

Code_Aster

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General advices of use of the operator DYNA_NON_LINE

Summary:

This document presents the use of transitory methods of resolution (implicit or explicit) for the digital simulation of nonlinear dynamic problems on physical basis.

The operator general practitioner of reference for this kind of calculations names itself `DYNA_NON_LINE` and its correct use will be facilitated by the compliance with some rules of good practice described in this document.

These advices of use cover:

- the correct definition of the model to the dynamic direction (of which initial conditions and in extreme cases),
- the definition of the discretization of which the choice of the diagram in time ([R5.05.05], to see bibliography),
- the choice of the models of damping,
- some advices of postprocessing.

Being given the great diversity of the nonlinear problems, the user will be able very usefully to supplement his reading with other more specific references:

- U2.06.03: on damping,
- U2.06.05: for the interaction ground - structure (linear and not - linear),
- U2.06.09: for the mono one and multi-supports in seismic calculation,
- U2.06.10: on specificities of the studies of type civil engineer under seismic loading,
- U2.06.11: for the fluid use of models - structures coupled with `DYNA_NON_LINE`,
- U2.04.07: use of `DYNA_NON_LINE` to solve strongly nonlinear problems in evolution slow but and which have difficulty converging with `STAT_NON_LINE` (see bibliography),
- U2.06.32: for the revolving machines.

Reading of the U2.04.01 documentation, which gives advices of use for the operator `STAT_NON_LINE`, is too strongly recommended because especially will look further into here specificities related to dynamics. All aspects common to `STAT_NON_LINE` and `DYNA_NON_LINE` and which is detailed in U2.04.01 documentation, like the choice of the parameters of the algorithm of Newton, remains valid into dynamic and are thus not taken again here.

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1 Introduction

In order to have the finest possible evaluation of the response of a given mechanical system to a set of requests, it can prove to be essential to take account of non-linearities and dynamic phenomena.

Usually, one can distinguish two evolutions from the digital methods to lead to that:

- on the one hand classical linear dynamic calculations (often on modal basis) are called into question by the possible appearance of non-linearities (material, contact/friction or great transformations) which require the employment of the operator `DYNA_NON_LINE` (the operator `DYNA_VIBRATED` in transient on modal basis accept only local non-linearities, of which of nodes type of shocks),
- in addition, one can put the question of the validity and the limits of applications of the nonlinear static approaches quasi - (operator `STAT_NON_LINE`), when the evolution of the solution can be subjected to transitory phenomena whose scale of time becomes small compared to the particular characteristics of the structure.

In the first case, the user thus leaves a relevant model for the linear dynamics and which it is necessary to correctly enrich for the taking into account by non-linearities.

In the second case, it is a little the reverse: the user starts from a relevant model into non-linear quasi - static and that it is advisable to adapt to dynamics.

This documentation will try to guide the users who are potentially confronted with these two situations.

Being given the very large variety of the problems likely to be approached, the rules suggested here are inevitably rather general and it is strongly probable that installations on a case-by-case basis for specific problems are essential. It is completely illusory to think that even by respecting all the specifications of this document, nonlinear calculation will proceed without any surprise... The work of expert testimony remains impossible to circumvent!

In precondition to the reading of this documentation it is strongly recommended to have read the reference materials of the operator `STAT_NON_LINE` and `DYNA_NON_LINE` : [R5.03.01] and [R5.05.05]. Indeed, the theoretical aspects will be not very detailed here because they are already approached in these two reference documents.

This documentation is supplemented by other more specific references:

1. U2.06.03: on damping,
2. U2.06.05: for the interaction ground - structure (linear and not - linear),
3. U2.06.09: for the mono one and multi-supports in seismic calculation,
4. U2.06.10: on specificities of the studies of type civil engineer under seismic loading,
5. U2.06.11: for the fluid use of models - structures coupled with `DYNA_NON_LINE`,
6. U2.04.07: use of `DYNA_NON_LINE` to solve strongly nonlinear problems in evolution slow but and which have difficulty converging with `STAT_NON_LINE` (see bibliography),
7. U2.06.32: for the revolving machines.

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2 Definition of a problem adapted to transitory dynamics

In this chapter, we will propose advices for the clarification of the modeling of a problem of mechanics which one wishes to make the transitory digital simulation (non-linear) with *Code_Aster*. The following chapter will treat discretization in space and time, whereas in this chapter only the continuous model is approached.

2.1 Model

The first stage is the definition of a coherent model with the transitory assumption of evolution.

Thus, any mechanical system not having to have mode with worthless kinetic energy, it is appropriate to make sure that the density is defined in any point of the continuous model. In the same way, if one wants to introduce discrete elements to simulate solid bodies, a mass should be associated to them.

In the same way, for the finite elements of hulls type or plates, the user can have to make sure that all the degrees of freedom, in particular rotations, have an associated term of mass. To check this condition, one advises to specify the option `INER_ROTA=' OUI '` at the time of the call to `AFFE_CARA_ELEM` for all the models `HULL`.

Certain modelings not having mass (this calculation was not programmed) cannot be used directly with `DYNA_NON_LINE`, like the THM, the models nonlocal... This difficulty can be circumvented in their superimposing a classical model making it possible to represent the kinetic energy (mass): two models being based on the same nodes.

The use of certain artifices static currents in quasi -, as very stiff zones (fictitious material having a very large Young modulus) to take account of reinforcements which one does not wish to represent finely geometrically, for example, can generate disturbances in dynamics. Indeed, this very stiff material can generate oscillations high frequencies and wave propagations whose celerity is not physics. Moreover, with a diagram in explicit time, these very stiff zones are likely to make fall the step value of time criticizes (condition CFL, cf. [R5.05.05]).

Besides the inertial contribution, the system will have a dissipative contribution, therefore damping. Into nonlinear, dissipation can be due to the relation of behavior (plasticity...), with friction in the connections...

Usually, the detailed ignorance of all the dissipative mechanisms in the system is compensated by a simplified representation which makes it possible to define a total damping. In *Code_Aster*, one has two total models of viscous damping (which one can couple with discrete shock absorbers dashpot type): the model of Rayleigh and modal damping.

In practice, in both cases, it is essential to have carried out a preliminary modal calculation. Indeed, modal damping is defined on the modes which are then arguments of `DYNA_NON_LINE`. For the damping of Rayleigh, the only manner of apprehending its physical direction simply, it is to readjust it on values of modal damping (which can come from experimental results). We will reconsider more in detail damping of Rayleigh in the chapter following on the discretization.

