Implementation of procedure of modal retiming

Summary:

In this document, one presents techniques of modal retiming. Modal retiming consists in exploiting the clean modes of the structure in order to obtain a digital model which as well as possible reflects the dynamic behavior of the studied structure.

Contents

1 Introduction

Modal retiming consists in adjusting the parameters of the digital model starting from the knowledge of the clean modes identified in experiments on the real structure.

By using techniques of optimization, one tries to find the model digital where the associated clean modes are close to in experiments identified clean modes.

One presents in this document three techniques of modal retiming. The two first are dedicated to the conservative system and third is dedicated to the dissipative system.

2 Modal retiming of a conservative system

Two techniques of modal retiming are presented. The first exploits the sensitivity of the clean modes and the second exploits the diagonality of the generalized matrices.

2.1 Exploitation of the difference between modal deformations and the difference between Eigen frequencies

One tries to find the parameters of the digital model such as the modal deformation calculated restricted at the points of observation is colinéaire to the modal deformation obtained in experiments and that the associated Eigen frequency is equal in experiments identified Eigen frequency.

For that, one calculates the scalar product normalized between the measured clean deformation and the clean deformation calculated restricted with the ddl associated with the points of measurement. This calculation corresponds to the calculation of MAC (Modal Criterion Insurance).

If one indicates respectively by $(y_{i_{num}}, f_{i_{num}})$ and $(y_{i_{mes}}, f_{i_{mes}})$ calculated clean modes and identified clean modes, one a:

$$MAC(y_{i_{num}}, y_{i_{mes}}) = \frac{(y_{i_{num}}^T y_{i_{mes}})^2}{(y_{i_{num}}^T y_{i_{num}})(y_{i_{mes}}^T y_{i_{mes}})}$$

The vectors are defined V and F_r such as:

$$V = \begin{pmatrix} \vdots \\ MAC(y_{i_{num}}, y_{i_{mes}}) - 1 \\ \vdots \end{pmatrix}$$

$$F_r = \begin{pmatrix} \vdots \\ f_{i_{num}} - f_{i_{mes}} \\ \vdots \end{pmatrix}$$

The functional calculus to be minimized is formulated as follows:

$$\epsilon = V^T W_{MAC} V + F_r^T W_{freq} F_r$$

It is obviously necessary to evaluate the difference between two similar modes. Thus, this technique is not adapted to a structure where the modal density is important.

This procedure is used by `MACR_RECAL` (option `DYNAMICS`) in the case test sdls121a [V2.03.121].

2.2 Exploitation of the orthogonality of the measured clean modes

From the modal deformations raised at the points of observation, O N carries out an expansion on the digital model support. One tries, thereafter, to find the parameters of the digital model so that matrices of mass and of stiffness generalized relating to in experiments identified modes are diagonal and that the difference between the measured own pulsation and the calculated own pulsation either minimal.

One considers the pulsation clean (or more exactly the square of the pulsation) by calculating the relationship between the generalized stiffness and masses it generalized of the identified mode.

This technique requires, neither a pairing between the experimental mode and the digital mode, nor a modal calculation. It is thus adapted to the structures where the modal density is high. It requires nevertheless the expansion of the modes identified on the digital model.

The expansion of the 2rd mode identified on the digital model can be done in the following way:

- One chooses a base of expansion made up of modal deformations calculated with the digital model support:

$$Y=[y_1 \dots y_n]$$

- One calculates then the coordinates η_i identified modal deformation Φ_i^{mes} on the basis Y , restricted at the points of observation. These coordinates can be obtained by a minimization of the least type squares.

$$\varepsilon=\left(\Phi_i^{mes}-Y\,\eta_i\right)^T W_i\left(\Phi_i^{mes}-Y\,\eta_i\right)$$

- One carries out thereafter an expansion of the deformation identified on the ddl of the digital model: $\Phi_i=Y\,\eta_i$

The following stage consists in calculating the standardized generalized matrices:

$$MAC_W(i\,,j)=\frac{\left(\Phi_i^T W\,\Phi_j\right)^2}{\left(\Phi_i^T W\,\Phi_i\right)\left(\Phi_j^T W\,\Phi_j\right)}$$

If the matrix of weighting W is equal to the matrix of mass M or with the matrix of rigidity K , MAC_W becomes a diagonal matrix.

