

## Operator POST\_FATIGUE

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### 1 Goal

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To calculate, in a point, the damage of tiredness of a structure subjected to a history of loading.

Unlike `CALC_FATIGUE`, `POST_FATIGUE` do not operate on a field but on a “signal” extracted beforehand from a calculation or defines in addition.

The various methods available [R7.04.01] are:

<b>methods based on uniaxial tests: methods of Wöhler, Manson-Whetstone sheath and Taheri</b>
These methods have as a common point to determine a value of damage starting from the evolution during the characterizing time of a scalar component, for the calculation of the damage, the state of constraints or structural deformations. With this intention, it is necessary to extract, by a method of counting of cycles, the elementary cycles of loading undergone by the structure, to determine the elementary damage associated with each cycle and to determine the total damage by a rule of linear office plurality,
<b>method of Lemaître generalized</b>
This method makes it possible to calculate the damage (of Lemaître or Lemaître-Sermage) starting from the data of the tensor of the constraints and the cumulated plastic deformation,
<b>criteria of tiredness multiaxial</b>
These criteria apply to uniaxial or multiaxial loadings periodic or not-periodicals. They provide a value of criterion indicating if there is damage or not, and also values of the damage and number of cycle to the rupture.

The order produces a concept of the type `table`.

## 2 Syntax

```
tabl_post_fatig = POST_FATIGUE (
# if purely uniaxial loading (or regarded as uniaxial)
  ♦ / LOADING = 'UNIAXIAL' ,
    ♦ HISTORY = _F (
      ♦ / SIGM = histsigm / [function]
        / [formula]
      / EPSI = histepsi / [function]
        / [formula]
    ),
    ♦ COUNTING = / 'RAINFLOW' ,
      / 'RAINFLOW_MAX ' ,
      / 'RCCM' ,
      / 'NATURAL' ,
    ♦ DELTA_OSCI = / delta, [R]
      / 0. , [DEFECT]
    ♦ COEF_MULT = _F ( ♦ KT = kt ), [R]
    ♦ CORR_KE = 'RCCM' ,
    ♦ TOO_BAD = / 'WOHLER' ,
      / 'MANSON_COFFIN' ,
      / 'TAHERI_MANSON' ,
      / 'TAHERI_MIXTE' ,
    ♦ MATER = to subdue,
    ♦ CORR_SIGM_MOYE = / 'GOODMAN' ,
      / 'TO STACK' ,
    ♦ TAHERI_NAPPE = fnappe, / [tablecloth]
      / [formula]
    ♦ TAHERI_FONC = ffonc , / [function]
      / [formula]
    ♦ OFFICE PLURALITY = 'LINEAR' ,
```

```
# if periodic loading (for tiredness with great numbers of cycles and for periodic cycles)

♦ / LOADING = 'MULTIAXIAL' ,

♦ TYPE_CHARGE = / 'PERIODIC' ,
                / 'NON_PERIODIQUE' ,

♦ HISTORY = _F (
    ◇ SIGM_XX = fxx , / [function]
                    / [formula]
    ◇ SIGM_YY = fyy , / [function]
                    / [formula]
    ◇ SIGM_ZZ = fzz , / [function]
                    / [formula]
    ◇ SIGM_XY = fxy , / [function]
                    / [formula]
    ◇ SIGM_XZ = fxz , / [function]
                    / [formula]
    ◇ SIGM_YZ = fyz , / [function]
                    / [formula]

    ◇ EPS_XX = fxx , / [function]
                    / [formula]
    ◇ EPS_YY = fyy , / [function]
                    / [formula]
    ◇ EPS_ZZ = fzz , / [function]
                    / [formula]
    ◇ EPS_XY = fxy , / [function]
                    / [formula]
    ◇ EPS_XZ = fxz , / [function]
                    / [formula]
    ◇ EPS_YZ = fyz , / [function]
                    / [formula]

    ◇ EPSP_XX = fxx , / [function]
                    / [formula]
    ◇ EPSP_YY = fyy , / [function]
                    / [formula]
    ◇ EPSP_ZZ = fzz , / [function]
                    / [formula]
    ◇ EPSP_XY = fxy , / [function]
                    / [formula]
    ◇ EPSP_XZ = fxz , / [function]
                    / [formula]
    ◇ EPSP_YZ = fyz , / [function]
                    / [formula]
)

    ◇ MATER = to subdue , [to
subdue]

    ◇ TOO_BAD = / 'WOHLER' ,
                / 'MANSON_C' ,
                / 'FORMES_VIE' ,

# If DAMAGE = 'FORMES_VIE'
♦ FORMULE_VIE = for_vie, / [formula]
                    / [function]
```

```
# Finsi

◇ COEF_CORR = / corr , [R]
◇ COEF_PREECROU = / co_pre , [R]
                  / 1. , [DEFECT]

# If TYPE_CHARGE = 'PERIODIC'
  ◆ CRITERION = / 'MATAKE_MODI_AC' ,
                / 'DANG_VAN_MODI_AC' ,
                / 'FORMULE_CRITERE' ,
                / 'CROSSLAND' ,
                / 'PAPADOPOULOS' ,

  ◇ METHOD = / 'CERCLE_EXACT' ,

# If CRITERION = 'FORMULE_CRITERE'
  ◆ FORMULE_GRDEQ = for_grd, / [formula]
  ◇ FORMULE_CRITIQUE = for_grd, / [formula]

# Finsi
# Finsi

# If TYPE_CHARGE = 'NON_PERIODIQUE'
  ◆ CRITERION = / 'MATAKE_MODI_AV' ,
                / 'DANG_VAN_MODI_AV' ,
                / 'FATESOCI_MODI_AV' ,
                / 'FORMULE_CRITERE' ,

  ◆ PROJECTION = / 'UN_AXE' ,
                 / 'DEUX_AXES' ,

  ◇ DELTA_OSCI = / delta, [R]
                 / 0. , [DEFECT]

# If CRITERION = 'FORMULE_CRITERE'
  ◆ FORMULE_GRDEQ = for_grd, / [formula]
# Finsi
# Finsi
# Finsi
```

```
# if unspecified loading (damage of Lemaitre or Lemaitre-Sermage)
♦ / LOADING = 'UNSPECIFIED' ,
    ♦ HISTORY = _F (
        ♦ SIGM_XX = fxx , / [function]
        / [formula]
        ♦ SIGM_YY = fyy , / [function]
        / [formula]
        ♦ SIGM_ZZ = fzz , / [function]
        / [formula]
        ♦ SIGM_XY = fxy , / [function]
        / [formula]
        ♦ SIGM_XZ = fxz , / [function]
        / [formula]
        ♦ SIGM_YZ = fyz , / [function]
        / [formula]
        ♦ EPSP = p , / [function]
        / [formula]
        ♦ TEMP = temp , / [function]
        / [formula]
    )
    ♦ TOO BAD = 'LEMAITRE' ,
    ♦ MATER = to subdue ,
    ♦ OFFICE PLURALITY = 'LINEAR' ,
# Finsi
    ♦ INFORMATION = / 1, [DEFECT]
    / 2,
    ♦ TITLE = title [l_Kn]
)
```

## 3 Operands

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### 3.1 Operand **LOADING**

This keyword makes it possible to the user to specify the type of treated loading. The loading can be 'UNIAXIAL', 'MULTIAXIAL' or 'UNSPECIFIED'. To each loading corresponds its (or its) method (S) of evaluation of the damage by tiredness.

**Note:** When the loading is multi-axial, it is enough to give the history of the loading over one period or a block of the neck-cycles. If the loading is unspecified, it is necessary to provide the whole of the history of the loading.

### 3.2 Operands specific to the calculation of the type **UNIAXIAL**

#### 3.2.1 Operand **HISTORY**

The history of loading can be the evolution of a value of constraint or uniaxial deformation in the course of time,

**Note:**

*That does not mean that the loading cannot be multi-axial, but only that for the calculation of the damage, the loading is characterized by the evolution of a scalar component, in the course of time (Von-Put signed, invariant of a signed nature 2,...). It is the evolution of this scalar component which the user must provide to the order POST\_FATIGUE .*

##### 3.2.1.1 Operand **SIGM**

◇ SIGM = histsigm,

Name of the function or the formula describing the history of the loading in constraints in a point. It is a function or a formula of the parameter INST, which gives the evolution during the time of a scalar component characterizing the state of stresses of the structure.

This operand is obligatory for the calculation of the damage by a method of WOHLER.