Generally, it is necessary to keep in mind that all the models of total damping were create for the linear cases and that they are then used to take into account certain dissipative phenomena like friction or of the imperfect connections. Therefore, if one adds with the model the taking into account of these non-linearities, whereas one keeps the values of total damping readjusted on a linear approach, that can result in having too much dissipation in the nonlinear system.

Ideally, the taking into account of all non-linearities, associated with a fine discretization of the system, should make it possible not to have to use in more one contractual total damping. In reality, one is obliged to neglect certain non-linear aspects for reasons of cost CPU and/or ignorance on the mechanisms implemented, and in this case, total damping plays its part to play. Documentation [U2.06.03] presents more in details its use.

2.2 Boundary conditions

In quasi - static, it is current to impose boundary conditions whose evolutions in time are functions simple to define, like slopes or continuous functions closely connected per pieces.

In dynamics, if one wants to avoid the parasitic oscillations of the solution, it is absolutely necessary to impose sufficiently regular quantities in time: thus at least continuously derivable, or better C^2 . To arrive at that simply, one can choose to be defined polynomial functions of time instead of functions closely connected. Possible irregularities in the imposed loadings can be partially compensated by the use of a model of adapted damping (in particular thanks to a diagram of integration in times of the type HHT, to see bibliography). Nevertheless, of the excessive oscillations into nonlinear can compromise the continuation of calculations irremediably because the solution (of which the internal variables) depends on its history.

On the diagram below one shows a possible regularization of a curve closely connected (in dotted lines) by a curve adapted to dynamic calculation. One can note the need, on this example, to have to make begin the dynamic evolution in a negative time, if one wants to keep comparable values for the positive moments (apart from the zones of regularization).



Figure 2.2-a : smoothing of an evolution forced to adapt it to a transitory calculation.

In all the cases where one can calculate the spectrum of the imposed signals, the rule of Shannon to define sampling is not strict enough for non-linear calculations. In practice, one advises to have between 4 and 10 pas de time over the shortest period of all the imposed signals.

Any insufficient discretization in time is likely to result in over-oscillations high frequencies of the solution. There then exists a palliative with that by choosing certain diagrams in time, but with the risk to introduce too much damping into the system.

2.2.1 Mono-support

The loadings of type mono-support, which are usually employed during seismic calculations, are characterized by characteristics which it is advisable to point out here.

First of all, the resolution, according to the assumption mono-support, is done natively in the relative reference mark related to the foundation raft. In postprocessing, it is thus necessary to recombine with the movement of training to obtain the fields kinematics absolute. For acceleration, it is in general immediate because the acceleration of training is the input datum known in the form of the accélérogrammes employed.

On the other hand, to rebuild absolute displacement, it is necessary to have the displacement of training. In the classical case where one has only accélérogrammes, one is obliged to integrate twice in time to calculate displacement corresponding (this integration can be done with the operator `CALC_FONCTION`, keyword `JUST`, cf. documentation [U4.32.04]). The user must then be very attentive with the problems of the digital integration of sampled signals such of the constants of integration which can induce drift or of the oscillations to very low frequency. The operator `CALC_FONCTION` have options to correct these drifts (keyword `CORR_DEP` and `CORR_ACCE`), but one very strongly recommends to control the quality of the final integrated result well because, in certain cases, these corrections are insufficient, including the correction by filtering - passes high of `CALC_FONCTION` (with `CORR_ACCE`, `METHOD = 'FILTERING'`). A complementary specific correction is then essential. The passage by the FFT to make integration in frequency can then constitute a good alternative making it possible to lead to more correct integrated signals or at least easier to correct (just a shift of a constant, for example, but without drift).

2.2.2 Multi-supports

Among the types of loadings which can require an adaptation to `DYNA_NON_LINE`, one can evoke the case of the multi-supports (in imposed acceleration) for the seismic studies.

This method, originally developed for linear transitory calculations, rests on the definition of static modes to the supports. Into non-linear, as long as its modes remain relevant, i.e. non-linearities are sufficiently low not to require their reactualization, the approach multi-supports classic remains valid.

On the other hand, in the case general, one can call into question the use of statics mode calculated initially. In this case, to avoid their use, the request multi-supports can be taken into account by imposing on the supports displacements corresponding to the integration of the accélérogrammes which are data input of the linear method multi-supports.

The method is then rigorous, whatever the type of non-linearity, but the integration of the accélérogrammes must be carried out with precaution, not to lead to incorrect signals. Apart from sampling, it is essential, inter alia, to check the absence of drift of the signal (a correction can be made with the option `CORR_ACCE` of `CALC_FONCTION`). In *Code_Aster*, one can easily carry out this integration while making use of the FFT which is available in `CALC_FONCTION`. It is also the occasion to analyze the frequential contents of the signal. There is then the relation, in the frequential field: $U = -A/\omega^2$.

One can also make use of the option `JUST` of `CALC_FONCTION`. For more information on the implementation of the requests of the mono type or multi-supports, the reading of documentation [U2.06.09] is very indicated.

2.2.3 Contact

To finish, the loadings resulting from the contact by penalization can also disturb the solution by oscillations high frequencies, which are related to the value of the coefficient of penalization. In the same way, if one wants to use an algorithm in explicit time, a too large coefficient of penalization will involve a fall of the critical step of time. Compared to statics, it can prove to be obligatory to lower the coefficient of penalization, with the disadvantage of increasing the interpenetration at the time of the contact.

Notice important for the diagrams in explicit times:

if one imposes boundary conditions in displacement which evolves in the course of time, it should be taken account owing to the fact that these conditions in fact are imposed in acceleration into explicit (because it is the primal unknown factor). That means that one must enter `DYNA_NON_LINE` the derivative second of the signal in displacement which one wants to impose. This evolution of imposed displacement must thus be derivable at least twice in time...

2.3 Initial conditions

In same logic as for the boundary conditions, one recommends to avoid the singular initial conditions, as what can generate of the imposed loadings having of the evolutions in times of the type Dirac or Heaviside.

One will thus privilege the regular conditions with derivative (first and if possible second) in time in t_0 worthless.

If one wants to start from a prestressed state (not virgin), it completely possible and is recommended to connect a dynamic calculation with a static calculation quasi -. In this case, the initial state is then balanced perfectly and, subject to regularity under the conditions imposed at the time of the rocker, the transitory solution must not have nonphysical oscillations. This method of static sequence quasi - - dynamic allows taken into the account easy one of all the preloads, like gravity. If one had wanted to take into account this initial state prestressed with only one dynamic resolution, it would have been necessary to introduce a strong initial damping and to wait until the oscillations were dissipated to have the prestressed static state. Then, one can continue calculation with physical damping. One is obliged to proceed thus when one uses a computer code which does not treat to it quasi - static non-linear (cf. EUROPLEXUS, to see bibliography).

