

## Note of use of the boundary conditions treated by elimination

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### Summary

Treatment of the boundary conditions of the Dirichlet type by elimination (AFFE\_CHAR\_CINE) do not offer the same general information as by dualisation (AFFE\_CHAR\_MECA for example).

This treatment is to be used when one searches to improve the execution times of a calculation or if one wishes to work with positive definite matrices.

Let us note that the boundary conditions available in AFFE\_CHAR\_\* (\* = MECA/THER/ACOU) cannot all be eliminated and treated by AFFE\_CHAR\_CINE.

In this document, it is shown how to use the "loads kinematics" in the command sets Aster.

There are 3 cases (simpler with most complicated):

- One uses a "total" ordering of calculation (THER\_LINEAIRE, STAT\_NON\_LINE,...). In this case, the loads kinematics are used like the other loads.
- One wishes to do a calculation of clean modes. It is then necessary to add an argument in the order ASSE\_MATRICE.
- One wishes to do a calculation "step by step" and to solve the linear systems with the orders TO FACTORIZE and TO SOLVE. In this case, the order should be used CALC\_CHAR\_CINE.

## 1 Principle of elimination

One seeks to solve in  $\mathbb{R}^n$  the problem of minimization under constraint (Pb1) according to:

$$\min_{u \in U_G} \left( \frac{1}{2} u^T K u - u^T f \right) \quad \text{with} \quad U_G = \{ u \in \mathbb{R}^n, u|_G = u_0 \}$$

where

- $u_0 \in \mathbb{R}^p$  is known ( $1 \leq p \leq n$ )
- $G$  is the subset of  $N = \{1, \dots, n\}$ , of cardinal  $p$  :  $G = g_1 \dots g_p$
- $u|_G$  is the projection of  $u$  on under space generated by  $\{u_i\}_{i \in G}$
- where  $(u_i)_j = \delta_{ij}, \forall j \in N$
- $K$  is a symmetrical matrix  $n \times n$ ,
- $f \in \mathbb{R}^n$  is fixed.

The constraint  $u|_G = u_0$  or not represent boundary conditions of the homogeneous Dirichlet type.

If one notes  $L = C_N \setminus G$  the complementary one to  $G$  in  $N$ , one can, using  $u_i$  defined previously, to break up  $\mathbb{R}^n$  all in all direct of  $V_G =$  vector space generated by  $\{u_i\}_{i \in G}$  and of  $V_L =$  vector space generated by  $\{u_i\}_{i \in L}$ ;

Consequently, we have  $\mathbb{R}^n = V_G \oplus V_L$

and one notes  $u = u_G \oplus u_L$  where  $u_G = u|_G$  and  $u_L = u|_L$

that is to say still in vectorial notation  $u = \begin{pmatrix} u_G \\ u_L \end{pmatrix}$

The problem (Pb1) can thus be written in the form of the problem (Pb2):

$$\begin{cases} \min \left( \frac{1}{2} u_G^T K_{GG} u_G + \frac{1}{2} u_L^T K_{LL} u_L + u_L^T K_{LG} u_G - u_L^T f_L - u_G^T f_G \right) \\ u_G \in V_G \\ u_L \in V_L \\ u_G = u_0 \end{cases}$$

What amounts writing:

$$(Pb1) \Leftrightarrow (Pb2) \quad \begin{cases} \min \left( \frac{1}{2} u_L^T K_{LL} u_L + u_L^T K_{LG} u_0 - u_L^T f_L \right) \\ u_L \in V_L \\ u = u_0 \oplus u_L \end{cases}$$

One then eliminated  $u_G$  problem of minimization.

We now will search the matrix problem associated with (Pb3).

One searches  $u_L$  minimizing

$$\frac{1}{2} u_L^T K_{LL} u_L + u_L^T K_{LG} u_0 - u_L^T f_L$$

what amounts solving the following matrix problem:

$$K_{LL} u_L = F_L - K_{LG} u_0$$

One can thus write:

$$(Pb1) \Leftrightarrow (Pb2) \Leftrightarrow (Pb3) \Leftrightarrow \begin{bmatrix} K_{LL} & 0 \\ 0 & I_G \end{bmatrix} \begin{bmatrix} u_L \\ u_G \end{bmatrix} = \begin{bmatrix} f_L - K_{LG} u_0 \\ u_0 \end{bmatrix}, \text{ that is to say } K' \begin{bmatrix} u_L \\ u_G \end{bmatrix} = f'$$

## 2 Treatment in Aster

### 2.1 The loads kinematics

A kinematic load (standard Aster : char\_cine\_\* [\* = meca/ther/acou]) makes it possible to characterize the unit G of the imposed ddl and them  $(u_0)_i$  for  $i \in G$  who are the values assigned to these ddl.