##### 3.2.1.2 Operand **EPSI**

◇ EPSI = histepsi,

Name of the function or the formula describing the history of the loading in deformations in a point. It is a function or a formula of the parameter INST, which gives the evolution during the time of a scalar component characterizing the state of structural deformations.

This operand is obligatory for the calculation of the damage by the methods of MANSON\_COFFIN or TAHERI\_MANSON or TAHERI\_MIXTE .

### 3.2.2 Operand **COUNTING**

◆ COUNTING =

To be able to calculate the damage undergone by a structure, it is necessary beforehand to extract the elementary cycles from the history of loading. For that of many methods are available. In Code\_Aster, three methods were programmed.

/ 'RAINFLOW' ,

Method of counting of the extents in cascade or method of RAINFLOW (recommendation AFNOR A03-406 of November 1993) for the extraction of the elementary cycles of the history of loading [R7.04.01].

/ 'RAINFLOW\_MAX' ,

This method is similar to that of Rainflow excludes the fact that the elementary cycle of which amplitude is maximum is placed at the beginning of the history of loading to take in account of the effects of the overloads.

/ 'RCCM' ,  
Method of the RCC-M [R7.04.01].

/ 'NATURAL' ,  
Method known as natural which consists in generating the cycles in the order of their application [R7.04.01].

In the special case where the history of loading is constant (for example, average loading applied), Code\_Aster will count the whole history of loading like a cycle of amplitude worthless.

### 3.2.3 Operand DELTA\_OSCI

◇ DELTA\_OSCI = delta,

Filtering of the history of the loading. In all the cases, if the function remains constant or decreasing on more than two consecutive points one removes the intermediate points to keep only the two extreme points. Then, one removes history of loading the points for which the variation of the value of the constraint is lower than the value *delta*. By default *delta* is equal to zero, which amounts keeping all the oscillations of the loading, even those of low amplitude.

It is noted that if the keyword **COEF\_MULT** and **DELTA\_OSCI** are all present, Code\_Aster will apply initially **COEF\_MULT** and then **DELTA\_OSCI**.

**Example:** Let us consider the following history of loading:

N° not	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Moment	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Loading	4.	7.	2.	10.	9.6	9.8	5.	9.	3.	4.	2.	2.4	2.2	12.	5.
N° not	16	17	18	19	20	21	22	23	24	25	26	27	28	29	
Moment	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	
Loading	11.	1.	4.	3.	10.	6.	8.	12.	4.	8.	1.	9.	4.	6.	

The extraction of the peaks of this history of loading, with a value of *delta* of 0,9 conduit to destroy all the oscillations of amplitude lower than 0.9. What leads to the following history of loading:

N° not	1	2	3	4	7	8	9	10	11	14	15	16	17	18	19
Moment	0.	1.	2.	3.	6.	7.	8.	9.	10.	13.	14.	15.	16.	17.	18.
Loading	4.	7.	2.	10.	5.	9.	3.	4.	2.	12.	5.	11.	1.	4.	3.
N° not	20	21	23	24	25	26	27	28	29						
Moment	19.	20.	22.	23.	24.	25.	26.	27.	28.						
Loading	10.	6.	12.	4.	8.	1.	9.	4.	6.						

One removed:

- item 5 because  $\Delta \sigma = |\sigma(5) - \sigma(4)| < 0,9$ ,
- item 6 because  $\Delta \sigma = |\sigma(6) - \sigma(4)| < 0,9$ ,
- item 12 because  $\Delta \sigma = |\sigma(12) - \sigma(11)| < 0,9$ ,
- item 13 because  $\Delta \sigma = |\sigma(13) - \sigma(11)| < 0,9$ .

In the same way the item 22 is removed because the history of loading is increasing between items 21.22 and 23 and thus one keep only the extreme points.

## 3.2.4 Keyword COEF\_MULT

◇ COEF\_MULT = \_F

This keyword factor gathers the coefficients of performance of the history of loading. For the moment, only one multiplying coefficient of the history of loading is available: the coefficient of stress concentration  $K_T$ .

Values of the coefficient of stress concentration are available in the RCC\_M.

### 3.2.4.1 Operand $K_T$

◇  $K_T = kt$

$kt$  is the coefficient of stress concentration which depends on the geometry of the part, the geometry of a possible defect and type of loading. This coefficient is used to apply to the "filtered" history of loading a homothety of report  $kt$ .

It is noted that if the keyword COEF\_MULT and DELTA\_OSCI are all present, Code\_Aster will apply initially COEF\_MULT and then DELTA\_OSCI.

## 3.2.5 Operand CORR\_KE

◇ CORR\_KE = 'RCCM',

This operand makes it possible to take account of an elastoplastic coefficient of concentration,  $K_e$  who is defined by the RCC-M as being the relationship between the amplitude of real deformation and the amplitude of deformation determined by an elastic analysis.

$$\begin{cases} K_e = 1 & \text{si } D_s < 3S_m \\ K_e = 1 + (1-n) \left( \frac{\Delta\sigma}{3 \cdot S_m} - 1 \right) / (n(m-1)) & \text{si } 3S_m < D_s < 3mS_m \\ K_e = 1/n & \text{si } 3mS_m < D_s \end{cases}$$

where  $S_m$  is the acceptable maximum constraint and  $n$  and  $m$  two constants depending on material.

Values  $S_m$ ,  $n$  and  $m$  are provided in the operator DEFINI\_MATERIAU [U4.43.01] under the keyword factor TIREDNESS and operands SM\_KE\_RCCM, N\_KE\_RCCM and M\_KE\_RCCM.

## 3.2.6 Operand TOO\_BAD

To calculate the damage undergone by a structure in a point, various methods are available [R7.04.01]. Methods based on uniaxial tests: method of Wöhler, method of Manson-Whetstone sheath, methods of Taheri. These methods have as a common point to determine a value of damage starting from the evolution in the course of the time of one **scalar component** characterizing the state of constraint or structural deformation.

That does not mean that the state of stresses cannot be multiaxial, but only that for the calculation of the damage one chose a uniaxial component characterizing the state of constraint or deformation (forced Von-Put signed, invariant of a nature 2 signed of the tensor of the deformations,...).

The methods of Manson-Whetstone sheath and Taheri use the deformations generated by the loading.

The method of Wöhler uses the constraints generated by the loading.

◇ DAMAGE = 'WOHLER',

For a history of constraints associated with a uniaxial loading, the number of cycles to the rupture is given using the curve of Wöhler of material  $\left( N_{rupt} = \text{WOHLER} \left( \frac{\Delta \sigma}{2} \right) \right)$ .

The curve of Wöhler of material must be introduced into the operator `DEFI_MATERIAU [U4.43.01]` under one of the three possible mathematical forms [R7.04.01]:

- point by point discretized function (keyword factor `TIREDNESS`, operand `WOHLER`),
- analytical form of Basquin (keyword factor `TIREDNESS`, operands `A_BASQUIN` and `BETA_BASQUIN`),
- form "zones current" (keyword factor `TIREDNESS`, operands `E_REFE`, `A0`, `A1`, `A2`, `A3` and `SL` and keyword factor `ELAS` operand `E`).

### Notice on the curves of tiredness:

*For the small amplitudes of constraints, the difficulty of the prolongation of the curve of tiredness can arise: for example, for the curves of tiredness of the RCC-M beyond  $10^6$  cycles, the corresponding constraint, 180 MPa is regarded as limit of endurance, i.e. very forced lower than 180 MPa must produce a factor of null use or an infinite number of cycles acceptable.*

*The method adopted here corresponds to this concept of limit of endurance: if the amplitude of constraint is lower than the first X-coordinate of the curve of tiredness, then one takes a factor of null use i.e. an infinite number of cycles acceptable.*

◇ `DAMAGE = 'MANSON_COFFIN'`,

For a uniaxial history of loading of deformations type, the number of cycles to the rupture is given using the curve of Manson-Whetstone sheath of material  $\left( N_{rupt} = \text{MANSON\_COFFIN} \left( \frac{\Delta \varepsilon}{2} \right) \right)$ .

The curve of Manson-Whetstone sheath of material must be introduced into the operator `DEFI_MATERIAU [U4.43.01]` (keyword factor `TIREDNESS`, operand `MANSON_COFFIN`).

◇ `DAMAGE = 'TAHERI_MANSON'`,

This method of calculating of the damage applies only to loadings in deformations.

Are  $n$  elementary cycles (extracts by a method of counting) of half-amplitude  $\frac{\Delta \varepsilon_1}{2}, \dots, \frac{\Delta \varepsilon_n}{2}$ .

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-Whetstone sheath of material.

