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Titre : Adaptation de maillage en non-linéaire
Responsable : DELMAS Josselin

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Adaptation of grid into non-linear

Summary:

This document presents the methodology of realization of a non-linear study with adaptation of grid. This adaptation of the grid during the transient is obtained using the software LOBSTER called via the order MACR_ADAP_MAIL [U7.03.01].

An example illustrates the possibilities and the implementation of strategies of mending of meshes.

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1   Introduction

The adaptation of grid brings a help to the user to provide the most reliable possible calculations with respect to the errors of discretization induced by the method of resolution to the finite elements.

We will not detail in this document the principle of the adaptation and we return the reader to the document [U2.08.01]. We point out just some elements.

Schematically, an iteration of adaptation of grid is presented in the form of on the figure below. The software calculates the digital solution on the network number k, then from of deduced the values from the indicator of error on all the grid. Starting from the knowledge of the grid number k and of the indicating number k, software of adaptation of grid `LOBSTER`, creates the new grid number $k+1$.
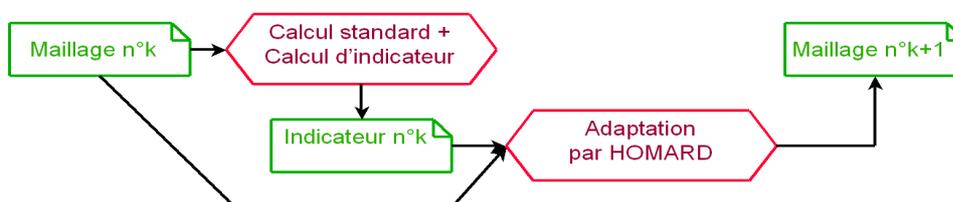

Illustration 1: Iteration of adaptation of grid

For a static calculation, that amounts seeking to improve the solution by a succession of calculations on different grids. For a transitory calculation in time, calculation is suspended at a given moment; the grid is adapted; calculation begins again on the new grid. It is with this case that we attack ourselves here. Indeed, in the case of the resolution of a non-linear transient, the restarting of calculation is not inevitably easy matter to achieve. The objective of this document is to help the user to implement this kind of modeling.

2   Adaptation into non-linear

In this part, one describes the standard problem which one seeks to solve as well as the essential difficulties that it presents.

2.1   Problem considered

The standard problem in which one is interested is a transient implying a model with a non-linear law of behavior. One wishes to adapt the grid support during the resolution according to the diagram of the figure 2. The principle is the following:
   •   one solves the transient on one or more step of time
   •   one adapts the grid on the basis of calculation of an indicator of error
   •   the transient is taken again

In fact, this diagram dissimulates a great part of the complexity of the problem. Let us take the example of the use of an elastoplastic law of behavior of von-Put with isotropic work hardening. With each step of time, one stores the state of the field of calculation Ω, given by the field of displacement, the stress field and the field of internal variables (u, σ, α). It is this triplet carried by the grid M who determines the balance completely of Ω. However in the case of the use of the adaptation of grid, the grid becomes an evolutionary element. If it is supposed that the grid is adapted to each step of time, the triplet also should be transferred (u, σ, α) from one grid to another. It is the phase of projection. Unfortunately, once projection carried out, nothing ensure that new triplet ensures the balance of field on the new grid. It is necessary for that to correct the new triplet. It is the phase of rebalancing.
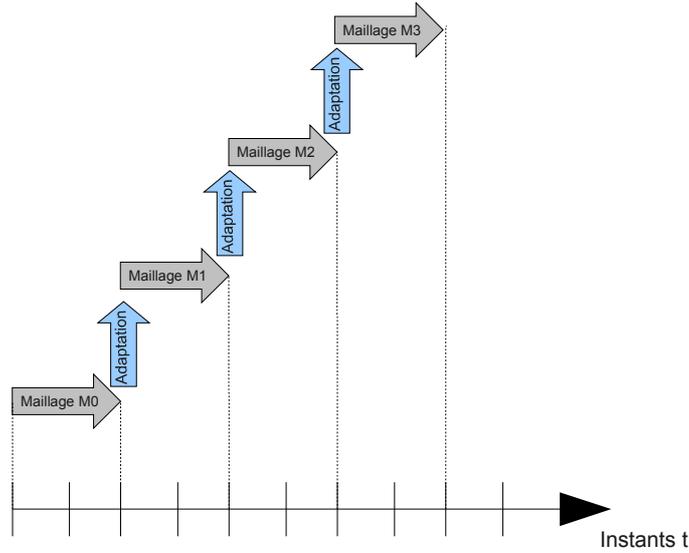
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Illustration 2: Walk in time with adaptation of grid

