

Note of use of modeling FOURIER

1 Goal

The analysis of Fourier is intended to calculate the answer of structures with axisymmetric geometry requested by nonaxisymmetric loadings broken up into Fourier series.

Limitations:

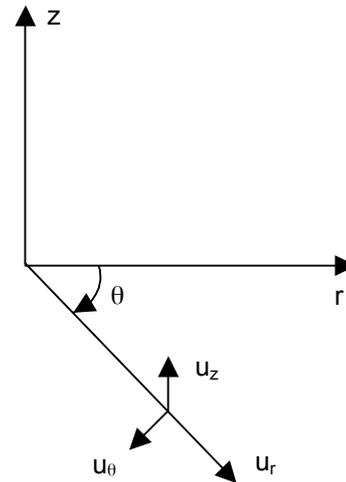
- the decomposition of the loading in Fourier series is supposed to be made by the user,
- establishment *Aster* relate to only isotropic or orthotropic materials,
- in thermics, there does not exist total order making it possible to solve a problem on several harmonics. Calculation must be done harmonic by harmonic.

2 Notations

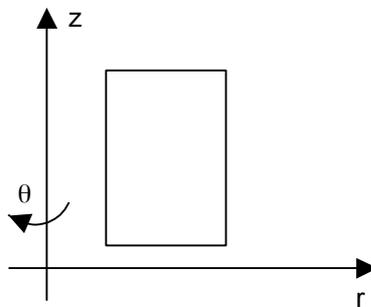
All the fields considered (forces, displacements, strains, stresses, flow) are expressed in cylindrical coordinates with following convention on the order of the components:

radial component according to r
axial component according to z
component tangential (or circumferential) according to θ

Example: (u_r, u_z, u_θ)
 (f_r, f_z, f_θ)



The grid is localised in the plan (r, z) , the symmetry of revolution being done around the axis Oz .
The trihedron (r, z, θ) is directed in the direct direction.



3 Modeling, loadings

To affect the axisymmetric finite elements Fourier on the grid, the operator is used `AFFE_MODELE` in the following way:

```
Mo = AFFE_MODELE ( GRID = my,
                  AFFE = _F ( ALL = 'YES',
                              PHENOMENON = 'MECHANICS' or 'THERMICS',
                              MODELING = 'AXIS_FOURIER' )
                  );
```

The decomposition in Fourier series of the loading must be made as a preliminary by the user

$$\text{that is to say } f = \sum_{l=0}^N \left[\begin{pmatrix} \cos l\theta & 0 \\ 0 & \cos l\theta \\ 0 & -\sin l\theta \end{pmatrix} F_l^s(r, z) + \begin{pmatrix} \sin l\theta & 0 \\ 0 & \sin l\theta \\ 0 & \cos l\theta \end{pmatrix} F_l^a(r, z) \right]$$

with $F_l^{s(a)} = (f_{rl}^{s(a)}, f_{zl}^{s(a)}, f_{\theta l}^{s(a)})$

Loads F_l^s and F_l^a are introduced harmonic by harmonic and type by type by the operator `AFFE_CHAR_MECA`. One does not specify the mode nor the type on this level.

Example: one supposes a loading in pressure distributed symmetrical mode 1 and pure torsion (antisymmetric mode 0).

One will write:

```
ch1sym = AFFE_CHAR_MECA ( Model = Mo,
                          PRES_REP = _F ( GROUP_MA = 'grma',
                                          CLOSE = p));
ch0anti = AFFE_CHAR_MECA ( Model = Mo,
                           FORCE_NODALE = _F ( FZ = F,
                                                NODE = ' N1 ' ) );
```

The boundary conditions of the Dirichlet type will be introduced into a load except for:

```
to chdir = AFFE_CHAR_MECA ( Model = Mo,
                           DDL_IMPO = _F ( GROUP_NO = 'grno',
                                           DX = 0. , DY = 0. , DZ = 0. ,));
```

The acceptable loadings by the elements of Fourier are:

in elasticity:

Elements	Nature of the loading	Keyword <code>AFFE_CHAR_MECA</code>
TRIA3 - TRIA6	Forces of volume	FORCE_INTERNE
QUAD4 - QUAD8 - QUAD9	Rotation	ROTATION
	Gravity	GRAVITY
	Specific forces	FORCE_NODALE
SEG2 - SEG3	Pressure	PRES_REP
	Surface forces	FORCE_CONTOUR

in thermics:

Elements	Nature of the loading	Keyword <code>AFFE_CHAR_THER</code>
Surface	Source of heat	SOURCE
Edge	Imposed normal flow	FLUX_REP
	Exchange	EXCHANGE

3.1 Typical case of gravity

One considers the cylinder below of which the hatched part corresponds to the part with a grid on which one carries out an axisymmetric calculation Fourier (plan rZ).