The definition of a kinematic load is done via the operator AFPE\_CHAR\_CINE for  $(U_0)_i$  constant or functions of the geometry or time.

### 2.2 The vectors kinematics

The kinematic vector is a cham\_no\_\* which represents the vector  $\begin{bmatrix} 0 \\ u_0 \end{bmatrix}$ .

With each kinematic load corresponds a kinematic vector.

This operation is carried out by the operator CALC\_CHAR\_CINE.

### 2.3 Calculation of K'

K' is directly calculated at the assembly time by the operator ASSE\_MATRICE provided naturally that one provides in argument a list of loads kinematics.

The structure of data MATR\_ASSE\_\* was modified in order to be able to store K' when that is necessary.

### 2.4 Calculation of f'

After L" operator TO\_FACTORIZE the concept of the matr\_asse\_\* type produced, contains factorized of K' and the matrix  $K_{LG}$  unchanged.

The calculation of f' is carried out at the time of the resolution: it is necessary to provide the operator TO SOLVE in argument the kinematic vector corresponding to  $\begin{bmatrix} 0 \\ u_0 \end{bmatrix}$  via the keyword CHAM\_CINE.

This operator calculates then  $f'$  before solving  $\text{fact}(K') \begin{bmatrix} u_L \\ u_G \end{bmatrix} = f'$ .

## 3 Examples of command files

### 3.1 Mechanical calculation with a total order (STAT\_NON\_LINE):

```
DEPIMP=AFFE_CHAR_CINE (MODELE=MOD,  
                        MECA_IMPO=_F ( GROUP_MA = 'LCD1', DY = -2.0))  
  
RESU=STAT_NON_LINE (MODELE=MOD, CHAM_MATER=CHMAT,  
                    EXCIT=_F ( LOAD = DEPIMP, FONC_MULT = FONC),  
                    ...)
```

### 3.2 Loads kinematics for a calculation of clean modes:

```
CHARCINE=AFFE_CHAR_CINE (MODELE=MODEL,  
                        MECA_IMPO=_F (GROUP_MA=' GM2', DX=0.0, DY=0.0))  
  
KASS=ASSE_MATRICE (MATR_ELEM=KELEM,  
                  NUME_DDL=NUMÉRIQUE,  
                  CHAR_CINE=CHARCINE, );  
  
MASS=ASSE_MATRICE (MATR_ELEM=MELEM,  
                  NUME_DDL=NUMÉRIQUE,  
                  CHAR_CINE=CHARCINE, );  
  
# calculation of the clean modes of the structure  
MODES=CALC_MODES (MATR_RIGI=KASS,  
                 MATR_MASS=MASS,  
                 CALC_FREQ=_F (NMAX_FREQ=10))
```

### 3.3 Calculation “step by step” by using the orders TO FACTORIZE and TO SOLVE :

```
CHCINE=AFFE_CHAR_CINE ( MODELE=MO, MECA_IMPO= (  
    _F (GROUP_NO = 'SUPY', DY = 0.),  
    _F (GROUP_NO = 'LOAD', DX = -1.)))  
  
MEL=CALC_MATR_ELEM ( MODELE=MO, CHAM_MATER=CHMAT, OPTION=' RIGI_MECA')  
  
NU=NUMÉRIQUE_DDL ( MATR_RIGI=MEL)  
  
MATAS=ASSE_MATRICE (MATR_ELEM=MEL, NUME_DDL=NU, CHAR_CINE=CHCINE)  
  
SCMBRE=CRÉA_CHAMP (...)  
  
VCINE=CALC_CHAR_CINE (NUME_DDL=NU, CHAR_CINE=CHCINE )  
  
MATAS=FACTORISER (reuse=MATAS, MATR_ASSE=MATAS)
```

DEP=TO SOLVE (MATR=MATAS, CHAM\_NO=SCMBRE, CHAM\_CINE=VCINE)