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## Propagation of uncertainties and calculation of curves of brittleness

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### Summary:

This document gives the elements for the implementation of digital simulations of Monte Carlo via the distribution of calculations with Code\_Aster.

The five principal ingredients are:

- the creation of the experimental design: pulling of the random variables of  $N_s$  case of studies;
- the setting under Code\_Aster format for the launching of  $N_s$  mechanical studies (file `.distr`);
- Code\_Aster calculations;
- readings of the writing and computation results in a single file;
- statistical postprocessing: calculation of classical statistical estimators like the average, the standard deviation, etc; the estimate of the parameters of curves of lognormal brittleness (POST\_DYNA\_ALEA); other postprocessings...

## Contents

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1 Introduction.....	3
2 Simulation of Monte Carlo with Code_Aster.....	3
2.1 Realization of the experimental design.....	3
2.2 Definition and launching of the studies distributed with Code_Aster.....	4
2.3 To recover the results of calculations in a file.....	4
3 Statistical postprocessings.....	4
3.1 Calculation of the statistical sizes.....	4
3.2 Calculation of curves of brittleness.....	5
3.2.1 Definitions.....	5
3.2.2 Evaluation of the parameters by maximum of probability.....	6
3.2.3 Evaluation by linear regression.....	7
4 Bibliography.....	9

## 1 Introduction

The digital method of Monte Carlo consists in making  $N_S$  digital simulations of the mechanical model in order to evaluate the statistical sizes of the computation results. The 3 principal stages of a propagation of uncertainties by simulation of Monte Carlo are:

- The modeling of uncertainties: Generation of a sample of  $N_S$  achievements of the random data as starter of the mechanical model (random parameters and stochastic process),
- Propagation of uncertainties: Calculation of  $N_S$  sizes results corresponding to these data,
- The calculation of the statistical estimators of the required sizes: average, standard deviation, fractiles...; evaluation of curves of seismic brittleness.

Within the framework of a mechanical study with Code\_Aster, one can call on the features of distribution of parametric studies in order to carry out such a probabilistic analysis. Indeed, in the case running where them  $N_S$  mechanical pullings and simulations are independent, one can as a preliminary draw up an experimental design of the cases of study to launching. One launches then them  $N_S$  corresponding studies, and Lbe statistical sizes are calculated in postprocessing of the results.

The approach and its implementation with Code\_Aster are described in this document.

## 2 Simulation of Monte Carlo with Code\_Aster

Within the framework of the simulation of Monte Carlo, the modeling of uncertainties passes by the generation of random variables and possibly of stochastic processes. In what follows, it is supposed that the user chose laws characterizing them well  $m$  dubious parameters. With this intention, one can resort to test results, the judgement of expert or the principle of the maximum of entropy (see [R4.03.05]). Very often, one can suppose that the various sources of uncertainties are independent. One can then carry out random pullings according to  $m$  marginal laws of the random variables. In certain cases, one can also include more complex sizes (not scalars) like the random matrices or the stochastic processes.

One also speaks about experimental design to indicate the method of pulling and the sample which results from it. In Code\_Aster, the simulation of Monte Carlo according to an experimental design is possible using the option distributed calculations [U2.08.07].

### 2.1 Realization of the experimental design

All the methods of which pullings are independent can be put in work via the distribution of calculations with Code\_Aster. In particular, one can call on random pullings according to the method of the Latin hypercube (Latin Hypercube Sampling – LHS) or quasi Monte Carlo (quasi-random continuations like the continuation of Sobol,...) [bib4, bib5].

- Random variables: uncertainties on the parameters of the model finite elements (parameters materials, values of a game, a stiffness of thrust rubber bands, a modulus Young, etc).
- Random matrices: In dynamics of the structures on modal basis, the random variables can be the values of the generalized matrices of mass, stiffness and damping of the model to the finite elements. In this approach, one makes pulling of these random matrices using the operator `GENE_MATR_ALEA` [U4.36.06]. In this approach, one models at the same time uncertainties of model and modeling by a probabilistic approach known as “not-parametric”.
- Random excitations the operator `GENE_FONC_ALEA` allows to generate random signals stationary starting from the spectral concentration of power. The operator `GENE_ACCE_SEISME` allows to generate seismic signals nonstationary in amplitude and frequential contents. In other cases, one can have a database containing of the signals which model a random physical phenomenon. That can be the case in seismic analysis where a set of accélérogrammes measured in situ can be provided by seismologists.