The calculation of the elementary damage of the following cycles is carried out by the algorithm described below:

- If  $\frac{\Delta \varepsilon_{i+1}}{2} \geq \frac{\Delta \varepsilon_i}{2}$

the calculation of the elementary damage of the cycle  $(i+1)$  is determined by interpolation on the curve of Manson-Whetstone sheath of material,

- If  $\frac{\Delta \varepsilon_{i+1}}{2} < \frac{\Delta \varepsilon_i}{2}$

one determines:

$$\frac{\Delta \sigma_{i+1}}{2} = \text{Fnappe} \left( \frac{\Delta \varepsilon_{i+1}}{2}, \text{Max}_{j < i} \left( \frac{\Delta \varepsilon_j}{2} \right) \right)$$
$$\frac{\Delta \varepsilon_{i+1}^*}{2} = \text{Ffonc} \left( \frac{\Delta \sigma_{i+1}}{2} \right)$$

where:

Fnappe is a tablecloth introduced under the operand TAHERI\_NAPPE,

Ffonc is a function introduced under the operand TAHERI\_FONC.

The value of the damage of the cycle  $(i+1)$  is obtained by interpolation of  $\frac{\Delta \varepsilon_{i+1}^*}{2}$  on the curve of Manson-Whetstone sheath of material.

$N_{rupt_{i+1}}$  is the number of cycles to the rupture of the cycle  $(i+1)$

$$N_{rupt_{i+1}} = \text{MANSON\_COFFIN} \left( \frac{\Delta \varepsilon_{i+1}^*}{2} \right)$$

and  $Dom_{i+1}$  is the damage of the cycle  $(i+1) = \frac{1}{N_{rupt_{i+1}}}$ .

The curve of Manson-Whetstone sheath of material must be introduced into the operator DEF1\_MATERIAU [U4.43.01] (keyword factor TIREDNESS, operand MANSON\_COFFIN).

◇ DAMAGE = 'TAHERI\_MIXTE',

This method of calculating of the damage applies only to loadings in deformations.

Are  $n$  elementary cycles (extracts by a method of counting) of half-amplitude  $\frac{\Delta \varepsilon_1}{2}, \dots, \frac{\Delta \varepsilon_n}{2}$ .

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-whetstone sheath of material.

The calculation of the elementary damage of the following cycles is carried out by the algorithm described Ci - below:

- If  $\frac{\Delta \varepsilon_{i+1}}{2} \geq \frac{\Delta \varepsilon_i}{2}$

the calculation of the elementary damage of the cycle  $(i+1)$  is determined by interpolation on the curve of Manson-Whetstone sheath.

- If  $\frac{\Delta \varepsilon_{i+1}}{2} < \frac{\Delta \varepsilon_i}{2}$

one determines:

$$\frac{\Delta \sigma_{i+1}}{2} = \text{Fnappe} \left( \frac{\Delta \varepsilon_{i+1}}{2}, \text{Max}_{j < i} \left( \frac{\Delta \varepsilon_j}{2} \right) \right)$$

where Fnappe is a tablecloth introduced under the operand of TAHERI\_NAPPE.

The value of the damage of the cycle  $(i+1)$  is obtained by interpolation of  $\frac{\Delta \sigma_{i+1}}{2}$  on the curve of Wöhler of material.

$N_{rupt_{i+1}}$  is the number of cycles to the rupture of the cycle  $(i+1)$

$$N_{rupt_{i+1}} = \text{WOHLER} \left( \frac{\Delta \sigma_{i+1}}{2} \right)$$

and  $Dom_{i+1}$  is the damage of the cycle  $(i+1) = \frac{1}{N_{rupt_{i+1}}}$ .

This method requires the data of the curve of Wöhler and the curve of Manson - Whetstone sheath of material which must be introduced into the operator `DEFI_MATERIAU` [U4.43.01] (keyword factor `TIREDDNESS`).

### 3.2.7 Operand `MATER`

♦ `MATER = to subdue,`

Allows to specify the name of material `to subdue` created by `DEFI_MATERIAU` [U4.43.01].

The material `to subdue` must contain the values of all the data materials necessary to calculation of the damage.

### 3.2.8 Operand `CORR_SIGM_MOYE`

♦ `CORR_SIGM_MOYE = / 'GOODMAN',`  
`/ 'TO STACK' ,`

This operand is used only in the case of the calculation of the damage by the method of `WOHLER`.

If the part is not subjected to pure or symmetrical alternate constraints, it is - with - to say if the average constraint of the cycle is not worthless, one can balance the curve of Wöhler to calculate the number of effective cycles to the rupture using the diagram of Haigh [R7.04.01].

From a cycle  $(S_{alt}, \sigma_m)$  identified in the signal, one calculates the value of the corrected alternate constraint  $S'_{alt}$ .

If the line of Goodman is used

$$S'_{alt} = \frac{S_{alt}}{1 - \frac{\sigma_m}{S_u}}$$

If one uses the parabola To stack

$$S'_{alt} = \frac{S_{alt}}{1 - \left( \frac{\sigma_m}{S_u} \right)^2}$$

The value of the limit to the rupture of material  $S_u$  must be introduced into the operator `DEFI_MATERIAU` [U4.43.01] (keyword factor `RCCM`, operand `KNOWN`).

### 3.2.9 Operand `TAHERI_NAPPE`

♦ `TAHERI_NAPPE = fnappe,`

This operand makes it possible to specify the name of a tablecloth.

$\text{Fnappe} = \left( \frac{\Delta \varepsilon}{2}, \varepsilon_{max} \right)$  necessary to the calculation of the damage by the methods

`TAHERI_MANSON` and `TAHERI_MIXTE`.

The tablecloth must have as parameters  $X$  and  $EPSI$ . The parameter  $X$  corresponds to the maximum deformation reached during a possible pre-work hardening.

Tablecloth introduced under the operand `TAHERI_NAPPE` is the cyclic curve of work hardening with pre-work hardening of material.

The cyclic curve of work hardening without pre-work hardening, given under the keyword `TAHERI_FONC`, must be obligatorily one of the curves defining the tablecloth. This curve must be given for  $X=0$ .

### 3.2.10 Operand `TAHERI_FONC`

◇ `TAHERI_FONC = ffonc,`

This operand makes it possible to specify the name of a function  $F_{\text{fonc}} = \left(\frac{\Delta \sigma}{2}\right)$  necessary to the calculation of the damage by the method `TAHERI_MANSON`.

The parameter of this function must be `SIGM`.

This function is the cyclic curve of work hardening of material.

### 3.2.11 Operand `OFFICE_PLURALITY`

◇ `OFFICE_PLURALITY = 'LINEAR',`

Methods of `WOHLER`, `MANSON_COFFIN` and `TAHERI` calculate a value of damage for each elementary cycle extracted the uniaxial loading introduced by the user.

The operand `OFFICE_PLURALITY` allows to ask the calculation of the total damage undergone by the structure in a point.

The only rule available is the rule `To mine`, which consists in summoning all the elementary damage  $D = \sum_i D_i$ .

## 3.3 Operands specific to the calculation of the type `MULTIAXIAL`

### 3.3.1 Operand `TYPE_CHARGE`

This operand makes it possible to specify the type of loading applied to the structure:

- `PERIODIC`, the loading is periodic;
- `NON_PERIODIQUE`, the loading is not periodical.

### 3.3.2 Operand `HISTORY`

This keyword gathers all the phase of definition of the history of loading. History of loading can be the evolution of the tensor of constraints, total deflection and deformation plastic in the course of time.

It is noted that at least a type of the loading (forced, total deflection, deformation plastic) must be provided. For a type of the tensor, it is necessary to provide you `S` the very six components.

In this operator, elastic strain = total deflection - plastic deformation. For the criterion which requires the elastic strain, the request of the total deflection is obligatory. If one does not inform the plastic deformation, one will take zero value.

#### 3.3.2.1 Operands `SIGM_XX/SIGM_YY/SIGM_ZZ/SIGM_XY/SIGM_XZ/SIGM_YZ`

Names of the functions or the formulas describing the history of each component of the tensor of the constraints in the course of time. Each function or formula depends on the parameter `INST`. All the functions or formulas must be defined for the same moments.

#### 3.3.2.2 Operands `EPS_XX/ EPS_YY/ EPS_ZZ/ EPS_XY/ EPS_XZ/ EPS_YZ`

Names of the functions or the formulas describing the history of each component of the tensor of the total deflections in the course of time. Each function or formula depends on parameter INST. All the functions or formulas must be defined for the same moments.

