

## Note of modeling of the gyroscopy

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### Summary:

The linear and non-linear analyses modal or dynamic of the lines of trees require to integrate gyroscopic concept of damping. The analysis of a transient number of revolutions (gone up of speed or deceleration) requires, as for it, to integrate the concept of stiffness gyroscopic.

The codes of revolving machines dedicated use a preferential axis for the modeling of the rotors (  $Y$  for ROTORINSA,  $Z$  for CADYRO, etc). In *Code\_Aster*, it is possible, with the help of some precautions in the setting in data, to model a rotor with any axis of rotation. Once the axis of rotation is chosen, ON defines the positive direction along this axis as being the direction **trigonometrical** usual of rotation.

This note of use constitutes a help with the use of the features gyroscopic damping and stiffness. It must enable him to conclude a calculation of revolving machines coherent and correct and this whatever the axis of rotation of the line of trees.

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## 1 Gyroscopic damping and stiffness

### 1.1 Characteristics of the gyroscopy

Currently available modelings in Code\_Aster for the revolving machines utilize the structural elements of standard beam (for the rotors) and the discrete elements (for the discs, the stages, the supports, etc).

Calculations of the kinetic energy of the element beam of Timoshenko (by considering the deformations of inflection) or of the discrete element reveal the terms of gyroscopic effect.

For a turning system, the classical equation of the dynamics of the structures depends on the number of revolutions as follows:

$$M \ddot{\delta} + C_{gyro}(\Omega) \dot{\delta} + (K + K_{gyro}(\dot{\Omega})) \delta = 0$$

Where  $M$  and  $K$  are the classical matrices of stiffness and mass of the mechanical system turning.  $C_{gyro}(\Omega)$  is a nonsymmetrical matrix, function number of revolutions  $\Omega$ , including the characteristics of gyroscopic damping. This matrix of gyroscopic damping is antisymmetric, and its contribution is proportional to the angular velocity.

$K_{gyro}(\dot{\Omega})$  is a nonsymmetrical matrix, function of the acceleration of rotation  $(\dot{\Omega})$ , including the characteristics of gyroscopic stiffness. This matrix of gyroscopic stiffness is full. Its contribution is proportional to the angular acceleration and is, therefore, worthless in the event of number of constant revolutions. The gyroscopic matrix of stiffness has direction only in transient speed.

Gyroscopic damping and the stiffness are currently available in Code\_Aster for the right beams (Elements of Timoshenko `POU_D_T` and of Euler `POU_D_E`) of constant or variable section and circular form like for the discrete elements. The assumptions retained for the beams are:

- Assumption of Timoshenko: transverse shearing and all the terms of inertia are taken into account. This assumption is to be used for weak twinges (Elements `POU_D_T`).
- Assumption of Euler: transverse shearing is neglected. This assumption is checked for strong twinges (Elements `POU_D_E`).

The number of revolutions `clean` (along the axis of the beam) can be constant or variable (case of a transient number of revolutions: rise of speed or deceleration).

The discs are cylinders of cross-section whose axis is confused with the axis of the beam. These discs are supposed to be indeformable and are modelled by discrete elements. The material is homogeneous and isotropic.

Finally and for more details, the reader will be able to refer to the reference material of gyroscopic damping and the stiffness in Code\_Aster [R5.05.07].

### 1.2 Calculation of the gyroscopic matrices

It is possible to build the matrices of gyroscopic damping and stiffness starting from each element of the model as for rigidity and the mass. For the structural finite elements of standard beam or the discrete elements, it is possible to calculate the real elementary matrices corresponding to the options of calculation `'MECA_GYRO'` and `'RIGI_GYRO'`. The call to these operations is done as follows:

```
AMGYELEM [ matr_elem_DEPL_R ] = CALC_MATR_ELEM
( / ♦ OPTION: 'MECA_GYRO'
  ♦ MODEL: Mo [model]
  ♦ CHAM_MATER: chmat [cham_mater]
  ♦ CARA_ELEM: will cara [cara_elem]
)
```

```
RIGYELEM [ matr_elem_DEPL_R ] = CALC_MATR_ELEM
```

```
( / ♦ OPTION: 'RIGI_GYRO'  
♦ MODEL: Mo [model]  
♦ CHAM_MATER: chmat [cham_mater]  
♦ CARA_ELEM: will cara [cara_elem]  
)
```

The assembly of the elementary gyroscopic matrices is obtained with the operator `ASSE_MATRICE` usual [U4.61.22].

```
AMGYASS=ASSE_MATRICE (MATR_ELEM=AMGYELEM,  
NUME_DDL=NUMEDDL, )  
RIGYASS=ASSE_MATRICE (MATR_ELEM=RIGYELEM,  
NUME_DDL=NUMEDDL, )
```

It will be noted that one must use same classifications and the same mode of storage as for the matrices of rigidity and mass (operator `NUME_DDL` [U4.61.11]).

