Examination of the random answers

Summary :

The introduction of a “stochastic approach of seismic calculation” to solve a vibratory problem of mechanics under random excitation requires a particular postprocessing.

The order POST_DYNA_ALEA [U4.76.02] allows, starting from the spectral concentration of power of an interspectre-answer, to evaluate its standard deviation, its apparent frequency, the distribution of its peaks. It also allows, in a first approach, to calculate the useful function of Vanmarcke in the case of a seismic analysis.

NB:

This order also makes it possible to carry out the statistical estimates for any type of interspectre of answer to a random excitation not necessarily seismic (for example: effect of the swell or a turbulent flow).

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1 Introduction

For a structure subjected to a random excitation of type swells, turbulent flow, or earthquake… the loading is not known in a deterministic way, but is generally described by probabilistic or spectral information like the spectral concentration of power. For the linear structures it is possible to use a stochastic method of calculating which makes it possible to determine the spectral concentrations of power of answer to these random excitations.

The operator `POST_DYNA_ALEA` has as a function to carry out the statistical analyses of the spectral concentration of power of answer. It thus provides probabilistic information of the answer of the structure. Statistical calculations of parameters are carried out on the basis of calculation of the spectral moments of the spectral concentration of power considered.

These statistical parameters are: the standard deviation, the frequency connects, the factor of peak according to the definition of Vanmarcke, the average maximum and the factor of irregularity.

Note:

The operator `POST_DYNA_ALEA` , conceived initially for the seismic approach, after a calculation with the operator `DYNA_ALEA_MODAL` [U4.56.06] ([bib1], [bib2]) , phad also to carry out postprocessings of the operator `DYNA_SPEC_MODAL` developed by department TTA within the framework of the resorption of FLUSTRU. This operator carries out the calculation of the answer of a structure of the type uniformly tubes Steam Generator excited by a transverse flow.

2 Spectrum - Interspectre - Interspectrale Matrix

2.1 Treatment of the signal - Conventions selected

2.1.1 Introduction

A signal can have two representations: a temporal representation of the form $x=f(t)$ or a frequential representation of the form $X=F(f)$. These two representations are connected between them by **Transformation of Fourier.**

There exists in the digital field and the experimental field various manners of calculating the spectral sizes relative to a temporal signal $x(t)$ (dimensional representation or not, factor 1/2 π or not for the Transformation of FOURIER).

However, if the various definitions of the DSP (cf [§2.2.2] and [Annexe1]) starting from the Transformation of Fourier of the signal do not change anything with the calculation carried out by `CALC_INTE_SPEC` [U4.56.03], it is important on the other hand, in the calculations carried out by the operator of postprocessing `POST_DYNA_ALEA`, that the data are coherent so that the results produced by this operator are with the physical dimension of the starting signal.

It is also necessary to know, for a quantitative comparison between calculation and experiment, which are the conventions adopted for the calculation of the spectral quantities. The whole of these conventions is recalled in [Annexe1] for each type of signals. We give again only the general formulas here.

2.1.2 Transformation of Fourier

For **Transformation of FOURIER in frequency** (f) **of a signal** (of unit U), expressed in u/Hz we adopt the following definition: $X(f)=\int_{-\infty}^{+\infty} x(t)\, e^{-2i\pi ft}\, dt$

The reverse transformation is expressed then by: $x(t)=\int_{-\infty}^{+\infty} X(f)\, e^{+2i\pi ft}\, df$

One can also express **Transformation of Fourier into pulsation** $(\omega=2\pi f)$, by the following definition:

$$X^p(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} x(t)\, e^{-i\omega t}\, dt$$

The reverse transformation is expressed by: $x(t)=\int_{-\infty}^{+\infty} X^p(\omega)\, e^{+i\omega t}\, d\omega$

What leads to equivalence: $X^p(\omega)=X^p(2\pi f)=\frac{1}{2\pi}X(f)$

2.2 Concept of Power - Spectral concentration of Power

2.2.1 Power of a signal - Spectrum of Power of a signal

Just like the signal itself, the power of the signal can be expressed according to time or of the frequency:

- the instantaneous temporal power is simply called power:

$$p(t)=x(t).\, x^*(t)$$

  where $x^*(t)$ is the complex quantity combined of $x(t)$.

- the frequential power is commonly called spectral concentration of power or spectrum:

$$\underline{S_{xx}(f)=X(f).\, X^*(f)=|X(f)|^2}$$

This definition is not possible that when the transform of Fourier of the signal exists.

One can then express the total energy of the signal by $E=\int_{-\infty}^{+\infty} S_{xx}(f)\, df=\int_{-\infty}^{+\infty}|X(f)|^2\, df$

The expression of this DSP for the various types of signals is given in [Annexe1]. One will see later on [§3.3] another definition - equivalent according to the theorem of Wiener - Kinchine - but more general, of the spectral concentration of power based on the statistical approach.