### 3.3.2.3 Operands EPSP\_XX/ EPSP\_YY/ EPSP\_ZZ/ EPSP\_XY/ EPSP\_XZ/ EPSP\_YZ

Names of the functions or the formulas describing the history of each component of the tensor of the total deflections in the course of time. Each function or formula depends on parameter INST. All the functions or formulas must be defined for the same moments.

### 3.3.3 Operand CHAM\_MATER

◇ CHAM\_MATER = cham\_mater

Allows to specify the name of the field of material `cham_mater` created by AFFE\_MATERIAU [U4.43.03].

The material to subdue defined with the order DEFI\_MATERIAU and which is used for the assignment of material to the grid with the order AFFE\_MATERIAU must contain the definition of the curve of Wöhler as well as the necessary information with the implementation of the criterion, see the keywords factors TIREDNESS and CISA\_PLAN\_CRIT order DEFI\_MATERIAU [U4.43.01].

The keyword CHAM\_MATER is not obligatory when one uses a formula for the damage.

### 3.3.4 Operand COEF\_PREECROU

◇ COEF\_PREECROU = / coef\_pre,  
/ 1.0,

This coefficient is used to take into account the effect of a possible pre-work hardening.

### 3.3.5 Operand COEF\_CORR

◇ COEF\_CORR = corr,

The criteria of Crossland and Dang Van-Papadopoulos make it possible for periodic loadings to calculate a value  $R_{crit}$  who indicates if there is damage or not for the number of cycles associated with the limits with endurance  $\tau_0$  and  $d_0$ .

These criteria do not give a value of the damage, which can however be interesting.

With this intention, one proposes to use the value of the criterion and the curve of Wöhler of material, by defining an equivalent constraint:

$$\sigma^* = (R_{crit} + b) \times \text{corr}$$

Most curves of Wöhler are obtained with tests of alternate pure traction and compression. However the criterion of Dang-Van-Papadopoulos is a criterion in shearing. Consequently, it is necessary "to correct" the equivalent constraint  $\sigma^*$  before applying it to a curve of Wöhler obtained with tests of traction and compression; it is the role of the operand COEF\_CORR.

The value of the damage is obtained while applying  $\sigma^*$  on the curve of Wöhler of material.

So that there is coherence between the criterion and the curve of Wöhler, it is necessary that:

$$\left\{ \begin{array}{l} \sigma^* \leq \tau_0 \quad \text{pas de dommage} \\ \sigma^* > \tau_0 \quad \text{dommage} \end{array} \right\}$$

for a curve of Wöhler defined in shearing and that:

$$\left\{ \begin{array}{l} \sigma^* \leq d_0 \quad \text{pas de dommage} \\ \sigma^* > d_0 \quad \text{dommage} \end{array} \right\}$$

for a curve of Wöhler defined in traction and compression.

The user can thus specify a value `corr`, by taking account of the type of curve of Wöhler it has.

The value taken by default for `COEF_CORR` is  $\frac{d_0}{\tau_0}$ , in coherence with curves of Wöhler in traction

- compression.

**Note:**

If  $R_{crit} < 0$ , if the prolongation on the left of the curve of Wöhler is linear (in `DEFI_FONCTION (... PROL_GAUCHE = 'LINEAR' ...)`), the user will obtain a damage different from zero. To obtain a null damage when  $R_{crit} < 0$ , it is necessary that the prolongation on the left is equal to `'EXCLUDED'` or `'CONSTANT'`.

### 3.3.6 Operand CRITERION

- ◆ `CRITERION =` / `'MATAKE_MODI_AC'`,  
/ `'DANG_VAN_MODI_AC'`,  
/ `'MATAKE_MODI_AV'`,  
/ `'DANG_VAN_MODI_AV'`,  
/ `'FATESOCI_MODI_AV'`,  
/ `'FORMULE_CRITERE'`,  
/ `'CROSSLAND'`,  
/ `'PAPADOPOULOS'`,

The user introduces the values of each component of the tensor of the constraints in various moments  $(t_0, \dots, t_N)$ , and it is supposed that  $[t_0, t_N]$  is one period of the loading.

The loadings can be constraints, total deflections, plastic deformations or combinations of these parameters.

The following table lists criteria of starting available for two types of loadings.

TYPE_CHARGE = 'PERIODIC'	TYPE_CHARGE = 'NON_PERIODIQUE'
'MATAKE_MODI_AC'	'MATAKE_MODI_AV'
'DANG_VAN_MODI_AC'	'DANG_VAN_MODI_AV'
'FORMULE_CRITERE'	'FATESOCI_MODI_AV'
'CROSSLAND'	'FORMULE_CRITERE'
'PAPADOPOULOS'	

For the loading with constant amplitude, the operand `CRITERION` allows to specify the criterion which the half-amplitude will have to satisfy with maximum shearing. For the loading with variable amplitude, the operand `CRITERION` allows to specify the criterion which will have to satisfy the maximum damage.

The criteria of starting in Code\_Aster can be called by a name for the well established criteria. It is also possible for the user to build a criterion of starting by itself like a formula of predefined sizes.

Notation:

- $\mathbf{n}^*$  : normal with the plan in which the amplitude of shearing is maximum;
- $\Delta \tau(\mathbf{n})$  : amplitude of shearing in constraint in a plan of normal  $\mathbf{n}$  ;
- $\Delta \gamma(\mathbf{n})$  : amplitude of shearing in deformation in a plan of normal  $\mathbf{n}$  ;
- $N_{max}(\mathbf{n})$  : normal maximum constraint as regards normal  $\mathbf{n}$  ;
- $\tau_0$  : limit of endurance in alternate pure shearing;
- $d_0$  : limit of endurance in alternate pure traction and compression;
- $P$  : hydrostatic pressure;
- $c_p$  : coefficient being used to take into account a possible pre-work hardening;

$\sigma_y$  : elastic limit.

### Criterion MATAKE\_MODI\_AC

The initial criterion of MATAKE is defined by the inequation [éq.3.12-1]:

$$\frac{\Delta \tau}{2}(\mathbf{n}^*) + a N_{max}(\mathbf{n}^*) \leq b \quad \text{éq 3.12-1}$$

where  $a$  and  $b$  are two constant data by the user under the keywords MATAKE\_A and MATAKE\_B keyword factor CISA\_PLAN\_CRIT of DEFI\_MATERIAU, they depend on characteristic materials and are worth:

$$a = \left( \tau_0 - \frac{d_0}{2} \right) / \frac{d_0}{2} \quad b = \tau_0$$

If the user has the results of two tensile tests compression, alternated and the other not, the constant ones  $a$  and  $b$  are given by:

$$a = \frac{\Delta \sigma_2 - \Delta \sigma_1}{(\Delta \sigma_1 - \Delta \sigma_2) - 2 \sigma_m},$$

$$b = \frac{\sigma_m}{(\Delta \sigma_2 - \Delta \sigma_1) + 2 \sigma_m} \times \frac{\Delta \sigma_1}{2},$$

with  $\Delta \sigma_1$  the amplitude of loading for the alternate case ( $\sigma_m = 0$ ) and  $\Delta \sigma_2$  the amplitude of loading for the case where the average constraint is nonworthless ( $\sigma_m \neq 0$ ).

We modify the initial criterion of MATAKE by introducing the definition of an equivalent constraint, noted  $\sigma_{eq}(\mathbf{n}^*)$  :

$$\sigma_{eq}(\mathbf{n}^*) = \left( c_p \frac{\Delta \tau}{2}(\mathbf{n}^*) + a N_{max}(\mathbf{n}^*) \right) \frac{f}{t},$$

where  $f/t$  represent the report of the limits of endurance in inflection and alternating torsion, and must be well informed under the keyword COEF\_FLEX\_TORS keyword factor CISA\_PLAN\_CRIT of DEFI\_MATERIAU.

### Criterion DANG VAN MODI\_AC

The initial criterion of DANG VAN is defined by the inequation [éq 3.12-2]:

$$\frac{\Delta \tau}{2}(\mathbf{n}^*) + a P \leq b \quad \text{éq 3.12-2}$$

where  $a$  and  $b$  are two constant data by the user under the keywords D\_VAN\_A and D\_VAN\_B keyword factor CISA\_PLAN\_CRIT of DEFI\_MATERIAU, they depend on characteristic materials. If the user has two tensile tests compression, alternate other not constants  $a$  and  $b$  are worth:

$$a = \frac{3}{2} \times \frac{\Delta \sigma_2 - \Delta \sigma_1}{(\Delta \sigma_1 - \Delta \sigma_2) - 2 \sigma_m} \quad b = \frac{\sigma_m}{(\Delta \sigma_2 - \Delta \sigma_1) + 2 \sigma_m} \times \frac{\Delta \sigma_1}{2}$$

with  $\Delta \sigma_1$  the amplitude of loading for the alternate case ( $\sigma_m = 0$ )  $\Delta \sigma_2$  and for the case where the average constraint is nonworthless ( $\sigma_m \neq 0$ ).