There exist two possibilities to carry out the experimental design: 1) either one creates the sample of the sets of parameters as a preliminary according to the method of pulling wished and one stores it in a data file, for that one can use a software dedicated like OpenTURNS or the dedicated toolbox of Python and Matlab) 2) is one uses one of the operators of *Code\_Aster* to generate a realization of a stochastic process or a random matrix. In the case of a transitory dynamic calculation, one can thus use random temporal loadings generated by *Code\_Aster*. One can of course use the two options jointly.

**Notice:** If, within the framework of a seismic analysis, one has a sample of  $N_S$  accélérogrammes, one can be brought back, for the experimental design, with a discrete random variable taking values of 1 with  $N_S$ . Each entirety indicates one of the achievements of the random process (i.e. a accélérogramme). Pulling by Latin Hypercube then consists in traversing the entireties of 1 with  $N_S$ .

**Caution:** Orders *GENE\_MATR\_ALEA* and *GENE\_FONC\_ALEA* generate the same continuation of random variables inside the same execution of *Code\_Aster*. Thus, from one execution to another, a strictly identical command file (same calls to the orders in the same order with the same arguments) will provide the same results exactly. This is due to the fact that the generator of random variables used by *Code\_Aster* is always initialized with the same value. If one wishes to generate results different from one execution to another, then the keyword should be used *INIT\_ALEA*. On the other hand, *GENE\_ACCE\_SEISME* generate random independent seismic signals.

## 2.2 Definition and launching of the studies distributed with Code\_Aster

It is necessary, in the window *astk*:

1	to define the deterministic study (file <code>.comm</code> , grid, etc) nominal
2	to define, in <code>.comm</code> , face values of the parameters (dubious) which will be modified then according to the experimental design
3	to define, always in <code>.comm</code> , variables of interest and to write these variables in a file result (under format python)
4	to define a repertoire output (standard <code>repe</code> ) where these results will be written
5	to define the experimental design in <code>.distr</code> (sets of parameters for the studies with launching)
6	to go in the mitre "Option", to choose <code>distribution = yes</code> (cf [U1.04.00] §8)
7	in the mitre "Option", to choose <code>class = distr</code> if calculations on the centralized waiter
8	if useful, to add a file of the type <code>hostfile</code> defining the machines available (cf [U1.04.00] §9)

For more detail, one can consult the tutorial [U2.08.07].

## 2.3 To recover the results of calculations in a file

Results of the calculations distributed with *Code\_Aster* are written in output files that one finds in  $N_S$  named repertoires  $CALC_i$ ,  $i=1, N_S$  (cf [U2.08.07]). In order to proceed to postprocessing by a software of statistics (like OpenTURNS or the dedicated toolbox of Python and Matlab, R), it is necessary to gather the sample of exits in a single file of results. This can be carried out in a very easy way via a script python.

## 3 Statistical postprocessings

### 3.1 Calculation of the statistical sizes

Within the framework of an approach by simulation of Monte Carlo, one is interested not only in one deterministic computation result, but one wishes to evaluate average, median, standard deviation, fractile and other statistics describing the variable of exit. This in particular makes it possible to associate confidence intervals with the computation results.

After having recovered the sample of the exits, one can call on the software preferred (like OpenTURNS, dedicated toolbox of Python and Matlab, R) to carry out these statistical postprocessings. Moreover, of the lognormal curves of brittleness can be evaluated using the operator `POST_DYNA_ALEA` of `Code_Aster`. This is described more in detail in the following paragraph.

## 3.2 Calculation of curves of brittleness

### 3.2.1 Definitions

The curves of brittleness give the conditional probability of failure of a structure or a component according to the seismic level of excitation. The modeling and the propagation of uncertainties described above also allow the determination of the curves of brittleness. It is enough to introduce a criterion of failure and to check with each calculation of Monte Carlo if the failure is reached or not.

