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Responsable : VOLDOIRE François

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Note of use for the calculation of limiting load

Summary

The objective of this note is to give the necessary information so that user can carry out calculations of load limits with *Code_Aster*.

The first part points out the broad outlines of the method and its properties.

The second part presents the various stages necessary to the implementation in *Code_Aster*.

Finally, the third part presents some remarks of implementation while being based on the CAS-test *Aster* [V6.04.124] and on the industrial case aiming at determining the pressure limits for a joint of Canopy tank.

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1   Introduction

This document aims at guiding the user for the implementation of a calculation of limiting analysis with and supplements documentation available in on the limiting analysis which consists of:

- reference document: [R7.07.01];
- documents of validation SSNV124A [V6.04.124] and SSNV146 [V6.04.124];
- document of formation: Heading Formation of the www.code-aster.org site

With this intention *code_aster* use:

- a kinematic approach regularized (method of regularization of Norton-Hoff-Friaâ, cf [1]) for the criterion of resistance of Von Mises (adjustment by a coefficient of regularization of which the limiting value led to convergence);
- quasi-incompressible finite elements;
- a nonlinear static resolution by parametric piloting;
- postprocessing to obtain a higher estimate of the values $\lambda_{\lim}^{\sup}$ and lower $\lambda_{\lim}^{\inf}$ who frame the limiting value $\lambda_{\lim}$.

This document is composed of three parts:

1. the first part which points out the broad outlines of the method and its properties;
2. the second part who presents the various stages necessary to the implementation in **C ode_ has ster** ;
3. the third part which presents some remarks of implementation while being based on the CAS-test [V6.04.124] and the industrial case aiming at determining the limiting pressure for a joint of Canopy tank.

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2 Broad outlines of the method

2.1 Présentation

The objectives of the analysis limits are:
- analysis of security vis-a-vis an extreme behavior (State Ultimate limit E.L.U.);
- fast dimensioning without expensiveexpensive to describe the whole of the process of ruin;
- the energy characterization of the ruin and the comprehension of the modes of ruin;
- obtaining simple information on the nonlinear evolution of structural material.

The limiting analysis is with a problem which one can deal in two manners, cf [8], [9], [10]:
- "plastic" calculation of ruin of elastoplastic structures with ductile plate. The way of loading and the model of behavior of material must be entirely described;
- calculation of loss of potentiality of balance to criterion of resistance given, for a direction of loading given. It is about a problem of optimization (under constraint) of the parameter of load λ . This approach is called "design the collapse".

For standard materials, these two methods give the same result.

The "design the collapse" or "analyzes limit" (term indicating the design the collapse in the case of an elastoplastic material with rule of normal flow) aims at determining directly, in a simplified way and without having recourse to the description of the way of loading by an expensive elastoplastic incremental calculation, the border of the field of plastic ruin (and by deduction the field of the bearable loads) for a structure Ω , of geometry and limits of resistance of materials given, subjected to a loading given by its direction $\mathbf{F}$, and of amplitude parameterized by positive reality λ . A "permanent" loading $\mathbf{F}_0$, such as for example gravity, can possibly be present in more (without being amplified by λ).
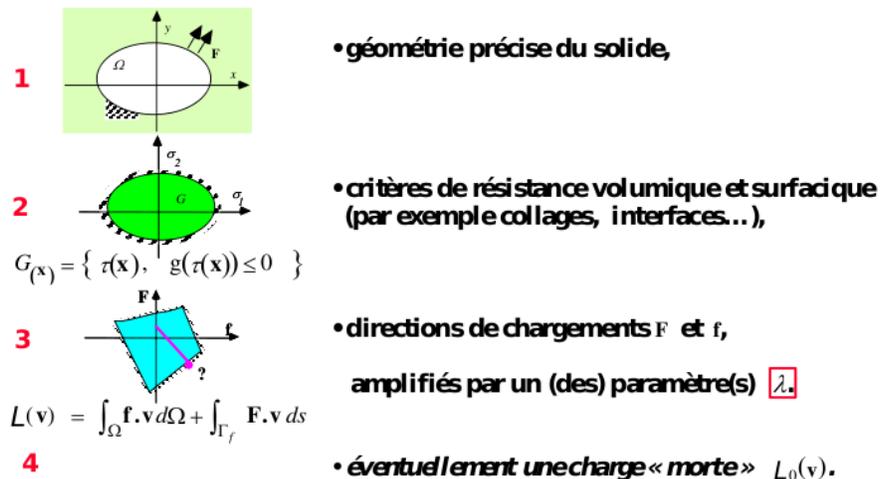

Figure 2.1-a: Ingredients of the design the collapse.

Note: ON cannot take account of any change of geometry by the design the collapse, as that arrives at the time of a ruin by buckling, or for a solid very soupl That constitutes the assumptions of the method: the configuration of the solid is that of its initial geometry, the connections of the

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structure are supposed to be given and fixed until the ruin; in the same way the loadings, in efforts only, are fixed directions.

Two approaches of the design the collapse are accessible:
* the static approach which considers the value of limiting load by the interior and which requires the construction of the statically acceptable stress fields, which is delicate in general by finite elements. It consists in maximizing to it (or them) parameter (S) of loading in the condition that the equations (linear) of statics remain checked and that the criterion in constraints is not violated;
* kinematic, dual approach of the preceding one, which considers the value of limiting load by the outside and which requires a minimization by a method of regularization of (or) the parameter (S) of loading under the condition which the power of the external efforts remains higher than the resistant power, (definite starting from the criterion of resistance), of a nonregular functional calculus, which must consequently be regularized in the case general.