2.2.2 Power of interaction - Spectral concentration of interaction of two signals - Interspectre

- One also defines **instantaneous power of interaction of two signals** $x(t)$ et $y(t)$:

$$p_{xy}(t)=x(t).\,y^*(t) \text{ et } p_{yx}(t)=x^*(t).\,y(t)$$
$$\text{reliées par } p_{xy}(t)=p_{yx}*(t)$$

- If the two signals admit a transform of Fourier $X(f)$ et $Y(f)$, one can express **frequential power of interaction** or **interspectre** by $S_{XY}(f)=X(f).\,Y^*(f)$

- If them **two signals are real** then power of interaction $p_{xy}(t)=p_{yx}(t)=x(t).\,y(t)$ is real. But there is no reason so that $S_{XY}(f)$ that is to say also real; on the other hand $S_{XY}(f)$ is complex with square symmetry, namely:

  even real part and odd imaginary part or even module and odd phase

- If $X(f)=Y(f)$, one speaks then about**autospectre.**

2.2.3 Matrix interspectrale

A matrix interspectrale of order N is a matrix $N\times N$ complex, whose each term depends on the frequency in the form of a function on f . The diagonal terms are the autospectres, the extra-diagonal terms are the interspectres between the points considered (each line or column representing a point in physical grid or a mode in modal calculation). Interspectres handled in practice being square, only them $\frac{N(N+1)}{2}$ terms of triangular higher (or lower) are sufficient to define the matrix interspectrale completely.

2.3 Establishment in *Code_Aster*

The matrices interspectrales handled by the operator `POST_DYNA_ALEA` consist of complex functions of the frequency: $S_{\mathbf{XY}}(f)$.

These matrices are stored in tables of concept `interspectre`.

3 Recalls on the statistical laws [bib4]

3.1 Definitions

t discrete parameter $(t_n)_{n=1,\mathrm{N}}$ or continuous (time or a variable of space).

$X(t)$ random process.

At every moment t_n a random variable is associated X_n, random variable of realization x_n.

Then $x(t)=\left(x_n=x(t_n)\right)_{n=1,N}$ is a realization of the process $X(t)$, process made up of N random variables a priori independent.

Each variable X_n is characterized by its **function of distribution** $F_n(x,t_n)=\mathrm{Prob}(X_n\leq x)$ or by its **density of probability** $p(x,t_n)=\dfrac{\partial F_n}{\partial x}(x,t_n)$.

The random process is also characterized by its **functions moments**, the first two moments have a particular importance. It is about **the expectation** or **average** $m(t)$ noted too $E[X(t)]$ and for any couple (t_1,t_2) **function of autocorrelation** $R(t_1,t_2)$ or $R_{XX}(t_1,t_2)$ noted too $E[X(t_1)X(t_2)]$.

$$m(t)=E[X(t)]=\int xp(x,t)dx$$
$$R_{XX}(t_1,t_2)=E[X(t_1)X(t_2)]=\int\int x_1x_2p(x_1,t_1;x_2,t_2)dx_1dx_2$$

One defines also one **function of intercorrelation** for two processes $X(t)$et $Y(t)$.

$$R_{XY}(t_1,t_2)=E[X(t_1)Y(t_2)]=\int\int x_1y_2p(x_1,t_1;y_2,t_2)dx_1dy_2$$

The “spreading out” of the process is characterized by **variance** :

$$\sigma^2(t)=E\left[\left(X(t)-\mu(t)\right)^2\right]$$

For a process with worthless average (μ = 0.), **variance** who then characterize the “intensity of the phenomenon” (square of the standard deviation or average quadratic value) is equal to the function of autocorrelation at time $t=t_1=t_2$:

$$\sigma^2(t)=E[X(t)X(t)]=R_{XX}(t,t)=\int x^2p(x,t)dx$$

3.2 Assumptions in random dynamics

Very classically several assumptions are posed within the framework of random dynamics. One admits thus that the studied processes are **stationary, with average worthless and ergodic**.

3.2.1 Stationary processes with worthless average - variance

A process is known as **stationary** if the whole of its “probabilistic characteristics” is invariant during a translation t_0 parameter t. What implies:

$$\mu(t)=Cte$$
$$R_{XX}(t_1, t_2)=R_{XX}(t_2-t_1)=R_{XX}(\tau)=R_{XX}(-\tau)$$

For a process with worthless average $\sigma^2=R_{XX}(0)$.

3.2.2 Ergodicity

This concept comes from a reasoning of Gibbs (1839-1903) for whom time from observation from a physical phenomenon can be regarded as infinite in front of the scale of time at the molecular level. The system passes then by all the possible states while remaining more possible for a long time, or while generally passing, in the states which are most probable, so that **temporal average** becomes **equalize with the statistical average on the states**, i.e. the expectation. This is prolonged for the functions of correlation and intercorrelation.

$$\mu=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2} x(t)\,dt$$
$$R_{\mathrm{XX}}(t)=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2} x(t-\tau)\,x(t)\,dt$$

Note:

For the continuation of the document one will suppose that the random process is stationary with worthless average. The whole of the developments carried out in Code_Aster checks these assumptions.

