Operator POST_ERREUR

1 Goal

To calculate the relative error between a solution obtained by a calculation finite elements and an analytical solution of reference in terms of a given standard, with an aim of evaluating the convergence of the finite elements.

The currently available standards are:
- the standard in energy,
- the standard L^2 displacement,
- the standard L^2 contact pressure.

The turned over concept is a table.

Contents

2 Syntax

```
[*] = POST_ERREUR   (

      ♦  MODEL     =  Mo,                                    [model]
      ♦  GROUP_MA  =  grma   ,                               [l_grma]
      ♦  OPTION    =  /   'ENER_RELA',
                      /   'DEPL_RELA',
                      /   'LAGR_RELA',

      /  OPTION =            'ENER_RELA',
      #  ================================

         ♦  CHAM_GD      =  CH,                          [cham_elem]
         ♦  CHAM_MATER   =  chmater,                        [cham_mater]
         ♦  DEFORMATION  =  /   'SMALL',
         ◊  SIXX         = lcmp ,                           [l_formule]
         ◊  SIYY         = lcmp ,                           [l_formule]
         ◊  SIZZ         = lcmp ,                           [l_formule]
         ◊  SIXY         = lcmp ,                           [l_formule]
         ◊  SIXZ         = lcmp ,                           [l_formule]
         ◊  SIYZ         = lcmp ,                           [l_formule]

      /  OPTION =            'DEPL_RELA',
      #  ================================

         ♦  CHAM_GD   =  CH,                             [cham_no]
          ◊ DX        = lcmp ,                              [l_formule]
         ◊  DY        = lcmp ,                              [l_formule]
         ◊  DZ        = lcmp ,                              [l_formule]

      /  OPTION =            'LAGR_RELA',
      #  ================================

         ♦  CHAM_GD   =  CH,                             [cham_no]
          ◊ LAGS_C    = lcmp ,                              [l_formule]

         ),
```

3 Operands

3.1 Operands generals

3.1.1 Keyword OPTION

This keyword makes it possible to determine the type of standard to use:
- 'ENER_RELA' for the standard in energy (FEM and X-FEM).
- 'DEPL_RELA' for the standard L^2 displacement (FEM and X-FEM).
- 'LAGR_RELA' for the standard L^2 contact pressure (FEM only).

This keyword also determines the type of field corresponding to the keyword CHAM_GD :
- the stress field at the points of Gauss in the case OPTION='ENER_RELA'.
- The field of displacement to the nodes in the cases OPTION=' DEPL_RELA' and OPTION=' LAGR_RELA' .

3.1.2 Operand MODEL

Name of the model on which the option is calculated. It must be a question of the same model as that which was used to carry out mechanical calculation from which the field given by the keyword comes CHAM_GD .

3.1.3 Operand GROUP_MA

The operand GROUP_MA allows to specify the groups of meshs for which calculations of energy where standards L^2 will be carried out.

3.2 Operands for OPTION = 'ENER_RELA'

The option 'ENER_RELA' allows to estimate the difference between the stress field obtained by calculation finite elements $\boldsymbol{\sigma}_h$ and the stress field of reference $\boldsymbol{\sigma}$.
For each group of meshs of the list given by the operand GROUP_MA , the macro-order POST_ERREUR calculate:
- The elastic energy of the field difference $\boldsymbol{\sigma}_h - \boldsymbol{\sigma}$

$$\frac{1}{2}\int_{\Omega_i}(\boldsymbol{\sigma}_h-\boldsymbol{\sigma}):\boldsymbol{D}^{-1}:(\boldsymbol{\sigma}_h-\boldsymbol{\sigma})\,dV \quad ,$$

where Ω_i is the field obtained by concaténant all the meshs of the group of meshs considered and $\boldsymbol{D}$ is the tensor of Hooke.
- The elastic energy of the stress field of reference $\boldsymbol{\sigma}$

$$\frac{1}{2}\int_{\Omega_i}\boldsymbol{\sigma}:\boldsymbol{D}^{-1}:\boldsymbol{\sigma}\,dV \quad .$$

Finally, the relative error in terms of the standard in energy is obtained by:

$$e=\sqrt{\frac{\sum_i \frac{1}{2}\int_{\Omega_i}(\boldsymbol{\sigma}_h-\boldsymbol{\sigma}):\boldsymbol{D}^{-1}:(\boldsymbol{\sigma}_h-\boldsymbol{\sigma})\,dV}{\sum_i \frac{1}{2}\int_{\Omega_i}\boldsymbol{\sigma}:\boldsymbol{D}^{-1}:\boldsymbol{\sigma}\,dV}} \quad ,$$

where the sum is taken on the whole of the groups of meshs.