The principal ingredients for the establishment of curves of brittleness by digital simulation are the following:

- Determination of the seismic excitation to consider (database of accélérogrammes),
- Definition of criteria of failure,
- Modeling of uncertainties and propagation by digital simulation (Monte Carlo)
- Estimate of the parameters (median and standard deviations logarithmic curve) of the curve of lognormal brittleness.

The curve of brittleness of a component can be defined starting from the concept of "capacity". Capacity of a component is the value of the parameter representative of the seismic action from which the component is failing. The current approach consists in modelling the capacity by a random variable according to a lognormal law, such as  $A = A_m \varepsilon$ , where  $A_m$  is the median capacity and  $\varepsilon$  indicate a lognormal random variable of median unit and standard deviation logarithmic curve  $\beta$ . Too, capacity  $A$  of a component (and  $A_m$  thus its curve of brittleness), is characterized by two parameters which are the median ("median capacity") and the standard deviation  $\beta$ .

Thus, the probability of ruin for a level of acceleration  $a$  given can be written [bib3, bib2]:

$$P_f(a) = \int_0^a \underbrace{p(x)}_{\text{lognormal law}} dx = \Phi\left(\frac{\ln(a/A_m)}{\beta}\right)$$

where  $\Phi(\cdot)$  indicate the fonction of distribution of a centered Gaussian random variable reduced. The use of the model lognormal has the advantage of requiring a reduced number of simulations compared to a direct calculation of the probabilities of failure. Indeed, in a direct calculation, without assumption of law a priori, it is necessary to determine rare probabilities of events (probabilities in tail of distribution), which asks for a number of very important simulations of Monte Carlo. In what follows, we present two methods, the estimate by maximum of probability and the linear regression, to evaluate these two parameters.

If the damage or the failure of a structure is characterized by a variable of interest  $Y$  continue, accompanied with a threshold  $Y_s$ , one can express the probability of failure (level of damage) for an earthquake of level  $a$  like

$$P_f(a) = P(Y > Y_s | a)$$

Variables of damage  $Y$  classics for a concrete structure are the drift transistor (differential displacement between two stages of a building) or the fall of the first Eigen frequencies. The notations introduced here will be used for the evaluation of a curve of brittleness by regression lineaire (§3.2.2).

### 3.2.2 Evaluation of the parameters by maximum of probability

In what follows, it is considered that maximum acceleration was selected to characterize the seismic level of excitation and thus the capacity. The followed approach then consists in modelling the result of the digital experiments by a random variable of Bernoulli  $X$ . Indeed, for each digital simulation  $i$ , there are two possible exits: either the critical level was reached and there is failure ( $x_i=1$ ) either there is not failure ( $x_i=0$ ). In the same way for each simulation, one can determine the value of maximum acceleration  $a_i$ . The estimate of the parameters of a curve of brittleness can be done by the method of the maximum of probability. The function of probability to be maximized for this problem is written:

$$L = \prod_{i=1}^N \left( P_{f|a}(a_i) \right)^{x_i} \left( 1 - P_{f|a}(a_i) \right)^{1-x_i}$$

In this expression, the realization  $x_i$  of  $X$  thus takes value 1 if there are failure or 0 if there is not failure for the loading (maximum acceleration)  $a_i$ . These events arrive with the probability  $P_{f|a}$  data by the expression (1). Estimates of the parameters  $\beta$  and  $A_m$  are those which minimize  $-\ln(L)$  :

$$(\beta^e, A_m^e) = \arg \min_{\beta, A_m} [-\ln(L)]$$

This stage can be put in work in postprocessing of the computation results. In the command file *Code\_Aster*, it is necessary, for each simulation, to check if there is failure or not. By way of an example, if the criterion of failure consists of a working stress, it is checked if the calculated maximum constraint exceeds this constraint.