The combined use (ideal case!) of these two approaches provides framings of the limiting load.

2.2   An analytical example [7]

One considers a hyperstatic system with three bars: to see [fig. 2.2-a], the bars have an identical criterion of resistance, expressed in term of normal effort (or tension) N : $g(N)=|N|\leq \bar{N}$.
The point D is subjected to a force $\vec{F}$ components $\left(F_x, F_y\right)$ nonworthless, amplified by a multiplicative factor λ .
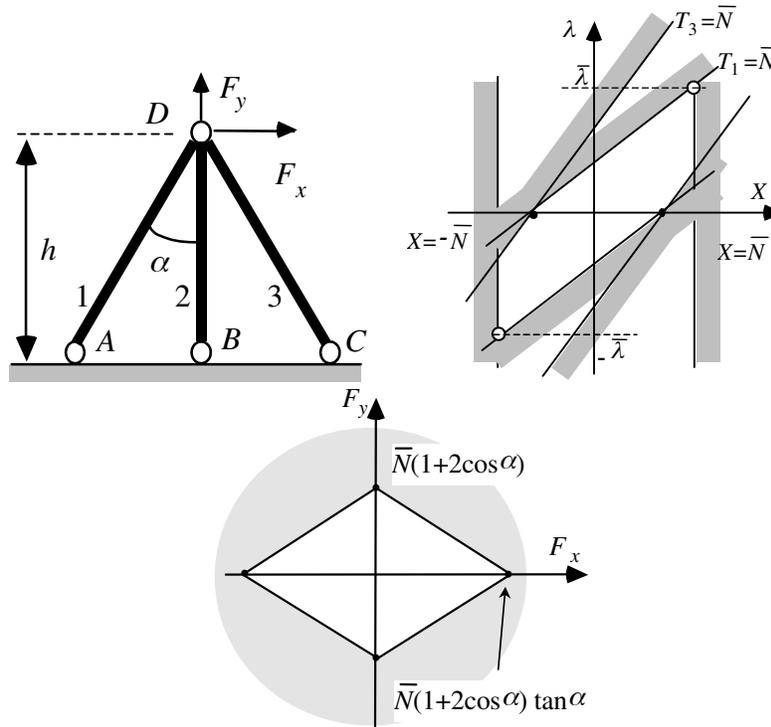
Figure 2.2-a: on the left: system with three bars; in the medium: field of the bearable tensions; on the right field of the bearable loadings.

The space of statically acceptable solutions (tensions T_i in the bars) is defined by the equations:

$$\begin{cases} T_1 = \dfrac{\lambda\left(F_x\cos\alpha + F_y\sin\alpha\right) - X\sin\alpha}{\sin 2\alpha} \\ T_2 = X \\ T_3 = \dfrac{\lambda\left(F_y\sin\alpha - F_x\cos\alpha\right) - X\sin\alpha}{\sin 2\alpha} \end{cases} \tag{1}$$

For λ positive, it is noted that it is in $T_1 = T_2 = \overline{N}$ that the extremum is reached, to see [fig. 2.2-a]. One finds thus that the bearable maximum value of the load factor λ (or limit charges), obtained by the static approach, is:

$$\overline{\lambda} = \overline{N} \frac{\left(1 + 2\cos\alpha\right)\sin\alpha}{F_x\cos\alpha + F_y\sin\alpha} \tag{2}$$

The kinematic approach gives the same result [6]: it is well the limiting load of this problem.

2.3   Some useful properties of calculation of limiting load

In the ideal case, the hight delimiters and lower of the limiting load must be equal to the limiting value. With the digital approach, there will be always a variation and it is the lower limit which is

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more penalizing. It is however to note that, for the structures which one can lead sufficiently calculation far (as in the cases tests), the upper limit is in practice that which is closest to the exact value.

One frequently chooses like threshold of resistance the elastic limit: that goes in the direction of security.

One points out hereafter some useful properties of the calculation of limiting load (see [4], [8]):

- the limiting load is proportional to the value of the limit of resistance or threshold σ_y in a homogeneous solid. It does not depend on the history of the loading undergone by the first structure;

- as the criterion of resistance is convex (criterion of von Mises), the field of the bearable loadings (thus the border of the limiting loadings) within the space of loadings is convex. One can thus approach the field of the acceptable loadings by the generalized polyhedron built on the tops, corresponding each one to a direction chosen within the space of loadings;

- the conditions of Dirichlet (an imposed displacement) which are applied to the part Γ_u edge $d\Omega$ structure, or an initial unelastic deformation – thermics, plastic… – do not have an effect on the field of the chargbe acceptable, (the ruin being the impossibility of satisfaction of balance, the mode of ruin corresponds to a speed-direction of flow);

- the limiting load does not depend on the possible presence of a stress field auto--balanced (residual stress);

- for a two-dimensional solid, the material and direction of loading being given, a terminal **lower** obtained by the static approach in **plane constraints** is necessarily lower than the exact limiting load obtained in **plane deformations** : $\lambda^-_{C_{PLAN}} \leq \lambda^{\lim}_{D_{PLAN}}$;

- reciprocally, Udo not limit **higher** obtained by the kinematic approach in **plane deformations** is necessarily lower than the exact limiting load obtained in **plane constraints** : $\lambda^+_{D_{PLAN}} \geq \lambda^{\lim}_{C_{PLAN}}$. This result thus provides one raising. If one wishes to deal with a problem in plane constraints, it is necessary then to make the kinematic approach on a three-dimensional modeling of a "slice" of solid ;