3.3 Spectral concentration of power

Within the framework of this statistical approach, one can give a very general definition of **spectral concentration of power** or **DSP.** One will retain for *Code_Aster* definitions following expressed in frequency or pulsation:

$$S_{\mathbf{XX}}(f)=\int_{-\infty}^{+\infty} R_{XX}(\tau)\,e^{-2\mathrm{i}\pi f\tau}\,d\tau\,;\,G_{\mathbf{XX}}(f)=\int_{0.}^{+\infty} R_{XX}(\tau)\,e^{-2\mathrm{i}\pi f\tau}\,d\tau$$
$$S^{p_{\mathrm{xx}}}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{XX}(\tau)\,e^{-i\omega\tau}\,d\tau\,;\,G^{p_{\mathrm{xx}}}(\omega)=\frac{1}{2\pi}\int_{0.}^{+\infty} R_{XX}(\tau)\,e^{-i\omega\tau}\,d\tau$$

who lead to the following relations:

$$G^{p_{\mathrm{xx}}}(\omega)=\frac{1}{2\pi}G_{\mathbf{XX}}(f)$$
$$S_{\mathbf{XX}}(f)=2\,G_{\mathbf{XX}}(f)\,S^{p}_{\mathbf{XX}}(\omega)=2\,G^{p_{\mathrm{xx}}}(\omega)$$

One can show that $G_{\mathbf{XX}}(f)$, which is equal to the Transformation of Fourier of $R_{XX}(t)$, is real, positive. One will refer to [Annexe1] who contains all conventions adopted to ensure the coherence of the results.

3.4 Spectral moments

One calls spectral moments the following quantities (which one defined in pulsation):

$$\lambda_i=\int_{-\infty}^{+\infty}|\omega|^i S^{p_{xx}}(\omega)\,d\omega=\int_{-\infty}^{+\infty}|\omega|^i S_{XX}(f)\,df$$

One has in particular: $\lambda_0=\sigma^2\ \lambda_2=\sigma^2_{\dot{X}\dot{X}}\ \lambda_4=\sigma^2_{\ddot{X}\ddot{X}}$ who are the standard deviations of X and of its first derived.

These moments are systematically calculated until order 4; using the keyword `MOMENT` it is possible to ask the calculation of the higher modes. In *Code_Aster*, calculation is carried out for a DSP expressed according to the frequency f.

4 Measurements of going beyond threshold and reliability

The classical methods give access only the maximum of displacement (or acceleration) by summation "adapted" of the maxima on each mode. The essential interest of the stochastic approach of random vibratory calculation lies in the statistical knowledge of the answer of the structure which can thus be converted into statistical data of reliability. For this reason, two modes of ruin can be taken into account:

- ruin by going beyond threshold: this kind of ruin occurs when the answer of the system exceeds a limiting value. That amounts seeking the probability that the values of the process remain below a value threshold during the duration of observation T. the factor of peak (**peak Factor)** allows to estimate, in manner approached, the average maximum, over one duration of observation of the signal.
- ruin by tiredness or accumulation of damage.

This second approach could also be treated starting from the first statistical elements calculated in `POST_DYNA_ALEA`. It is carried out in the order `POST_FATI_ALEA` [U4.67.05] [R7.04.02].

Within the framework of the studies under seismic excitations, we are interested primarily in the problem of going beyond of threshold. From where initially the calculation of a certain number of statistical parameters which make it possible to characterize the signal to be studied (spectral moments and formulas of Rice [§4.1]), provided with these characteristics we will be able to then estimate the probabilities of going beyond threshold using classical models of probability [§4.2], as well as a criterion of reliability (law of Vanmarcke [§4.3]).

4.1 Spectral moments and characteristic parameters

The spectral moments are defined by: $\lambda_i=\int_{-\infty}^{+\infty}|\omega|^i S_{XX}(f)\,df$

The infinite whole of these spectral moments characterize the interspectre perfectly and thus make it possible to establish a certain number of digital results. In the typical case of an oscillator with 1 ddl or a signal with only one peak, the first three spectral moments are enough to find the autospectre S_{XX}. It is the case which we retain in *Code_Aster* since it is supposed that the values are distributed according to a law of GAUSS.

4.1.1 Formulas of Rice

For a random signal such as definite previously:: stationary with **worthless average (centered)** and ergodic, one supposes moreover than the measured values are distributed according to one **normal law profile of the Gauss type** (cf [§4.2.1]).

The analysis of a stationary Gaussian random loading has the advantage of leading to simple analytical expressions - known under the name of formulas of Rice - and to represent many real phenomena.

The following statistical parameters are obtained as from the various spectral moments connected to different the derivative from X (cf [§3.4]):

- Standard deviation: $\sigma_X = \sqrt{\lambda_0}$

Note:

If only the positive part of the spectrum is provided, Code_Aster multiplies by 2 the 1er spectral moment $\sigma_X = \sqrt{2\lambda_0}$.

A extremum (maximum or minimum) of amplitude X is defined by the probability of having a worthless derivative $\dot{X}=0$ associated with a derivative second $\ddot{X}$ unspecified.