At the time of the initialization of the diagram of integration for nonlinear transitory calculation, one thus seeks to reverse the matrix of mass. If it is singular, then a message informs the user and one arbitrarily puts initial acceleration at zero. Generally the non-inversibility of the matrix of mass must lead the user to check his model, except if it is voluntary.

At the time of continuations, two aspects are sensitive.

On the one hand, it is to better avoid having too abrupt variations of step of time: the coherence of the solution on the way of the continuation would be affected. A strong change of step of time can be seen like a filter. A great step of time constitutes a low-pass filter of a solution initially calculated with a fine step.

In addition, if one wants to change type of diagram in time, certain rules are to be respected. If one wants to pass from an implicit method (quasi - static or dynamic) to an explicit diagram, the continuation will be mathematically valid because the initial state will be balanced (with the residue near). On the other hand, the direct opposite rocker introduces an error because the initial state, coming from an explicit calculation, will not respect balance with the implicit direction. Indeed, this operator will seek to reverify balance: one solves balance by reversing the matrix of mass, which gives initial acceleration. If this field is nonnull, that represented the not-checking, with the static direction, of balance, at the initial moment. In practice, this inaccuracy can generate oscillations of the solution. In a more subtle way, the rocker of θ - diagram towards a diagram of the second order like the average acceleration or complete HHT will introduce a small error on the terms of acceleration, which can disturb the digital solution. If, despite everything, one wants to make these rockers which induce errors, one can minimize the inaccuracies while choosing to make these rockers at the time of phases where the solution evolves little.

3 Discretization of the continuous problem

In complement of the advices previously given for the continuous model, this chapter will list the aspects most important to respect to obtain a discretized model adapted to `DYNA_NON_LINE`.

3.1 Grid

Comme preliminary to transitory calculation, it is strongly recommended to carry out a modal calculation (for example with `CALC_MODES`), in order to obtain modal information which will make it possible to qualify the quality of the model in dynamics and to adjust certain parameters. The objective not being to return in the details of the modal analysis, one can nevertheless point out some rules.

In general, one can be defined a cut-off frequency for the problem to be studied, and thus an associated modal truncation. The good representation of all the modes of this truncated base can give indications on the sizes of meshes to be used, besides the considerations already taken into account for static calculations quasi -. Approximately, about ten meshes by smallest wavelength is sufficient (to be adjusted according to the wealth of the elements, of course).

The modal analysis as will make it possible to check as the model is free from problems like contributions nondefinite to inertia or the stiffness.

Lastly, it is essential for the use of modal damping in `DYNA_NON_LINE` or to readjust the damping of Rayleigh, as one will see it in what follows (*cf.* § 3.3).

In a way more marked much than for calculation quasi - static, the dynamic resolution will put up rather badly with grids presenting of brutal variations of sizes of elements. Indeed, these zones can be assimilated to interfaces which will disturb the wave propagation. One can then see appearing considered waves which are superimposed on the "physical" wave trains. In the same way, if one wants a good representation of the waves, the grid must be fine on all the way of the waves: one can be satisfied to refine only in certain non-linear zones. If a wave leaves a zone with a grid finely to go towards a zone with a grid more coarsely, it will undergo a filter and the wave reflected by the opposite edge is likely to be strongly disturbed, to even disappear. If one is interested only in short durations, therefore before any return of wave on the zone with a grid finely, then this error on the considered wave is not penalizing. On the other hand, if one wants to calculate solutions over longer durations, the made mistake could be considerable.

Lastly, contrary to the static case quasi -, time has a physical direction and the scales of time that one wants to analyze are strongly coupled on the scales in space of the discretized problem. Thus, the step of time is dependent in keeping with mesh, which one can immediately perceive with the concept of condition CFL for the explicit diagrams.

3.2 Diagram in time (R5.05.05)

Time having a physical direction in dynamics, the quality of its discretization is all the more sensitive.

One can state some rules:

- the evolution of the imposed loadings must be sampled in a sufficiently fine way (between 5 and 10 pas de time per the shortest period of the signals considered),
- the modal behavior of the structure must be well represented (like above, one must have between 5 and 10 pas de time per the weakest period of the modes considered).

Being given the character low frequency, at best intermediate frequency, majority of the problems which one can tackle here, these two rules are not, in general, very penalizing.

Into explicit, it is moreover necessary to observe the condition of Current (CFL cf. [R5.05.05] and bibliography) under penalty of digital divergence ("explosion" of the kinetic energy). For a diagram of integration of centered the differences type, the critical step of time is worth $2/\omega$ with ω who is the highest own pulsation of the system.

One can calculate this pulsation with `CALC_MODES` by choosing the option '`PLUS_GRANDE`' (or with the option '`PLUS_PETITE`' by reversing the roles of the matrix of mass and stiffness).

For the diagram of Tchamwa-Wielgosz (see bibliography), the critical step of time is slightly weaker and decrease when one increases damping related to the diagram (parameter `PHI`).

The condition of Current can also be approximate, at least on massive models, by $\Delta t = l_{min}/c$ with l_{min} who is the smallest length of the discrete model and C the celerity of the waves of traction at the point considered. The operator `DYNA_NON_LINE` makes use of this formula to give an approximation of the condition of Current. There exist however certain limitations:

1. one cannot calculate automatically the condition of Current associated with the presence with discrete springs (it is not programmed),
2. one does not correct the formula for the elements of structures (hulls, plates and beams).

It can thus exist cases where the value returned by `Code_Aster` is not one undervaluing of the true condition of Current. Therefore, in the event of divergence, it is advisable to decrease the step of time.

Moreover, the calculation of the condition of Current is not reactualized and is done only at the beginning of calculation, basing itself on the celerity of the elastic waves for the initial state. If the elastic module drops (damage), the initial condition of Current can become too severe. There is no risk of divergence (except for materials whose elastic module could increase), but time CPU could be a little decreased by reactualizing the condition of Current (as it is made in the codes dedicated to fast dynamics, to see bibliography).

For most structures, the condition of Current is very penalizing: the celerity of the waves being often about a few thousands of m/s , one arrives at steps of time of less $10^{-5}s$, for sizes of usual structures.

