

Operator CALC_G_XFEM

1 Goal

This operator calculates the following sizes of breaking process, in 2D and 3D for cracks nonwith a grid (method X-FEM):

- the rate of refund of energy by the method theta in the case of a linear thermoelastic problem or not linear [R7.02.01] and [R7.02.03], in statics or dynamics [R7.02.02], and elastoplasticity [R7.02.07];
- stress intensity factors $K1$, $K2$ and $K3$ by the method of singular displacements in the case of a linear thermoelastic problem [R7.02.05];
- the maximization of G and of KI under constraints terminals.

Before a first use, it is advised to refer to the reference documents and of corresponding advices of use, in particular the document [U2.05.01].

This operator generates a concept of the type `table_sdaster`.

2 Syntax

```
[table_sdaster] = CALC_G_XFEM

(
# Recovery of the result of mechanical calculation
  ◆ RESULT = resu, / [evol_elas]
                    / [evol_noli]
                    / [dyna_trans]
                    / [mode_meca]

# If RESULT of the evol_elas type, evol_noli or dyna_trans
  ◇ / TOUT_ORDRE = 'YES', [DEFECT]
    / NUME_ORDRE = l_ordre, [l_I]
    / LIST_ORDRE = read , [listis]
    / INST = l_inst, [l_R]
    / LIST_INST = l_reel, [listr8]

# If RESULT of the mode_meca type
  ◇ / TOUT_MODE = 'YES', [DEFECT]
    / NUME_MODE = l_ordre, [l_I]
    / LIST_MODE = read , [listis]
    / FREQ = l_inst, [l_R]
    / LIST_FREQ = l_reel, [listr8]

  ◇ CRITERION = / 'RELATIVE', [DEFECT]
                ◇ PRECISION = / prec, [R]
                    / 1.E-6 , [DEFECT]
                / 'ABSOLUTE',
                ◆ PRECISION = prec, [R]

# Creation of the field theta

  ◆ THETA =_F ( ◆ CRACK = ffx, [fiss_xfem]
                ◇ / R_INF = R, [R]
                  / R_INF_FO = rz, [function]
                  / R_SUP = R, [R]
                  / R_SUP_FO = rz, [function]
                ◇ NUME_FOND = / 1, [DEFECT]
                    / N, [I]
                ◇ NB_POINT_FOND = N, [I]

                ),

# Recovery of the field theta

  ◇ CHAM_THETA = /CO ('cham_theta') [evol_noli]

# Loading

  ◇ EXCIT =_F ( ◆ LOAD = load , [char_meca]
                [char_cine_meca]
                ◇ FONC_MULT = fmult, [function]
                [formula]
                ),
```

```
# Behavior
  ◇ / BEHAVIOR = _F (
    ◆ RELATION = / 'ELAS',
                  / 'ELAS_VMIS_LINE',
                  / 'ELAS_VMIS_TRAC',
                  / 'ELAS_VMIS_PUIS',
    ◇ DEFORMATION = / 'SMALL', [DEFECT]
                  / 'GROT_GDEP',
                  / 'PETIT_REAC',
    ◇ / ALL = 'YES', [DEFECT]
      / GROUP_MA = lgrma, [l_gr_maille]
    ),
# if relation=' ELAS'
  ◇ ETAT_INIT = _F (SIGM = siefelga, [cham_elem, cham_no]
    ),
# Method of discretization of theta in bottom of crack (local 3D)
  ◇ SMOOTHING = _F (◇ LISSAGE_THETA= / 'LEGENBRE' [DEFECT]
                  / 'LAGRANGE'
    ◇ LISSAGE_G = / 'LEGENBRE' [DEFECT]
                  / 'LAGRANGE'
                  / 'LAGRANGE_NO_NO'
    # If LISSAGE_THETA = 'LEGENBRE' or LISSAGE_G = 'LEGENBRE':
    ◇ DEGREE = / 5 [DEFECT]
              / 0,1,2,3,4,6,7,
    ),
# Option requested
  ◆ OPTION = / 'CALC_G',
             / 'CALC_K_G',
  ◇ TITLE = title, [l_Kn]
# Impression of information
  ◇ INFORMATION = / 1, [DEFECT]
                 / 2,
  )
```

3 Operands

3.1 Operand RESULT

```
/ RESULT = resu
```

Name of a concept result of the type `evol_elas`, `evol_noli`, `dyna_trans` or `mode_meca`. This operand makes it possible to recover the field of displacement (and speed and of acceleration for a calculation in dynamics).