It is then a question of finding the terms of the matrices K and M who minimize at the same time:

$MAC_K(i\,,j)$ for $i\neq j$
$MAC_M(i\,,j)$ for $i\neq j$

Variation enters the 2rd identified own pulsation $\hat{\omega}_i^2$ and $\omega_i^2=\dfrac{\Phi_i^T K\,\Phi_i}{\Phi_i^T M\,\Phi_i}$

The matrix MAC_W is symmetrical, one can arrange his lower triangular part in a named vector $MAC_{W(i<j)}$. The functional calculus to be minimized can be formulated then as follows:

$$\varepsilon=MAC_{K(i<j)}^T W_K\,MAC_{K(i<j)}+MAC_{M(i<j)}^T W_M\,MAC_{M(i<j)}+\sum_i\left(\hat{\omega}_i^2-\frac{\Phi_i^T K\,\Phi_i}{\Phi_i^T M\,\Phi_i}\right)^T W_i\left(\hat{\omega}_i^2-\frac{\Phi_i^T K\,\Phi_i}{\Phi_i^T M\,\Phi_i}\right)$$

One can choose the matrices of following weighting:

W_K = nb_modes_identifiés * I_d
W_M = nb_modes_identifiés * I_d
W_i = 0.5 * nb_modes_identifiés * (nb_modes_identifiés – 1) * I_d

Where I_d is the matrix identity

This choice of weighting makes it possible to assign the same weight to the equations on the frequencies and the equations on the extra-diagonal terms of the generalized matrices.

The implementation of this approach of modal retiming is illustrated in modeling D of the case test sdls121 [V2.03.121].

3 Modal retiming of a dissipative system

In the case of a dissipative system, one exploits the relation of standard of the complex modal deformations.

Modal retiming used here consists in finding the parameters of the model so that in experiments identified clean modes check the relations of standard associated with the digital model.

The dissipative structure is modelled as follows:
$M\ddot{y}+B\dot{y}+Ky=0$

Where: M indicate the matrix of mass
B indicate the matrix of damping
K indicate the matrix of rigidity
y indicate displacement

The assumption is made that the matrices of the system are symmetrical.

In the majority of the cases, the modal matrix of the conservative system associated with this dissipative system diagonalise not simultaneously three matrices M , B and K . One brings back oneself then to a first order differential connection within the space of dimension 2N.

The vector of state is introduced: $x=\begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

One transforms the NR equations of the second order into 2N first order equations in the following way:

$$\begin{bmatrix} B & M \\ M & 0 \end{bmatrix}\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix}-\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} y \\ \dot{y} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

That one can write: $U\dot{x}-Ax=0$

With this transformation, the clean solutions are form: $x_\nu=\begin{bmatrix} y_\nu \\ s_\nu y_\nu \end{bmatrix}$

Where s_ν corresponds to the eigenvalue associated with x_ν .

The eigenvalues can be real or complex. If the structure is slightly deadened, all the eigenvalues complex are combined.

The spectral matrix is put then in the following form: $\underset{(2N,2N)}{S}=\begin{bmatrix} S_2 & 0 \\ 0 & \bar{S}_2 \end{bmatrix}$

And the associated modal matrix: $\underset{(2N,2N)}{X}=\begin{bmatrix} Y & \bar{Y} \\ YS_2 & \bar{Y}\bar{S}_2 \end{bmatrix}$

With: $\underset{(N,N)}{Y}=[y_\nu]$

The modal matrix X check the following relations of orthonormality: $\begin{cases} X^T UX=N_0 \\ X^T AX=N_0 S \end{cases}$

Where N_0 is a diagonal matrix which defines the standard of X : $N_0=\begin{bmatrix} N_2 & 0 \\ 0 & \bar{N}_2 \end{bmatrix}$

Taking into account cuttings in submatrices, the development of the first line of the relations of orthonormality are written as follows:

$$\begin{cases} Y^T BY+S_2 Y^T MY+Y^T MYS_2=N_2 \\ Y^T B\bar{Y}+S_2 Y^T M\bar{Y}+Y^T M\bar{Y}\bar{S}_2=0 \end{cases} \quad (1)$$

and: $$\begin{cases} S_2 Y^T MYS_2-Y^T KY=N_2 S_2 \\ S_2 Y^T M\bar{Y}\bar{S}_2-Y^T K\bar{Y}=0 \end{cases} \quad (2)$$

The diagonal terms of the first line of the systems of equations (1) and (2) lead to the following relations:

$$\begin{cases} y_\nu^T B y_\nu+2 s_\nu y_\nu^T M y_\nu=n_\nu \\ s_\nu^2 y_\nu^T M y_\nu-y_\nu^T K y_\nu=s_\nu n_\nu \end{cases}$$

The combination of these two equations leads to:

$$s_\nu^2 y_\nu^T M y_\nu+s_\nu y_\nu^T B y_\nu+y_\nu^T K y_\nu=0 \quad (3)$$

In the same way, the diagonal terms of the second-row forward of the two systems of equations (1) and (2) lead to the following equations:

$$y_\nu^T B\bar{y}_\nu+2\Re(s_\nu) y_\nu^T M\bar{y}_\nu=0 \quad (4)$$

$$s_\nu \bar{s}_\nu y_\nu^T M\bar{y}_\nu-y_\nu^T K\bar{y}_\nu=0 \quad (5)$$

These three equations (3) (4) and (5) must also be checked for all the clean modes identified on the real structure. The technique of retiming presented here consists in finding the parameters of the digital model which make it possible to check the three equations.