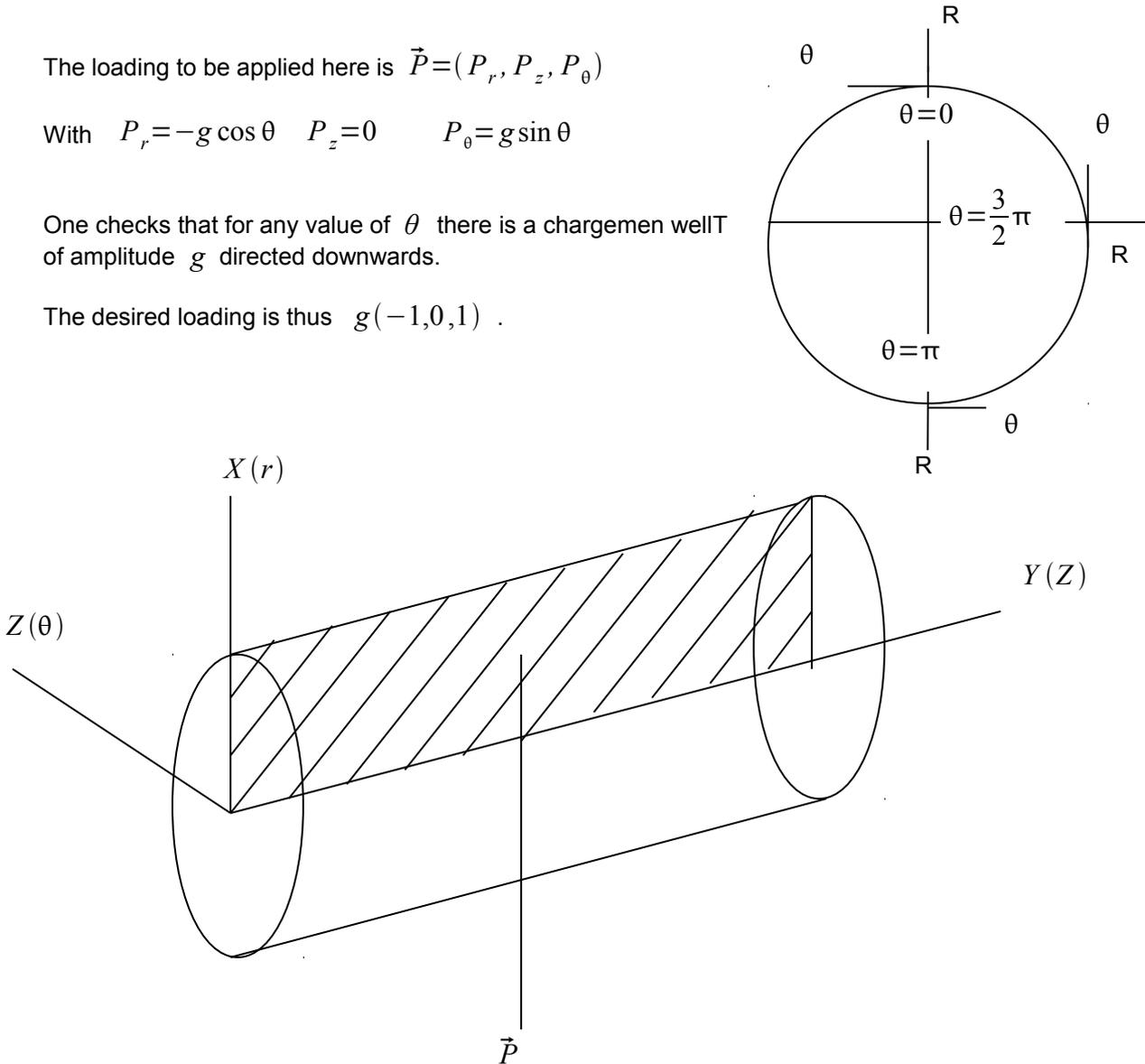
This cylinder is subjected to its actual weight P directed along the axis Oz .

The loading to be applied here is $\vec{P} = (P_r, P_z, P_\theta)$

With $P_r = -g \cos \theta$ $P_z = 0$ $P_\theta = g \sin \theta$

One checks that for any value of θ there is a charge g directed downwards.

The desired loading is thus $g(-1, 0, 1)$.



Nevertheless, the vector direction of the loading being normalized in `AFPE_CHAR_MECA`, it is necessary to multiply by this standard to obtain the desired loading.