2.2   Phase of rebalancing

In the continuation, the following assumptions will be used:
  • use of an elastoplastic law of behavior of von-Put with isotropic work hardening
  • adaptation of the grid to each step of loading

The following notations are posed:
  • M_i indicate the grid with the step of loading i
  • $(u_i, \sigma_i, \alpha_i)$ indicate the triplet field of displacement, stress field and field of internal variables to the step of loading i
  • $(x)_{M_i}$ indicate the size x range by the grid M_i

One details the computing process now. One starts from a balanced state $(u_i, \sigma_i, \alpha_i)_{M_i}$ carried by the grid M_i with the step of loading i. One proceeds to the resolution of the problem to the step of loading $i+1$. The balanced state is obtained $(u_{i+1}, \sigma_{i+1}, \alpha_{i+1})_{M_i}$. On the basis of calculation of an indicator of error, one produces a new grid M_{i+1}.

To continue calculation, should be projected the balanced state $(u_{i+1}, \sigma_{i+1}, \alpha_{i+1})_{M_i}$ on the new grid M_{i+1}. We will detail this projection in the continuation. The state then is obtained $(u_{i+1}^{proj}, \sigma_{i+1}^{proj}, \alpha_{i+1}^{proj})_{M_{i+1}}$ but which is not inevitably balanced. Indeed, the phase of projection can induce disturbances on each of the 3 fields and remove their coherence. It is thus a question of rebalancing the mechanical state at the moment $i+1$ field Ω discretized by M_{i+1}.

With this intention, one refers to documentation [R5.03.01] "quasi-static non-linear Algorithm" and by not taking into account the variables of orders, one finds that the system that one solves with the first iteration of each step of time is the following:

$$\begin{cases} \mathbf{K}_i . \Delta \, \mathbf{u}_{i+1}^0 + \mathbf{B}^T . \Delta \, \lambda_{i+1}^0 = \mathbf{L}_{i+1}^{\text{méca}} - \mathbf{Q}_i^T . \sigma_i \\ \mathbf{B} . \Delta \, \mathbf{u}_{i+1}^0 = \mathbf{u}_{i+1}^d - \mathbf{B} . \mathbf{u}_i \end{cases} \qquad (1)$$

Second members $\mathbf{L}_{i+1}^{\text{méca}} - \mathbf{Q}_i^T . \sigma_i$ and $\mathbf{u}_{i+1}^d - \mathbf{B} . \mathbf{u}_i$ measure respectively:
  • the variation with balance between the loading and a mechanical state of stresses
  • the variation with the checking of the boundary conditions

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The resolution of the subjacent non-linear system brings to the cancellation of these variations and the production of a balanced mechanical state. It is exactly what one seeks to not do on the basis of the mechanical state of the previous moment $(\boldsymbol{u}_i, \boldsymbol{\sigma}_i, \boldsymbol{\alpha}_i)_{M_i}$ as in (1) but starting from the mechanical state project $(\boldsymbol{u}_{i+1}^{proj}, \boldsymbol{\sigma}_{i+1}^{proj}, \boldsymbol{\alpha}_{i+1}^{proj})_{M_{i+1}}$.