Moreover, we define an equivalent constraint within the meaning of DANG VAN, noted  $\sigma_{eq}(\mathbf{n}^*)$  :

$$\sigma_{eq}(\mathbf{n}^*) = \left( c_p \frac{\Delta \tau}{2}(\mathbf{n}^*) + a P \right) \frac{c}{t}$$

where  $c/t$  represent the report of the limits of endurance in alternated shearing and traction, and must be well informed under the keyword `COEF_CISA_TRAC` keyword factor `CISA_PLAN_CRIT` of `DEFI_MATERIAU`.

For more information, to consult the document [R7.04.04].

## Criterion `MATAKE_MODI_AV`

The criterion `MATAKE_MODI_AV` is an evolution of the criterion of `MATAKE`. Contrary to the two preceding criteria, this criterion selects the critical plan according to the damage calculated in each plan. It is the plan in which the damage is maximum which is retained. This criterion is adapted to the nonperiodic loadings, which induces the use of a method of counting of cycles in order to calculate the elementary damage. To count the cycles, we use method `RAINFLOW`.

The once known elementary damage is cumulated linearly to determine the damage.

To calculate the elementary damage we project the history of shear stresses on one or two axes in order to reduce this one to a unidimensional function of  $\tau_p = f(t)$  time. After having extracted the elementary under-cycles from  $\tau_p$  with method `RAINFLOW` we define an elementary equivalent constraint for any elementary under-cycle  $i$  :

$$\sigma_{eq}^i(\mathbf{n}) = \alpha \left( c_p \frac{\text{Max}(\tau_{p1}^i(\mathbf{n}), \tau_{p2}^i(\mathbf{n})) - \text{Min}(\tau_{p1}^i(\mathbf{n}), \tau_{p2}^i(\mathbf{n}))}{2} + a \text{Max}(N_1^i(\mathbf{n}), N_2^i(\mathbf{n}), 0) \right)$$

éq 3.12-3

with  $\mathbf{n}$  the normal of the plan running,  $\tau_{p1}^i(\mathbf{n})$  and  $\tau_{p2}^i(\mathbf{n})$  values of projected shear stresses of the under-cycle  $i$  and  $N_1^i(\mathbf{n})$  and  $N_2^i(\mathbf{n})$  normal constraints of the under-cycle  $i$ . From  $\sigma_{eq}^i(\mathbf{n})$  and of a curve of tiredness we determine the number of cycles to the elementary rupture  $N^i(\mathbf{n})$  and corresponding damage  $D^i(\mathbf{n}) = 1/N^i(\mathbf{n})$ . In [éq 3.12 - 3]  $\alpha$  is a corrective term which makes it possible to use a curve of tiredness in traction - compression. Constants  $a$  and  $\alpha$  must be well informed under the keywords `MATAKE_A` and `COEF_FLEX_TORS` keyword factor `CISA_PLAN_CRIT` of `DEFI_MATERIAU`.

We use a linear office plurality of damage. That is to say  $k$  the number of elementary under-cycles, for a normal  $\mathbf{n}$  fixed, the cumulated damage is equal to:

$$D(\mathbf{n}) = \sum_{i=1}^k D^i(\mathbf{n})$$

éq 3.12-4

To determine the normal vector  $\mathbf{n}^*$  corresponding to the maximum cumulated damage we vary  $\mathbf{n}$ , the normal vector  $\mathbf{n}^*$  corresponding to the maximum cumulated damage is then given by:

$$D(\mathbf{n}^*) = \underset{\mathbf{n}}{\text{Max}}(D(\mathbf{n}))$$

## Criterion `DANG_VAN_MODI_AV`

The approach and the techniques put in work to calculate this criterion are identical to those used for the criterion `MATAKE_MODI_AV`. The only difference lies in the definition of the elementary equivalent constraint where hydrostatic pressure  $P$  replace the maximum normal constraint  $N_{max}$  :

$$\sigma_{eq}^i(\mathbf{n}) = \alpha \left( c_p \frac{\text{Max}(\tau_{p1}^i(\mathbf{n}), \tau_{p2}^i(\mathbf{n})) - \text{Min}(\tau_{p1}^i(\mathbf{n}), \tau_{p2}^i(\mathbf{n}))}{2} + a \text{Max}(P_1^i(\mathbf{n}), P_2^i(\mathbf{n}), 0) \right)$$

Constants  $a$  and  $\alpha$  are to be informed by the user under the keywords `D_VAN_A` and `COEF_CISA_TRAC` keyword factor `CISA_PLAN_CRIT` of `DEFI_MATERIAU`.

For more information to consult the document [R7.04.04].

## Criterion FATESOCI\_MODI\_AV

The criterion of FATEMI and SOCIE is defined by the relation:

$$\varepsilon_{eq}(n) = \frac{\Delta \gamma(n)}{2} \left( 1 + k \frac{N_{max}(n)}{\sigma_y} \right)$$

where  $k$  is a constant which depends on characteristic materials. Contrary to the other criteria, it uses shearing in deformation instead of shearing in constraint. Moreover, the various quantities which contribute to the criterion are multiplied and not added. The criterion of FATEMI and SOCIE is usable after an elastic design or elastoplastic. This criterion selects the critical plan according to the damage calculated in each plan. It is the plan in which the damage is maximum which is retained.

This criterion is adapted to the nonperiodic loadings, which leads us to use the method of counting of cycles RAINFLOW to calculate the elementary damage. The elementary damage is then cumulated linearly to determine the damage.

In order to calculate the elementary damage we project the history of shearing in deformation on one or two axes in order to reduce this one to a unidimensional function of time  $\gamma_p = f(t)$ . After having extracted the elementary under-cycles with method RAINFLOW we define an elementary equivalent deformation for any elementary under-cycle  $i$  :

$$\varepsilon_{eq}^i(\mathbf{n}) = \alpha c_p \left( \frac{\text{Max}(\gamma_{p1}^i(\mathbf{n}), \gamma_{p2}^i(\mathbf{n})) - \text{Min}(\gamma_{p1}^i(\mathbf{n}), \gamma_{p2}^i(\mathbf{n}))}{2} \right) \left( 1 + a \text{Max}(N_1^i(\mathbf{n}), N_2^i(\mathbf{n}), 0) \right)$$

éq 3.12-5

with  $a = \frac{k}{\sigma_y}$ ,  $\mathbf{n}$  the normal with the plan running,  $\gamma_{p1}^i(\mathbf{n})$  and  $\gamma_{p2}^i(\mathbf{n})$  values of shearings in deformation projected of the under-cycle  $i$ ,  $N_1^i(\mathbf{n})$  and  $N_2^i(\mathbf{n})$  being two values of the normal constraint of the under-cycle  $i$ . From  $\varepsilon_{eq}^i(\mathbf{n})$  and of a curve of Manson-Whetstone sheath we determine the number of cycles to the elementary rupture and  $N^i(\mathbf{n})$  corresponding damage  $D^i(\mathbf{n}) = 1/N^i(\mathbf{n})$ .

**It will be noted that the shearing strains used in the criterion of FATEMI and SOCIE are distortions  $\gamma_{ij}$  ( $i \neq j$ ). If one uses the shearing strains of the tensorial type  $\epsilon_{ij}$  ( $i \neq j$ ), they should be multiplied by a factor 2 because  $\gamma_{ij} = 2\epsilon_{ij}$ .**

In the equation [éq 3.12-5],  $\alpha$  is a corrective term which to use a curve of Manson-Whetstone sheath obtained in traction and compression.  $c_p$  is a coefficient which makes it possible to take into account a possible pre-work hardening.

Constants  $a$  and  $\alpha$  must be well informed under the keywords FATSOC\_A and COEF\_CISA\_TRAC keyword factor CISA\_PLAN\_CRIT order DEFI\_MATERIAU.

It is noted that a rigorous approach is to use the curve of Manson-Whetstone sheath obtained directly in torsion (which is not always available). The use of the curve of Manson-Whetstone sheath obtained in traction and compression with the corrective term  $\alpha$  (which is the relationship between two limits of endurance), as programmed in Code\_Aster, is thus an approximation.