- limiting loads obtained in plane deformations with the criterion of Tresca are worth $\sqrt{3}/2$ time those found with the criterion of vone Settings ;

- with geometry and direction of loading given, if one replaces in a given zone of the structure material of field of resistance G_1 by a material of field of resistance $G_2 \subset G_1$ ($g_2(\tau) \leq g_1(\tau)$) (for example: criterion of Tresca included in that of von Mises), then the functions of support (maximum resistant powers) are: $\pi_2(\varepsilon(v)) \leq \pi_1(\varepsilon(v))$ and thus: $\lambda^{\lim}_2 \leq \lambda^{\lim}_1$;

- in particular if a defect is replaced 1 (hole, crack) present in the structure by the defect 2 containing the defect 1 , then one a: $\lambda^{\lim}_2 \leq \lambda^{\lim}_1$. In the same way, if the structure is heterogeneous, with two zones whose limits of resistance are $\sigma_{y1} \leq \sigma_{y2}$, the load limits will be higher than that of the same homogeneous situation for the threshold σ_{y1} and lower than that for the threshold σ_{y2} ;

- in the presence of a direction of loading $f = \lambda(\alpha f_1 + (1-\alpha) f_2)$ combining two directions f_1 and f_2 , $\alpha \in [0, 1]$, then the exact limiting load checks:

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$$\lambda^{\lim}(\alpha) \geq \bar{\lambda} = \frac{\lambda_1^{\lim} \lambda_2^{\lim}}{(1-\alpha)\lambda_1^{\lim} + \alpha\,\lambda_2^{\lim}}$$. This result remains valid for approximations by the interior of the limiting loads.

Whereas the general three-dimensional situations are inaccessible analytically, in 2D plane deformations (D_PLAN) and plane constraints (C_PLAN) it is possible to build with the hand and to calculate solutions for the static approach and the kinematic approach, using fields built per blocks, which give framings of the limiting load: this proves useful to corroborate a result got by finite elements.

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3 The implementation of the limiting analysis

3.1 Stages of calculation

To carry out a limiting analysis, it is necessary:
- to use a grid 2D (plan or axis) or 3D compatible with the incompressible finite elements;
- to define the model with the incompressible finite elements;
- to define the limit of resistance of material σ_y ;
- to define the permanent loading $\mathbf{F_0}$ and the variable loading $\mathbf{F}$ who is controlled by λ ;
- to define the condition of incompressibility;
- to carry out a non-linear calculation with the incremental behavior and piloting planned for the limiting analysis;
- post-to treat calculation to obtain the values higher and lower estimated limiting load.

3.2 Grid

For the choice of the grid and the use of the éléments incompressible, one returns to documentation [R3.06.08]

Note: the use of element with three fields UPG will require *to impose a boundary condition on swelling (GONF), the nodes tops must thus be perfectly identified in the grid:*
- *maybe as of the phase of design and realization of the grid;*
- *maybe, more simply and a posteriori, by calling in on the order DEFI_GROUP [U4.22.01] with the keyword CREA_GROUP_NO by specifying the group of incompressible meshs of elements and by using the option CRIT_NOEUD = 'TOP' .*

3.3 Model

3.3.1 Options of modeling

It is necessary to select incompressible elements, one thus returns to the order AFFE_MODELE [U4.41.01].

3.3.2 Condition of incompressibility

To express the condition of incompressibility for incompressible elements UPG with three chams, the order is used AFFE_CHAR_MECA [U4.44.01] with the keyword DDL_IMPO to impose on the component GONF group of the nodes tops of the incompressible elements to remain worthless.

3.4 Material

The material used for the limiting analysis with the incompressible elements is a material with a criterion of vone Settings, elastoplastic perfectly plastic.

The data of the material characteristics are provided under the keyword factor ECRO_LINE order DEFI_MATERIAU [U4.43.01].

The slope of the traction diagram (E_T) is selected worthless (operand D_SIGM_EPSI), the only data necessary to provide is thus the elastic limit (i.e. the threshold of resistance in our case) (operand SY).

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3.5 Loading

•

The variable loading $\mathbf{F}$ who is controlled by λ must necessarily be of standard effort (force, pressure, gravity) and declared in the order AFFE_CHAR_MECA [U4.44.01].

If the structure is also subjected to a permanent loading $\mathbf{F}_0$, it is necessary to think of recalling it at the time it postprocessing (see [§3.8]).

3.6 List of moments

The list of moments is used to control the method of regularization as Norton-Hoff, cf [1], [5], via a coefficient m, and not evolution of the loading as during an ordinary calculation:

$$m = 1 + 10^{1-t} \tag{3}$$

so that, when the moment becomes sufficiently large, m tends towards 1, and the behavior approaches a behavior perfect rigid-plastic, to see the uniaxial curve [fig. 3.6-a].
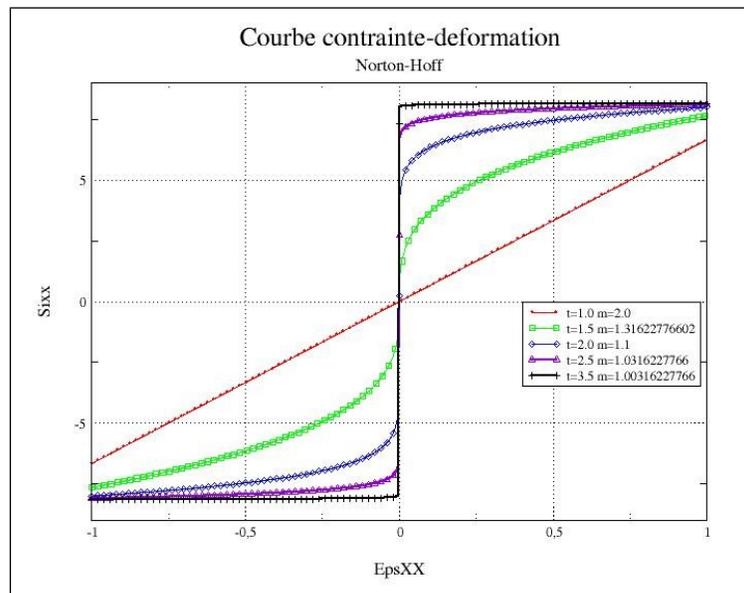

Figure 3.6-a. Stress-strain curve for various values of the moment t

In practice, one will choose at the beginning a list of moments with constant steps (see Table 2.6-a) before refining the steps of calculation in the event of nonconvergence.

t	1.0	1.5	2.0	2.5	3.0	…
$m = 1 + 10^{1-t}$	2.00	1.30	1.10	1.03	1.01	

If the document of use of the order POST_ELEM with the keyword CHAR_LIMITE [U4.81.22] recommends, in practice, to limit itself to moments ranging between 1 and 2 not to have too long

calculations while allowing to obtain an upper limit of the load limits sufficiently precise, we observe that the load limits lower requires at least 2 to 3 additional iterations (moment higher than 3) to converge towards the values of reference at the time of the cases tests.

3.7   Calculation

Modeling with incompressible elements must necessarily use the order STAT_NON_LINE [U4.51.03] and the keyword BEHAVIOR.

For the limiting analysis, it is obligatory to call on the operands following of the order STAT_NON_LINE :
   • In BEHAVIOR one chooses RELATION=' NORTON_HOFF' . This operand is used to describe the relation of behavior of viscosity (independent of the temperature) in the calculation of limiting loads of structures, with threshold of von Mises [U4 . 51.11]
   • In PILOTING , one chooses TYPE=' ANA_LIM' . This mode of piloting is specific to the calculation of load limits (law NORTON_HOFF) by kinematic approach. It must be only applied to the load declared via the keyword factor EXCIT , operand TYPE_CHARGE=' FIXE_PILO' .

The calculation of the limiting load can require much iterations of linear research and iterations of Newton. It also is thus strongly advised to employ the various options of calculation of the order STAT_NON_LINE to improve convergence, as the linear research whose practice shows that it is enough to resort to 2 or 3 iterations.

Note: *if the intensity of the loading is amplified $L \rightarrow \beta L$ (whereas one does not consider a permanent loading $L_0 = 0$), the solutions depend on the factor β according to the following relations:*

$$u_m(\beta) = \beta^{-1} u_m(1) \quad ; \qquad \sigma^D(u_m(\beta)) = \beta^{1-m} \sigma^D(u_m(1)) \tag{4}$$

With convergence for $m \rightarrow 1^+$, the limiting loading given by the solution $u_m(\beta)$ is well the same one as that given by $u_m(1)$, since $\lambda_{\lim}(\beta) = \lambda_{\lim}(1)/\beta$.

3.8   Postprocessing

From the result of nonlinear calculation carried out, the operator POST_ELEM [U4.81.22] and the keyword CHAR_LIMITE produce a table which gives, for each moment of calculation, the estimate of the upper limit CHAR_LIMI_SUP ($\hat{\lambda}_m$) limiting load supported by the structure; this continuation is monotonous decreasing when $m \rightarrow 1^+$, i.e. when $t \rightarrow +\infty$.

Moreover, in the absence of permanent loading F_0 (operand CHAR_CSTE = 'NOT' who is the option by default), the table also contains the estimate CHAR_LIMI_ESTIM (λ_m) lower limit of the limiting load. This value (approximation of the gauge of convex of resistance) is not calculated that at the points of Gauss of the finite elements. Also the value λ_m obtained for each m , lower than $\hat{\lambda}_m$ [6], cannot be considérée that as an indication (this continuation is not necessarily monotonous). It allows, with the value by excess $\hat{\lambda}_m$, to provide a framing of the load limits discretized problem.

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On the other hand, if a permanent loading F_0 is present (operand CHAR_CSTE = 'YES'), such an estimate of the lower limit is not available any more and the table indicates the power then PUIS_CHAR_CSTE constant loading in the field speed solution of the problem.

The visualization of the field of displacement obtained for a value of the coefficient $m \rightarrow 1^+$ give a "idea" of the mode of ruin of the studied structure. One finds more details on postprocessing in [R7.07.01].

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4 Some remarks of implementation

4.1 Case representative test: SSNV146 has

4.1.1 Description

These remarks are based on the implementation of the case test SSNV146A who is drawn from the benchmark of the European project Brite EuRam BE97-4547 "LISA" [8]. It is the calculation of the limiting load of a tank with a spherotoric bottom in 2D axisymmetric, under internal pressure, to see Figure 3.a. the internal ray of the cylindrical part is: $49\ mm$, while the thickness is: $2\ mm$. The ray of the spherical part to the apex is $98\ mm$, while the ray of the torus of connection is of $20\ mm$.