- Median number of extrema a second: $N_e = \frac{1}{\pi}\frac{\sigma_{\ddot{X}}}{\sigma_{\dot{X}}} = \frac{1}{\pi}\sqrt{\frac{\lambda_4}{\lambda_2}}$

The going beyond a level X_0 is defined by the probability of having $X=X_0$ with a slope $\dot{X}$ unspecified: one thus counts the passages of this level with the positive slopes **and** negative. Taking into account the assumptions of Gaussian laws, the number of passage by X_0 and a second expresses itself by: $N_{X_0} = \frac{1}{\pi}\frac{\sigma_{\dot{X}}}{\sigma_X} e^{-\frac{X_0^2}{2\sigma_X^2}}$

What leads to the following expressions:

- Many goings beyond level **with positive slope** a second: $N_{X_0^+} = \frac{1}{2} N_{X_0}$
- Many passages **by zero** (X_0 = 0) a second: $N_0 = \frac{1}{\pi}\frac{\sigma_{\dot{X}}}{\sigma_X} = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$
- Many passages **by zero with positive slope** a second:

$$N_{0^+} = \frac{1}{2} N_0 = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$$

N_{0^+} represent an average statistical frequency of passage by zero with positive slope.

In the case of a “simple” signal, i.e. with **only one peak**, N_{0^+} , many passages by zero can be comparable to one **apparent frequency** also noted $f_{\acute{}}$. In the case much more general of an unspecified signal, the physical interpretation of the value N_{0^+} is more prone to guarantee!

The factor of irregularity translates the frequential pace of the signal. Ranging between 0 and 1, it tends towards 1 when the process is with narrow band, on the other hand it tightens towards 0 for a broad band process. Its expression is:

$$I=\frac{N_0}{N_e}=\frac{\sigma_{\dot{X}}^2}{\sigma_X \sigma_{\ddot{X}}}=\sqrt{\frac{\lambda_2^2}{\lambda_0 \lambda_4}}$$

Three parameters - N_0, N_e, I - characterize the signal entirely. One can, in particular, estimate the median number of positive peaks a second: $N_{pic^+}=1/4(1+I)N_e$.

The whole of these parameters is calculated and stored in a "printable" table on the file `RESULT` using the order `IMPR_TABLE`.

4.2 Distributions of the positive peaks

One of principal knowledge interesting the originators of structures starting from his answer estimated at a random excitation is the determination of the goings beyond threshold and in particular **probabilities of goings beyond** certain critical points.

The formulas of Rice (preceding paragraph) make it possible to know the average rate of crossings of certain levels. The following approach makes it possible to give a law of probability of presence of such or such peak. One is thus interested in maximum positive of the answer.

A maximum occurs when $\dot{x}(t)=0\,\text{avec}\;\ddot{x}(t)<0$. One is thus interested in the density of joint probability $p(x,\dot{x}=0,\ddot{x},t)\,\text{de}\;X(t),\dot{X}(t),\ddot{X}($. (It is necessary thus that the process is twice derivable, which is acquired when one admits a Gaussian distribution of the signal.)

This density of probability of **positive peaks** allows for example to calculate the proportion of peaks ranging between has and B (or the probability that the next peak lies between has and b) which is worth:

$$\int_a^b p(x,0,\ddot{x},t)\,dx$$

The stationary Gaussian signal, being centered compared to its median value (worthless in seismic analysis), the distribution of the peaks is symmetrical compared to this average. One is thus interested in **distribution of the positive peaks**. In the case general, the distribution of the peaks of amplitude X positive is written in the form [bib5]:

$$p_{\text{pic}}^{+}(\boldsymbol{X})=\frac{2}{\sqrt{2\pi}\,\sigma_X(1+I)}\left[\sqrt{1-I^2}\,e^{\frac{X^2}{2\sigma_X^2(1-I^2)}}+\frac{I\,\boldsymbol{X}}{\boldsymbol{\sigma}_X}e^{\frac{X^2}{2\sigma_X^2}}\int_{-\infty}^{\alpha}e^{\frac{-t^2}{2}}dt\right]$$

$$\text{Si}\;X<0\;\text{alors}\;p_{\text{pic}}^{+}(X)=0 \qquad \text{avec}\;\begin{cases} I=\dfrac{\sigma_{\dot{X}}^2}{\sigma_X\sigma_{\ddot{X}}} \\ \alpha=\dfrac{X}{\sigma_X}\dfrac{I}{\sqrt{1-I^2}} \end{cases}$$

It is the so known formula under the name of LONGUET-HIGGINS [bib6]. We present, Ci after, the chart of this formula for 4 values of I .

