Note of modeling of damping mechanics

Summary

The linear and non-linear dynamic analyses, for the study of the vibratory answer with an excitation in force or imposed or for the modal analysis complexes, require to add characteristics of mechanical cushioning to the characteristics of rigidity and mass.

One has several classical modelings, applicable to all the types of finite elements available. These modelings are available directly by the operators.

- the model of viscous damping,
- the model of damping hysteretic (known as also “structural damping”) for the harmonic analysis and the calculation of mode.

For finer calculations, it is also possible to model materials having a realistic viscoelastic behavior, whose behavior is described by a set of internal variables.

For the analyses using a modal base of real clean modes, it is possible to introduce modal damping coefficients. These coefficients can come as well from tests carried out on site as of a fascinating modal calculation of account any of the types of depreciation suggested.

Contents

1 Model of viscous damping

The model of viscous damping is most usually used. It corresponds to modeling of a dissipated energy proportional to the vibratory speed:

$$E_d=\frac{1}{2}v^T Cv=\frac{1}{2}u^T Cu \qquad \text{éq 1-1}$$

where C is matrix of viscous damping, with real coefficients.

It leads to the classical equations of the dynamics of the structures:

$$Mu+Cu+Ku=f(t) \qquad \text{éq 1-2}$$

with K matrix of rigidity and M matrix of mass.

1.1 Viscous damping proportional “total”, or of Rayleigh

This modeling, easy to implement, corresponds to a linear combination of the matrices of mass and stiffness:

$$C=\alpha K+\beta M \qquad \text{éq 1.1-1}$$

It is currently available, by using the operator `DEFI_MATERIAU` [U4.43.01 §3.1] and `ASSEMBLY (OPTION=' AMOR_MECA')` [U4.61.21]. One can also employ `COMB_MATR_ASSE` [U4.72.01], after having assembled the matrices of rigidity and mass with real coefficients. The option `SANS_CMP=' LAGR'` must be used during the combination of the matrices in order to preserve the boundary conditions of the system (imposed by relations of Lagrange in the matrix of rigidity).

This approach allows validation of algorithms of resolution,
It is not realistic for the industrial studies, because it does not make it possible to represent the heterogeneity of the structure compared to damping (dissipation with the supports or the assemblies). Moreover total identification of the coefficients α and β is not possible, in experimental modal analysis, that for two Eigen frequencies $[f_1, f_2]$ distinct; it gives, for the Eigen frequencies $\omega \notin [\omega_1, \omega_2]$ with $\omega=2\pi f$, a law of evolution of the reduced damping of the form (see [R5.05.04]):

$$2\xi=\alpha\ \omega+\frac{\beta}{\omega}$$

For the discrete elements, the parameters of damping of Rayleigh not being definable in the operator `AFFE_CARA_ELEM`, if one wishes to take account of the contribution of these elements with the total matrix of damping of Rayleigh, it is necessary obligatorily in to pass by method of assembly of this matrix with the operator `COMB_MATR_ASSE`. Without thus proceeding, the total matrix of damping will take account only of the voluminal contribution of the elementsS, surface or of standard beam. If the operator is used `DYNA_NON_LINE`, the damping of Rayleigh thus does not take account of the contribution of the discrete elements.

1.2 Viscous damping proportional of the elements of the model

1.2.1 Characteristics of damping

It is possible to build a matrix of damping starting from each element of the model, as for rigidity and the mass.

Two features are usable:

- the assignment of discrete elements, on meshs POI1 or SEG2, by the operator AFFE_CARA_ELEM [U4.42.01]. This one makes it possible to define, with several possible modes of description, a matrix of damping for each degree of freedom.
- the definition of a characteristic of damping for any elastic material by the operator DEFI_MATERIAU [U4.43.01] by:

  AMOR_ALPH With = α [R]
  AMOR_BETA = β

  this material being then affected with the meshs concerned.