Geometry	The axisymmetric spherotoric bottom reserve (see Figure 3.1-a) has the following characteristics: • ray interns cylindrical part: $49\ mm$; • thickness: $2\ mm$; • ray of the spherical part to the apex: $98\ mm$; • ray of the torus of connection: $20\ mm$
Material properties	Elastic limit: $\sigma_y = 100\ MPa$
Boundary conditions	Conditions of symmetry
Loadings	Pressure interns $1\ MPa$

Table 4.1-a: data of test SSNV146 (benchmark LISA) [8].

The following table recapitulates the results got by the participants in the benchmark using the same grid which contains 34 elements `QUAD8` (including two elements in the thickness) and 141 nodes.

Modeling				Higher value estimated	Lower value estimated
EDF [1]	$m=1,0476$	$n=21$	$t=2,3222$	3.9514	3.6049
	$m=1,0322$	$n=31$	$t=2,4914$	3.9456	3.7090
	$m=1,0141$	$n=71$	$t=2,8513$	3.9404	3.8372
	$m=1,0099$	$n=101$ [1]	$t=3,0043$ [2]	3.9396	3.8673
Univ. of Liège/LTAS				3.931	nothing
Research centre FZJ				nothing	3,997

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Table 4.1-b: Results of the benchmark LISA [8]

Foot-note [1]   : the coefficient of regularization by the law of Norton-Hoff $n = (m-1)^{-1}$ (old method of EDF)

Foot-note [2]   : $m = 1 + 10^{1-t}$, the 'urgent' parameter t was not explicit in this old method, one adds here to better be able to compare with results in sections 4.2 and 4.3.

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[Figure 4.1-a] the evolution of the terms of the two continuations presents $\underline{\lambda}_m$ and $\hat{\lambda}_m$ according to the coefficient of regularization $n=(m-1)^{-1}$ with $m=1+10^{1-t}$, cf [7]. The hight delimiters and lower of the estimated limiting load are calculated with the list of moments, exploiting directly the coefficient of regularization by the law of Norton-Hoff. This makes it possible to carry out this convergence directly and to simplify the use.
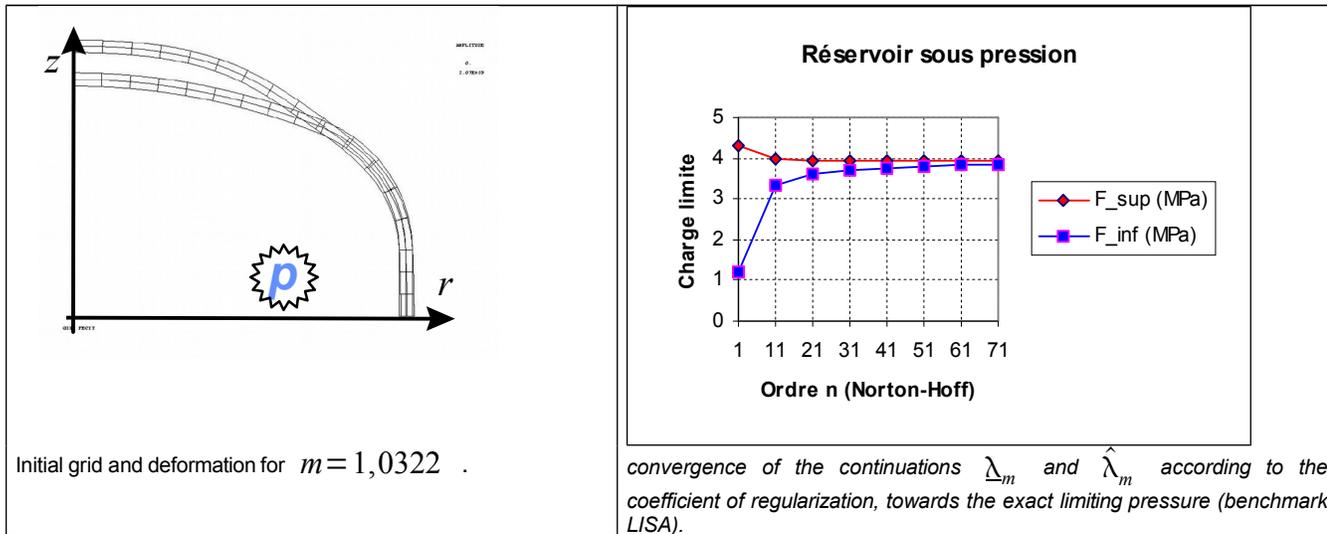

Initial grid and deformation for $m=1,0322$.

convergence of the continuations $\underline{\lambda}_m$ and $\hat{\lambda}_m$ according to the coefficient of regularization, towards the exact limiting pressure (benchmark LISA).

Figure 4.1-a : results of EDF calculation carried out within the framework of the benchmark LISA with the old method

Note:

> *For a pipe of same dimensions:* $P_{\lim}=4,0005\,MPa$.
> *For a sphere of same dimensions:* $P_{\lim}=4,040\,MPa$.
> *It is noted that this test is "not very discriminating" from a mechanical point of view.*

4.1.2   Results of the case test SSNV146

The grid is that used for the benchmark "LISA". Calculation was carried out until the moment $t=2\,s$. The version used is the version of exploitation STA 8.3.

```
 ---- TABLE: ECHL1     NOM_PARA: CHAR_LIMI_SUP


REFERENCE: NON_DEFINI
  OK    ECHL1              RELA        0.690%  VALE:
3.9581130295563D+00
CHAR_LIMI_SUP            SHEET        1.000%  REFE:
3.9310000000000D+00
```

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The declared value of reference (3,931) is a value estimated for the upper limit and it was provided by the university of Liege for the benchmark "LISA", to see [Tab.3.1-b].