In experiments, the modal deformation is measured only on the directions of observation (significant directions of the sensors). An expansion of this deformation on the digital model is necessary in order to obtain y_ν .

One agrees to make the various equations associated with each mode with the same dimension. One leads then to the system of equations according to:

$$\frac{y_\nu^T B\,\bar{y}_\nu + 2\,\Re(s_\nu)\,y_\nu^T M\,\bar{y}_\nu}{|n_\nu|} = z_{1\nu}$$

$$\frac{s_\nu \bar{s}_\nu\, y_\nu^T M\,\bar{y}_\nu - y_\nu^T K\,\bar{y}_\nu}{|n_\nu s_\nu|} = z_{2\nu}$$

$$\frac{s_\nu^2\, y_\nu^T M y_\nu + s_\nu\, y_\nu^T B y_\nu + y_\nu^T K y_\nu}{n_\nu s_\nu} = z_{3\nu}$$

For more convenience, one chooses n_ν equalize to the euclidian norm of y_ν.

One deposits then these quantities in the vectors Z_1, Z_2 and Z_3 in order to be able to define the general form of the functional calculus ε to minimize.

$$Z_1 = \begin{bmatrix} \vdots \\ z_{1\nu} \\ \vdots \end{bmatrix} = Z_{1\mathrm{r}}$$

$$Z_2 = \begin{bmatrix} \vdots \\ z_{2\nu} \\ \vdots \end{bmatrix} = Z_{2\mathrm{r}}$$

$$Z_3 = \begin{bmatrix} \vdots \\ z_{3\nu} \\ \vdots \end{bmatrix} = Z_{3\mathrm{r}} + j\,Z_{3\mathrm{i}}$$

$$\varepsilon = \left(Z_{1\mathrm{r}}^T W_1 Z_{1\mathrm{r}}\right) + \left(Z_{1\mathrm{i}}^T W_1 Z_{1\mathrm{i}}\right) + \left(Z_{2\mathrm{r}}^T W_2 Z_{2\mathrm{r}}\right) + \left(Z_{3\mathrm{r}}^T W_3 Z_{3\mathrm{r}}\right)$$

Where W_1, W_1 and W_1 are weightings associated with the various blocks with equations.

It is noted however that the modal deformation must be expressed on the digital model. It is obtained by expansion of measurement on the digital model. One generally uses the modes of the digital model as bases expansion.

An illustration of this technique is in sdld21e.

4 Practical advices

Before carrying out a procedure of retiming, it is paramount to choose the parameters well to be readjusted. That requires a reflection on behalf of the user in order to seize the good parameters which have a physical direction with respect to the study to carry out.

It is also necessary to carry out a study of sensitivity of the functional calculus to be minimized compared to the parameters to adjust. Indeed, it is useless to adjust a parameter which does not make vary the functional calculus. If the parameter is paramount for the study but the functional calculus is insensitive to this parameter, then a new functional calculus should be found more adapted much.

It is also necessary to limit the number of parameters to be readjusted.

One can distinguish two main categories of methods.

The first category consists in directly exploiting the measured sizes. That relates to the methods of type sensitivity.
The second category consists in making check on the digital model the index properties of the clean modes, in substituent the clean modes calculated by the identified clean modes. One carries out an expansion of the modal deformation identified in order to obtain a size defined on the digital model.

The advantage of the first category resides on the fact that the measured data are directly exploited. The disadvantage is that one is obliged to carry out a pairing between the measured mode and the calculated mode similar to each iteration of the procedure of retiming. This technique is thus not suitable when the modal density is high.

The advantage of the second category resides on the fact that one does neither a modal calculation on the digital model during iterations of calculation nor a pairing between the calculated mode and the measured mode. The disadvantage is that one is obliged to carry out an expansion of measurement on the digital model. This technique is thus not suitable when that the number of points of measurement is reduced. A second iteration of retiming after reactualization of the digital model support is perhaps necessary in order to refine the results. Indeed, the reactualization of the model support makes it possible to improve quality of the modal expansion.

All in all, one can draw the following conclusion:

- If the modal density is high, the exploitation of the properties of the modal matrices generally gives better results. That requires nevertheless a good distribution of the points of observation in order to be able to carry out a better expansion of the modal deformation.
- If the modes of the structure are isolated modes, and that one is limited of many points of observation, then the technique of type sensitivity is adapted much for the adjustment of the initial digital model.

5 Bibliography

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