Newton. This fact each point of crack can lead to the rupture by propagation of an element per iteration of Newton. In addition, the implementation of the behavior of material is analytical, which guaranteed the stability of the converged states. Indeed, during a step of loading, each point of crack can lead to the rupture by propagation of as much of element than it is necessary to arrive at balance. So the model presented makes it possible to represent the propagation of macro-cracks even in unstable phase. Let us consider the case of a structure subjected to an increment of loading containing n_{pf} points of numbered cracks of 1 with n_{pf} . So during the step of loading no new crack starts but that each point of crack *I* must break n_{elt} elements so that a stable condition is reached, then the iteration count total of necessary Newton to solve the problem on the increment of loading is of $n_{it}=max(n_{eli})$. One can deduce from it that the duration of simulations depends only little amongst cracks present in the structure.

4.4 Parameters materials necessary to the law

4.4.1 Keywords

In short, 5 parameters materials are necessary to the law to be defined in the keyword ENDO_HETEROGENE of DEFI_MATERIAU :

1) WEIBULL : parameter m of the model of Weibull,
2) SY : nominal constraint of starting,
3) KI : stress intensity factor K_i,
4) EPAI : thickness E_{ps}
5) GR.: Seed of random pulling defining the initial defects

The characteristic length returned under the keyword LONG_CARA of DEFI_MATERIAU.

4.4.2 The Councils of identification

The classical model of Weibull is based on the assumption of the weak link: the total rupture of the structure corresponds to the local rupture, and thus directly with heterogeneities. The parameters of the distribution of Weibull are found in experiments with tests of splitting, characterized by an unstable propagation of the first defect which suddenly takes along the test-tube to the rupture. The assumption of the weakest link is thus valid in this case. As we saw, the model ENDO_HETEROGENE consider more complex types of crackings, with the formation of a network of cracks. For this reason one introduces the concept of starting, by distinguishing it from the propagation; the assumption of the weak link and the application of the model of Weibull stop indeed with the finite element. However, the procedure of experimental chock of the parameters of Weibull by tests of splitting is still valid because it is about a completely experimental procedure and that it applies to a case where the assumption of the weak link is valid; on the other hand, that corresponds only to the starting of cracking for ENDO_HETEROGENE.

A more complicated stage relates to the determination of the remaining parameters: tenacity in mode I and the characteristic length. The two parameters are used by the model ENDO_HETEROGENE for the calculation of the threshold of propagation. The law of breaking process used is precisely nonlinear because of the presence of l_c. On the contrary, in the experimental tests the propagation is associated with tenacity alone, when one refers to a model of linear breaking process. The two concepts of tenacity used thus do not seem to coincide, because they are not associated with the same ideal model. It is thus necessary to take with precaution the values of tenacity resulting from the classical tests.

Concerning the characteristic length only, one can see this value like a key for the passage between the small scale, that of micro the defects, on the great scale, that of the macro-cracks. An empirical rule to choose it consists in taking a length ranging between the length of the smallest defect and that of a macro-crack. One could thus make a choice a priori characteristic length, and then choose tenacity consequently.

4.5 Description of the internal variables

The model has 12 internal variables which require to be stored at previous time:

1) $VI(1)$: damage d,
2) $VI(2)$: indicator (0 healthy element, 1 element pointed, 2 broken by starting, 3 broken by propagation),
3) $VI(3)$: constraint of starting σ_a,
4) $VI(4)$: constraint of propagation σ_p,
5) $VI(5)$: number of the element pointed number 1,
6) $VI(6)$: number of the element pointed number 1 if starting,
7) $VI(7)$: iteration of Newton of rupture,

8) $VI(8)$: current iteration of Newton,
9) $VI(9)$: coordinate X of the point of crack (after rupture by propagation),
10) $VI(10)$: coordinate Y of the point of crack (after rupture by propagation),
11) $VI(11)$: coordinate X point of crack 2 during starting,
12) $VI(12)$: coordinate Y point of crack 2 during starting,

5 Features and checking

The law of behavior is indissociable modeling D_PLAN_GRAD_SIGM. One can thus make only plane modeling. The description of the parameters as well as the advices of identification are given in section 4.4.

The law of behavior ENDO_HETEROGENE as well as modeling D_PLAN_GRAD_SIGM are checked by the cases following tests:

SSNV147	Modeling of the starting of crack with the model ENDO_HETEROGENE	[V6.03.147]
SSNV148	Calculation of the stress intensity factor by the regularization of the constraints with ENDO_HETEROGENE	[V6.03.148]

Note:

In parallel calculation, L orsqu' one carries out a calculation on several processors with the law ENDO_HETEROGENE , nothing guarantees the unicity of the solution because of generated thresholds of starting in a random way. One can have a sensitivity machine about 1.E-3 %. In certain cases, it can there to have digital problems of not-convergence. This is why it is advised to disable the distribution of the data with AFFE_MODELE/DISTRIBUTION= ' CENTRALIZE '.

6 Bibliography

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7 Description of the versions of the document

Index document	**Version Aster**	**Author (S) Organization (S)**	**Description of the modifications**
With	10/02/14	S. Granet EDF-R&D/AMA D. Seyedi BRGM	Initial text