The material model and field, necessary to calculation, are also extracted from the structure of data result. The possible options of calculation for each type of modeling are recalled in the table below.

	Calculation of G	Calculation of K
D_PLAN/C_PLAN	CALC_G	CALC_K_G
AXIS	CALC_G	CALC_K_G
3D	CALC_G	CALC_K_G

Table 3.1: Availability, by modeling, of the options of calculation.

Remarks on the properties materials:

Characteristics of material, recovered in the structure of data `resu`, are the following ones:

- Young modulus E ,
- Poisson's ratio `NAKED`,
- thermal dilation coefficient `ALPHA` (for a thermomechanical problem),
- elastic limit `SY` (for a nonlinear elastic problem),
- slope of the traction diagram `D_SIGM_EPSI` (for a nonlinear elastic problem with linear isotropic work hardening).

For the calculation of restitution of energy, these characteristics can depend on the geometry (option '`CALC_G`') and of the temperature (option '`CALC_G`'). They must be independent of the temperature for the calculation of the stress intensity factors.

Characteristics `SY` and `D_SIGM_EPSI` are treated only for one nonlinear elastic problem with work hardening of Von Mises. The calculation of the coefficients of intensity of constraints is treated only in linear elasticity.

Note:

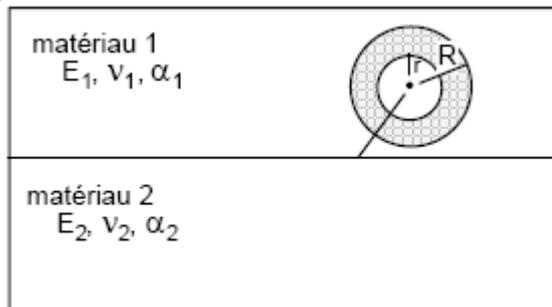
For the calculation of the stress intensity factors (option '`CALC_K_G`'), the characteristics must be defined on all materials, including on the elements of edge, because of method of calculating [R7.02.05]. To ensure itself so it is advised to do one `AFFE = _F` (`ALL = 'YES'`) in the order `AFFE_MATERIAU [U4.43.03]`, since it is necessary to using the rule of overload then.

Factors of intensity of the constraints obtained with the option `CALC_K_G` are calculated by evaluating the bilinear form of G with a purely mechanical singular solution (asymptotic solution of Westergaard). If a thermomechanical problem is solved, one then does not take counts the singularity due to the thermal field of it.

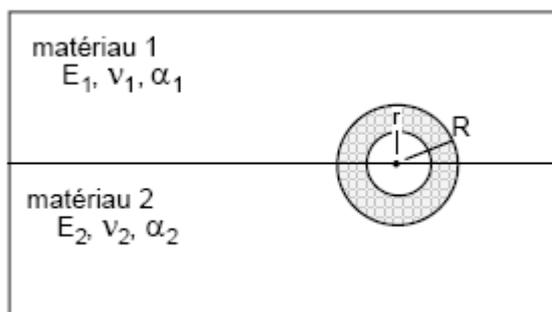
An indicator of the error due to this approximation can be obtained by evaluating the difference enters G and G_{IRWIN} . In practice, one evaluates in any point of the bottom of crack the quantity $\frac{|G - G_{irwin}|}{|G|}$, and one makes of it then the arithmetic mean. If this average exceeds 50 %, it is estimated whereas one leaves the perimeter of validity of the approach, and a message of alarm is transmitted.

Problem of bi--material:

1^{er} case : There is a bi--material but the point of crack is in only one material, cf Appears 3.1-a. If one is assured that the crown, definite enters the rays inferior R_INF and superior R_SUP , has like support of the elements of same material, calculation is possible whatever the selected option. If not only L'option 'CALC_G' is possible.

Figure 3.1-a: Bi--material: 1^{er} case

2nd case : There is a bi--material where the point of crack is with the interface, cf Appears 3.1-b. To date, only L'option of calculation of the rate of refund of energy (option 'CALC_G') is available. The calculation of coefficients of intensity of constraints is not possible in this case.

Figure 3.1-b: Bi--material: 2nd case

Calculation of factors of intensity of the constraints equivalents of a model with cohesive forces: For the studies of propagation of cracks in the presence of cohesive forces, one can have to calculate factors of intensity of the constraints equivalents, according to a quite specific procedure of surface integrals on the cohesive zone, detailed in the reference material [R7.02.19]. Some installations of syntax are then necessary:

- keywords R_INF / R_SUP do not have to be well informed, because it is not necessary to define a torus for the field θ ;
- the keyword BEHAVIOR does not have to be well informed, all information are extracted from the result;
- only smoothings LAGRANGE and LAGRANGE_NO_NO are available;
- only the option CALC_K_G is available.