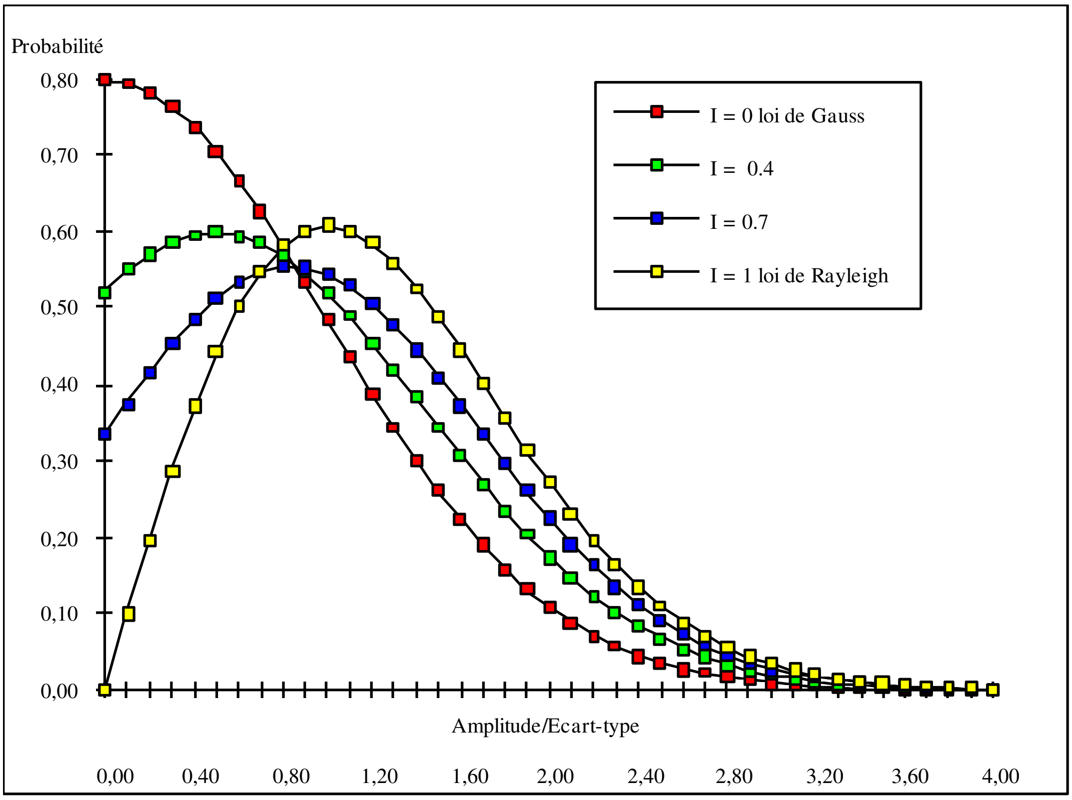

Figure 4.2-a: Distribution of peaks of standardized positive amplitude compared to the standard deviation of the signal

This distribution of the positive peaks is in the case of simplified the signals for which the factor of irregularity is worth $I=0 \text{ ou } I=1$.

4.2.1 $I=0$ Signal with broad band: law of Gauss or normal law

In the case of a broad band signal, the positive peaks are distributed according to a law of GAUSS:

$$p_{pic}^{+}(X)=\frac{2}{\sqrt{2\pi\sigma_{X^2}}}e^{-\frac{X^2}{2\sigma_{X^2}}}$$

4.2.2 $I=1$ Signal with narrow band: law of Rayleigh

In the case of a signal with narrow band, the positive peaks are distributed according to a law of RAYLEIGH:

$$p_{pic}^{+}(X)=\frac{X}{\sigma_{X^2}}e^{-\frac{X^2}{2\sigma_{X^2}}}$$

4.2.3 Calculation of the values in *Code_Aster*

The values of these two laws are calculated in *Code_Aster* under the keywords RAYLEIGH factor or GAUSS.

From the standard deviation $\sigma_X=\sqrt{\lambda_0}$ calculated previously one calculates the values of probability of the peaks $p^+_{pic}(X)\,\mathrm{pour}\, X\in[0{,}6\,\sigma_X]$ with a step by default of $\dfrac{6\sigma_X}{200}$.

If the user wishes to refine his analysis, it can provide the values `VALMIN` and `VALMAX` field of variation of X. It can also provide the step value of calculation, if not this one will be taken to the 200ème band selected.

The figure [Figure 4.2-a] shows that the selected field by default, until $6\sigma_X$, the totality of the values covers well of X with nonworthless probabilities.

4.3 Seismic answer: law of Vanmarcke

In the case of the answer to an earthquake of a primary structure (i.e. excited at its base by the ground) having one **dominating mode**, IE which answers (taking into account the exiting frequencies) on only one mode, one uses the law of reliability of VANMARCKE [bib10] which makes it possible to estimate, on one **operation life** T **probability that the process exceeds the threshold of ruin**.

The concept of dominating mode is very important here, if the structure answers on several modes the formula in its current expression is not appropriate more.

That is to say $X(t)$ the answer to a Gaussian white vibration, of a slightly deadened linear oscillator. The probability is defined $W(T)$ that the process remains in the field of security. $W(T)$ represent the fraction of sample which did not cross the threshold of ruin after one duration T ; it is a measurement of reliability.

It can be written in the form $W(T)=\mathrm{Prob}\{|X(t)<X_0;0\le t<T|\}: p_1(T)=-\dfrac{\mathrm{dW}(T)}{dT}$ is the density of probability of crossing of the threshold.

For the high values of T one will take: $p_1(T)=A\alpha e^{-\alpha T}$ where A depends on the initial conditions and α is the limiting rate of the reduction.