As we use a linear office plurality of damage, if  $m$  is the number of elementary under-cycles, then for a normal  $\mathbf{n}$  fixed, the cumulated damage is equal to:

$$D(\mathbf{n}) = \sum_{i=1}^m D^i(\mathbf{n})$$

To find the vector normal  $\mathbf{n}^*$  corresponding to the maximum cumulated damage we vary  $\mathbf{n}$ . The normal vector  $\mathbf{n}^*$  associated with the maximum cumulated damage is then given by:

$$D(\mathbf{n}^*) = \underset{\mathbf{n}}{\text{Max}}(D(\mathbf{n}))$$

## Criterion FORMULE\_CRITERE

This kind of criterion makes it possible to the user to build a criterion like a formula of the sizes predefined. This criterion is based on a general relation:

“Equivalent size” = “Curve of life”

where the “equivalent Size” is a formula provided under the operand FORMULE\_GRDEQ (to see 3.4.6) and “Curve of life” is provided under the operand COURBE\_GRD\_VIE (see 3.4.7) that is to say by a function (counted or formulates, under the operand of 'FORMULE\_VIE', to see 3.4.8), that is to say by a name of curve 'WOHLER' or 'MANSON\_C' defined beforehand in DEFI\_MATERIAU.

## Criterion of Crossland

The criterion is written:

$$R_{crit} = \tau_a + a \cdot P_{max} - b$$

where

$$\tau_a = \frac{1}{2} \underset{0 \leq t_0 \leq T}{\text{Max}} \underset{0 \leq t_1 \leq T}{\text{Max}} \|\tilde{S}(t_1) - \tilde{S}(t_0)\| \text{ is the amplitude of cission}$$

with  $\tilde{S}$  diverter of the tensor of the constraints  $\sigma$

$$P_{max} = \underset{0 \leq t \leq T}{\text{Max}} \left( \frac{1}{3} \text{trace } \sigma \right) \text{ is the maximum hydrostatic pressure}$$

$$a = \frac{\left( \tau_0 - \frac{d_0}{\sqrt{3}} \right)}{\frac{d_0}{3}} \text{ and } b = \tau_0$$

with  $\tau_0$  limit of endurance in alternate pure shearing

and  $d_0$  limit of endurance in alternate pure traction and compression

## Criterion of Dang Van-Papadopoulos

The criterion is written:

$$R_{crit} = k^* + a \cdot P_{max} - b$$

where

$$k^* = R$$

$R$  ray of the smallest sphere circumscribed with the way of loading within the space of diverters of constraints  $\tilde{S}$

$$R = \underset{0 \leq t \leq T}{\text{Max}} \sqrt{\frac{1}{2} \cdot (\tilde{S}(t) - C^*) : (\tilde{S}(t) - C^*)}$$

$$C^* = \underset{\text{Min}}{\text{Max}} \sqrt{(\tilde{S}(t) - C) : (\tilde{S}(t) - C)} \text{ is the center of the hypersphère}$$

$P_{max} = \text{Max}_{0 \leq t \leq T} \left( \frac{1}{3} \text{trace } \sigma \right)$  is the maximum hydrostatic pressure

$$a = \frac{\left( \tau_0 - \frac{d_0}{\sqrt{3}} \right)}{\frac{d_0}{3}} \quad \text{and} \quad b = \tau_0$$

with  $\tau_0$  limit of endurance in alternate pure shearing

and  $d_0$  limit of endurance in alternate pure traction and compression

**Note:**

The initial goal of these criteria Crossland and Dang Van-Papadopoulos is not to determine a value of damage, but a value of criterion  $R_{crit}$  such as:

$$\begin{cases} R_{crit} \leq 0 & \text{pas de dommage} \\ R_{crit} > 0 & \text{dommage possible} \end{cases}$$

One can also however to determine a value of damage.

### 3.3.7 Opéranof FORMULE\_GRDEQ

◆ FORMULE\_GRDEQ = for\_grd, [formula]

Allows to provide the formula of the criterion like a function of the sizes available. Lists sizes available for each type of loading are in the following table:

TYPE_CHARGE = 'PERIODIC', CRITERION = 'FORMULE_CRITERE'
The sizes available are:
'DTAUMA' : half-amplitude of maximum shear stress ( $\Delta \tau(\mathbf{n}^*)/2$ )
'PHYDRM' : hydrostatic pressure ( $P$ )
'NORMAX' : maximum normal constraint on the critical level ( $N_{max}(\mathbf{n}^*)$ )
'NORMOY' : average normal constraint on the critical level ( $N_{moy}(\mathbf{n}^*)$ )
'EPNMAX' : maximum normal deformation on the critical level ( $\varepsilon_{Nmax}(\mathbf{n}^*)$ )
'EPNMOY' : average normal deformation on the critical level ( $\varepsilon_{Nmoy}(\mathbf{n}^*)$ )
'DEPSPE' : half-amplitude of the equivalent plastic deformation ( $\Delta \varepsilon_{eq}^p/2$ )
'EPSPR1' : half-amplitude of the first principal deformation (with the taking into account of the sign)
'SIGNM1' : maximum normal constraint on the level associated with $\varepsilon_1$
'DENDIS' : density of dissipated energy ( $W_{cy}$ )
'DENDIE' : density of energy of the elastic distortions ( $W_e$ )
'APHYDR' : half-amplitude of the hydrostatic pressure ( $P_a$ )
'MPHYDR' : average hydrostatic pressure ( $P_m$ )
'DSIGEQQ' : half-amplitude of the equivalent constraint ( $\Delta \sigma_{eq}/2$ )
'SIGPR1' : half-amplitude of the first principal constraint (with the taking into account of the sign)
'EPSNM1' : maximum normal deformation on the level associated with $\sigma_1$
'INVA2S' : half-amplitude of the second invariant of the deformation $J_2(\varepsilon)$
'DSITRE' : half-amplitude of the Tresca half-constraint ( $(\sigma_{max}^{Tresca} - \sigma_{min}^{Tresca})/4$ )
'DEPTRE' : half-amplitude of the Tresca half-deformation ( $(\varepsilon_{max}^{Tresca} - \varepsilon_{min}^{Tresca})/4$ )

'EPSPAC' : plastic deformation accumulated  $p$   
'RAYSPH' : the ray of the smallest sphere circumscribed with the way of loading within the space of diverters of the constraints  $R$   
'AMPCIS' : amplitude of cission  $\tau_a$   
'DEPSEE' : half-amplitude of the equivalent elastic strain (  $\Delta \varepsilon_e^p / 2$  )

There exist sizes depending on the orientation of the plan which pass through a point of material. For these sizes, one defines criteria of the standard critical plan. The critical plan is the plan which makes maximum a formula criticizes (see Opéranof FORMULE\_CRITIQUE ).

'DTAUCR' : half-amplitude of constraint shearing as regards normal  $\mathbf{N}$  (  $\Delta \tau(\mathbf{n})/2$  )  
'DGAMCR' : half-amplitude of deformation (of engineering) shearing as regards normal  $\mathbf{N}$  (  $\Delta \gamma(\mathbf{n})/2$  )  
'DSINCR' : half-amplitude of normal constraint as regards normal  $\mathbf{N}$  (  $\Delta N(\mathbf{n})/2$  )  
'DEPNCR' : half-amplitude of normal deformation as regards normal  $\mathbf{N}$  (  $\Delta \varepsilon_n(\mathbf{n})/2$  )  
'MTAUCR' : maximum constraint shearing as regards normal  $\mathbf{N}$  (  $\tau_{max}(\mathbf{n})$  )  
'MGAMCR' : deformation (of engineering) maximum shearing as regards normal  $\mathbf{N}$  (  $\gamma_{max}(\mathbf{n})$  )  
'MSINCR' : maximum normal constraint as regards normal  $\mathbf{N}$  (  $N_{max}(\mathbf{n})$  )  
'MEPNCR' : maximum normal deformation as regards normal  $\mathbf{N}$  (  $\varepsilon_{nmax}(\mathbf{n})$  )  
'DGAMPC' : half-amplitude of plastic deformation (of engineering) shearing as regards normal  $\mathbf{N}$  (  $\Delta \gamma^p/2$  )  
'DEPNPC' : half-amplitude of normal plastic deformation as regards normal  $\mathbf{N}$  (  $\Delta \varepsilon_e^p/2$  )  
'MGAMPC' : plastic deformation (of engineering) maximum shearing as regards normal  $\mathbf{N}$  (  $\gamma_{max}^p(\mathbf{n})$  )  
'MEPNPC' : maximum normal plastic deformation as regards normal  $\mathbf{N}$  (  $\varepsilon_{nmax}^p(\mathbf{n})$  )