A standard syntax is proposed at the end of this documentation.

3.2 Operands

TOUT_ORDRE/NUME_ORDRE/LIST_ORDRE/INST/LIST_INST/TOUT_MODE/NUME_MODE/LIST_MODE/FREQ/LIST_FREQ
/PRECISION/CRITERION

These operands are used with the operand RESULT.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (<http://www.gnu.org/copyleft/fdl.html>)

Operands TOUT_ORDRE, NUME_ORDRE, LIST_ORDRE, INST, LIST_INST are associated with the results of the type evol_elas, evol_noli or dyna_trans. See [U4.71.00].

Operands TOUT_MODE, NUME_MODE, LIST_MODE, FREQ, LIST_FREQ are associated with the results of the type mode_meca.

3.3 Keyword EXCIT and operands CHARGE/FONC_MULT

```
◇ EXCIT = _F ( ◇ LOAD = load  
              ◇ FONC_MULT = fmult)
```

The keyword EXCIT allows to recover a list of loadings load, resulting from the orders AFFE_CHAR_MECA or AFFE_CHAR_MECA_F [U4.44.01], and possible associated multiplying coefficients fmult.

The keyword EXCIT is optional and does not have to be well informed in the case general.

SI the keyword EXCIT is absent from order, the loading taken into account is that extracted from resu. If the loading is provided via EXCIT, then it is this loading which will be used in CALC_G_XFEM. If the loading provided in EXCIT is different from that present in resu (coherence of the name and amongst loads, of the couples load-function), an alarm is emitted and calculation continues with the loadings indicated by the user. Attention, this use is valid only when the result is created via the operator CREA_RESU. Indeed, CREA_RESU allows to define a result with or without application loads. In this last case, EXCIT give the possibility of defining the loading directly in CALC_G_XFEM.

The loadings currently supported by various modelings and being able to have a direction in breaking process are the following ones:

- Voluminal effort: ROTATION, FORCE_INTERNE, GRAVITY.
- Surface effort on the lips of the crack: FORCE_CONTOUR (2D), FORCE_FACE (3D), PRES_REP.
- Thermal dilation: the temperature is transmitted via AFFE_MATERIAU/AFFE_VARC

In the event of thermomechanical problem, the temperature is transmitted via the properties materials (AFFE_MATERIAU/AFFE_VARC/EVOL). Thermal dilation thus is automatically taken into account in calculation with CALC_G.

Note:

*The loadings not supported by an option are **been unaware of**. To date, the following loadings being able to have a direction in breaking process are not treated:*

- FORCE_NODALE
- FORCE_ARETE
- DDL_IMPO *on the lips of the crack*
- FACE_IMPO
- PRE_EPSI

It is important to note that the only loadings taken into account in a calculation of breaking process with the method θ are those supported by the elements inside the crown, where the field of vectors theta is nonnull (enters R_INF and R_SUP [R7.02.01 §3.3]). **Only types of load likely to influence the calculation of G are thus the voluminal loadings (gravity, rotation), a nonuniform field of temperature or efforts applied to the lips of the crack.**

Caution:

- *If several loadings of comparable nature (for example voluminal force) appear in calculation, they are combined between them for postprocessing. The weather is however not possible to date to be this combination so loadings of the type FORMULA are present : calculation finishes then in error.*
- *One observes also a rule of exclusion at the time of the simultaneous presence of a field of predeformations (via 'PRE_EPSI') and of an initial stress field.*

- It is not possible to date to associate a load defined from one `FORMULA` and a multiplying coefficient (`FONC_MULT`). In this case, calculation finishes in error.
- The loads kinematics (`AFFE_CHAR_CINE` and `AFFE_CHAR_CINE_F`), cannot be taken into account in calculation.
- For the option `CALC_K_G`, if a loading is imposed on the lips of the crack (`PRES_REP` or `FORCE_CONTOUR`), then it is necessary **obligatorily** to correctly direct the meshes of those (while using `ORIE_PEAU_2D` or `ORIE_PEAU_3D`) prior to calculation of `K` (case only fissures with a grid).
- If one does a calculation in great transformations (keyword `DEFORMATION = 'GROT_GDEP'` or `'PETIT_REAC'`) the loadings supported near the face of crack (more exactly in the torus of integration) must be died loads, typically an imposed force and not a pressure [R7.02.03 §2.4]; these loads must be declared like nonfollowing in `STAT_NON_LINE`. Nevertheless, of the following loads can be defined far from the crack, because they then do not intervene in the calculation of `G`.
- It is noted that the calculation of `CALC_G_XFEM` with modeling `AXIS` is not available for the results of calculations thermo - mechanical in great deformation and great rotation.