1.2.2 Calculation of the matrices of damping

For all the types of finite elements (of continuous, structural or discrete mediums), it is possible of to calculate the real elementary matrices corresponding to the option of calculation 'AMOR_MECA', after having calculated the elementary matrices corresponding to the options of calculation 'RIGI_MECA' and 'MASS_MECA' or 'MASS_MECA_DIAG'. Each elementary matrix is then of the form:

- when the material i, characteristics of viscous damping proportional (α_i, β_i), is affected with the element elem

$$\mathbf{C}_{\mathrm{elem}} = \alpha_i \mathbf{K}_{\mathrm{elem}} + \beta_i \mathbf{m}_{\mathrm{elem}}$$

- for a discrete element

$$\mathbf{C}_{\mathrm{elem}} = \mathbf{has}_{\mathrm{discrete}}$$

This operation is possible with:

```
mel  [matr_elem_DEPL_R] = CALC_MATR_ELEM
    (  /  ♦  OPTION:    'AMOR_MECA'
          ♦  MODEL:    Mo              [model]
          ♦  CHAM_MATER:  chmat            [cham_mater]
          ◊  CARA_ELEM:   will cara           [cara_elem]
     );
```

The assembly of all the elementary matrices of damping is obtained with the operator ASSE_MATRICE usual [U4.61.22]. It will be noted that one must use same classifications and the same mode of storage as for the matrices of rigidity and mass (operator NUME_DDL [U4.61.11]). One can as recall as the use of the macro-order ASSEMBLY [U4.61.21] makes it possible to gather avantageusemen these stages.

It is noticed that the matrix of damping obtained can be nonproportional:

$$C \neq \alpha K + \beta M$$

1.2.3 Use of the matrix of viscous damping

The matrix C for the dynamic analysis lnéaire is usable direct (keyword `MATR_AMOR`) with the operators of linear dynamic response:

- transient **DYNA_VIBRA** [U4.53.03]
- harmonic **DYNA_VIBRA** [U4.53.03]

It is essential for the modal analysis complexes with the operator of research of the eigenvalues:

CALC_MODES [U4.52.02]

For the analyses in modal base, one must project this matrix in the subspace defined by a unit Φ real clean modes. This operation is possible with the operator **PROJ_MATR_BASE** [U4.63.12]. Let us note that in the case general (C nonproportional), the projected matrix is not diagonal. It remains nevertheless usable (keyword `AMOR_GENE`) for the calculation of the dynamic response in force or imposed in modal space, with the operator of dynamic response linear:

- transient **DYNA_VIBRA** [U4.53.21]

1.2.4 Use of viscous modal damping

For the analyses in modal base of real clean modes, the dynamic differential equation in generalized coordinates:

$$\ddot{q}_i + 2\ \xi_i\ \omega_i\ \dot{q}_i + \omega_i^2\ q_i = \frac{\Phi_i^T}{\mu_i} f(t) \qquad \text{éq 1.2.4-1}$$

fact of appearing a modal damping coefficient ξ_i expressed like a fraction of critical damping and the generalized mass of the mode μ_i , which depends on the mode of standardisation of the clean mode.

In the case of a matrix of damping C strictly proportional, coefficients ξ_i result from the diagonal terms of the matrix of damping generalized $\Phi^T C\ \Phi$ by:

$$2\ \xi_i\ \omega_i = \frac{\Phi_i^T C\ \Phi_i}{\Phi_i^T M\ \Phi_i}$$

and, in the case of own standards modes with the unit modal mass,

$$2\ \xi_i\ \omega_i = \Phi_i^T C\ \Phi_i$$

One can use this relation in the case of a matrix of damping C nonproportional, by applying the assumption of BASILE, who is acceptable for weak depreciation (in particular if there is no damping localised dominating) and of the real clean modes sufficiently uncoupled.

The modal damping coefficients can be provided by order (keyword `AMOR_REDUIT`) with two operators for:

- transitory analysis in modal space **DYNA_VIBRA** [U4.53.03]
- transitory analysis in physical space **DYNA_VIBRA** [U4.53.03]
- seismic analysis by spectrum of oscillator **COMB_SISM_MODAL** [U4.84.01]

Let us note that there does not exist any tool for automatic extraction of these coefficients, starting from the matrix of amndtortissement generalized $\Phi^T C\ \Phi$, concept produced by the operator **PROJ_MATR_BASE** [U4.63.12].