If one transposes it to the case which interests us, the system to be solved is thus the following:

$$\begin{cases} \mathbf{K}_i . \Delta \mathbf{u}_{i+1}^0 + \mathbf{B}^T . \Delta \lambda_{i+1}^0 = \mathbf{L}_{i+1}^{\text{méca}} - \mathbf{Q}_{i+1}^T . \boldsymbol{\sigma}_{i+1}^{proj} \\ \mathbf{B} . \Delta \mathbf{u}_{i+1}^0 = \mathbf{u}_{i+1}^d - \mathbf{B} . \mathbf{u}_{i+1}^{proj} \end{cases} \qquad (2)$$

From the point of view of modeling, that thus amounts starting again the resolution of the step of loading $i+1$ by using as initial state the mechanical state project $(\boldsymbol{u}_{i+1}^{proj}, \boldsymbol{\sigma}_{i+1}^{proj}, \boldsymbol{\alpha}_{i+1}^{proj})_{M_{i+1}}$.

The use of the mechanical state project $(\boldsymbol{u}_{i+1}^{proj}, \boldsymbol{\sigma}_{i+1}^{proj}, \boldsymbol{\alpha}_{i+1}^{proj})_{M_{i+1}}$ as initial state is capital. It initially makes it possible to start from a starting point nearest possible to the point of arrival and thus to increase the chances to see converging the method of Newton. But there is a reason much more important. Within the framework of use of laws of irreversible behavior, the internal variables, such as the cumulated plastic deformation, carry the mechanical history of each point of the structure. It is necessary to start from a state of internal variables most faithful possible at the real state of the structure for correctly translating its mechanical history.

That brings us to a new observation: it is essential to produce a mechanical state project $(\boldsymbol{u}_{i+1}^{proj}, \boldsymbol{\sigma}_{i+1}^{proj}, \boldsymbol{\alpha}_{i+1}^{proj})_{M_{i+1}}$ most faithful possible. It is what we will approach now.

2.3 Projection

The projection of the fields is done with the operator PROJ_CHAMP ; for any information on his complete syntax and his features, the reader will refer to [U4.72.05].

We are interested in projection of the mechanical state which is composed of 3 fields. We will examine the projection of each one of these fields.

2.3.1 Projection of the field of displacement

The field of displacement DEPL is a continuous field with the nodes. Its projection does not pose a particular problem. The method of collocation, which is used by default for this kind of field, is adapted perfectly.

2.3.2 Projection of the stress field

The stress field (and/or of efforts) SIEF_ELGA is natively calculated at the points of integration. So as to minimize the disturbance induced by projection, it is preferable to directly exploit the field at the point of integration. It is the method which the operator of projection will automatically choose for this field.

It is possible to make projection by using a field by elements with the nodes or a field with the nodes. That implies rather technical additional handling. This approach is disadvised.

2.3.3 Projection of the field of internal variables

The field of internal variables VARI_ELGA is natively calculated at the points of integration. So as to minimize the disturbance induced by projection, it is preferable to directly exploit the field at the point of integration. It is the method which the operator of projection will automatically choose for this field.

As for the stress field, it is possible to use a field by elements with the nodes or a field with the nodes but that poses more problem. Indeed, if one considers the case of plasticity of Von Mises, cumulated

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plastic deformation p must always be positive, $p \geqslant 0$. Let us consider the figure 3. It represents an element SEG2 with 2 nodes $N1$ and $N2$ and at 2 points of integration $PG1$ and $PG2$. This element carries a field at its points of integration of value $VG1$ and $VG2$, which is both positive. However, following extrapolation at the tops, one notes that the value $VN1$ at the top $N1$ remain quite positive while the value $VN2$ at the top $N2$ becomes negative. So as to be used as initial value, cumulated plastic deformation p must be positive or worthless; the value $VN2$ will thus be put at zero.