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4.2   More thorough calculation

To improve the estimate of the values higher and lower of the estimated limiting load, calculation was thorough further until the moment $T = 2,45\ S$ (let us recall that it is not a physical time, to see § 3.6) beyond whose calculation does not converge any more.

INST	M	CHAR_LIMI_SUP $\hat{\lambda}_m$	CHAR_LIMI_ESTIM $\underline{\lambda}_m$
$t=1$	$m=2$	4.30410	1.27329
$t=2$	$m=1,1$	3.94863	3,27091
$t=2,125$	$m=1,0750$	3.93504	3,40992
$t=2,25$	$m=1,0562$	3.9 2505	3,52215
$t=2,450$	$m=1,0355$	3.9 1 429	3.65061

Table 4.2-a: Évolution of the estimated terminals sup and inf limiting load. Initial grid.

When one compares the got results [. 4.2-a] with those provided by EDF the old version [. 4.1-b], one observes that equivalence " *moment of calculation* " and " *Norton-Hoff coefficient* " do not seem to be identical if one bases oneself on the load limits higher or the load limits lower.

Indeed, if the decreasing tendency (resp. increasing) of the load limits higher (resp. lower) is confirmed ([fig. 4.1-a]), then, with $t=2,45\ s$, the Norton-Hoff coefficient are equivalent would be higher than $n=101$ for the higher value whereas it would be lower than $n=21$ for the lower value.

4.3   Influence of the smoothness of the grids

The calculation of the case test with the initial grid does not converge afterwards any more $t=2.45\ s$. As for any other calculation into non-linear, the grid was refined to try to improve convergence.

This calculation would allow especially crumbux to approach the load limits by a better estimate of the lower value.

With a discretization twice finer, that is to say a grid of 136 elements QUAD8 (either 4 elements in the thickness), calculation was brought to convergence until $t=2,153\ s$.

INST (S)		CHAR_LIMI_SUP $\hat{\lambda}_m$	CHAR_LIMI_ESTIM $\underline{\lambda}_m$
1.00000	$m=2,000$	4.32038	1.18088
2.00000	$m=1,100$	3.97963	3.26240
2.0625	$m=1,074$	3.97297	3.34124
2.125	$m=1,07499$	3.96726	3.41252
2.1531	$m=1,07029$	3.96498	3.44220

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Table 4.3-a: Évolution of the estimated terminals sup and inf limiting load. Fine grid.

The relative variation $\dfrac{\lambda_{\text{lim}}^{\text{sup}} - \lambda_{\text{lim}}^{\text{inf}}}{1 \big/ 2 \cdot \left(\lambda_{\text{lim}}^{\text{sup}} + \lambda_{\text{lim}}^{\text{inf}}\right)}$ master key of 6.97% (initial grid) with 14.11% with the finer grid.

It is also noted that at the same moments ($t=1\ s$ and $2\ s$), one does not have exactly any more the same values higher and lower of the limiting load estimated with the initial grid [Table 4.2-a].

The influence of the smoothness of the grid thus appears at the same time in total convergence and in the precision Dbe estimated values at the saved moments. One concludes from it that LE calculation of limiting load by the finite elements is sensitive to the grids. The finer grid inevitably does not give a limit more pushed because of a stop of calculation earlier. This is not nevertheless a problem, knowing that analysis of limiting load by the law Norton-Hoff tends towards a singular problem which posesspoke difficulties of convergence. The objective of the analysis is to find one continuation of values of $\hat{\lambda}_m$ converging towards a limiting load λ_{lim} .

4.4 Évolution of the estimate with the list of momentS

For the case test SSNV 146 considered, the value of reference is not a theoretical value but resulting itself from a digital calculation.

The case test SSNV124, describing a situation of a solid 2D or 3D in charge of homogeneous manner, allows to compare the values estimated compared to an analytical solution, to see [V6.04.124] (for which $\hat{\lambda}_m = \lambda_{\text{lim}}$, $\forall\ m$).

INST (S)	relative error compared to the value of reference (in %)	
	Value sup	**Value inf**
1.00	0.00	50.00
2.00	0.99	0.99
3.00	0.00	0.99
4.00	0.00	0.10
5.00	0.00	0.01

Table 4.4-a : Case test SSNV124
Évolution of the relative error for the values sup and inf estimated for the limiting load

[Table 4.4-a] watch the convergence of the values higher and lower estimated towards the analytical value of reference. This convergence is slow for the estimated lower value and it is much faster for the higher value.

These results tend to show that if one considers the average of the values higher and lower one would obtain a conservative value of the estimated limiting load.

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4.5 Problems presenting of symmetries

On the way let us note an important facility in the implementation. Although the limiting load $\lambda_{\lim}$ that is to say calculated in the form of an integral on the field, it is not necessary to multiply the value obtained if one calculates on a under-part of the solid, cell of symmetry of the problem.

We illustrate it with an alternative 3D of axisymmetric modeling C of the case test SSNV124.

The case represents an internal cylinder of ray thus $a=1\,mm$ and Externe $b=3\,mm$ subjected to a pressure interns $1\,MPa$ in internal wall.