3.4 Keyword THETA

The field theta is calculated in `CALC_G_XFEM` starting from the keywords `R_INF/R_INF_FO`, `R_SUP/R_SUP_FO` and `CRACK`.

The various cases are described in the table below according to the option of calculation, modeling (2D or 3D) and the type of crack (crack with a grid or not).

	CALC_G	CALC_K_G
2D	<ul style="list-style-type: none"> ◇ R_INF, R_SUP... ◆ CRACK 	<ul style="list-style-type: none"> ◇ R_INF, R_SUP... ◆ CRACK
3D	<ul style="list-style-type: none"> ◇ R_INF, R_SUP... ◆ CRACK 	<ul style="list-style-type: none"> ◇ R_INF, R_SUP... ◆ CRACK

The Councils on the choice of the crowns (in `CALC_G_XFEM`) :

- To avoid using a field theta defined with a lower ray `R_INF` no one. The fields of displacements are singular in bottom of crack and introduce results vague in post - treatment of breaking process.
- It is advised to use the order successively `CALC_G_XFEM` with at least three fields theta of different crowns to make sure of the stability of the results. In the event of important variation (higher than 5-10%) it is necessary to wonder about the good taking into account of all modeling.
- For the option `CALC_K_G` into 2D-axisymmetric, the ruffleyon of the crowns must be small in front of the ray of the bottom of crack to have the best possible precision. It is interdict to have crowns of ray larger than the ray of the bottom of crack.

3.4.1 Operand CRACK

◆ `CRACK = fiss,`

ItT operand allows of informs the relative information to the crack defined by the order `DEFI_FISS_XFEM` [U4.82.08].

This keyword is obligatory.

In the few cases where one carries out a study of propagation with cohesive elements, this crack is of type COHESIVE (keyword TYPE_DISCONTINUITE in DEFI_FISS_XFEM), and the operator carries out then automatically a calculation of factors of intensity of the constraints equivalents by a specific method surface integrals on the cohesive zone, detailed in [R7.02.19], §6.3.

3.4.2 Operands R_INF, R_INF_FO, R_SUP, R_SUP_FO

These operands make it possible to calculate the field theta when this one was not beforehand given. They correspond respectively the rays inferior and to superior of the crowns (scalar or function, in 3D, of the curvilinear X-coordinate).

The two rays can be introduced either by constant actual values which are arguments of the simple keywords R_INF and R_SUP ; maybe by functions of the curvilinear X-coordinate on the bottom of crack directed, which are arguments of the simple keywords R_INF_FO and R_SUP_FO.

Some advices are given below.

In 3D, when the rays are not function of the curvilinear X-coordinate, operands R_INF and R_SUP are optional. If they are not indicated, they are automatically calculated starting from the maximum H sizes of meshes connected to the nodes of the bottom of crack. These sizes of meshes in each node of the bottom are calculated in the order DEFI_FISS_XFEM and are present in the concept or fiss_xfem [D4.10.01]. It was selected to pose R_SUP = 4:00 and R_INF = 2:00. If one chooses the value automatically calculated for R_SUP and R_INF, it is advisable however to make sure that these values (displayed in the file .mess) are coherent with dimensions of the structure.

In the case of a defect initially opened and whose bottom is not plan, it is at present not possible to calculate the rate of refund of energy.

3.4.3 Operand NUME_FOND

◇ NUME_FOND = N,

This keyword is optional.

It can arrive, for certain structures, that the bottom of crack is discontinuous. In the case of a crack defined by DEFI_FISS_XFEM the bottom of crack is then cut out in several parts.

The operand NUME_FOND allows to indicate on which of these under-parts of the bottom of crack one wishes to carry out calculation. By default, calculation is done on the first bottom of crack.

3.4.4 Operand NB_POINT_FOND

◇ NB_POINT_FOND = nbnofo,

This keyword is optional.

Prear defect, calculation is done on all the points of the bottom of crack, i.e. all the points of intersection between the bottom of crack and the edges of the grid, for a crack nonwith a grid. Lbe points of the bottom of crack can be very irregularly spaced, which can lead to awkward oscillations on the parameters $G(s)$ or $K(s)$ calculated.