Then one makes write, with ugly `CREA_TABLE`, results of simulations in a table of results. This table must contain two columns, `PARA_NOCI` and `DEFA`, which informs the value respectively  $a_i$  (value PGA or any other indicator) and the value  $x_i$  (1 or 0).

```
TAB1=CRÉA_TABLE (LISTE= (
    _F (PARA=' PARA_NOCI', LISTE_R = PGA, ),
    _F (PARA=' DEFA', LISTE_I =XI, ),
), );
```

**Notice** : If a column is informed `REQUEST` instead of `DEFA`, then this column must contain the values of the seismic request (variable of interest describing the failure or the damage, for example a maximum constraint or displacement).

```
TAB2=CRÉA_TABLE (LISTE= (
    _F (PARA=' PARA_NOCI', LISTE_R = PGA, ),
    _F (PARA=' DEMANDE', LISTE_I =DEMANDE, ),
), );
```

The keyword `THRESHOLD` must then be well informed in `POST_DYNA_ALEA` for to determine the continuation of  $x_i$ .

The operator `POST_DYNA_ALEA [U4.84.04]` allows to estimate the parameters of the curve of brittleness lognormale starting from the table `TAB1` via the keyword `BRITTLNESS`:

```
TAB_POST=POST_DYNA_ALEA (FRAGILITE= (_F (TABL_RESU=TAB1,
    AM_INI =0.3,
    BETA_INI=0.1,
```

```
METHODE= 'EMV',
FRACTILE = (0.0, 0.05, 0.5, 0.95, 1.0),
NB_TIRAGE =Ns,
),),
TITLE = 'curve 1',
INFO=2,);
```

One gives initial values of the parameters  $A_m$  and  $\beta$  to estimate (not starting for the algorithm of optimization) via `AM_INI` and `BETA_INI`. If the keyword is informed `FRACTILE`, one also determines the fractiles (confidence intervals) of the curve of brittleness by a method of D-sampling (known as method of “bootstrap”). D-sampling consists in drawing from new samples starting from the values of the original sample. These pullings are carried out with handing-over, to see [bib1] for more details. Then, one determines the parameters of the curve of brittleness for each sample “bootstrap”, which gives a sample of curves of brittleness. One then determines the fractiles for the sample of curves of brittleness obtained. In general, one as many draws samples “bootstrap” than one has values in the original sample. It is nevertheless possible to work with a lower number of pullings while informing `NB_TIRAGE` (by default the number of pullings corresponds in keeping with the original sample, here  $N_s$ ). In the example above, one thus determines the curved envelopes (fractiles 1.0 and 0.0) as well as the median (0.5) and fractiles with 0.05 and 0.95.

In the graph below, one gives an example of curve of brittleness for  $A_m=0.68$  and  $\beta=0.25$  :

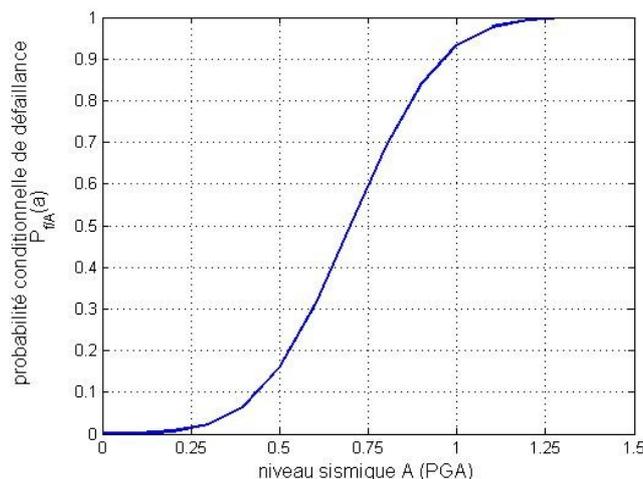


Figure 3.2.2-a : Conditional probability of failure according to the PGA

### 3.2.3 Evaluation by linear regression

The regression is a very widespread method to determine the parameters of a curve of lognormal brittleness. It is practised especially in the United States to evaluate the brittleness of the civil structures like the roads, the ouvfume of art and public buildings.