4.3.1 Assumption of independent crossings

With the assumption that the goings beyond threshold with a positive slope are **independent events**, the number of crossings on $[0,T[$ constitute a process of Fish of rate of arrival $N_{X_0}=2N^+_{X_0}$ (number of going beyond X_0 defined in [§4.1.1]). Probability that n passages occur over the duration T is written by application of the Fish law (see [§4] [bib8]):

$$P\{n\,\text{passages sur}\;[0,T[\}=e^{-N_{X_0}T\frac{(N_{X_0}T)^n}{n!}}$$

The structure is “reliable” if the threshold is not exceeded during the duration T. Reliability $W(T)$ thus corresponds to $n=0$ passage d' where $W(T)=e^{-N_{X_0}T}$.

The limiting rate of the reduction is thus worth here $\alpha=N_{X_0}=2N^+_{X_0}$.

4.3.2 Law of Vanmarcke

For a Gaussian stationary process, probability of exceeding the value X is worth [§4.1.1]: $N_X = N_0 e^{-\frac{X^2}{2\sigma_{X^2}}}$; one from of deduced that probability that the initial value of the envelope is lower than the threshold X is: $1 - \frac{N_X}{N_0} = 1 - e^{-\frac{X^2}{2\sigma^2}}$.

One then combines this expression with the limiting law of the reduction obtained with the assumption of independent crossings, which leads to **the expression of reliability** :

$$W(T) = Ae^{-\alpha T} = \left(1 - e^{-s^2/2}\right) e^{-N_0 T \frac{\left(1-e^{-hs}\right)}{e^{\frac{s^2}{2}} - 1}}$$

with $N_0 = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$ rate of passage by 0 and T duration of observation

$$\text{où} \quad s = \frac{X}{\sqrt{\lambda_0}} \quad h = d^{1.2}\sqrt{\frac{\pi}{2}} \quad \delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}}$$

δ is an estimator of bandwidth of the DSP of X .

This relation has the immense advantage of providing an explicit estimator of reliability according to the reduced value of the threshold *S*, amongst equivalent semi-cycles N_0 , and of the parameter of bandwidth δ .

NB:

"The agreement between the estimator and simulations can be improved if one replaces δ by $\delta^{1.2}$ " [bib6] "correction" introduced into the formula written above compared to the expression of the rate of the reduction limits given in the preceding paragraph.

The following chart is carried out in the case of a process of apparent frequency $15\,Hz$, that is to say $N_0=30\,Hz$, for one duration of observation of $T=1\text{s}$. The estimator of bandwidth δ is taken equal to 0.30. As in the preceding illustrations, the amplitude is standardized compared to the standard deviation.

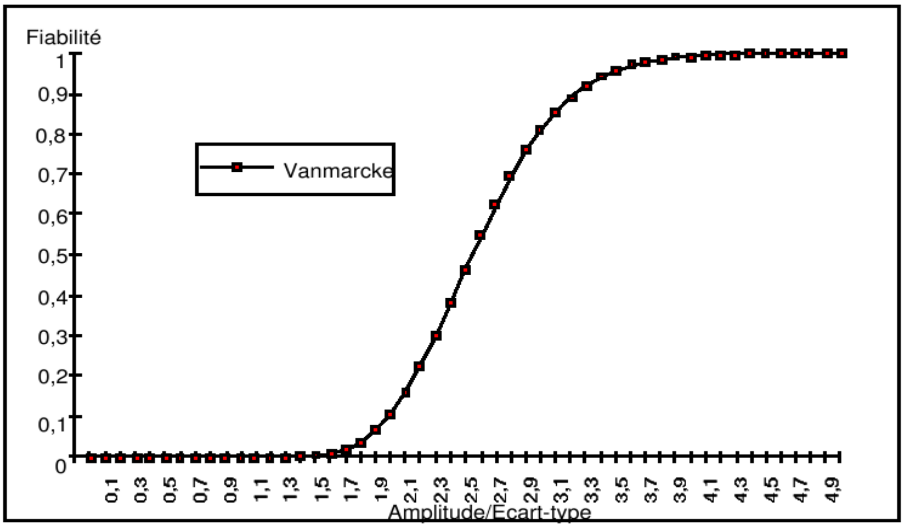

Figure 4.3.2-a: Evolution of reliability according to the law of Vanmarcke according to the amplitude of the standardized process compared to the standard deviation of the signal

Recall:

This statistical analysis is carried out starting from relatively restrictive assumptions, namely that the process must be with "narrow band"; it will thus have to be checked that the factor of irregularity I is not too different from 1 and that the signal comprises only one principal peak.

4.4 Factor of peak established in *Code_Aster*

The operator POST_DYNA_ALEA allows to calculate the factor of peak of Vanmarcke (FACT_PIC) and the maximum average D (MAX_MOY). This last is obtained like the product of the factor of peak with the standard deviation: $d=\sigma\,\eta_{T_{SM},p}$

More generally, the factor of peak makes it possible to estimate them p - fractiles of the distribution of maximum of a Gaussian process starting from the standard deviation. The factor of peak, due to Vanmarcke [bib10], is written:

$$\eta^2_{T_{SM},p}=2\ln\left(2N_\eta\left[1-\exp\left(-\delta^{1.2}\sqrt{\pi\ln\left(2N_\eta\right)}\right)\right]\right).$$

In this expression, δ is the bandwidth of the process (cf §4.3.2) and N_η is determined starting from the centre frequency $\nu_0^+=0.5N_0$ like $N_\eta=0.5N_0T\left(-\ln p\right)^{-1}$.