3.2.1 Operand CHAM_GD

The stress field σ_h extracted from a computation result finite elements.

3.2.2 Operand CHAM_MATER

Name of the material field to be used for calculations of energy. It is recommended that it is the same model as that which was used to carry out mechanical calculation from which the field given by the keyword comes CHAM_GD.

3.2.3 Operand DEFORMATION

This keyword makes it possible to define the assumptions of used for the calculation of the deformations *(cf.* [U4.51.11], §4.5). The only authorized value is 'SMALL', which corresponds to small displacements and small deformations.

3.2.4 Operands SIXX, SIYY, SIZZ, SIXY, SIXZ and SIYZ

These keywords make it possible to define the components of stress field of reference σ in the shape of objects formula. These operands are optional because all the components which are not specified are put at zero.

The value of each keyword is a list of formulas to be put in correspondence with the list of groups of meshs specified by the operand GROUP_MA .

A source of frequent error is not to inform SIZZ for a problem plan, by forgetting that, in the case general (*i.e.* Poisson's ratio not no one), the component σ_{zz} tensor of the constraints σ is worthless only in the case of the plane constraints (C_PLAN).

3.2.5 Example

Analysis the error in terms of the standard in energy for an opening of crack in pure mode I, for a problem plan. It is to note that the component σ_{zz} is worthless for this problem and is thus not specified.

```
# retrieval of the stress field of the structure of data result
Scal=CRÉA_CHAMP (OPERATION=' EXTR',
                 TYPE_CHAM=' ELGA_SIEF_R',
                 RESULTAT=UTOT,
                 NOM_CHAM=' SIEF_ELGA',
                 NUME_ORDRE=1)

# analysis the error in terms of the standard in energy
tabNRJ=POST_ERREUR (OPTION=' ENER_RELA',
                    CHAM_GD=Scal,
                    MODELE=MODELK,
                    DEFORMATION=' PETIT',
                    CHAM_MATER=CHMA,
                    GROUP_MA=' SURF',
                    SIXX=SXX,
                    SIYY=SYY,
                    SIXY=SXY,
                    )
)
```

3.2.6 Produced table

The produced table contains, for each mesh of the group of mesh, the energy of the field difference $\sigma_h - \sigma$ and the energy of the field reference σ. It contains also the sum on all the groups of meshs of the energy of the field difference $\sigma_h - \sigma$, the sum on all the groups of meshs of the energy of the field reference σ and the relative error in terms of the standard in energy.

GROUP_MA	DIFFERENCE	REFERENCE	RELATIVE ERROR
SURFING	1.53608E-09	3.50518E-06	-
TOTAL	1.53608E-09	3.50518E-06	2.09340E-02

3.3 Operands for OPTION = 'DEPL_RELA'

The option 'DEPL_RELA' allows to estimate the difference between the field of displacement obtained by calculation finite elements u_h and the field of displacement of reference u.
For each group of meshs of the list given by the operand GROUP_MA, the macro-order POST_ERREUR calculate:

- The standard L^2 field difference $u_h - u$

$$\sqrt{\int_{\Omega_i} \|u_h - u\|^2 dV} \quad ,$$

where Ω_i is the field obtained by concaténant all the meshs of the group of meshs considered.

- The standard L^2 field of displacement of reference u

$$\sqrt{\int_{\Omega_i} \|u\|^2 dV} \quad .$$

Finally, the relative error in terms of the standard L^2 displacement is obtained by:

$$e = \sqrt{\frac{\sum_i \int_{\Omega_i} \|u_h - u\|^2 dV}{\sum_i \int_{\Omega_i} \|u\|^2 dV}} \quad ,$$

where the sum is taken on the whole of the groups of meshs.

3.3.1 Operand CHAM_GD

The field of displacement u_h extracted from a computation result finite elements.

3.3.2 Operands DX, DY and DZ

These keywords make it possible to define the components of field of displacement of reference u in the shape of objects formula. These operands are optional because all the components which are not specified are put at zero.

The value of each keyword is a list of formulas to be put in correspondence with the list of groups of meshs specified by the operand GROUP_MA.