One can classify the implicit schemes in three categories (one puts side, voluntarily, the diagrams order 1 and/or of speed which are more specifically adapted to the very irregular problems):

- average acceleration (NEWMARK) of order 2 and which does not bring digital dissipation: to use in first,
- θ - diagram (THETA_SCHEMA) who is of a nature 1 and dissipative: one recommends it for the irregular problems like the shocks,
- Complete HHT (MODI_EQUI = 'YES') who remains of order 2, contrary to the case of the modified average acceleration (MODI_EQUI = 'NOT', option by default). This diagram is specifically developed to introduce a digital damping high frequency and thus not to disturb the physical answer low frequency. Damping is directly controlled by the parameter ALPHA diagram.

If one observes oscillations high frequencies in solution digital (approximately, of the oscillations the period is about some steps of time), one can choose complete diagram HHT, to start with a value about $-0,1$ for the parameter ALPHA. A value of $-0,3$ constitute a high limit still usable. It is strongly recommended to conduct a parametric study on the value of the parameter ALPHA in order to choose the value nearest to zero which makes it possible to obtain a correct solution without on - digital damping.

If one wishes more damping on average frequency, then the diagram of average acceleration modified can be employed.

The implicit schemes are to be used, firstly, with a formulation in displacements:

FORMULATION = 'DISPLACEMENT'.

Into explicit, one has two diagrams:

- centered differences (DIFF_CENT) who is nondissipative,
- Tchamwa-Wielgosz (TCHAMWA) who is dissipative, in a way comparable to HHT.

Here still, one recommends to start by using a nondissipative diagram.

Lastly, into explicit, it is recommended to use a matrix of diagonal mass (lumpée), which is obtained by the keyword MASS_DIAG = 'YES' of DYNA_NON_LINE. This option not being available for all the finite elements, the user can be constrained to use the consistent mass, as into implicit. The lumpée mass makes it possible to correct a share of the frequential drift over long lives coming from the error in time induced by the diagram of integration.

If one solves the problem on modal basis, then the disadvantage of the very weak step of critical time for an explicit diagram disappears. Indeed, it not of time limits will be directly proportional to the smallest clean period of the truncated modal base. There is the relation: $\Delta t = 2/\omega$ with ω who is the highest own pulsation of the system. The more the modal base will be truncated, the more the step of time criticizes associated will be large.

Moreover, the calculation of the condition of Current is then immediate because one explicitly knows all the pulsations of the base, therefore highest in particular. In Code_Aster, the automatic calculation of the condition of modal Current on basis is exact and will be always valid some is the type of finite element used.

The explicit diagrams are associated with a formulation in acceleration (one wants to solve by reversing the mass): FORMULATION = 'ACCELERATION'.

Remarks important for the explicit diagrams:

if one imposes boundary conditions in displacement which evolves in the course of time, it should be taken account owing to the fact that these conditions in fact are imposed in acceleration into explicit (because it is the primal unknown factor). That means that one must enter DYNA_NON_LINE the derivative second of the signal in displacement which one wants to impose.

This evolution of imposed displacement must thus be derivable at least twice in time...

The use of the quadratic elements is not recommended into explicit: indeed, parasitic oscillations can appear on the fields solutions (displacement or constraints). This phenomenon can also be amplified by the joint use of the matrix of consistent mass.

For the checking of the assumption of plane constraints, if one uses the method of Borst, the associated total iterative process is then blocked with the first iteration (contrary to the implicit case). That can involve a light inaccuracy of the results, moderated by the fact that the step of time being in addition very small, the not-flatness of the constraints can only be very moderate. However, it is completely possible to use the local version of the algorithm of Borst by the means

of the keyword `ITER_CPLAN_MAXI` under `BEHAVIOR`. Indeed, these local iterations take place into explicit well, with a overcost of associated calculation.

To finish, it is advisable to announce that analytical results on the characteristics (convergence, error...) diagrams in time are obtained for a linear framework. The demonstrations in non-linear mode are very rare and are confined with typical cases. In practice, certain characteristics of the diagrams can be degraded into non-linear. That can explain why it is not inevitably essential to use the diagram which, into linear will have exceptional performances (order 4...), but that it is to better privilege simpler and more robust diagrams, in particular with digital dissipation high frequency. For example, on irregular problems, like the shocks, of the dissipative diagrams of order 1 are advised (like θ - diagram).

In the same way, into non-linear, the evaluation specifies condition of Current asks for a reactualization of its calculation. Indeed, the condition of Current calculated initially can not be conservative (for example so certain elements see their size decreasing, or so of the shocks occur, with a modeling by penalization). `Code_Aster` do not reactualize this calculation and in the event of divergence, it is recommended to decrease the step of time to start again calculation.

The results of linear analysis on the diagrams constitute a solid however bases for their analysis (cf. [R5.05.05] and bibliography), while knowing that non-linearities can disturb the behavior of the diagrams.

Concerning the choice of the diagram in time, the user must pay attention to the definition of the step of computing time. Into explicit, one recommends to place oneself slightly below the condition of Current: between 0.5 and 0.7 times the condition of Current. Into explicit, there will be no subdivision of the step of time due to nonconvergence: the user only remains main step of time all the way along calculation.

Into implicit, the classical diagrams are unconditionally stable, but that does not mean that one can take an unspecified step of time! A too large step will not bring a divergence, but the error on the solution will be obviously important.

The step of time to impose could be judiciously limited:

- in higher value, by the step of time to respect for discretizing well the evolutions of the loading imposed and for representing well the highest Eigen frequency of the system which one wants to take account,
- in lower value, by the condition of Current, to the direction which it has of shorter time for which information can pass from a node of the mesh to another.

Between these terminals, it is essential to conduct a parametric study to make sure of the good convergence of the digital solution.

The lower limit gives crucial information as for the maximum subdivisions of the step of time than it is necessary to authorize when the algorithm does not converge. To leave the step of time to subdivide itself until going clearly under the condition of Current can be used for nothing because the solution calculated risk then to undergo a digital pollution which will not facilitate convergence.

Always in link with the iterations for the checking of balance to each step, one can notice that, in most case, if the step of time is sufficiently fine, the maximum number of iterations with convergence remains moderate: often about 10, whereas in quasi - static, one can usually exceed these values.

Therefore, the idea consists in saying that the step of time is of a good order of magnitude if the iteration count to convergence remains moderate. If this number increases, one can try to reduce the step slightly, by always respecting the terminals defined above. There exist nevertheless cases or one can have, punctually, need to authorize more iterations at the time of some steps.