The operand NB_POINT_FOND allows to fix a priori the number of points of postprocessing, in order to improve the regularity of the results. *nbnofo* points are équi-distribute along the bottom of crack. Some advices are given in the §3.7.

This keyword is usable only for one smoothing of LAGRANGE-LAGRANGE.

3.5 Keyword BEHAVIOR

◇ BEHAVIOR = _F (

This keyword factor makes it possible to redefine the behavior of material. But the use of this keyword **does not have to be systematic** : indeed, by defect, the law of behavior used in CALC_G_XFEM is identical to that used for mechanical calculation (via MECA_STATIQUE or STAT_NON_LINE). The fact of informing the key word behavior creates an Alarm, but calculation continues, it is up to the user to check that the behaviors at the time of the mechanical resolution and the calculation of G are identical.

The calculation of the rate of refund of energy G direction has only in **elasticity** linear or not linear. Lastly, it onlyE variable of order (see [U4.43.03]: operator AFFE_MATERIAU, keyword AFFE_VARC) authorized for the calculation of the rate of refund of energy is the temperature 'TEMP'.

Note:

- *Nothing prohibits to affect a behavior different during calculation from displacements (for example elastoplastic) then to carry out this postprocessing with another relation (for example elastic non-linear). A checking of coherence is carried out on the behaviors used for calculation and postprocessing, and a message of alarm is transmitted if there is a difference; the user is responsible for the interpretation of the got results [R7.02.03].*
- *For example, if the loading is perfectly radial monotonous, calculations in nonlinear elasticity and elastoplasticity lead to the same results.*

For more precise details, to refer to [U2.05.01].

3.5.1 Operand RELATION for the elastic laws of behavior

• RELATION =

Possible elastic relations of behavior ('ELAS', 'ELAS_VMIS_LINE', 'ELAS_VMIS_TRAC', 'ELAS_VMIS_PUIS') are detailed in [U4.51.11].

/ 'ELAS'

Relation of elastic behavior linear i.e. the relation between the strains and the stresses considered is linear [R7.02.01 §1.1].

It is possible to define a state not no one initial constraints (see keyword ETAT_INIT), which results in regarding the elastic behavior as incremental.

/ 'ELAS_VMIS_LINE'

Relation of nonlinear elastic behavior, Von Mises with linear isotropic work hardening. The data materials necessary of the field material are provided in the operator DEFI_MATERIAU (cf [R7.02.03 §1.1] and [R5.03.20]).

/ 'ELAS_VMIS_TRAC'

Relation of nonlinear elastic behavior, Von Mises with nonlinear isotropic work hardening. The data materials necessary of the field material are provided in the operator DEFI_MATERIAU (cf. [R7.02.03 §1.1] and [R5.03.20]).

/ 'ELAS_VMIS_PUIS'

Relation of nonlinear elastic behavior, Von Mises with nonlinear isotropic work hardening defined by a law power. The data materials necessary of the field material are provided in the operator DEFI_MATERIAU (cf [R7.02.03 §1.1] and [R5.03.20]).

3.5.2 Operand ETAT_INIT

◇ ETAT_INIT=_F (SIGM = siefelga)

In the case of a relation of elastic behavior incremental, it is possible to define a state of initial stresses not no one.

The use of this keyword requires as a preliminary the definition of an initial state in the operator `STAT_NON_LINE` at the time of the resolution of calculation post-to be treated by `CALC_G_XFEM`.

`STAT_NON_LINE` this definition in two manners allows (see u4.51.03):

- with an initial state of standard result, well informed under the keyword `EVOL_NOLI` keyword factor `ETAT_INIT`
- with an initial state of fields type, well informed under the keywords `SIGM/DEPL/VARI/STRX/COHE` keyword factor `ETAT_INIT`

With this day, the post treatment with `CALC_G_XFEM` calculations using the keywords `EVOL_NOLI/DEPL/VARI/STRX/COHE` is not possible. Only the post treatment with `CALC_G_XFEM` calculations using the keywords `SIGM` is possible.

The initial stress field provided can be of type `SIEF_ELGA` only (possibility of creating them from `CREA_CHAMP` in particular).

In all the cases, this initial stress field must be **auto--balanced**, in absence of crack, with the only boundary conditions. The user must check that its initial stress field is licit by applying it in the keyword `ETAT_INIT` of the operator `STAT_NON_LINE`, with a linear elastic behavior (`RELATION = 'ELAS'`), with the only boundary conditions; the mechanical result must be the same stress field without additional deformations (see Figure 3.1).