**It will be noted that there exist two types of shearing strain measurement: distortions of shearing  $\gamma_{ij}$  ( $i \neq j$ ) and shearing strains  $\varepsilon_{ij}$  ( $i \neq j$ ). Let us note that  $\gamma_{ij} = 2\varepsilon_{ij}$ . For 'DGAMCR', 'MGAMCR', 'MGAMPC', the distortions of shearing were used  $\gamma_{ij}$ .**

TYPE\_CHARGE = 'NON-PERIODIQUE', CRITERION = 'FORMULE\_CRITERE'

The sizes available are:

'TAUPR\_1' : projected shear stresses of the first top of the under-cycle (  $\tau_{p1}(\mathbf{n})$  )  
'TAUPR\_2' : projected shear stresses of the second top of the under-cycle (  $\tau_{p2}(\mathbf{n})$  )  
'SIGN\_1' : normal constraint of the first top of the under-cycle (  $N_1(\mathbf{n})$  )  
'SIGN\_2' : normal constraint of the second top of the under-cycle (  $N_2(\mathbf{n})$  )  
'PHYDR\_1' : hydrostatic pressure of the first top of the under-cycle  
'PHYDR\_2' : hydrostatic pressure of the second top of the under-cycle  
'EPSPR\_1' : shearing in deformation (tensorial type  $\varepsilon_{ij}$  ( $i \neq j$ )) projected first top of the under-cycle (  $\gamma_{p1}(\mathbf{n})$  )  
'EPSPR\_2' : shearing in deformation (tensorial type  $\varepsilon_{ij}$  ( $i \neq j$ )) projected second top of the under-cycle (  $\gamma_{p2}(\mathbf{n})$  )  
'SIPR1\_1' : first principal constraint of the first top of the under-cycle (  $\sigma_1(1)$  )  
'SIPR1\_2' : first principal constraint of the second top of the under-cycle (  $\sigma_1(2)$  )  
'EPSN1\_1' : normal deformation on the level associated with  $\sigma_1(1)$  first top of the under-cycle  
'EPSN1\_2' : normal deformation on the level associated with  $\sigma_1(2)$  second top of the under-cycle

```

`ETPR1_1' : first principal total deflection of the first top of the under-cycle (  $\epsilon_1^{tot}(1)$  )
`ETPR1_2' : first principal total deflection of the second top of the under-cycle (  $\epsilon_1^{tot}(2)$  )
`SITN1_1' : normal constraint on the level associated with  $\epsilon_1^{tot}(1)$  first top of the under-cycle
`SITN1_2' : normal constraint on the level associated with  $\epsilon_1^{tot}(2)$  second top of the under-cycle
`EPPR1_1' : first principal plastic deformation of the first top of the under-cycle (  $\epsilon_1^p(1)$  )
`EPPR1_2' : first principal plastic deformation of the second top of the under-cycle (  $\epsilon_1^p(2)$  )
`SIPN1_1' : normal constraint on the level associated with  $\epsilon_1^p(1)$  first top of the under-cycle
`SIPN1_2' : normal constraint on the level associated with  $\epsilon_1^p(2)$  second top of the under-cycle
`SIGEQ_1' : equivalent constraint of the first top of the under-cycle (  $\sigma_{eq}(1)$  )
`SIGEQ_2' : equivalent constraint of the second top of the under-cycle (  $\sigma_{eq}(2)$  )
`ETEQ_1' : equivalent total deflection of the first top of the under-cycle (  $\epsilon_{eq}^{tot}(1)$  )
`ETEQ_2' : equivalent total deflection of the second top of the under-cycle (  $\epsilon_{eq}^{tot}(2)$  )

```

**Note:**

- 1) For the periodic loading, the formula of criterion is used to determine the plan of maximum shearing if the parameter 'DTAUMA' is introduced into the formula.
- 2) For the loading not-periodical, after having extracted the elementary under-cycles with method RAINFLOW, we calculate an elementary equivalent size by the formula of criterion for any elementary under-cycle. It is noted that the under-cycle is represented by two states of stress or deformation, noted by the first and the second tops of the under-cycle.
- 3) Parameters of entries of the order FORMULA must be among those listed in the table above.
- 4) Expressions of certain sizes are in the document [R7.04.04].
- 5) One stresses that the thermal deformation was not taken into account, i.e.,  $\epsilon^{tot} = \epsilon^e + \epsilon^p$ .
- 6) The operators used in the formula must be in conformity with the syntax of Python as indicated in the note [U4.31.05].

**3.3.8 Opéranof FORMULE\_CRITIQUE**

```
◇ FORMULE_CRITIQUE = for_grd, [formula]
```

It keyword makes it possible to define a critical size that the critical plan makes maximum. It is necessary that this formula contains at least a parameter depend on the orientation of the plan.

**3.3.9 Opéranof TOO\_BAD**

```
◆ TOO_BAD = / 'WOHLER',
             / 'MANSON_C',
             / 'FORMES_VIE'
```

This keyword makes it possible to provide a curve of connecting the size equivalent to the number of cycles to the rupture.

In Code\_Aster, the limit of endurance is fixed at 10 million cycles. If the calculated equivalent size is lower than the limit of endurance, the calculated damage is 0.

If TOO\_BAD = 'WOHLER', one will take the curve of Wohler ( $N_f = f(SIGM)$ ) defined in AFFE\_MATERIAU.

If TOO\_BAD = 'MANSON\_C', one will take the curve of Manson-Whetstone sheath ( $N_f = f(EP SN)$ ) defined in AFFE\_MATERIAU.

If `DAMAGE = 'FORMES_VIE'`, one will provide a function defining the curve of life.

### 3.3.9.1 Opéranof FORMULE\_VIE

◆ `FORMULE_VIE = for_vie,` / [formula]  
/ [function]

Allows to specify the curve connecting the equivalent size and the lifetime.

If `for_vie` is provided by a tabulée function, it must be in the form:

$$N_f = f(\text{grandeur\_équivalente}).$$

If `for_vie` is provided by a formula, it must be in the form:

$$\text{grandeur équivalente} = f(N_f).$$

In this case, the parameter of entry for the order `FORMULA` must be 'NBRUPT' (i.e.,  $N_f$ ).

### 3.3.10 Operand METHOD

◆ `METHOD = 'CERCLE_EXACT'`

Allows to specify the name of the method which will be used to calculate to it half amplitude of maximum shearing.

Method of 'CERCLE\_EXACT' is used to determine the circle circumscribed at the points which are in plans of shearing. This method rests on the process which consists in obtaining the circle which passes by three points, cf document [R7.04.04].

### 3.3.11 Operand PROJECTION

◆ `PROJECTION = / 'UN_AXE',`  
`/ 'DEUX_AXES',`

If the loading is not periodical, it is necessary to project the history of shearing on one or two axes, cf document [R7.04.04].

- `UN_AXE`, the history of shearing is projected on an axis;
- `DEUX_AXES`, the history of shearing is projected on two axes.

### 3.3.12 Operand DELTA\_OSCI

◇ `DELTA_OSCI = / delta,`  
`/ 0.0,`

Filtering of the history of the loading. In all the cases, if the function remains constant or decreasing on more than two consecutive points one removes the intermediate points to keep only the two extreme points. Then, one removes history of loading the points for which the variation of the value of the constraint is lower than the value `delta`. By default `delta` is equal to zero, which amounts keeping all the oscillations of the loading, even those of low amplitude. For more information to see the documentation of the order `POST_FATIGUE`, [U4.83.01], even operand.

## 3.4 Operands specific to the calculation of the type UNSPECIFIED

The history of loading can be the evolution of the tensor of constraints, the cumulated plastic deformation and the temperature in the course of time.

### 3.4.1 Operand EPSP

◇ `EPSP = p,`

Name of the function describing the history of the plastic deformation cumulated in the course of time, only for the calculation of the damage of `LEMAITRE`.

This function or formula depends on the parameter `INST` and must be defined for the same moments as the functions or formulas describing the history of the components of the tensor of the constraints.

The operand `EPSP` must be used jointly with the operands `SIGM_XX`,...

## 3.4.2 Operand `TEMP`

◇ `TEMP = temp`,

Name of the function or the formula describing the history of the temperature in the course of time, only for the calculation of the damage of `LEMAITRE`. It is used in this case to determine the value of the mechanical characteristics (Young modulus  $E$ , Poisson's ratio  $\nu$  and parameter material  $S$ ) at the moments of calculation of the damage.