In this approach, the variable of interest is connected  $Y$  (the drift transistor for example), with the value of the seismic indicator  $a$  (for example the PGA) via a relation of the type:

$$Y = b a^c \eta$$

In the expression above,  $\eta$  is a lognormal random variable which one can define starting from the reduced centered normal variable  $U$  :  $\eta = \exp(\beta U)$ , and variables  $b$  and  $c$  are the parameters of the model. The latter are obtained by a linear regression using the following expression:

$$\ln(Y) = \ln(b) + c \ln(a) + \epsilon$$

where  $\epsilon$  is a normal random variable (Gaussian) centered standard deviation  $\sigma$  such as  $\epsilon = \sigma U$ . With these notations, the probability of failure is written like a conditional probability, according to the seismic level  $a$  and for a threshold  $Y_s$ :

$$P_f(a) = P(Y > Y_s | a) = 1 - P(Y < Y_s | a).$$

By taking account of the lognormal expression of the variable  $Y$ , one obtains the analytical expression of the curve of brittleness:

$$P_f(a) = \Phi\left(\frac{\ln(b a^c / Y_s)}{\sigma}\right)$$

Using the straight regression line, one can express the median threshold  $Y_s$  starting from the median capacity like  $Y_s = b a_s^c \eta$ . If it is considered a threshold criticizes "best-estimate" that one compares to the value median in the approach uncertainties, one can note  $A_m = a_s$ . One leads thus to the expression of the curve of lognormal brittleness:

$$P_f(a) = \Phi\left(\frac{\ln(a / A_m)}{\beta}\right)$$

where  $\beta = \sigma / c$  with the standard deviation of the error of regression  $\sigma$ . In practice, the value of the median capacity can be deduced directly from the curve of regression and is expressed starting from the parameters of the latter like

$$A_m = \exp\left[\frac{\ln(Y_s / b)}{c}\right]$$

It is important to stress that this model makes it possible to take account of the heteroscedastic behavior (the variance of  $Y$  depends on the parameters of the model and is not constant).

The operator `POST_DYNA_ALEA [U4.84.04]` allows to estimate the parameters of the curve of brittleness lognormale by regression starting from the table `TAB2 §3.2.2` via the keyword `BRITTLENESS`:

```
TAB_POST=POST_DYNA_ALEA (FRAGILITE= ( _F (TABL_RESU=TAB2,  
                                         METHODE= 'REGRESSION',  
                                         SEUIL=0.05,  
                                         ), ),  
                           TITLE = 'curve 2',  
                           INFO=2, );
```

The experiment shows that the dispersion (variability) of  $Y$  increase with the seismic level. On the other hand, contrary to the method of maximum of probability, the standard deviation logarithmic curve does not depend on the threshold.

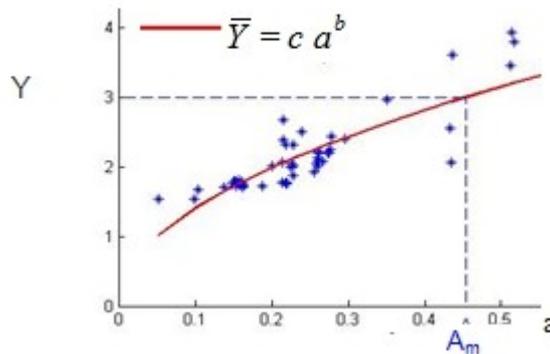


Figure 3.2.3-a : Example of and curve group of dots of regression: variable Y according to the PGA.

## 4 Bibliography

[bib1] Saporta G., *Probabilities, analyzes data and statistics*. Technip editions, 2006.

[bib2] Zentner I., Numerical computation of fragility curves for NP equipment. *Nuclear Eng. Design* **240:6** (2010) 1614-1621.

[bib3] EPRI, Seismic Probabilistic Risk Assessment Implementation Guides, Final Carryforward 1002989.2003.

[bib4] Lefebvre Y., of Rocquigny E., Duffoy A., Delcoigne F., Sudret B., Cagnac A., *Methodological guide for the treatment of uncertainties*. Note EDF R & D HT-56-2007-01798, 2007.

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[bib6] Ellingwood B.R., Kinali K. Quantifying and communicating uncertainty in seismic risk assessment. *Structural Safety* **31(2)**, 2009.

### Code\_Aster documentation

[R4.05.05] Generation of seismic signals

[U1.03.02] Access methods to the Aster objects

[U2.08.07] parametric Calculations – Distribution of calculations

[U1.04.00] Interface of access to Code\_Aster: astk