The theoretical result $\lambda_{\lim}=\dfrac{2\sqrt{3}}{3}.\sigma_y.\ln\dfrac{b}{a}$ give a load limits

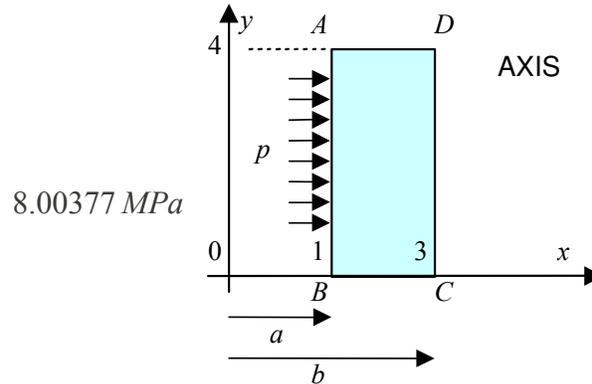**Figure 4.5-a: cylinder under internal pressure (SSNV124c test).**

With the grid of the ¼ of cylinder presented [fig. 4.5-b], one can read in the table below directly the results of the values higher and lower of the estimated limiting load.

```
#TABLE_SDASTER
  NUME_ORDRE INST        CHAR_LIMI_SUP CHAR_LIMI_ESTIM
          1 1.00000E+000  8.00361E+000    2.34638E+000
          2 1.69897E+000  8.00360E+000    5.91927E+000
          3 2.00000E+000  8.00360E+000    6.84900E+000
          4 3.00000E+000  8.00360E+000    7.87601E+000
          5 4.00000E+000  8.00360E+000    7.99071E+000
          6 5.00000E+000  8.00360E+000    8.00231E+000
          7 6.00000E+000  8.00360E+000    8.00347E+000
          8 7.00000E+000  8.00360E+000    8.00359E+000
          9 8.00000E+000  8.00360E+000    8.00360E+000
```

Table 4.5-a: Évolution of the estimated terminals sup and inf limiting load. Case SSNV124c test.
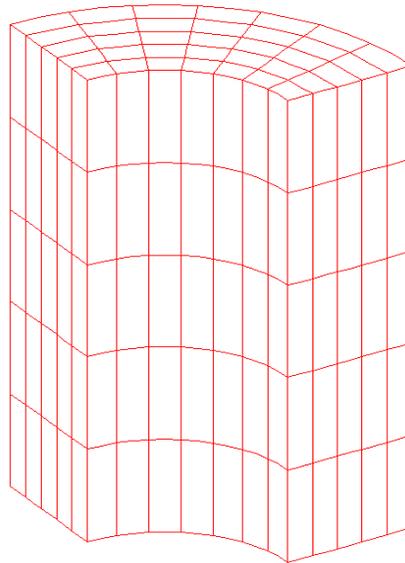
Figure 4.5-b: Grid of the quarter of cylinder considered (SSNV124).

4.6 Comparison analyzes limit and incremental elastoplastic calculation until the ruin on an example

It is known that in perfect elastoplasticity, the tangent operator has worthless eigenvalues starting from a certain level of loading in effort: that means that the elastoplastic solid reached the plastic ruin.

It can be interesting to do an elastoplastic calculation (with criterion of von Mises) incremental until the ruin, in order to obtain a lower "terminal" of the limiting load. As the algorithm of Newton used to solve nonlinear static balance diverges for this level from loading, it is necessary to use a piloting by length of arc [R5.03.80], or on a variable of displacement, being used to control the loading by the deformation of the solid undergone before the ruin.

Here an industrial example: it is calculation nearion interns limit of a Joint of Canopy tank. The problem is axisymmetric; the part is blocked on its higher border. The limit of resistance (contractual) is fixed at $100\,MPa$; a criterion of von Mises is admitted. Two calculations were carried out:

- a calculation of analysis limits with the method presented in the preceding paragraphs;
- a perfect elastoplastic incremental calculation (without and with work hardening), with criterion of von Mises in great transformations (Simo-Miehe, to see [R5.03.21]), in order to check that the change of geometry substantially does not modify the limiting prediction of pressure.
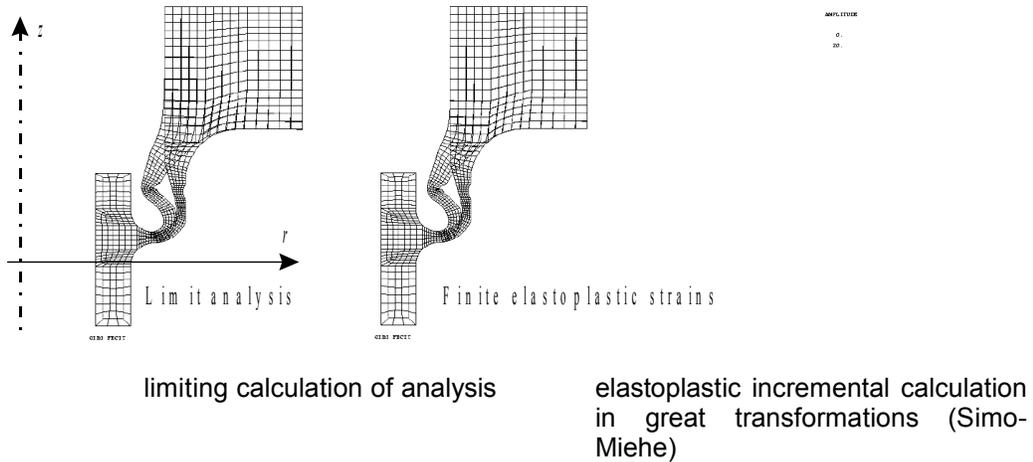
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limiting calculation of analysis

elastoplastic incremental calculation in great transformations (Simo-Miehe)

Figure 4.6-a. Deformed grid (amplified)

[Fig. 4.6-a] the watch which the calculated deformations are very close between the two methods and make it possible to predict the mode of ruin.