To finish, if one is obliged to change diagram into time to use a dissipative diagram, like HHT, it is essential to conduct a parametric study on this damping. Indeed, the risk to introduce a too large dissipation is not negligible, especially with the diagram of modified average acceleration. The following paragraph will reconsider this point.

3.3 Models of damping

The order of introduction and use of dissipation in the discretized model is the following:

1. intrinsic dissipation related on the relations of nonlinear behavior, the connections (friction),
2. total dissipation of standard damping structural (Rayleigh or modal, that one can use simultaneously),
3. digital dissipation of the diagram in time.

Ideally, the first category should be sufficient, but in practice, for reasons of simplification of the model, it is often essential to add structural damping, the damping brought by the diagram being the last recourse.

We will approach here only the use of structural damping and that related to the diagram (for more information, the reader will be able to refer to documentation [U2.06.03]).

3.3.1 Structural damping

Total dissipation of standard structural damping is taken into account by modal damping or the damping of Rayleigh.

It is possible to use at the same time the total damping of Rayleigh, as well as modal damping and even the digital damping of the diagram in time. In these cases of combinations of several sources of damping (not counting dissipation material due to the selected relation of behavior or the contact - friction), the user must check the total coherence of the dissipation and thus of not to risk to count several times damping on the same zone. For example, if one wishes to combine damping of total (of Rayleigh or modal) and dissipation material, in general the regulations imposesNT to compared lower the value of total damping to a purely elastic calculation.

Another example: the case of a building on paraseismic supports. If one wants to represent the dissipation of the supports by a local model (linear law not - or dashpots) and the damping of the building by modal damping, it will have to be checked that one does not introduce modal damping on the modes which make mainly work the supports, like the modes of horizontal rigid bodies (two translation and the rotation of vertical axis).

Let us recall finally that the more one will multiply the sources of dissipation, the more difficult their control and their physical interpretation will be.

3.3.1.1 Modal damping

With regard to modal damping, one insist on the respect of the three basic rules:

- to check that all the values of modal damping are quite positive, because any negative value is likely to involve a dynamic instability,
- to check that the associated modal base is sufficiently complete (container has minimum all the spectrum of the excitation) not to have of lack of damping following certain modes of answer of the structure,
- modal damping being clarified in `DYNA_NON_LINE`, it can be necessary to decrease the step of time so that integration in time remains stable, even with an unconditionally stable implicit scheme.

This base modal, being used to define modal depreciation, the same profile of classification as that D must imperatively have be matrices of the dynamic system used during the calculation of the answer itself. Before use modal depreciation modal base in the calculation of the answer, this rule requires thus possibly D E transform R beforehand this initial modal base in a new modal base, of which the definition D be boundary conditions is identical to that relative to calculation answer via `DYNA_NON_LINE`, thus ensuring the same number and the same position of Lagrange of dualisation. It is in particular the case during the application of nonworthless displacements imposed for the calculation of the answer. The cas-test zzzz113a details the corresponding procedure.

3.3.1.2 Damping of Rayleigh

This model makes it possible to define the total matrix of damping C as being a linear combination of the matrices of rigidity and mass (to have a diagonal matrix of damping on the basis of usual dynamic mode):

$$C = \alpha K + \beta M$$

Three cases of identification are presented here to illustrate the effects induced by this modeling:

- damping proportional to the characteristics of inertia: $\alpha=0 \quad \beta=\beta_i$.
This case was very much used of direct transitory resolution: if the matrix of mass is diagonal, that of damping is still and the saving space memory is obvious in it. The coefficient β can be identified with experimental reduced damping ξ_i clean mode (ρ_i, ω_i) who takes part more in the answer from where $\beta_i=2\xi_i\omega_i$. For any other pulsation one obtains a reduced modal damping $\xi=\beta_i\frac{\omega_i}{\omega}$. Modes of a high nature $\omega \gg \omega_i$ will be deadened very little and the low frequencies modes $\omega < \omega_i$ too much deadened.

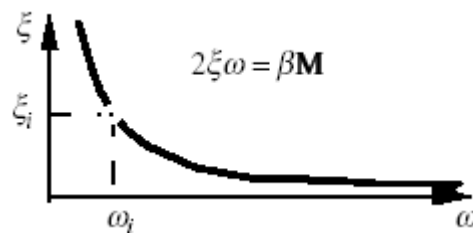


Figure 3.3.1.2-a : pace of damping proportional to the mass.

This damping proportional to the mass is the only damping of Rayleigh which is easily usable with an explicit diagram. Indeed, one can show that the introduction of a term proportional to the stiffness involves a fall of the critical step of time (condition CFL).

- damping proportional to the characteristics of rigidity: $\alpha=\alpha_j \quad \beta=0$.
The coefficient can be identified, like previously starting from modal damping ξ_j associated with the mode (ρ_j, ω_j) , from where $\alpha_j=2\frac{\xi_j}{\omega_j}$. For any other pulsation one obtains a reduced modal damping $\xi=\alpha_j\frac{\omega}{\omega_j}$. High modes $\omega \gg \omega_j$ thus are very deadened.

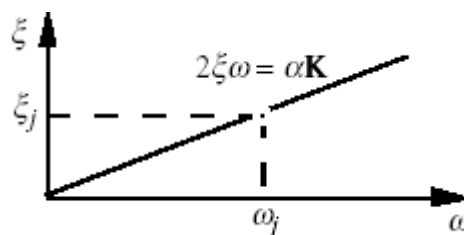


Figure 3.3.1.2-b : pace of damping proportional to the stiffness.

- damping proportional complete: $\alpha=\alpha_j \quad \beta=\beta_i$.
From an identification on two different modes (ρ_i, ω_i) and (ρ_j, ω_j) , we will obtain for any other pulsation a reduced modal damping $\xi=\alpha_j\frac{\omega}{\omega_j} + \beta_i\frac{\omega_i}{\omega}$. In the interval $[\omega_i, \omega_j]$ the variation of reduced damping is weak and outwards one finds the combination of the preceding disadvantages: the modes external with the interval are deadened too much.

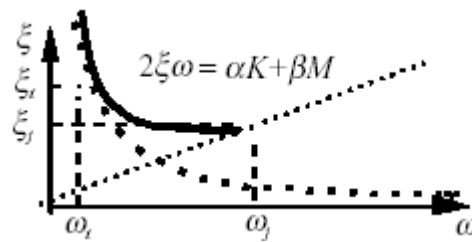


Figure 3.3.1.2-c : pace of the damping of complete Rayleigh.