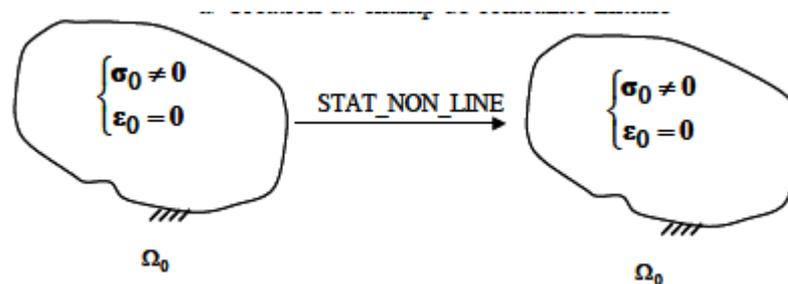


Figure 3.1 : Checking of the validity of the initial stress field.

Taking into account the difficulty of validation of the established formulation, it is not currently licit to cumulate a predeformation (*via* the keyword `PRE_EPSI` of the operator `AFFE_CHAR_MECA`) and an initial constraint.

3.5.3 Operand DEFORMATION

This keyword makes it possible to define the assumptions used for the calculation of the deformations. For more precise details on the formalisms of deformations, to see the paragraph `DEFORMATION` of [U4.51, 11].

To start, sizes calculated by `CALC_G_XFEM` are only defined in small deformations. It is thus not possible to use for `CALC_G_XFEM` another formalism that `SMALL`, `GROT_GDEP` or `PETIT_REAC`. If such is the case, calculation stops in error.

Then, one needs in theory a coherence between the formalisms of deformations used for mechanical calculation itself and the postprocessing of which it is question here. That would thus mean that mechanical calculations themselves must be realized with the formalisms `SMALL`, `GROT_GDEP` or `PETIT_REAC` only. However, we leave the possibility of carrying out postprocessing in small deformations (`SMALL`) starting from the result of a mechanical calculation carried out with another formalism (for example `GDEF_LOG`). In this case, an alarm is emitted, and charges with the user with deciding if the provided result has a direction or not. To carry out a more reliable calculation of the rate of refund of energy in great deformations, one advises to use the method of equivalence in opening presented briefly in U2.05.01.

- ◇ `DEFORMATION =`
/ `'SMALL'` : the deformations used in the relation of behavior are the linearized relations. That means that one remains in Assumption Small Disturbances: small displacements, small rotations and small deformations. In this case, calculations of the sizes licit and are validated. This option is the only possible one for the cracks nonwith a grid.

/ 'GROT_GDEP' : Allows to treat great rotations and great displacements, but while remaining in small deformations

/ 'PETIT_REAC' : available only in incremental behavior, it is an approximation of the great deformations for which the increments of deformations are calculated in the current geometry (reactualized). It is valid only for small increments and weak rotations [U4.51.11].

3.5.4 Operands ALL/GROUP_MA

◇ / ALL = 'YES' ,
/ GROUP_MA = lgrma ,

Specify the meshes or the nodes on which the relation of behavior is used.

3.5.5 Relation of behavior available for each option

BEHAVIOR	'ELAS'	'CALC_G'	'CALC_K_G'
		'SMALL'	'SMALL'
		'GROT_GDEP'	
	'ELAS_VMIS_LINE'	'SMALL'	not disp.
	'ELAS_VMIS_TRAC'	'GROT_GDEP'	
	'ELAS_VMIS_PUIS'		

Table 3.6.4-a: Availability, by option, of the relations of behavior.

3.6 Operand OPTION

◆ OPTION = / 'CALC_G' ,
/ 'CALC_K_G' ,

3.6.1 OPTION = 'CALC_G' [R7.02.01] and [R7.02.03]

It allows the calculation of the rate of refund of energy G by the method theta in 2D or local 3D for a linear thermoelastic problem or not linear.

In 2D, for modeling AXIS, it is necessary to divide the result got by the ray into bottom of crack, cf §4.2.

3.6.2 OPTION = 'CALC_K_G' [R7.02.05]

This option calculates in 2D and 3D the rate of refund G and coefficients of intensity of constraints K_1 , K_2 and K_3 in linear thermoelasticity planes by the method of the singular fields (use of the bilinear form of G , [R7.02.05]).

Note:

- For this option, only the linear elastic designs without initial or elastic state linear with initial constraint are available.
- For this option in 2D, if INFORMATION is worth 2, one generates calculation and the impression (in the file MESSAGE) angle of propagation of the crack. This angle, calculated according to 3 criteria ($K1$ or G maximum, $K2$ minimal) according to the formulas of AMESTOY, BUI and DANG-VAN [R7.02.05 §2.5], is given with a margin of 10 degrees.