[Fig. 4.6-b] the watch the convergence of the continuations of the terminals of limiting pressure $\underline{\lambda}_m$ and $\hat{\lambda}_m$ according to the coefficient of regularization of the method of calculating of analysis limiting, towards the exact limiting pressure. The arithmetic mean of the two terminals seems to constitute a good estimate of the limiting pressure. For the last value of the coefficient of regularization chosen, one obtained the values deferred to [tab.4.6-a]. One compares them with perfect elastoplastic incremental calculation in great transformations. It is noted that the values are very close.

limiting analysis: limit lower	limiting analysis: limit higher	perfect elastoplastic incremental calculation
$18,72\ MPa$	$23,84\ MPa$	$23,25\ MPa$

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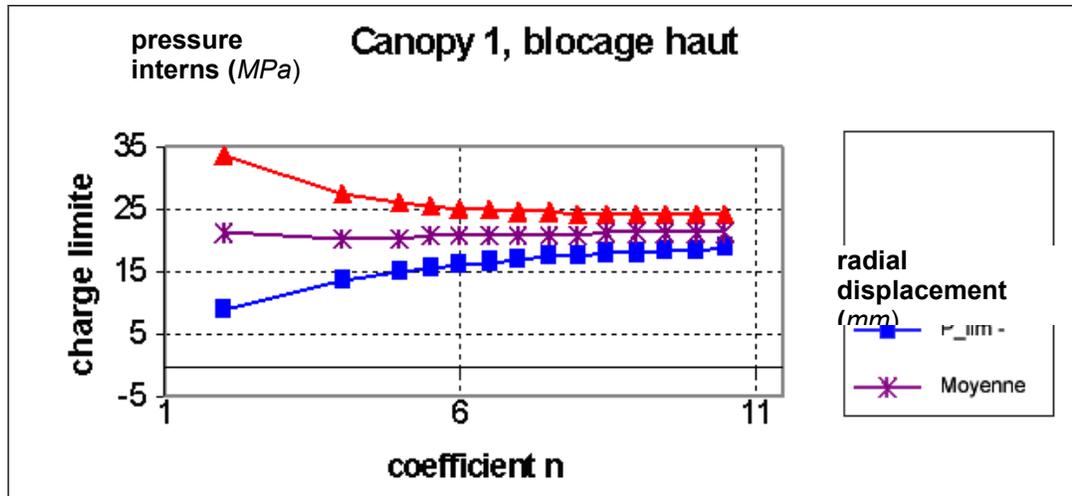
Table 4.6-a: Limiting pressures.

Figure 4.6-b. Limiting calculation of analysis: convergence of the continuations $\underline{\lambda}_m$ and $\hat{\lambda}_m$ according to the coefficient of regularization, towards the exact limiting pressure.

Perfect elastoplastic incremental calculation in great transformations was carried out with two types of piloting:

• by a radial displacement of a particular point;

• by length of arc (this last piloting having been more performing in time calculation);

and it was compared the results with those obtained by a calculation in small transformations.

One of the questions which installation is finally the effect of the work hardening of material, and thus of the choice of the threshold of resistance.

This is why one carried out an incremental elastoplastic calculation by taking account of the traction diagram of material considered (the principal parameters being: $\sigma_y = 195\,MPa$, $\sigma_u = 520\,MPa$).

[Fig. 4.6-c] allows to note that on this case two pilotings give an identical solution in great transformations, which is more "flexible" that in small transformations. The pressure of found ruin is higher than $56,4\,MPa$. Paid for a value $\sigma_y = 100\,MPa$, one "would have found": $28,9\,MPa$.

This example thus makes it possible to measure the conservative effect of the limiting method of analysis while having taken as threshold of resistance the elastic limit σ_y. On the other hand, to take the ultimate limit σ_u as threshold seems nonconservative, the structure undergoing of the substantial changes of geometry as soon as it plasticizes, before the ruin, in the way considered for incremental calculation.

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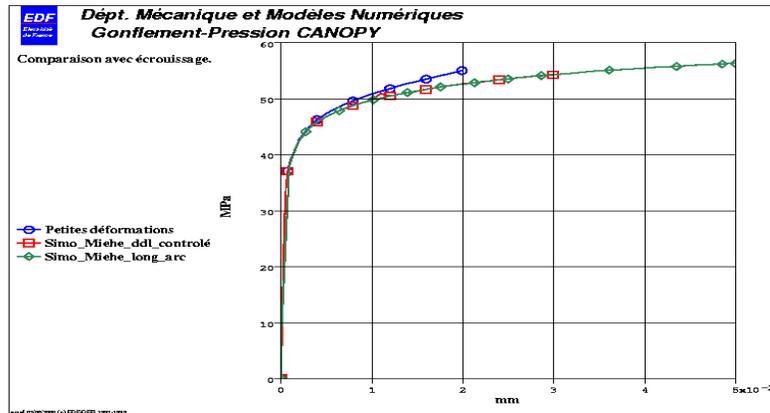
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Figure 4.6-c. Elastoplastic incremental calculation in great transformations (Simo-Miehe) with two types of piloting, compared with a calculation in small transformations.

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