2 Model of damping hysteretic

The model of damping hysteretic is usable to treat the harmonic answers of structures with viscoelastic materials. The damping coefficient hysteretic η is given starting from a test under harmonic cyclic loading with the pulsation ω for which one obtains a relation stress-strain which makes it possible to define:

- the energy dissipated by cycle in the form:

$$E_d=\int_{cycle}\sigma\, d\,\varepsilon$$

- the Young modulus complexE E^* starting from the relation stress-strains:

$\sigma=\sigma_0 e^{j\omega t}$ and $\varepsilon=\varepsilon_0 e^{j(\omega t-\varphi)}$ with σ_0 and ε_0 amplitudes, φ the phase

$E^*=\dfrac{\sigma}{\varepsilon}=\left(\dfrac{\sigma_0}{\varepsilon_0}\right)\ e^{j\varphi}=\left(\dfrac{\sigma_0}{\varepsilon_0}\right)\left(\cos\ \varphi+j\ \sin\ \varphi\right)$ where $E^*=E_1+j\ E_2=E_1\ \left(1+j\ \eta\right)$

with $E_1=\left(\dfrac{\sigma_0}{\varepsilon_0}\right)\left(\cos\varphi\right)$ = real part and $E_2=\left(\dfrac{\sigma_0}{\varepsilon_0}\right)\left(\sin\varphi\right)$ = imaginary part

$\eta=\dfrac{E_1}{E_2}=\tan\varphi$ = factor of dissipation

This led to the equations of the dynamics of the structures:

$$M\ \ddot{u}+K*\left(1+j\ \ \eta\right)\ u=f\left(\Omega\right) \qquad \text{éq 2-1}$$

with K matrix of real elastic rigidity, M matrix of mass and η the damping coefficient hysteretic. Let us note that one often speaks about complex matrix of rigidity.

This model is a simplified version of the model visco elastic standard, and presents several disadvantages:

- The model obtained cannot be transposed in the temporal field, since this model would not be causal,
- This model does not present dependence of damping to the frequency, like models it viscoelastic standard

It however constitutes a good approximation for the calculation of harmonic answers in a reasonably narrow waveband, and also has the advantage of being simple to put in œuvre.

2.1 “Total” damping hysteretic

This modeling, easy to implement, corresponds to:

$$\left(-M\omega^2+j\eta K+K\right)u=f\left(\Omega\right) \quad \text{éq 2.1-1}$$

It is currently available, by using the operator COMB_MATR_ASSE [U4.72.01], after having assembled the matrix of rigidity to real coefficients, but it is of a low utility:

- validation of algorithms of resolution,
- useless for the industrial studies, because it does not make it possible to represent the heterogeneity of the structure compared to damping (dissipation located in particular zones of the structure treated with viscoelastic materials).

2.2 Damping hysteretic of the elements of the model

2.2.1 Characteristics of damping

It is possible to build a complex matrix of rigidity starting from each element of the model, as for real rigidity and the mass.

Two features are usable:

- the assignment of discrete elements, on meshs POI1 or SEG2, by the operator AFFE_CARA_ELEM [U4.42.01]. This one makes it possible to define, with several possible modes of description, one **matrix of real rigidity** for each degree of freedom **and** a damping coefficient hysteretic to apply to this matrix.

```
AMOR_HYST =   eta                [R]
```

- the definition of a characteristic of damping for any elastic material by the operator DEFI_MATERIAU [U4.43.01] by the keyword:

```
AMOR_HYST =   eta              [R]
```

this material being then affected with the meshs concerned.

In LE case where certain materials would not be regarded as not affected by damping hysteretic, it is necessary to affect a damping to them hysteretic no one.