3.7 Keyword SMOOTHING

The scope of application of this keyword is limited to the case local 3D.

3.7.1 Operand LISSAGE_THETA

```
◇ LISSAGE_THETA = / 'LEGENBRE' [DEFECT]
                  / 'LAGRANGE'
```

The trace of the field theta on the bottom of crack can be discretized either according to the base of N first polynomials of Legendre ('LEGENBRE'), that is to say according to the functions of form associated with the discretization with the bottom with crack ('LAGRANGE') [R7.02.01].

LISSAGE_THETA = 'LEGENBRE' : $\theta(s)$ is discretized on a basis of polynomials of Legendre $\gamma_j(s)$ of degree j ($0 < j < Deg_{max}$) where Deg_{max} is the maximum degree given under the keyword DEGREE (between 0 and 7).

LISSAGE_THETA = 'LAGRANGE' : $\theta(s)$ is discretized on the functions of form of the node k bottom of crack: $\phi_k(s)$.

3.7.2 Operand LISSAGE_G

```
LISSAGE_G = / 'LEGENBRE' , [DEFECT]
             / 'LAGRANGE' ,
             / 'LAGRANGE_NO_NO'
```

$G(s)$ can be discretized either according to the polynomials of Legendre ('LEGENBRE'), that is to say according to the functions of form of the nodes of the bottom of crack ('LAGRANGE'). Method 'LAGRANGE_NO_NO' is resulting from the method LAGRANGE-LAGRANGE but it is simplified [R7.02.01].

If the smoothing of theta by polynomials of Legendre were retained with the preceding keyword, then the smoothing of G must also be of Legendre type.

Options available in Aster are summarized in the following table:

		Theta	
		Polynomials of LEGENDRE	Functions of form
$G(s)$	Polynomials of LEGENDRE	LISSAGE_THETA= 'LEGENBRE'	
		LISSAGE_G = 'LEGENBRE'	
	Functions of form		LISSAGE_THETA = 'LAGRANGE'
			LISSAGE_G = 'LAGRANGE'
			or 'LAGRANGE_NO_NO'

3.7.3 Operand DEGREE

```
◇ DEGREE = N
```

n is the maximum degree of the polynomials of Legendre used for the decomposition of the field θ in bottom of crack [§3.12] (when LISSAGE_THETA = 'LEGENBRE').

By default n is assigned to 5. The value of n must be ranging between 0 and 7.

If the discretizations are retained LISSAGE_THETA = 'LAGRANGE' and LISSAGE_G = 'LEGENBRE', one must have $n < NNO$, where NNO is the number of nodes in bottom of crack [R7.02.01 §2.3].

The Councils on smoothing:

- *it is difficult to give a preference to one or the other method of smoothing. In theory both give equivalent digital results. Nevertheless the smoothing of the type 'LAGRANGE' is little more expensive in time CPU than the smoothing of the type 'LEGENBRE' ;*
- *the smoothing of the type 'LEGENBRE' is sensitive to the maximum degree of the selected polynomials. The maximum degree must be defined according to the number of nodes in bottom of crack NO . If n is too large in comparison with NNO the results are poor [U2.05.01 §2.4];*
- *oscillations can appear with the smoothing of the type 'LAGRANGE', in particular if the grid comprises quadratic elements or if the crack is not with a grid. If the grid is radiant in bottom of crack (crack with a grid), it is then recommended to define crowns R_INF and R_SUP coinciding with the borders of the elements. A smoothing of the type 'LAGRANGE_NO_NO' allows to limit these oscillations;*
- *for the cracks with a grid and nonwith a grid, when a smoothing of the type is used 'LAGRANGE' it is recommended to use the operand NB_POINT_FOND to guarantee an equal distribution of the points of calculations in bottom of crack. The choice of a report of about 5 between the total number of points in bottom of crack (to be sought in the information printed in the file message by the order DEFI_FISS_XFEM) and the number of points of calculation seems suitable to limit the oscillations;*
- *the use of at least two types of smoothing with several crowns of integration and the comparison of the results are **essential** in order to check the validity of the model.*

3.8 Operand CHAM_THETA

```
◇ CHAM_THETA = /CO ('cham_theta') [evol_noli]
```

Concept of the type `evol_noli` who contains the field `theta` calculated in the operator (see the reference material to know how is calculated of field). This concept contains one `cham_no` (fields with the nodes) in 2D and several in 3D (according to the options used for smoothing). In the case 3D, there is one `cham_no` by moment in Lstructure of data results of the type has `evol_noli`. This time **does not have physical direction**, it only makes it possible "to arrange" the different ones `cham_no` in the structure of data `evol_noli`.