The damping of complete Rayleigh makes it possible to have a value of straight-line depreciation on a given plate of frequency, which makes it possible to control its effect on a frequential beach defined in coherence with the problem considered.

Application to the structure

The damping coefficients of Rayleigh are defined, on the level of the characteristics of the material (order `DEFI_MATERIAU`), by the parameters `AMOR_ALPHA` and `AMOR_BETA`. There is no specific keyword in `DYNA_NON_LINE` to activate the taking into account of this damping: if one does not want to use it, best is to withdraw all the occurrences of the keyword `AMOR_ALPHA` and `AMOR_BETA` of all materials of the model.

Values to force to obtain desired damping ξ , in the interval of the Eigen frequencies F_1 and F_2 result from the following equations:

$$\text{Equation 1: } \alpha = \frac{\xi}{\pi(f_1 + f_2)}$$

$$\text{Equation 2: } \beta = \frac{4\pi\xi f_1 f_2}{f_1 + f_2}$$

Where f_1 and f_2 are the two Eigen frequencies limiting the interval of study considered. Within the framework of this document, one seeks solutions low frequencies, therefore the frequencies f_1 and f_2 will be associated with the first frequencies of the model, whose modes are coherent with the imposed loading.

To give orders of magnitude, modal damping for the structures out of steel and generally about a few %, whereas for concrete structures, of standard civil engineer, one can go up up to 5, even 7% of damping, for overall linear calculations.

For the discrete elements, the parameters of damping of Rayleigh not being definable in the operator `AFFE_CARA_ELEM`, SI one uses the operator `DYNA_NON_LINE`, the damping of Rayleigh thus does not take account of the contribution of the discrete elements. Lhas total matrix of damping assembly for the resolution will take account only of the voluminal contribution of the elementsS, surface or of standard beam.

In the case of operators of linear dynamics, if one wishes to take account of the contribution of S elements discrete for the total matrix of damping of Rayleigh, it is necessary obligatorily in to pass by method of assembly of this matrix with the operator `COMB_MATR_ASSE`.

3.3.2 Damping due to the diagram in time

Documentation [R05.05.05] and especially the note [6] present this aspect. One here will restrict oneself to recall the main tendencies of them.

On a system with a degree of linear freedom (mass arises, of own pulsation ω), one can obtain the following characterization of the damping induced by the implicit scheme (average acceleration, modified average acceleration and complete HHT), according to the step of time and for various values of the parameter ALPHA :

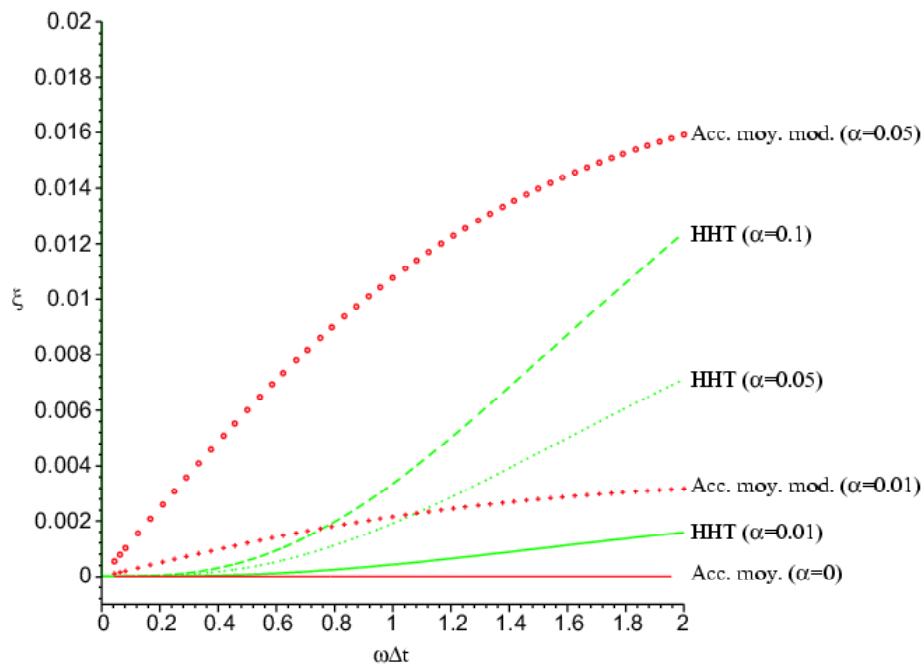


Figure 3.3.2-a : comparison of damping due to the diagram in time.

One finds although only the diagram of average acceleration does not dissipate.

When the two other diagrams are compared, one can notice that:

- only complete diagram HHT does not disturb the low frequency field,
- for the same value of the parameter ALPHA modified average acceleration introduced much more dissipation than diagram HHT.

Lastly, it is advisable to notice that the equivalent value of damping is dependent on the pulsation ω , and will thus depend on the finite element considered. On a complex problem, damping due to the diagram will thus not be homogeneous: the "stiffer" the element will be, the more it will see damping. In the same way, if the step of time is decreased, damping will drop.

In order to put forward the influence of damping high frequency of the implicit schemes, one will present some evolutions of acceleration solution of a simple linear problem of piping under earthquake.

On the following graph, one zoomed in on part of the answer in acceleration to compare various methods of resolution of the transitory problem. The reference solution in green dotted lines is obtained by calculation on modal basis (DYNA_VIBRATED). The truncation of the modal base naturally filters any disturbance high frequency.

Figure 3.3.2-b : local comparison of the behavior of the diagrams in time.

One compares this reference solution to a calculation with the diagram of average acceleration modified (red curve) which strongly oscillates in spite of the adjustment of the parameter ALPHA .
Lastly, one traces the answer obtained with a complete diagram HHT (black curve) which gives a result very close to the reference solution: the disturbances high frequencies (related to the step of time) are strongly attenuated.

On the following graph, one observes the response curves on a larger time interval. That makes it possible to seize the influence of the diagram on the “physical” answer: thus low or intermediate frequency. The various answers correspond to various diagrams of integration and step values of time. The reference solution (the problem being linear) is obtained by modal superposition (curve in mixed features thick chestnuts, named “modal cut 100 Hz ” and obtained with `DYNA_VIBRATED`).

Figure 3.3.2-c : total comparison of the behavior of the diagrams in time.