The loading to be introduced into `AFPE_CHAR_MECA` behind the keyword `GRAVITY` is thus $\sqrt{2} g(-1, 0, 1)$.

In `MACRO_ELAS_MULT`, this loading will be affected with symmetrical harmonic 1 (development in series cosine for the radial component).

4 Resolution with Aster

Once the loading broken up into harmonics of Fourier, the harmonics being uncoupled from/to each other (with number of different Fourier), it is necessary to assemble and solve as many linear systems there are harmonics.

Moreover, the nonanisotropic material being supposed, for the same number of harmonic, the symmetrical and antisymmetric modes are uncoupled. One will have to thus make resolutions as many (with the corresponding load) that there are couples (harmonic, mode) different.

Establishment in *Code_Aster* is different according to whether the phenomenon is thermal or mechanical.

4.1 Thermics

In thermics, there is no total order making it possible to calculate several harmonics directly. One must thus proceed harmonic by harmonic. Moreover, calculations of matrix and second members elementary can be done only with the orders `CALC_MATR_ELEM` and `CALC_VECT_ELEM` (and not by the order `THER_LINEAIRE`).

The mode of Fourier is to be introduced into `CALC_MATR_ELEM` by the simple keyword `MODE_FOURIER`. The type of the harmonic is not necessary, the matrices (and vectors) being independent of the type. The type is only taken into account with the recombination of Fourier.

It is important to assemble the matrices and vectors corresponding to the various harmonics with same classification in order to be able to recombine the fields results. The operator `NUME_DDL` who builds classification is thus used once for the first harmonic, classification thus created being re-used for all the other harmonics. This is possible if one differentiated the loads from Dirichlet of the loadings themselves (see example [§6.1]).

4.2 Mechanics

The order making it possible to treat several harmonics is `MACRO_ELAS_MULT` [U4.51.02]. In this macro, the harmonics are regarded as loading cases and one thus does as many resolutions as there are harmonics. As in thermics, it is necessary to differentiate the loads from Dirichlet, which must be identical for all the harmonics, of the loadings themselves, which can vary.

One obtains a structure of data `RESULT` containing all the fields corresponding to the calculated harmonics (see example [§6.2]).

5 Postprocessings

5.1 Thermics

The calculation of flows is done by the operator `CALC_CHAM_ELEM` by specifying the number of the harmonic by the simple keyword `MODE_FOURIER`.

The options of calculation of flow remain the standard options:

`FLUX_ELNO` to calculate flows with the nodes by element
`FLUX_ELGA` to calculate flows at the points of Gauss

The order of the components of the vector flow is $(\Phi_r, \Phi_z, \Phi_\theta)$.

The recombination of Fourier on the temperatures is done starting from the operator `CREA_CHAMP` [U4.72.04]. It makes it possible to obtain the temperatures in various angular sections introduced by the user.

The recombination of Fourier on flows is made in `CREA_CHAMP` [U4.72.04] according to the same principle.

5.2 Mechanics

The calculation of the strains and the stresses is done by the operator `CALC_CHAMP`.

The options of calculation remain the standard options:

`EPSI_ELNO` to calculate the deformations with the nodes by element
`SIEF_ELGA` to calculate the constraints at the points of Gauss
`SIGM_ELNO` to calculate the constraints with the nodes by element
`SIEQ_ELGA` to calculate the equivalents of constraints at the points of Gauss
`SIEQ_ELNO` to calculate the equivalents of constraints to the nodes by element
(for these 2 last options, it is necessary to have calculated `SIEF_ELGA` as a preliminary)

The order of the components of the tensor of the deformations (resp. constraints) is:

$(\varepsilon_{rr}, \varepsilon_{zz}, \varepsilon_{\theta\theta}, \varepsilon_{rz}, \varepsilon_{r\theta}, \varepsilon_{z\theta})$ (resp. $\sigma_{rr}, \sigma_{zz}, \sigma_{\theta\theta}, \sigma_{rz}, \sigma_{r\theta}, \sigma_{z\theta}$)

The recombination of Fourier is carried out by the operator `COMB_FOURIER` [U4.83.31] which makes it possible to recombine all the harmonics of the fields appearing in the structure of data `RESULT`. This recombination can be done on a list of angles.

The calculation of the equivalent constraints (von Mises, Tresca) is done by the operator `CALC_CHAMP` with the usual options `SIEQ_ELGA` and `SIEQ_ELNO`. It must be done **afterwards** recombination of Fourier on the constraints `SIGM_ELGA`.

6 Examples

6.1 Thermics: calculation on 2 harmonics

See the case test TPLV100 [V4.04.100].

6.2 Mechanics: calculation and recombination of Fourier on 2 harmonics

See the case test SSLV109 [V3.04.109].