This function or formula depends on the parameter `INST` and must be defined for the same moments as the functions or formulas describing the history of the components of the tensor of the constraints.

The operand `TEMP` must be used jointly with the operands `EPSP`, `SIGM_XX`,...

## 3.4.3 Methods of Lemaître and Lemaître-Sermage

These two methods make it possible to calculate the damage  $D(t)$  starting from the data of the tensor of the constraints  $\sigma(t)$  and of the plastic deformation cumulated  $p(t)$ .

They thus apply to unspecified loadings and are used only in post - treatment of a plastic or viscoplastic law having  $p$  like variable.

Evolution of  $D$  is defined by:

$$\begin{cases} \dot{D} = \frac{1}{(1-D)^{2s}} \left( \frac{1}{3ES} \cdot (1+\nu) \sigma_{eq}^2 + \frac{3}{2ES} (1-2\nu) \cdot \sigma_H^2 \right)^s \dot{p} & \text{si } p > p_d \\ D = 0 & \text{sinon} \end{cases}$$

where  $E$  : Young modulus,  $\nu$  : Poisson's ratio,  $S$  and  $s$  : parameters material,  $\sigma_{eq}$  : equivalent constraint of von Mises,  $\sigma_H$  : hydrostatic pressure,  $p$  : cumulated plastic deformation and  $p_d$  : threshold of damage.

◇ `DAMAGE = 'LEMAITRE'`,

Allows to calculate the damage of Lemaître or Lemaître-Sermage  $D(t)$  starting from the data of the tensor of the constraints  $\sigma(t)$  and of the cumulated plastic deformation  $p(t)$ . To note that the damage of Lemaître is obtained by assigning the value 1.0 with the exhibitor  $s$  ( $s=1$ ).

## 3.5 Operand `INFORMATION`

◇ `INFORMATION = / 1`,

Impression:

- elementary cycles determined by the method of counting chosen by the user,
- elementary damage associated with each cycle for the methods `WOHLER`, `MANSON_COFFIN` and `TAHERI`,
- damage of `LEMAITRE` in each point of calculation,
- total damage (if the user asked for his calculation).

◇ INFORMATION = / 2,

Impression:

- history of loading introduced by the user under the operands `SIGM` and `EPSI`,
- peaks extracted the history of loading (introduced under the operands `SIGM` and `EPSI`),
- elementary cycles determined by the method of counting chosen by the user,
- elementary damage associated with each cycle for the methods `WOHLER`, `MANSON_COFFIN` and `TAHERI`,
- damage of `LEMAITRE` in each point of calculation,
- total damage (if the user asked for his calculation).

The impressions are made in the file `message`.

## 3.6 Operand `TITLE`

◇ `TITLE` = `title`

Title associated with the table.

## 3.7 Produced table

The operator `POST_FATIGUE` create a table which is different according to calculations of post - treatment carried out:

- **Uniaxial loading** (Wöhler methods, Manson-Whetstone sheath and Taheri).

The table understands five parameters:

`NB_CYCL` : many elementary cycles extracted by the method of counting,  
`VALE_MIN` : values of the constraints or minimal deformations of each elementary cycle,  
`VALE_MAX` : values of the constraints or maximum deformations of each elementary cycle,  
`TOO_BAD` : values of the damage for each elementary cycle,  
`DOMM_CUMU` : value of the total damage after office plurality on all the elementary cycles.

- **Loading multiaxial**

The table understands all the parameters constituting the criteria used.

Moreover, for all the criteria, it table understands:

`CRITERION` : name of criterion  
`VALE_CRITERE` : value of criterion (size are equivalent)  
`NBRUP` : number of the cycle to the rupture (associate with a cycle or a block of the under-cycles)  
`TOO_BAD` : value of the damage of Wöhler (if requested by the user).

For the criteria of Crossland and Dang Van-Papadopoulos:

`AMPLI_CISSION` : amplitude of the cission  
`PRES_HYDRO_MAX` : value of the maximum hydrostatic pressure,  
`RAYON_SPHERE` : the ray of the smallest sphere circumscribed with the way of loading within the space of diverters of the constraints  $R$

- **Unspecified loading** (damage of Lemaître and Lemaître-Sermage).

The table understands two parameters:

TOO\_BAD : value of the damage in each point of discretization loading,

The order `IMPR_TABLE [U4.91.03]` allows to print the produced table.

## 4 Size and components introduced into Code\_Aster

The computed values are stored at the points of Gauss or the nodes according to the option selected. Size `FACY_R` (Cyclic Tiredness) was introduced into the catalogue of the sizes.

### For the periodic loading and the criteria of the type of plan criticizes maximum shearing

<code>DTAUM1</code>	<b>first</b> value of the half amplitude max of shearing in the critical plan
<code>VNM1X</code>	component $x$ normal vector with the plan criticizes related to <code>DTAUM1</code>
<code>VNM1Y</code>	component $y$ normal vector with the plan criticizes related to <code>DTAUM1</code>
<code>VNM1Z</code>	component $z$ normal vector with the plan criticizes related to <code>DTAUM1</code>
<code>SINMAX</code>	normal maximum constraint with the plan criticizes correspondent with <code>DTAUM1</code>
<code>SINMOY</code>	normal average constraint with the plan criticizes correspondent with <code>DTAUM1</code>
<code>EPNMAX</code>	normal maximum deformation with the plan criticizes correspondent with <code>DTAUM1</code>
<code>EPNMOY</code>	average maximum deformation with the plan criticizes correspondent with <code>DTAUM1</code>
<code>SIGE_Q</code>	Constraint equivalent within the meaning of the criterion selected correspondent to <code>DTAUM1</code>
<code>NBRUP</code>	many cycles before rupture (function of <code>SIGE_Q1</code> and of a curve of Wöhler)
<code>TOO_BAD</code>	damage associated with <code>NBRUP1</code> ( <code>ENDO1=1/NBRUP1</code> )
<code>VNM2X</code>	component $x$ normal vector with the plan criticizes related to <code>DTAUM2</code>
<code>VNM2Y</code>	component $y$ normal vector with the plan criticizes related to <code>DTAUM2</code>
<code>VNM2Z</code>	component $z$ normal vector with the plan criticizes related to <code>DTAUM2</code>

**Table 5.5-1: Components specific to multiaxial cyclic tiredness for the periodic loading**

### For the loading not-periodical and the criteria of the type of plan criticizes maximum damage

<code>VNM1X</code>	component $x$ normal vector with the plan criticizes dependent with the damage max
<code>VNM1Y</code>	component $y$ normal vector with the plan criticizes dependent with the damage max
<code>VNM1Z</code>	component $z$ normal vector with the plan criticizes dependent with the damage max
<code>TOO_BAD</code>	damage associated with the block with loading
<code>VNM2X</code>	component $x$ normal vector with the plan criticizes dependent with the damage max
<code>VNM2Y</code>	component $y$ normal vector with the plan criticizes dependent with the damage max
<code>VNM2Z</code>	component $z$ normal vector with the plan criticizes dependent with the damage max

**Table 5.5-2: Components specific to multiaxial cyclic tiredness for the loading not-periodical**

The parameter ' `DOMMAGE` ' is for operator `POST_FATIGUE`. The parameter `ENDO1/ENDO2` is for operator `CALC_FATIGUE`

For the loading not-periodical, if there exists only one critical plan of the maximum damage, `VNM2X`, `VNM2Y`, `VNM2Z` are identical to `VNM1X`, `VNM1Y`, `VNM1Z`. If several plans exist, one an alarm emits and leaves the two foregrounds.

## 5 Examples

### 5.1 Calculation of the damage of Wöhler (with correction of the average constraint)

One will refer to CAS-test SZLZ100 (see [V9.01.100]).

### 5.2 Calculation of the damage of Taheri

One will refer to CAS-test SZLZ108 (see [V9.01.108]).

## 5.3 Multiaxial calculation of the criteria of tiredness

One will refer to CAS-test SZLZ107 (see [V9.01.107]).

## 5.4 Calculation of the damage of Lemaître

One will refer to the CAS-test SZLZ109 (see [V9.01.109]).

## 5.5 Calculation of the damage of Lemaître-Sermage

One will refer to the CAS-test SZLZ109 (see [V9.01.109]).

One can find other examples in the tests:

SZLZ101 ([V9.01.101]): Calculation of the damage/Rainflow method.

SZLZ102 ([V9.01.102]): Tiredness with various methods counting.

SZLZ103 ([V9.01.103]): Tiredness counting by Rainflow method normalizes AFNOR.