It thus acts, for MAX_MOY,strictly known as of the median maximum, this being, the two statistics (average and median) are confused in the case of a Gaussian distribution.

5 Remarks

This postprocessing is carried out on interspectres stored in `interspectre`. It provides statistical elements of the answer of the structure which can thus be converted into statistical data of reliability, or to be used then for calculations as damage by tiredness (`POST_FATI_ALEA`).

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7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
4	A.DUMOND- EDF/R & D - AMV	Initial text
11.2	F.VOLDOIRE EDF/R & D /AMA	Some cosmetic corrections of formulas.
12.1	I.Zentner	Correction of the formula of Vanmarcke

Annexe 1 Conventions for the Spectral concentrations of Power

A1.1 Introduction

In order to preserve coherence necessary for the whole of calculations and the comparisons with the experiment (cf [§2.3] and [§3.3]), we develop hereafter the two coherent whole of definitions with calculations of random answer and of postprocessing such as they were retained for *Aster* :

- the first starting from spectral data expressed according to the frequency. It is this unit which is coherent with the calculation carried out in the operator `CALC_INTE_SPEC` [U4.56.03].
- the second starting from spectral data expressed according to the pulsation.

These two units return validates postprocessing such as it is expressed in `POST_DYNA_ALEA`.

We will each time specify the unit in which are expressed the various quantities handled according to the unit U of the signal of reference. The explanations given are brief. One will be able for more details to refer to the reference [bib10].

A1.2 Types of signals and definition of the power

We consider four types of signals:

- signals of finished energy,
- periodic signals,
- signals of finished power and deterministic signals,
- random satisfying the assumption with ergodicity and stationary signals.

In random dynamic calculation the signals are random. For the interpretation of experimental results, the signals are either periodic, or of finished power (deterministic).

We define for each type of signal an energy quantity which is either an energy, or a power and that we will indicate in the following paragraphs under the single term of power:

- **signals of finished energy** are defined by their energy E expressed in $u^2 s$:

$$E=\int_{-\infty}^{+\infty} x(t)^2 dt<+\infty \qquad \text{éq An1.2-1}$$

- **periodic signals** are defined by the power P signal expressed out of U2 :

$$P=\frac{1}{T}\int_{[T]} |x(t)|^2 dt \qquad \text{éq An1.2-2}$$

T indicate the period of the signal. $[T]$ is an interval length T .

- **signals of finished power** are defined by the average power P signal expressed in u^2 :

$$P=\lim_{T\to+\infty}\left(\frac{1}{T}\int_{-T/2}^{+T/2}|x(t)|^2 dt\right)<+\infty \qquad \text{éq An1.2-3}$$

- **random signals** are defined by the average power P signal expressed in u^2 :

$$P=E\left[|X(t)|^2\right]=\lim_{T\to+\infty}\left(\frac{1}{T}\int_{-T/2}^{+T/2}|x(t)|^2 dt\right)<+\infty \qquad \text{éq An1.2-4}$$

One makes use here of the assumption of ergodicity which underlies that the average statistics and temporal carried out on a realization of a process are identical.

A1.3 Autocorrelations

Taking into account the statistical recalls carried out in the body text one has for each type of signals previously definite:

- Autocorrelation of **signals of finished energy**, expressed in u^2/Hz :

$$R_{XX}(\tau)=\int \overline{x(t)}\,x(t+\tau)\,dt \qquad \text{éq An1.3-1}$$

- Autocorrelation of **periodic signals**, expressed in u^2 :

$$R_{XX}(\tau)=\frac{1}{T}\int_{[T]} \overline{x(t)}\,x(t+\tau)\,dt \qquad \text{éq An1.3-2}$$

- Autocorrelation of **signals of finished power**, expressed in u^2 :

$$R_{XX}(\tau)=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2} \overline{x(t)}\,x(t+\tau)\,dt \qquad \text{éq An1.3-3}$$

- Autocorrelation of **random signals**, expressed in u^2 :

$$R_{XX}(\tau)=E\left[\overline{X(t)}\,X(t+\tau)\right]=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{+T/2} \overline{x(t)}\,x(t+\tau)\,dt \qquad \text{éq An1.3-4}$$

A1.4 Definition of the spectral concentration of power

A1.4.1 Expression in frequency

One defines the spectral concentration of power by:

$$S_{XX}(f)=\int_{-\infty}^{+\infty} R_{XX}(\tau)\, e^{-2i\pi f\tau}\, d\tau \text{ ou } G_{XX}(f)=\int_{0}^{+\infty} R_{XX}(\tau)\, e^{-2i\pi f\tau}\, d\tau \qquad \text{éq An1.4.1-1}$$

The mechanic being interested only in the positive values of the frequency and time, the function G_{XX} is more often used.

One can show, if the Transformations of Fourier of the signals exist, that this definition is equivalent (theorem of Wiener-Kinchine) to the definitions of the spectral concentration of power following.