The use of a field of internal variables to the nodes is strongly disadvised because, besides the disturbances inherent in projection as in the preceding paragraph, it forces an additional treatment to make the field acceptable which disturbs it notably.
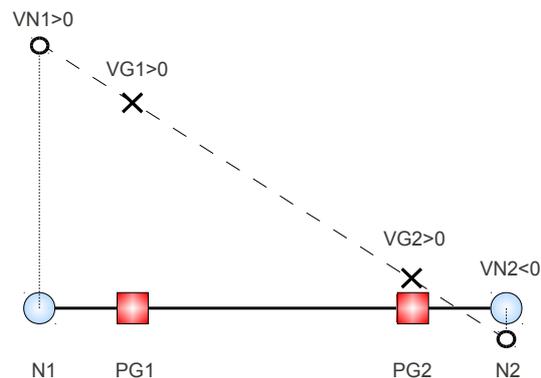
Illustration 3: Disturbance related to extrapolation

3   Implementation

3.1   Outline general

The outline general of the adaptation is presented on the figure 4. It includes the 4 phases described previously:
  •   resolution about one or more step of loading
  •   adaptation of the grid on the basis of calculation of an indicator of error
  •   projection of the fields at the end of the last step of time on the new grid
  •   balancing of the fields at the end of the last step of time on the new grid
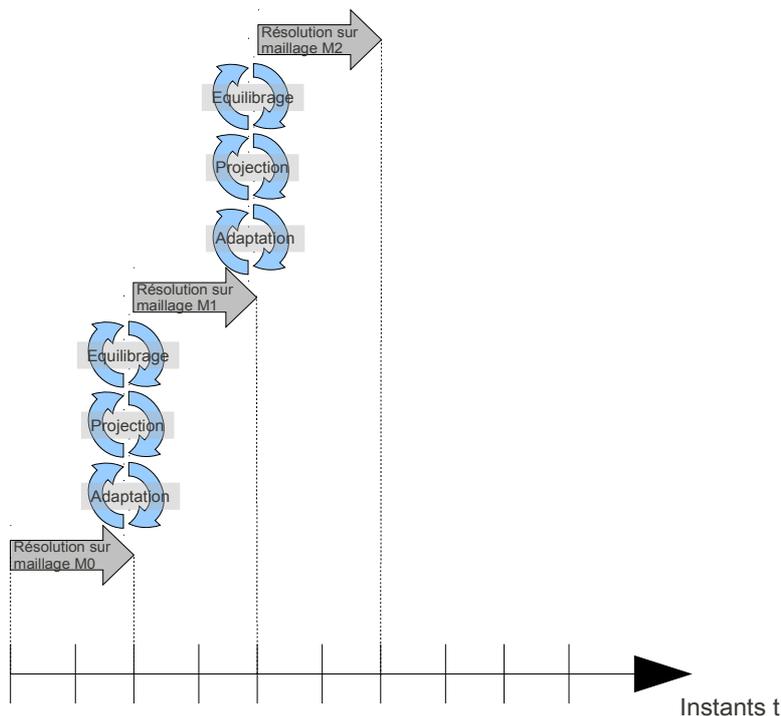
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Illustration 4: Complete process of adaptation

3.2 Example

An example of practical implementation is presented in the CAS-test *Code_Aster* SSNP158 [V6.03.158].

4 References

The documents [bib1] with [bib4] treat tool for adaptation of grid LOBSTER.
The documents [bib5] with [bib9] estimators of error treat.

[1]   G. NICOLAS & al. http://www.code_aster.org/outils/homard

[2]   G. NICOLAS, T. FOUQUET, "Software LOBSTER - Volume 1 – general Presentation", report EDF H-I23-2008-04107

[3]   "Macro-order MACR_ADAP_MAIL ", [U7.03.01]

[4]   "Macro-order MACR_INFO_MAIL ", [U7.03.02]

[5]   "Estimator of error of Zhu-Zienkiewicz in elasticity 2D", [R4.10.01]

[6]   "Estimator of error in residue", [R4.10.02]

[7]   "Indicating of space error in residue for transitory thermics", [R4.10.03]

[8]   "Detection of the singularities and calculation of a map of size", [R4.10.04]

[9]   J. DELMAS: "Estimators of error in quantities of interest", Report AMA 06-66