It is noted that:

- the nondissipative solution (HHT with `ALPHA=0`, therefore a diagram of type average acceleration) with a step of “large” time overestimates the amplitude and introduces a dephasing,
- solutions obtained with diagrams of type modified average acceleration (`MODI_EQUI= 'NOT '`), whatever the step of time, has a too strong dissipation,
- solutions obtained with complete diagram HHT (`MODI_EQUI= 'YES '`) with a step of fine time, allow to find the reference solution well.

To conclude on this part, being given the paces of the damping of Rayleigh, like that due to the diagram, one can build a relatively varied total damping.

To choose in an optimal way the value of the parameter `ALPHA` diagram HHT, it is essential to conduct a parametric study, in order to find the value nearest to zero adapted to the problem considered (one can advise to start with a value of -0.1, for example). To validate this choice, one can base oneself on the two types of analyses presented above: control of the oscillations in high frequencies on accelerations (*cf.* figure 3.3.2-b) and absence of on - digital damping on the answer in displacement (*cf.* figure 3.3.2-c). On this last aspect, one can compare with the answer in displacement obtained with a nondissipative diagram of type `NEWMARK`.

With complete diagram HHT, damping with low and intermediate frequency being weak, it is appropriate not to neglect to introduce into the model a modeling *ad hoc* real damping (total and/or due to the non-linearities).

The pace of the damping brought by the diagram of Tchamwa is qualitatively close to that of the modified average acceleration.

3.4 Modification of the matrix of mass

In certain specific cases, like calculations on hydraulic mixed models (HM or THM) or the cases having a very bad conditioning of the matrix of mass, it can be useful to modify the matrix of mass in order to make it invertible. It is even essential for the diagrams in explicit times.

This modification of the matrix of mass can be done with the keyword `COEF_MASS_SHIFT` (*cf.* R5.05.05 documentation) in the keyword `SCHEMA_TEMPS`. That makes it possible to introduce a “shift” of the matrix of mass \mathbf{M} who becomes: $\mathbf{M}' = \mathbf{M} + coef \cdot \mathbf{K}$.

This functionality makes it possible moreover to improve convergence of implicit calculations or to increase the step value of time criticizes into explicit. It should however be announced that amounts disturbing the mechanical system, in particular by shifting the Eigen frequencies of the system to the bottom (and by decreasing the celerity of the waves).

For example, U own pulsation of origin ω becomes then ω' such as: $\omega'^2 = \frac{\omega^2}{1 + coef \cdot \omega^2}$

One notes according to this expression that ω' tends towards a maximum value $coef^{-0.5}$: the modified system thus introduced a frequency cut-off controlled by the value of the coefficient of shift of mass. Thus, for example, for a maximum value in practice for $coef = 10^{-6}$, the corrected maximum own pulsation system ω' is worth 1000 r d.s^{-1} , and a value for an Eigen frequency of origin of 30 Hz is corrected by this method with $29,481 \text{ Hz}$. The Eigen frequencies located under the new cut-off frequency are thus slightly faded.

In practice, it is advisable not to use a too large value, except limiting itself to the calculation of solutions to very low frequency, even of static type quasi -. In this last case, the linear algorithm transitory not - can then be seen like a solvor regularized for quasi the - static, which can be useful when the static classical resolution does not converge.

In the same order of idea to use a transitory solvor to obtain a static answer whose convergence with a usual implicit method is very difficult, one can also strongly increase the mass of the system (for example while exploiting the density) and use a diagram in explicit time with a strong structural damping. In this case one combines the capacity of convergence of the explicit method with a step of time does not criticize too weak, because of increase in the mass (this step of time of stability will increase like the square root of the coefficient of increase in the total mass: if one multiplies the total mass by 100, it not of time limits will be multiplied by 10 because the celerity of the waves will be ten times weaker). This kind of method was already used for delicate cases like the simulation of excavation of underground galleries, where the surrounding ground can become unstable.

3.5 Instability and analyzes modal reactualized

During the resolution of the transitory problem it is possible to use tools for analysis to the eigenvalues on the reactualized total operators. One can carry out two types of analysis.

On the one hand, the calculation of the Eigen frequencies and oscillatory modes with the matrix of reactualized stiffness. That corresponds to the keyword `MODE_VIBR` (this option needing implicitly the matrix of mass, it is not available in `STAT_NON_LINE`). The matrix of stiffness can then be the elastic, secant or tangent stiffness. Thanks to this keyword one can follow the influence of non-linearities on the vibratory behavior of a structure. An example of application would be the case of structures out of reinforced concrete for which the damage varies the Eigen frequencies.

The graph below presents the possible choices for the matrix of stiffness (for a damaging material):

Figure 3.5-a : Schematic representation of the operators of stiffness daNS Code_Aster.

In addition, one can, thanks to the keyword `CRIT_STAB=_F` (`TYPE='FLAMBEMENT'`), to carry out an analysis of stability of the operator of stiffness. In the case of the small disturbances and where one can calculate the geometrical stiffness, then this option is assimilated to an analysis of buckling within the meaning of Euler on the matrix of brought up to date stiffness. In the other cases, when one cannot calculate the geometrical stiffness, then one rocks on the search for singularity of the operator of stiffness alone. In all the cases one obtains eigenvalues which will evolve during calculation.

In the case of the buckling of Euler, the eigenvalue is directly the multiplying coefficient of the loading which makes it possible to obtain the critical load.

In the second case, interpretation is less easy (the eigenvalues are not adimensional). If it is noted that an eigenvalue changes sign, that means that the calculated solution passed a junction and thus which one lost the unicity of the solution.

In all the cases, it is about analysis of stability to the "static" direction and, moreover, this option is available in `STAT_NON_LINE`. Currently, there does not exist in *Code_Aster* of operator allowing to lead an analysis of stability to the dynamic direction: therefore, for example, by calculating the damping of the system to detect when it becomes negative.

As these operations require a certain cost CPU (comparable to one `CALC_MODES` on some frequencies, with each step), one introduced the possibility of calculating these eigenvalues only for the list of moments of filing if it exists. To still reduce time CPU, it is also possible to make several continuations and to ask the calculation of the eigenvalues only on certain time intervals.

3.6 Filing and postprocessing

The number of steps of time which can be very large, it is strongly recommended to use the features of filing (keyword `FILING` of `DYNA_NON_LINE`) under penalty of having bases and enormous output files. The step of filing can go from some steps into implicit to 10 to 100 pas into explicit.

In complement, if one needs to precisely follow in the course of time the evolution of some parameters in some points, there exists the observation (keyword `OBSERVATION` of `DYNA_NON_LINE`) who comes to supplement filing.