One can as conduct this operation into same as time as the construction of the other matrices (mass, stiffness, damping,...) thanks to the macro-order `ASSEMBLY`

```
ASSEMBLY ( MODELE=MODELE,  
CHAM_MATER=CHMAT,  
CARA_ELEM=CARELEM,  
CHARGE=BLOCAGE,  
NUME_DDL=CO ('NUMEDDL'),  
MATR_ASSE= (_F (MATRICE=CO ('RIGIDITY'),  
OPTION=' RIGI_MECA',),  
_F (MATRICE=CO ('MASS'),  
OPTION=' MASS_MECA',),  
_F (MATRICE=CO ('AMOR'),  
OPTION=' AMOR_MECA',),  
_F (MATRICE=CO ('AMGY'),  
OPTION=' MECA_GYRO',),  
_F (MATRICE=CO ('RIGY'),  
OPTION=' RIGI_GYRO',),),),)
```

## 1.3 Use of the gyroscopic matrices

calculated matrices of gyroscopic damping and stiffness  $C_{gyro}$  and  $K_{gyro}$  are unit and must thus be multiplied by speed and the angular acceleration respectively to obtain the gyroscopic effect a given speed of rotation.

The matrix of gyroscopic damping is generally combined after assembly with the classical matrix of damping by the operator `COMB_MATR_ASSE` as follows :

```
OM=124.14*pi/30. # Number of revolutions out of turns/min transformed into  
rad/s  
GYOM=COMB_MATR_ASSE (COMB_R= (_F (MATR_ASSE=AMGY, COEF_R=OM,),  
_F (MATR_ASSE=AMOR, COEF_R=1.,),))
```

where `AMGY` and `AMOR` are the assembled matrices of gyroscopic and viscous damping.

Matrices gyroscopic are usable for the direct linear dynamic analysis with the operators of linear dynamic response:

- linear transient on physical basis `DYNA_LINE_TRAN` [U4.53.02] (cf modeling A in [V2.02.126])
- linear transient on modal basis `DYNA_TRAN_MODAL` [U4.53.21] (cf modeling B in [V2.02.126])
- harmonic `DYNA_LINE_HARM` [U4.53.11] (cf modeling C in [V2,02,126])
- `DYNA_VIBRA` [U4.53.03] (cf modeling C in [V2,02,126])

In the case of a transient number of revolutions treated on modal basis, the two gyroscopic matrices assemblies are combined with the classical matrices of damping and rigidity in a completely transparent way to the user. For that, it is enough to inform in **DYNA\_TRAN\_MODAL** or **DYNA\_VIBRA** parameters defining this transient speed (laws imposed speed and angular acceleration and matrices gyroscopic assemblies, cf [U4.53.21] and [U2.06.32]).

These matrices are essential for the modal analysis complexes of a system turning with the operator of research of the eigenvalues:

- **CALC\_MODES** [U4.52.02]

For the systems with a reduced number of degrees of freedom, a direct modal calculation can be carried out with the operator quoted above and one or the other of the algorithms of resolution (here algorithm 'SORENSEN' for example):

```
MODEG=CALC_MODES (MATR_RIGI=RIGIDITE,  
                  MATR_MASS=MASSE,  
                  MATR_AMOR=GYOM,  
                  OPTION=' CENTRE',  
                  CALC_FREQ=_F (FREQ=1.,  
                                NMAX_FREQ=16),  
                  SOLVEUR_MODAL=_F (METHODE=' SORENSEN'),  
                  VERI_MODE=_F (SEUIL=1.E-3),  
                  )
```

For a quadratic modal calculation (calculation of deadened modes) robust, one can use a strategy in two stages.

Method QZ is a reliable method for a quadratic modal calculation. However it is expensive. It is thus to restrict with systems with small number of degrees of freedom (a few thousands of degrees of freedom) or with problems reduced by modal truncation. It is necessary to carry out a calculation in two stages:

- 1a - to calculate a reduced not deadened modal base:

```
MODES=CALC_MODES (MATR_RIGI=RIGIDITE,  
                  MATR_MASS=MASSE,  
                  OPTION=' BANDE',  
                  CALC_FREQ=_F (FREQ= (3. , 3000.))  
                  );
```