2.2.2 Calculation of the matrices of damping

For all the types of finite elements (of continuous, structural or discrete mediums), it is possible to calculate the complex elementary matrices corresponding to the option of calculation 'RIGI_MECA_HYST', after having calculated the elementary matrices corresponding to the options of calculation 'RIGI_MECA'. Each elementary matrix is then of the form:

- when the material i, characteristics of damping hysteretic η_i, is affected with the element elem

$$k^*_{elem}=k_{elem}(1+j\eta_i)$$

- for a discrete element defined by a matrix of rigidity $\mathbf{K}_{\mathtt{discrete}}$ and a damping coefficient hysteretic η

$$k^*_{elem}=k_{discret}(1+j\eta_i)$$

This operation is possible with:

```
mel   [matr_elem_DEPL_C] = CALC_MATR_ELEM
```

```
( / ♦ OPTION:   'RIGI_MECA_HYST'
    ♦ MODEL:    Mo              [model]
    ♦ CHAM_MATER:  chmat           [cham_mater]
    ◊  CARA_ELEM:  will cara          [cara_elem]
     ♦  RIGI_MECA:  rigi           [matr_elem_*]
    ♦ LOAD   :  l_char         [l_char_meca]
);
```

Assembly of the complex matrix of rigidity $K*$, starting from the elementary matrices is obtained with the operator `ASSE_MATRICE` usual [U4.61.22]. It will be noted that one must use the same classification and the same mode of storage as for the matrix of mass (operator `NUME_DDL` [U4.61.11]).
The loading used for the calculation of the matrix of real rigidity (`OPTION 'RIGI_MECA'`) must be well informed by the keyword `'LOAD'` for the calculation of the matrix of elementary rigidity complex.

2.2.3 Use of the complex matrix of rigidity

The complex matrix of rigidity $K*$ is usable for the direct linear dynamic analysis (keyword `MATR_RIGI`) with the operator of dynamic response linear:

- harmonic answer **DYNA_VIBRA** [U4.53.03]

If the model to be taken into account for harmonic calculation is of important size, it can be interesting to resort to the methods of reduction of models. An effective approach for the strongly dissipative models is available in documentation [U2.06.04].

The search for eigenvalues can be done only with certain parameter settings of the operator of search for eigenvalues:

CALC_MODES [U4.52.02]

The reference material [R5.01.02] specifies the types of with problems which one can deal and possible parameter settings.

This research leads to complex clean modes, which could not thus be used for calculations in the temporal field.

Nevertheless, whenever introduced damping remains weak, it is possible to calculate the modes real associates with the associated nondissipative model, and to allot to them the rates of depreciation calculated by the calculation of complex modes to build a scale model on modal basis. This system can then be used to carry out a transitory calculation:

- transitory analysis in modal space **DYNA_VIBRA** [U4.53.03]

3 Viscoelastic model of damping to internal variables

The viscoelastic model of damping to internal variable is usable to treat the harmonic and transitory answers of structures with viscoelastic materials. This model rests on the existence of a law of behavior making it possible to determine the state of stress according to the history of the deformations:

$$\sigma=E_{\infty}\varepsilon(t)-\int_0^t E_v(t-\tau)\frac{\partial\varepsilon(\tau)}{\partial\xi}d\tau$$

where E_{∞} represent the Young modulus high frequency, and E_v the module of relieving. This module can be represented in the temporal field by a series of Prony

$$E_v(t)=E_r+\sum_{k=1}^{N} E_k \exp(-t/\tau_k) ,$$

or by a sum of first order rational fractions in the frequential field.

$$E_v(s)=E_r+\sum_{k=1}^{N} \frac{\omega_k E_k}{s+\omega_k} .$$

To take into account these laws of behavior, one introduces internal variables which make it possible to build a linear model are equivalent of order two, compatible with *Code_Aster*. These variables establish the link between the physical degrees of freedom impacted by the presence of a viscoelastic material and the sizes defining the behavior. Thus, as many vectors are introduced q_{vk} that parameters internal. These variables are controlled by the equations of evolution temporal

$$\tau_k \dot{q}_{vk}+q_{vk}-q=0$$

maybe, into frequential

$$s\, q_{vk}+\omega_k(q_{vk}-q)=0$$

One can illustrate the relations describing this behavior by considering a system masses “spring” for which the spring has a behavior which can be represented by N internal variables:

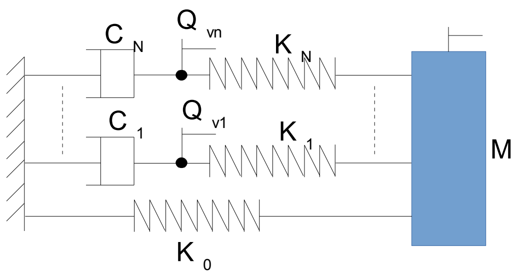

Figure 1: Representation of a system “1 DDL” in internal states

The equilibrium equations for this system are written

$$\begin{cases} M\ddot{q}+K_0 q+\sum_{k=1}^{N} K_k(q-q_{vk})=0 \\ C_k \dot{q}_{vk}+K_k(q_{vk}-q)=0 \qquad \forall k\in[1,N] \end{cases}$$

The dynamic system for this oscillator can thus be put in the classical form of a model of order 2

$$\begin{bmatrix} M & 0 & \dots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & 0 & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ q_{v1} \\ q_{vk} \\ q_{vN} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & C_1 & \ddots & \vdots \\ \vdots & \ddots & C_k & 0 \\ 0 & \dots & 0 & C_N \end{bmatrix} \begin{Bmatrix} \dot{q} \\ q_{v1} \\ q_{vk} \\ q_{vN} \end{Bmatrix} + \begin{bmatrix} K_0+\sum_{k=1}^{N} K_k & -K_1 & -K_k & -K_N \\ -K_1 & K_1 & 0 & 0 \\ -K_k & 0 & K_k & 0 \\ -K_N & 0 & 0 & K_n \end{bmatrix} \begin{Bmatrix} q \\ q_{v1} \\ q_{vk} \\ q_{vN} \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

In the state, *Code_Aster* does not allow to take into account of such laws of behavior for the linear dynamic analyses. The solution adopted to solve this problem consists in building a base of reduction adapted for the problem.

This base of reduction is built around clean modes and of the static answers of the structure to the viscoelastic efforts generated by the modes. Under these conditions, one has

$$[T]=\begin{bmatrix} \Phi_l & \Phi_e & T_p \\ \Phi_{lv1} & 0 & T_{v1} \\ \Phi_{lvk} & 0 & T_{kv} \\ \Phi_{lvN} & 0 & T_{vN} \end{bmatrix}$$

Under matrices T are built starting from the static problem

$$\begin{bmatrix} K_0+\sum_{k=1}^{N} K_k & -K_1 & -K_k & -K_N \\ -K_1 & K_1 & 0 & 0 \\ -K_k & 0 & K_k & 0 \\ -K_N & 0 & 0 & K_n \end{bmatrix}\begin{Bmatrix} T_p \\ T_{v1} \\ T_{vk} \\ T_{vN} \end{Bmatrix}=\begin{Bmatrix} 0 \\ F_{v1} \\ F_{vk} \\ F_{vN} \end{Bmatrix}$$

with the loading defined by

$$\begin{Bmatrix} 0 \\ F_{v1} \\ F_{vk} \\ F_{vN} \end{Bmatrix}=\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & C_1 & \ddots & \vdots \\ \vdots & \ddots & C_k & 0 \\ 0 & \dots & 0 & C_N \end{bmatrix}\begin{Bmatrix} \dot{\Phi} \\ \Phi_{v1} \\ \Phi_{vk} \\ \Phi_{vN} \end{Bmatrix}$$

One can then solve the problem project on the basis T, problem which takes into account the definite total viscoelastic behavior for materials.

Details for the setting in œuvre of these techniques in *Code_Aster* are detailed in documentation on the construction of the scale models for U2.06.04 dynamics.

This approach for the moment is reserved to the experienced users, since the reduction of model in the presence of internal states can lead to singular behaviors. In addition, the method suggested for the reduction is not necessarily sufficient, and sometimes it is necessary to enrich the base thus to build to build a reasonable model.

The search for eigenvalues can be done only with certain parameter settings of the operator of search for eigenvalues:

CALC_MODES [U4.52.02]

The reference material [R5.01.02] specifies the types of with problems which one can deal and possible parameter settings.
However, for the calculation of the clean modes of the scale model, it is recommended to use the full solvor (`METHODE= 'QZ'`) by searching all the eigenvalues (`OPTION=' TOUT'`), since if not, it is not possible to reach the real eigenvalues, representing relieving, which are important components of the behavior of this kind of materials.