3.3.3 Example

Error analysis in terms of the standard L^2 displacement for an opening of crack in pure mode I, for a problem plan.

```
# retrieval of the field of displacements of the structure of data result
Ucal=CRÉA_CHAMP (OPERATION=' EXTR',
                 TYPE_CHAM=' NOEU_DEPL_R',
                 RESULTAT=UTOT,
                 NOM_CHAM=' DEPL',
                 NUME_ORDRE=1)

# analysis the error in terms of the L2 standard of displacement
tabL2=POST_ERREUR (OPTION=' DEPL_RELA',
                   CHAM_GD=Ucal,
                   MODELE=MODELK,
                   GROUP_MA=' SURF',
                   DX=U1,
                   DY=U2)
```

3.3.4 Produced table

The produced table contains, for each mesh of the group of mesh, the standard L^2 field difference $\boldsymbol{u}_h - \boldsymbol{u}$ and normalizes it L^2 field reference $\boldsymbol{u}$. It contains also the sum on all the groups of meshs of the standard L^2 field difference $\boldsymbol{u}_h - \boldsymbol{u}$, the sum on all the groups of meshs of has standard L^2 field reference $\boldsymbol{u}$ and the relative error in terms of the standard L^2 displacement.

GROUP_MA	DIFFERENCE	REFERENCE	RELATIVE ERROR
SURFING	1.14688E-09	7.60569E-06	-
TOTAL	1.14688E-09	7.60569E-06	1.50793E-04

3.4 Operands for OPTION = 'LAGR_RELA'

The option 'DEPL_RELA' allows to estimate the difference between the contact pressure obtained by calculation finite elements λ_h and the field of pressure of reference λ.
For each group of meshs of the list given by the operand GROUP_MA, the macro-order POST_ERREUR calculate:

- The standard L^2 field difference $\lambda_h - \lambda$

$$\sqrt{\int_{\Gamma_i} (\lambda_h - \lambda)^2 dS} \quad ,$$

where Γ_i is the field obtained by concaténant all the meshs of the group of meshs considered.

- The standard L^2 field of displacement of reference λ

$$\sqrt{\int_{\Gamma_i} \lambda^2 dS} \quad .$$

Finally, the relative error in terms of the standard L^2 contact pressure is obtained by:

$$e = \sqrt{\frac{\sum_i \int_{\Gamma_i} (\lambda_h - \lambda)^2 dS}{\sum_i \int_{\Gamma_i} \lambda^2 dS}} \quad ,$$

where the sum is taken on the whole of the groups of meshs.

3.4.1 Operand CHAM_GD

The field of contact pressure λ_h extracted from a computation result finite elements.

3.4.2 Operand LAGS_C

This keyword makes it possible to define the contact pressure of reference λ in the shape of an object formula.

The value of the keyword is a list of formulas to be put in correspondence with the list of groups of meshs specified by the operand GROUP_MA .

3.4.3 Example

Error analysis in terms of the standard L^2 pressure in the case of the inclusion of two crowns.

```
# definition of the analytical contact pressure
PRES=FORMULE (NOM_PARA= ('X', 'Y'), VALE='-pres_cont*EXP (0.1) ')

# retrieval of the field of displacements of the structure of data result
Ucal=CRÉA_CHAMP (OPERATION=' EXTR',
                 TYPE_CHAM=' NOEU_DEPL_R',
                 RESULTAT=RESU1,
                 NOM_CHAM=' DEPL',
                 NUME_ORDRE=1)

# analysis the error in terms of the L2 standard of the contact pressure
tabL2=POST_ERREUR (OPTION=' LAGR_RELA',
                   CHAM_GD=Ucal,
                   MODELE=MO,
                   GROUP_MA=' S 2R 2 ',
                   LAGS_C=PRES)
```

3.4.4 Produced table

The produced table contains, for each mesh of the group of mesh, the standard L^2 field difference $\lambda_h-\lambda$ and normalizes it L^2 field reference λ . It contains also the sum on all the groups of meshs of the standard L^2 field difference $\lambda_h-\lambda$, the sum on all the groups of meshs of has standard L^2 field reference λ and the relative error in terms of the standard L^2 contact pressure.

GROUP_MA	DIFFERENCE	REFERENCE	RELATIVE ERROR
S2R2	1.24905E+03	1.79688E+05	-
TOTAL	1.24905E+03	1.79688E+05	6.95124E-03