- For the signals of finished energy:

$$G_{XX}(f)=|X(f)|^2 \text{ expressed in } u^2/Hz^2 \qquad \text{éq An1.4.1-2}$$

- For the periodic signals:

$$\text{If } X(f)=\sum_{n=-\infty}^{n=+\infty} C_n \delta(f-nf_0) \text{ then } G_{XX}(f)=\sum_{n=-\infty}^{n=+\infty} C_n^2 \delta(f-nf_0) \qquad \text{éq An1.4.1-3}$$

$G_{XX}(f)$ express yourself in u^2/Hz .

f_0 is the reverse of the period of the signal.

C_n coefficient of the Dirac functions.

- For the signals of finished power:

$$G_{XX}(f)=\lim_{T\to+\infty}\left(\frac{1}{T}|X_{[T]}(f)|^2\right) \text{ en } u^2/Hz \qquad \text{éq An1.4.1-4}$$

where $X_{[T]}$ indicate the restriction of $x(t)$ à $[-T/2 ; T/2]$.

- For the random signals:

$$G_{XX}(f)=\lim_{T\to+\infty} E\left[\frac{1}{T}|X_{[T]}(f)|^2\right] \text{ en } u^2/Hz \qquad \text{éq An1.4.1-5}$$

where $X_{[T]}$ indicate the restriction of $x(t)$ with $[-T/2 ; T/2]$.

Link between the DSP and the power.

With the definitions given above for the spectral concentrations of power, one has for all the signals, the relation:

$$P=\int_{-\infty}^{+\infty} G_{XX}(f)\,df \qquad \text{éq An1.4.1-6}$$

This relation is established by using the theorem of PARSEVAL.

A1.4.2 Expression in pulsation

In pulsation, one defines the spectral concentration of power by:

$$G'_{xx}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{XX}(\tau)e^{-i\omega\tau}\,d\tau \qquad \text{éq An1.4.2-1}$$

Just as for the expression in frequency, one can show, if the Transformations of Fourier of the signals exist, that this definition is equivalent (theorem of Wiener-Kinchine) to the definitions of the spectral concentration of power following

- For the signals of finished energy:

$$G'_{XX}(\omega)=2\pi|X'(\omega)|^2 \text{ expressed in } u^2/Hz^2 \qquad \text{éq An1.4.2-2}$$

- For the periodic signals:

$$\text{Si } X'(\omega)=\sum_{n=-\infty}^{n=+\infty} C_n\delta(\omega-n\omega_0) \text{ alors } G'_{XX}(\omega)=\sum_{n=-\infty}^{n=+\infty} C_n^2\delta(\omega-n\omega_0) \qquad \text{éq An1.4.2-3}$$

$G'_{XX}(\omega)$ express yourself in u^2/Hz , and $\omega_0=\frac{2\pi}{T}$ where T is the period of the signal.

C_n coefficient of the Dirac functions.

- For the signals of finished power:

$$G'_{XX}(\omega)=\lim_{T\to+\infty}\left(\frac{2\pi}{T}|X'_{[T]}(\omega)|^2\right) \text{ en } u^2/Hz \qquad \text{éq An1.4.2-4}$$

$X'_{[T]}$ indicate the restriction of $x(t)$ with $[-T/2;T/2]$.

- For the random signals:

$$G'_{XX}(\omega)=\lim_{T\to+\infty} E\left[\frac{2\pi}{T}|X'_{[T]}(\omega)|^2\right] \quad \text{in } u^2/Hz \qquad \text{éq An1.4.2-5}$$

$X'_{[T]}$ indicate the restriction of $x(t)$ with $[-T/2;T/2]$.

Link between the DSP and the power.

In the same way, there is for all the signals the relation - which rises from the theorem of PARSEVAL -:

$$P=\int_{-\infty}^{+\infty} G'_{XX}(\omega)\,d\omega \qquad \text{éq An1.4.2-6}$$

A1.4.1 Relation between DSP in frequency and DSP in pulsation

For the four types of signals:

$$G'_{XX}(\omega)=\frac{1}{2\pi}G_{XX}(f) \qquad \text{éq An1.4.3-1}$$

Annexe 2Transformation of Hilbert

That is to say $X(t)$ a real signal of transform of Fourier $X(\omega)$.

That is to say $H(\omega)$ the Transfer transfer function: $H(\omega)=j\,\text{sign}(\omega)=\begin{cases} j & \omega>0 \\ -j & \omega<0 \\ 0 & \omega=0 \end{cases}$

$H(\omega)$ transform $X(t)$ in its transform of Hilbert noted $\hat{X}(t)$. The system of transfer transfer function $H(\omega)$ product a dephasing of $+90°$ for the positive frequencies and of $-90°$ for the negative frequencies. It follows theorem of convolution that $\hat{X}(t)$ can also be defined like the convolution of $X(t)$ by the corresponding impulse response, that is to say $h(t)=1/\pi t$.

$\hat{X}(t)$ is also real, one second application of the transform of Hilbert restores the initial signal, changed of sign and amputee of his possible continuous component.

Example : $x(t)=A\cos\omega t\to\hat{x}(t)=\text{-}A\sin\omega t$

This property is at the base of the use of the transform of Hilbert to define the envelope of a process in narrow band.