3.9 Operand TITLE

```
◇ TITLE = title  
[U4.03.01].
```

3.10 Operand INFORMATION

```
◇ INFORMATION = /1, [DEFECT]  
/2,
```

Level of messages in the file 'MESSAGE'.

3.11 Produced table

The order `CALC_G_XFEM` generate a concept of the type `table`.

This table is defined as follows for the options `CALC_G` and `CALC_K_G` :

In all the cases	Depends on calculation carried out		In 2D and local 3D only			In local 3D		
NUME_FOND	NUME_CAS	NOM_CAS		COOR_X	COOR_Y	COOR_Z	NUM_PT	ABSC_CURV
	NUME_MODE							
	NUME_ORDR	INST						

Table 3.11-1: Table obtained with CALC_G (1)

In all the cases	CALC_K_G			CALC_K_G in local 3D
G	K1	K2	G_IRWIN	K3

Table 3.11-2: Table obtained with CALC_G (2)

G_{IRWIN} , is the rate of refund of energy obtained starting from the factors of intensity of the constraints K_1 and K_2 (and K_3) with the following formulas:

$$G_{IRWIN} = \frac{1}{E} (K_I^2 + K_{II}^2) \text{ in plane constraints}$$

$$G_{IRWIN} = \frac{(1-\nu^2)}{E} (K_I^2 + K_{II}^2) \text{ in deformations plane and axisymmetric}$$

$$G_{IRWIN} = \frac{(1-\nu^2)}{E} (K_I^2 + K_{II}^2) + \frac{K_{III}^2}{2\mu} \text{ in 3D}$$

with E Young modulus and ν Poisson's ratio and $\mu = \frac{E}{2(1+\nu)}$. The comparison enters G and

G_{IRWIN} allows to make sure of the coherence of the results: a too important variation must result in checking the parameters of calculation (refinement of the grid, choice of the crowns for theta, smoothing in 3D...).

The order IMPR_TABLE [U4.91.03] allows to print the results with the desired format.

4 Standardisation of the rate of refund G

4.1 2D plane constraints and plane deformations

In dimension 2 (plane constraints and plane deformations), the bottom of crack is tiny room to a point and the value $G(\theta)$ exit of the order CALC_G_XFEM is independent of the choice of the field θ :

$$G = G(\theta) \quad \forall \theta \in \Theta$$

4.2 Axisymetry

Into axisymmetric it is necessary to standardize the value $G(\theta)$ obtained with *Aster* for the option CALC_G :

$$G = \frac{1}{R} G(\theta)$$

where R is the distance from the bottom of crack to the axis of symmetry [R7.02.01 §2.4.4].

For the option CALC_K_G, values of G and of K provided in table result are directly the local values, it thus should not be standardized.

4.3 Symmetry of the model

If one models only half of the solid compared to the crack:

- that is to say to specify the keyword SYME = 'YES' in the orders concerned;
- that is to say not to forget to multiply by 2, values of the rate of refund of energy G or $G(s)$ and by 4 those of G_{Irwin} . Moreover values of the factors of intensity of the constraints corresponding to the mode of symmetry must also be multiplied by 2.

5 Examples

5.1 Example of use in 3D

Let us take the case of a crack nonwith a grid, and a calculation of factors of intensity of the constraints equivalents in the presence of cohesive forces.

The crack was defined by:

```
fiss=DEFI_FISS_XFEM (MODELE=MO,  
                    TYPE_DISCONTINUITE = 'COHESIVE',  
                    CHALLENGE_FISS=_F (FONC_LN = fonc1,  
                                        FONC_LT = fonc2,  
                                        GROUP_MA_BORD = group_ma,)),  
                    );
```

After mechanical calculation having given a result LMBO, postprocessing to obtain the factors of intensity of the constraints equivalents is done by:

```
table_k=CALC_G (OPTION=' CALC_K_G',  
               RESULT = LMBO,  
               INST = inst_fin,  
               THETA =_F (CRACK = fiss,  
                          NB_POINT_FOND = nb_pts,)),  
               SMOOTHING = F (LISSAGE_THETA = 'LAGRANGE',  
                              LISSAGE_G = 'LAGRANGE_NO_NO'),  
               )
```