As one could see it in the paragraph 3.3.2, if one wants to analyze the answers of speed or accelerations in the course of time, one can obtain kicked up a rumpus enough curves. These oscillations high frequencies can be the sign of an insufficient discretization in time (or of a too large irregularity in time) of the problem. It is also possible, by using a diagram in dissipative time standard HHT complete to smooth these disturbances. A compromise remains to be found between this smoothing and a too strong dissipation of the answer. Generally, it is necessary well to integrate that the instantaneous values of the least smoothed quantities as acceleration are to be handled with precaution. It is to better seek to carry out its analysis on quantities integrated more physically relevant in dynamics like energy.

In complement, all methods of analysis coming from the quasi - static to quantify the quality of a result (of which the various standards residue in balance) are available and relevant with `DYNA_NON_LINE`.

Lastly, one can as recall as if one wishes to carry out frequential analyses, it agrees to take care well to respect the field of validity of the FFT, for example the causality of the signal to be treated. For that, this signal in time must leave and finish to zero, with derivative worthless. Without the respect of these basic assumptions, the user is likely to get frequential results vague.

3.7 Energy assessment

For any transitory analysis, it is very useful to be able to have the energy assessment, which makes it possible to analyze the answer of the system and also to control the quality of the digital solution. Documentation [R4.09.01] presents all the energy features available with the operator `DYNA_NON_LINE`.

3.8 Features available in `STAT_NON_LINE` and not in `DYNA_NON_LINE`

Then, the methods of the type searches linear (mixed or not) are not authorized in dynamics. For the moment, this lack is to be relativized, knowing that the attempts at applications of these methods on studies of reinforced concrete structures in dynamics did not put forward significant contribution on convergence, as opposed to what one observes in quasi - static. Let us announce, nevertheless, that no theoretical argument would prohibit the use of these methods in dynamics.

Lastly, techniques of piloting available in `STAT_NON_LINE` (length of arc, for example) are prohibited in dynamics because they then do not have a direction: time has, in dynamics, a physical direction.

4 Specificities of the coupled problems fluid-structure

It is possible to use a vibroacoustic model coupled in `DYNA_NON_LINE`. This model is based on an approach (u, p, φ) with the following assumptions:

- the fluid is of acoustic type linear,
- the structure must be regarded as in small disturbances or Lagrangian being reactualized.

One can also take into account free surfaces.

Documentation [U2.06.11] presents in detail the implementation of a model nonlinear fluid-structure coupled for a calculation of tank. The note [5] the scope of application of the model fluid-structure analyzes coupled in `Code_Aster`.

5 Optimization of the performances

In quasi - static, it is not rare to have to carry out more than 10 iterations to have convergence within the meaning of the residue in balance. In implicit dynamics this value of 10 iterations constitutes, in general, a good starting value for the parameter `ITER_GLOB_MAXI` of `CONVERGENCE`. If one cannot converge in less than 10 to 20 iterations, it is then preferable to decrease the step of time rather than to increase the authorized maximum number of iterations.

Into explicit, there are no iterations for balance, the cost of calculation of each step of time will be thus constant, whatever the level of non-linearity (except, possibly, the local checking of the behavior).

The use, even current, of the explicit methods thus seems very tempting within sight of the time CPU which remains controlled. It is however necessary to moderate this optimism while keeping well with the spirit which one deprives of the parapet which is the precise checking of the balance and which, consequently, the quality of the explicit solution obtained must be analyzed with more precautions. The explicit algorithm will not diverge (if the condition of Current is observed), but the solution obtained is not guaranteed by a criterion of checking of balance. In particular a parametric study on the step of time is essential because the pace of the solution can strongly vary when this step becomes too large.

Moreover, `Code_Aster` is not a code optimized for explicit calculations and its performances into explicit modest, are compared with the specialized codes [2].

A solution to decrease the computing time is to project the problem on a reduced basis (bases modal or bases of Ritz). One then decreases largely the number by degrees of freedom and this kind of approaches is available in `DYNA_NON_LINE` (the resolution also gains with being explicit because the condition of Current on modal basis is little penalizing). To summarize, this kind of approach is particularly adapted to the problems where non-linearities remain moderate and localised. As soon as non-linearities become strong, one can put the question of the reactualization of the initial reduced base which loses its coherence with the current solution. The overcost of calculation due to the recalculation of the base and the reprojections then comes to decrease the interest of these methods.

Parallelism can bring a all the more notable profit as the system to be solved will comprise a large number of DDL and that the number of steps of time will be weak. Indeed, the algorithm of resolution imposes important communications on each step (one is not in a strategy of type multi-field in time and space). In practice, on problems comprising a few hundreds of thousands of DDL, the speedup remains good until 16 to 32 processors. In most case, solvor iterative PETSC will compared bring a profit to solvor MUMPS. Besides parallelism on the level solvor, the parallelism of the elementary stages (resolution of the behavior at the points of Gauss) will bring an important additional profit if integration behavior is expensive. In all the cases, it is very useful to consult in the file of message `.mess` measurements of time CPU by stage, which makes it possible to identify the stages most expensive of the transitory resolution to adapt parallelism consequently. U2.08.06 documentation gives all the details for the use of parallelism.

Conclusion

This document presents some general rules to facilitate the use of transitory dynamic methods for simulation of nonlinear systems [bib1].

The first stage is the adaptation of the model to the dynamic methods. They are mainly to make sure of the good regularity the imposed conditions, of the correct definition of the density and total damping (Rayleigh).

Then, it is recommended to start by using an implicit transitory method (`DYNA_NON_LINE` with a diagram in times of the type `NEWMARK` for the relatively regular problems or `THETA_SCHEMA` for the problems with shocks, then, if need be, `HHT`). Indeed, the implicit methods are developed and more the general practitioners in *Code_Aster*.

Lastly, for certain applications like fast dynamics, calculation on modal basis or certain calculation cases in slow evolution (cf. U2.04.07), the user has the possibility of using diagrams in explicit times. Performances in time CPU of *Code_Aster* into explicit are rather weak, if one compared to a code dedicated like `EUROPLEXUS` [2]. Moreover, all the features available into implicit are not it into explicit (like, for example, for the contact where only the penalization is authorized).

In order to qualify the quality of the digital solution obtained, it is essential to conduct certain parametric studies:

- as for static calculations quasi -, while exploiting the space discretization,
- by testing various steps of time,
- by testing various diagrams in time.

To validate this choice of parameters, it is relevant to analyze the answers in displacement and accelerations, like making use of the features of assessment of energy.

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