- 1b - to project the matrices of mass, stiffness and total damping GYOM on this basis (modal truncation):

```
PROJ_BASE (BASE=MODES,  
           PROFIL=' PLEIN',  
           MATR_ASSE_GENE= (_F (MATRICE=CO ('MAGUS'),  
                                MATR_ASSE=MASSE,)),  
                           _F (MATRICE=CO ('RIGE'),  
                                MATR_ASSE=RIGIDITE,)),  
                           _F (MATRICE=CO ('OMGE'),  
                                MATR_ASSE=GYOM,)),),  
           );
```

- 2 - to do a modal calculation on the matrices generalized obtained by projection:

```
MOD2ETAG= CALC_MODES (MATR_RIGI=RIGE,  
                      MATR_MASS=MAGE,  
                      MATR_AMOR=OMGE,  
                      OPTION=' TOUT',  
                      SOLVEUR_MODAL=_F (METHODE=' QZ'),  
                      );
```

One is thus brought to project the matrices antisymmetric and nonsymmetrical  $C_{gyro}$  and  $K_{gyro}$  in the subspace defined by a unit real clean modes. This operation is possible with the operator **PROJ\_MATR\_BASE** [U4.63.12]. Let us note that in the case general, the projected matrices are not symmetrical. They remain nevertheless usable for the calculation of the dynamic response in force or in movement imposed in modal space, with the operator of dynamic response linear:

- linear transient on modal basis **DYNA\_TRAN\_MODAL** [U4.53.21] (cf modeling B in [V2,02,126])

## 2 Example of calculation line of trees with axis according to X

In this section, we give a concrete example of a line of trees modelled in two different ways (axes of rotation  $X$  on the one hand and unspecified on the other hand). We base ourselves here on the example, documented in [V6.02.126], of a simple model of rotor. He is guided by 2 stages (respectively first and last node of the rotor), and supports 3 discs (cf appears below).

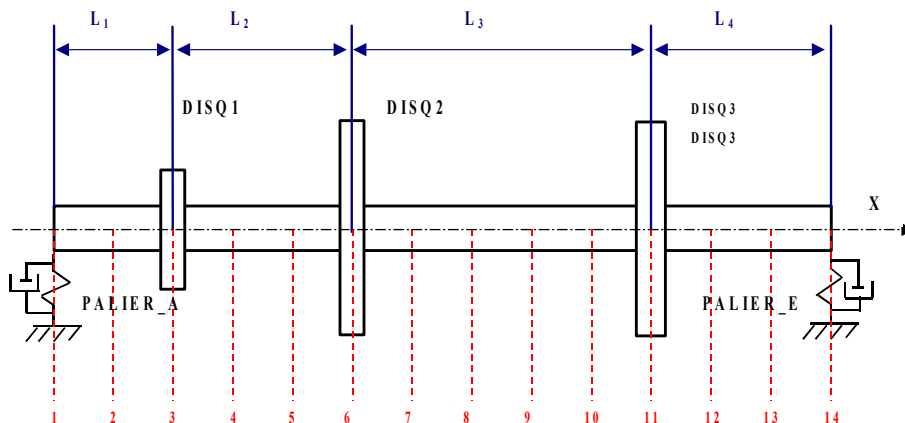


Figure 2-a: Model of rotor with 3 discs and 2 stages resulting from [bib1]

We consider two modelings for this CAS-test. In the first, L'axis of rotation is according to  $X$ , in the second, it is according to an unspecified axis defined by its nautical angles  $(\alpha, \beta, \gamma)$ . The stages of reading of grid and assignment of models and material being identical for two modelings (put except for the grid of the trees according to the desired axis of rotation), we focus ourselves on the stage which differentiates them, namely the assignment of the elementary characteristics by the order **AFFE\_CARA\_ELEM**.

The setting in data of the discrete elements is made in order to inform the characteristics in rigidity and damping (for the stages) and masses it and inertias (for the discs) directly in the total reference mark (keyword **REFERENCE MARK**, by default **REFERENCE MARK = 'TOTAL'**).

```
CARELEM=AFFE_CARA_ELEM (MODELE=MODELE,
  POUTRE= _F (GROUP_MA=' ROTOR',
    SECTION=' CERCLE',
    CARA=' R',
    VALE=.05, ),
  DISCRET= ( _F (CARA=' K_TR_D_N',
    GROUP_MA=' PALIER_A',
    REPERE=' GLOBAL',
    VALE= (0. , 5.E7,7.E7,0., 0. , 0. , ), ),
    _F (CARA=' K_TR_D_N',
    GROUP_MA=' PALIER_E',
```

```
        REPERE=' GLOBAL',
        VALE= (0. , 5.E7,7.E7,0., 0. , 0. ,),),
    _F (CARA=' A_TR_D_N',
        GROUP_MA=' PALIER_A',
        REPERE=' GLOBAL',
        VALE= (0. , 5000. , 7000. , 0. , 0. , 0. ,),),
    _F (CARA=' A_TR_D_N',
        GROUP_MA=' PALIER_E',
        REPERE=' GLOBAL',
        VALE= (0. , 5000. , 7000. , 0. , 0. , 0. ,),),
    _F (CARA=' M_TR_D_N',
        GROUP_MA=' DISQ1',
        REPERE=' GLOBAL',
        VALE= (14.58, 0.1232, 0.0646, 0.0646, 0. , 0. , 0. , 0.
, 0. , 0. ,),),
    _F (CARA=' M_TR_D_N',
        GROUP_MA=' DISQ2',
        REPERE=' GLOBAL',
        VALE= (45.94, 0.9763, 0.4977, 0.4977, 0. , 0. , 0. , 0.
, 0. , 0. ,),),
    _F (CARA=' M_TR_D_N',
        GROUP_MA=' DISQ3',
        REPERE=' GLOBAL',
        VALE= (55.13, 1.1716, 0.6023, 0.6023, 0. , 0. , 0. , 0.
, 0. , 0. ,),),
    );
```

### 3 Example of calculation of line of trees with unspecified axis

Below an example of setting in data of the same line of trees directed according to an unspecified axis of rotation defined by its nautical angles<sup>1</sup>  $(\alpha, \beta, \gamma)$ . The grid of the rotor is built according to this axis and the assignment of the elementary characteristics of the elements of type beam corresponding is done directly in the total reference mark. Like prear defect the values of the matrices of the discrete elements are expressed in the total reference mark, one precise explains `REFERENCE MARK = 'LOCAL'` in the assignment and one the local reference mark compared to the total reference mark by the keyword defines `ORIENTATION [U4,42,01]`.

```
CARELEM=AFFE_CARA_ELEM (MODELE=MODELE,
    POUTRE= _F (GROUP_MA=' ROTOR',
        SECTION=' CERCLE',
        CARA=' R',
        VALE=.05,),
    DISCRET= ( _F (CARA=' K_TR_D_N',
        GROUP_MA=' PALIER_A',
        REPERE=' LOCAL',
        VALE= (0. , 5.E7,7.E7,0., 0. , 0. ,),),
        _F (CARA=' K_TR_D_N',
        GROUP_MA=' PALIER_E',
        REPERE=' LOCAL',
        VALE= (0. , 5.E7,7.E7,0., 0. , 0. ,),),
        _F (CARA=' A_TR_D_N',
```

1 They are the angles making it possible to pass from the total reference mark of definition of the coordinates of the nodes to the local reference mark. For example:

- nautical angles allowing the passage of the axis of rotation  $X$  with the axis of rotation  $Z$  are  $(\alpha=0, \beta=-\pi/2, \gamma=0)$  ;
- nautical angles allowing the passage of the axis of rotation  $X$  with the axis of rotation  $Y$  are  $(\alpha=\pi/2, \beta=0, \gamma=0)$  .

```
GROUP_MA=' PALIER_A',
REPERE=' LOCAL',
VALE= (0. , 5000. , 7000. , 0. , 0. , 0. ),),),
_F (CARA=' A_TR_D_N',
GROUP_MA=' PALIER_E',
REPERE=' LOCAL',
VALE= (0. , 5000. , 7000. , 0. , 0. , 0. ),),),
_F (CARA=' M_TR_D_N',
GROUP_MA=' DISQ1',
REPERE=' LOCAL',
VALE= (14.58, 0.1232, 0.0646, 0.0646, 0. , 0. , 0. , 0.
, 0. , 0. ),),),
_F (CARA=' M_TR_D_N',
GROUP_MA=' DISQ2',
REPERE=' LOCAL',
VALE= (45.94, 0.9763, 0.4977, 0.4977, 0. , 0. , 0. , 0.
, 0. , 0. ),),),
_F (CARA=' M_TR_D_N',
GROUP_MA=' DISQ3',
REPERE=' LOCAL',
VALE= (55.13, 1.1716, 0.6023, 0.6023, 0. , 0. , 0. , 0.
, 0. , 0. ),),),
),
ORIENTATION=_F (GROUP_MA= ('DISQ1', 'DISQ2', 'DISQ3', 'PALIER_A',
'PALIER_E'),
CARA=' ANGL_NAUT',
VALE= ( $\alpha$ ,  $\beta$ ,  $\gamma$ ),),);
```



## 4 Bibliographical references

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- MR. LALANNE, G. FERRARIS, “ Rotordynamics Prediction in Engineering “, Second Edition, Wiley, 2001.
- ROTORINSA, software finite elements intended to envisage the dynamic behavior of rotors in inflection, LaMCoS UMR5259, INSA-Lyon.